On the Complexity of Optimal Electric Vehicles Recharge Scheduling

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***Abstract*—The massive introduction of Electric Vehicles (EVs) is expected to significantly increase the power load experienced by the electrical grid, but also to foster the exploitation of renewable energy sources: if the charge process of a fleet of EVs is scheduled by an intelligent entity such as a load aggregator, the EVs' batteries can contribute in flattening energy production peaks due to the intermittent production patterns of renewables by being recharged when energy production surpluses occur. To this aim, time varying energy prices are used, which can be diminished in case of excessive energy production to incentivize energy consumption (or increased in case of shortage to discourage energy utilization).**

In this paper we evaluate the complexity of the optimal scheduling problem for a fleet of EVs aimed at minimizing the overall cost of the battery recharge in presence of time-variable energy tariffs. The scenario under consideration is a fleet owner having full knowledge of the customers' traveling needs at the beginning of the scheduling horizon. We prove that the problem has polynomial complexity, provide complexity lower and upper bounds, and compare its performance to a benchmark approach which does not rely on prior knowledge of the customers' requests, in order to evaluate whether the additional complexity required by the optimal scheduling strategy w.r.t. the benchmark is worthy the achieved economic advantages. Numerical results show considerable cost savings obtained by the optimal scheduling strategy.

***Index Terms*—Optimization; Modeling; Electric Vehicles, Vehicle-to-Grid Interactions; Optimal Recharge Scheduling;**

I. Introduction

Battery/fuel cell-powered vehicles and hybrid automobiles propelled by either electricity generators and gasoline engines [1] are expected to play a pivotal role in the novel Smart Grid scenario, concurring in reducing carbon emissions by fostering the exploitation of Renewable Energy Sources (RESes). The potential consequences of a widespread introduction of Electric Vehicles (EVs) have been recently investigated [2]: on one hand the EVs plugged at charging stations significantly increase the power load experienced by the grid, on the other hand the storage capacities of their batteries could also be exploited to cope with the intermittent energy generation patterns of RESes by accumulating surpluses in energy production [3]. However, the scheduling flexibility and the battery capacity of a single EV is too limited to justify that an EV owner operates directly as a stakeholder on the electricity market. An entity which is responsible for intelligently scheduling the charging/discharging process of a large number of EVs [4] can coordinate such Vehicle-to-Grid (V2G) interactions according to various business models [5], possibly considering economic incentives to compensate the additional battery deterioration due to frequent charge/discharge or to encourage the owners to plug their EV when parked.

Therefore, V2G has recently gained increasing interest in various ICT-related disciplines, ranging from computer science to telecommunications and control theory: numerous optimal and heuristic scheduling strategies have been investigated, mostly aimed at reducing carbon emissions [6] or the overall recharge cost in presence of time-variable energy tariffs (see e.g. [7]–[9]). The telecommunications research community focused primarily on the characterization of the overall system in terms of interactions among the various stakeholders (EVs, load aggregator, energy utility), assuming either a centralized scheduler [10] or a set of aggregators operating in a distributed scenario [8], whereas the control-oriented community thoroughly investigated the development of on-board controllers for the recharging process [6] and the stochastic modeling of the individual EV behavior [11].

However, optimal strategies based either on Integer Linear Programming or Dynamic Programming are typically characterized by high computational complexity, both in terms of asymptotic number of operations and computational time. This paper aims at investigating whether the cost savings achieved by an optimal centralized scheduling strategy minimizing the overall recharging cost are worthy the additional complexity w.r.t. a straight heuristic. To do so, we define an optimal scheduling methodology for a service provider (e.g. car rental, logistics/transportation company) owning a fleet of EVs, with the aim of minimizing the overall cost of the battery recharge in presence of variable energy prices. The considered scenario is a day-ahead planning, which assumes that the details about the customers' traveling needs and the day-ahead energy price are available at the service provider to define the battery recharge scheduling for the next day. We prove that the problem has polynomial complexity, provide complexity lower and upper bounds, and evaluate the achieved cost savings w.r.t. an online approach which does not rely on prior knowledge of the users' requests nor aims to minimize the total recharge costs, but simply finds a feasible recharge plan fulfilling the users' traveling needs, with a computational complexity exhibiting linear dependency on the problem input size.

The remainder of the paper is structured as follows: Section II provides an overview of the related literature. Section III recalls some background notions, then defines the scheduling problem, proves that it can be solved to the optimum in a number of operations polynomially depending on the problem input size, describes an optimal algorithm for its solution, and provides complexity upper and lower bounds. Section IV evaluates the cost savings achieved by the optimal solution approach w.r.t. an online benchmark technique. Finally, we draw our conclusions in the last Section.

II. Related Work

The characterization of the interactions between EVs and power grid has been extensively addressed by the research community in the last decade: for a survey on the impact of the introduction of EVs in the Smart Grid environment the reader is referred to [11], whereas a detailed overview on the techno-economical models of load aggregation agents for EVs is proposed in [12].

A substantial body of work investigates optimal and heuristic policies for the battery recharge of a population of EVs. Among the most remarkable contributions, Han *et al.* [7] formulate a game model for V2G interactions in presence of a profit-driven recharging station and two coexisting sets of EVs, behaving respectively as selfish or cooperative: the recharge of the former set of EVs is decided by the customers themselves according to the real-time energy selling price, while the station can directly control the charging/discharging process of the EVs belonging to the latter set. A game theoretical framework is adopted also by Zou *et al.* [13], who design a distributed charging coordination method for EVs relying on an auction mechanism based on progressive second price. Conversely, since our scenario assumes that the entire fleet is owned by an unique entity, competitiveness among the vehicles is not considered and thus the objective function models only the aggregator's utility.

He *et al.* [8] propose a convex optimization model for a global scheduling optimization problem aimed at the minimization of the recharging cost assuming full knowledge of the future behavior of the customers. The authors compare it to a distributed version finding local optima for subsets of vehicles, which also takes into account dynamic arrivals, showing that the latter achieves close-to-optimum performance. The approach adopted in our work also compares the global optimum achievable by means of an Integer Linear Program (ILP) formulation run by an omniscient scheduler to the results obtained by a sub-optimal benchmark scheduling. However, that paper assumes that the periods in which each vehicle is plugged and available for recharging are problem inputs, while our model optimizes the assignment between vehicles and users to be served, with the aim of minimizing the recharge costs. Joe-Wong *et al.* [14] combine a convex optimization formulation for computing day-ahead energy prices and an algorithm for estimating and refining EVs' user reaction to the prices, which allow the provider to dynamically adjust the offered prices based on the EVs' behavior. Conversely, our optimization scenario does not allow for price negotiation, since it assumes day-ahead planning.

Li *et al.* [9] discuss a methodology for modeling the overall charging demand of EVs based on queuing theory, which allows for the differentiation of the fleet's behavior in case of a charging station and a local residential community. A queuing theory-based approach is used also by Alizadeh *et al.* [10] to provide a stochastic mathematical model for EV aggregate load aimed at short-term load forecasting. Paper [9] assumes a maximum limit on the number of vehicles contemporaneously charged, whereas paper [10] assumes an infinite number of available plugs. In our work, we adopt the latter assumption, which does not introduce additional waiting times in the recharging process.

Sundström and Binding [15] design two models for EVs' battery recharge, respectively exploiting a linear and a quadratic approximation of the non-linearities in the relationship between applied, external, charging power and the rate of change of the batterys state-of-energy. The authors conclude that the violations of the battery boundaries in case of linear approximation do not exceed 2% of the usable capacity, and that the improvements given by the quadratic approximation model do not justify the increase in computational complexity. In our paper, we also adopt a linear recharge model.

III. Problem Formulation

A. Background

We shortly describe the Minimum Weighted Matching Problem (MWMP). Let $G = (V, E)$ be a complete balanced bipartite graph, i.e. a graph in which vertexes can be divided in two disjoint subsets V_1, V_2 such that $|V_1| = |V_2| = \frac{|V|}{2}$. Each vertex $v \in V_1$ is connected to every vertex $v' \in V_2$ by an edge $e(v, v') \in E$. Let $w_{e(v,v')}$ be a non-negative weight associated to the edge $e(v, v')$. The problem goal is to individuate a perfect matching $\overline{E}$ of minimum weight for graph G, i.e. a set of $\frac{|V|}{2}$ edges connecting each node $v \in V_1$ to a different node $v' \in V_2$ such that each node $v \in V$ is vertex of exactly one edge belonging to the set $\overline{E}$ and the sum of the weights associated to the edges of $\overline{E}$ is minimum.

B. The Scheduling Problem (SP)

We consider a scenario in which a service provider owns a fleet of vehicles $\mathcal{V}$ and a recharging station where the vehicles are parked and can be recharged after usage. We assume that the station is equipped with a sufficient number of plugs to recharge the vehicles at any time without queuing. Each vehicle is characterized by the same battery charging rate r. Note that the recharge operations are assumed to be non-interruptible, meaning that, once the recharge process has started, it must be completed without intermediate interruptions. We also assume that time is divided in epochs of duration T. Let, $\mathcal{U}$ and $\mathcal{T}$ be the set of users and the set of discretized epochs within the optimization time span, respectively. Before the beginning of the scheduling horizon, every user $u \in \mathcal{U}$ specifies a service request in terms of the triplet $(t_u^d, t_u^a, e_u) \in \mathcal{T} \times \mathcal{T} \times \mathbb{R}^+$, indicating the user's departure and arrival time ($0 < t_u^d < t_u^a < |\mathcal{T}|$) and the amount of discharged energy during the travel, respectively.[1] Without loss of generalization, we assume that each user expresses a single service request during the optimization period. Moreover, we assume that the batteries of all the vehicles are fully charged at the beginning of the scheduling period. A vehicle returning to the charging station after serving a customer must be fully recharged before being assigned to a new user.

Let $c_\tau \geq 0$ be the energy price for each epoch $\tau \in \mathcal{T}$. A possible schedule for the recharge of the set of vehicles $\mathcal{V}$ to serve the set of users $\mathcal{U}$ during the set of time epochs $\mathcal{T}$ is defined by the set $\overline{\mathcal{Z}} \subseteq \mathcal{Z} = \mathcal{U} \times \mathcal{V} \times \mathcal{T}$ of $|\mathcal{U}|$ triplets (u, v, t), indicating the starting epoch t for the recharge of vehicle v serving user u. The goal of the problem is to individuate the feasible scheduling ensuring the lowest overall recharge cost experienced by the service provider.

Formulation:

$$\min_{\overline{\mathcal{Z}} \subseteq \mathcal{Z}} \sum_{(u,v,t) \in \overline{\mathcal{Z}}} \sum_{k=t}^{t+\lceil \frac{e_u}{r} \rceil - 1} c_k \tag{1}$$

subject to:

$$|\mathcal{Z}_u| = 1 \quad \forall\, u \in \mathcal{U} \tag{2}$$

where $\mathcal{Z}_u = \{(u', v, t) \in \overline{\mathcal{Z}} \colon u' = u\}$.

$$t \geq t_u^a \quad \forall\, (u, v, t) \in \overline{\mathcal{Z}} \tag{3}$$

$$t \leq t_{u'}^d - \lceil \frac{e_u}{r} \rceil \;\; \forall\, ((u, v, t), (u', v, t')) \in \overline{\mathcal{Z}} \times \overline{\mathcal{Z}} \colon t_u^a \leq t_{u'}^d \tag{4}$$

The objective function minimizes the overall recharging cost experienced by the service provider. Constraint (2) ensures that the set of vehicles serving a given user has cardinality 1, i.e. that each user service request is associated to exactly one vehicle. Constraint (3) imposes that a vehicle can be charged only after its return to the charging station and constraint (4) states that vehicle v associated to a given user u can serve a second user u' only in case u' departs after the return of u and there is sufficient time to complete the battery recharge before the departure of u'.

In the following Subsections we prove the computational equivalence of SP and MWMP by showing that SP is polynomially reducible to MWMP and vice versa. Moreover, we provide upper and lower bounds for the complexity of SP.

C. Polynomial-time Reduction of SP to MWMP

Theorem 1: The SP problem is polynomially reducible to the MWMP problem, i.e. $SP \leq_P MWMP$.

Proof: The proof consists in a Cook reduction. We start describing how to map a generic instance of SP onto an instance of MWMP.

Given a generic instance of the SP problem, let $N_1^{\mathcal{U}}$,$N_2^{\mathcal{U}}$ be two disjoint sets of nodes of cardinality $|\mathcal{U}|$ each, representing the users' arrivals and departures, respectively. Moreover, let $N_1^{\mathcal{V}}$, $N_2^{\mathcal{V}}$ be two disjoint sets of nodes of cardinality $|\mathcal{V}|$, representing the fleet of EVs parked at the charging station at times $t = 0$ and $t = |\mathcal{T}|$, respectively. Finally, we define the sets $N_1 = N_1^{\mathcal{U}} \cup N_1^{\mathcal{V}}$, $N_2 = N_2^{\mathcal{U}} \cup N_2^{\mathcal{V}}$ and consider the complete balanced bipartite graph $G = (V, E)$ with $V = N_1 \cup N_2$, $E = \{e(n, n') \colon n \in N_1, n' \in N_2\}$. Links have associated non-negative weights $w_{e(n,n')} = C(n, n')$, where the cost function $C : \mathcal{V} \cup \mathcal{U} \times \mathcal{V} \cup \mathcal{U} \rightarrow \mathbb{R}^+$ is defined as follows:

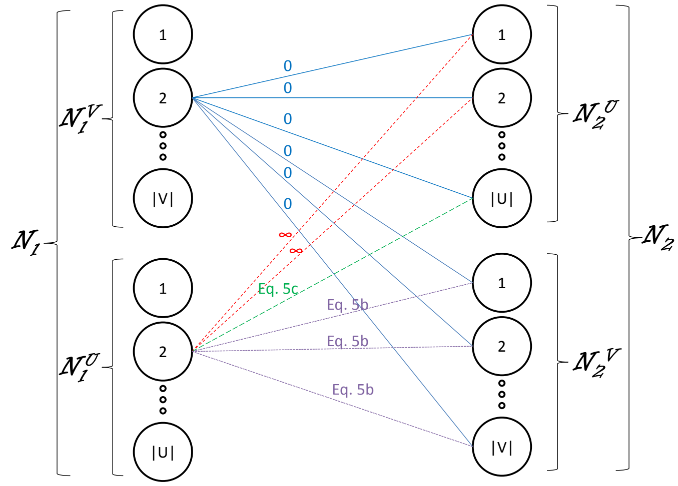

Fig. 1. Complete balanced bipartite graph obtained from the SP problem. Weights of the outgoing edges for the nodes corresponding to vehicle $v = 2$ and user $u = 2$ are shown as example.

$$C(n, n') = \begin{cases} 0 \text{ if } n \in N_1^{\mathcal{V}} & \text{(5a)} \\ \min_{t_n^a \leq t \leq |\mathcal{T}| - \lceil \frac{e_n}{r} \rceil} \sum_{k=t}^{t+\lceil \frac{e_n}{r} \rceil - 1} c_k & \text{(5b)} \\ \qquad \text{if } n \in N_1^{\mathcal{U}}, n' \in N_2^{\mathcal{V}} & \\ \min_{t_n^a \leq t \leq t_{n'}^d - \lceil \frac{e_n}{r} \rceil} \sum_{k=t}^{t+\lceil \frac{e_n}{r} \rceil - 1} c_k & \text{(5c)} \\ \qquad \text{if } n \in N_1^{\mathcal{U}}, n' \in N_2^{\mathcal{U}} \wedge t_n^a \leq t_{n'}^d - \lceil \frac{e_n}{r} \rceil & \\ \infty \text{ otherwise} & \text{(5d)} \end{cases}$$

A pictorial view of G is proposed in Figure 1. Each link corresponds to a possible scheduling choice and its weight is the corresponding recharging cost. An edge connecting a node $n \in N_1^{\mathcal{V}}$ to a node $n' \in N_2^{\mathcal{V}}$ corresponds to a vehicle that is not used during the scheduling period, and then its weight is 0 according to Eq. (5a). An edge between a node $n \in N_1^{\mathcal{V}}$ and a node $n' \in N_1^{\mathcal{U}}$ denotes that n' is the first user of vehicle n. The vehicle does not need to be charged (because all vehicles are fully charged at the begin) and then the associated weight is 0 (Eq. (5a)). Conversely, an edge (n, n') with $n \in N_1^{\mathcal{U}}$, $n' \in N_2^{\mathcal{V}}$ corresponds to user n being the last user for vehicle n' and its weight is then the minimum energy cost associated to a recharge period of $\lceil \frac{e_n}{r} \rceil$ slots, chosen during

[1]The energy usage associated to the travel can be computed as a function of the expected traveling distance. Note that the proposed scheduling approach is agnostic w.r.t. the technique adopted for the computation of such amount.

Algorithm 1 Mapping a solution of MWMP to a solution of SP in Theorem 1

$\overline{\mathcal{Z}} \leftarrow \oslash$
for all $e(n,n') \in \overline{E}\colon n \in N_1^{\mathcal{V}}$ **do**
  $\tilde{t} \leftarrow t_n^a, v \leftarrow n$
  while $n' \notin N_2^{\mathcal{V}}$ **do**
    while $\sum_{k=\tilde{t}}^{\tilde{t}+\lceil e_n/r \rceil -1} c_k \neq w_{e(n,n')}$ **do**
      $\tilde{t} \leftarrow \tilde{t}+1$ {A feasible solution exists by construction, which breaks the cycle before $\tilde{t}$ reaches $t_{n'}^d - \lceil \frac{e_n}{r} \rceil$}
    end while
    $\overline{\mathcal{Z}} \leftarrow \overline{\mathcal{Z}} \cup \{(n', v, \tilde{t})\}$
    $\tilde{t} \leftarrow t_{n'}^a, e(n,n') \leftarrow e(n'', n''') \in \overline{E}\colon t_{n''}^a = \tilde{t}$
  end while
end for

the interval $[t_n^a, |\mathcal{T}|]$, i.e. the time window between the return of the customer n and the end of the scheduling horizon (the vehicle n' must be charged after serving its last customer, in order to be fully charged before the beginning of the next scheduling period). This cost is expressed by Eq. (5b). Finally, an edge between $n \in N_1^{\mathcal{U}}$ and $n' \in N_2^{\mathcal{U}}$ denotes that the same vehicle is consecutively used for users n and n'. This is not possible if the time interval between the return of user n and the departure of user n' is not long enough to guarantee that the vehicle can be fully recharged (i.e. $t_n^a > t_{n'}^d - \lceil \frac{e_n}{r} \rceil$). In such case, Eq. (5d) sets the weight of edge (n, n') to ∞. Otherwise, if the interval is long enough, Eq. (5c) sets the cost of the edges (n, n') to the minimum energy cost associated to a recharge period of $\lceil \frac{e_n}{r} \rceil$ slots chosen in the period $[t_n^a, t_{n'}^d]$ (which is the time span between the return of user n and the departure of user n'). The asymptotic complexity of computing the cost function C is $O(|\mathcal{T}|(|\mathcal{U}| + |\mathcal{V}|)^2)$, which shows a polynomial dependency on $|\mathcal{T}|$, $|\mathcal{U}|$, and $|\mathcal{V}|$.

We observe that any possible matching $\overline{E}$ over graph G can be mapped into a corresponding scheduling $\overline{\mathcal{Z}}$ by means of Algorithm 1, which runs in in $O(|\mathcal{U}||\mathcal{T}|)$ operations. The Algorithm considers the vehicles one at a time, sequentially visits the edges of the matching $\overline{E}$ which correspond to the recharge periods scheduled for vehicle v (in chronological order), and adds to the set $\overline{\mathcal{Z}}$ the associated triplets.

Moreover, if the matching $\overline{E}$ has finite weight, the cost of the corresponding scheduling $\overline{\mathcal{Z}}$ has the same cost. If the matching has infinite cost, then it contains at least one edge with infinite weight and we can conclude that the corresponding schedule is not feasible.

The set of schedules corresponding to all the possible matchings over G is in general a subset of the set of all possible schedules, but we are going to show that for any schedule $\hat{\mathcal{Z}}$ there exists a matching $\overline{E}$ whose corresponding schedule $\overline{\mathcal{Z}}$, obtained by Alg. 1, has cost not larger than the cost of $\hat{\mathcal{Z}}$. It follows that the schedule corresponding to the optimal solution of MWMP is the optimal solution of SP. Consider a schedule $\hat{\mathcal{Z}}$ and then apply Algorithm 2, which

Algorithm 2 Mapping a solution of SP to a solution of MWMP in Theorem 1

1: $\overline{E} \leftarrow \oslash$
2: **for all** $v \in \mathcal{V}$ **do**
3:   $\mathcal{Z}_v \leftarrow \{(u, v', t) \in \overline{\mathcal{Z}}\colon v' = v\}$
4:   **if** $\mathcal{Z}_v \neq \oslash$ **then**
5:     sort $(u, v, t) \in \mathcal{Z}_v$ over t in ascending order and store the ordered list in $\mathcal{Z}_v^S$ {Let (u_i, v_i, t_i) be the ith element of the list $\mathcal{Z}_v^S$}
6:     $\overline{E} \leftarrow \overline{E} \cup \{w_{e(n,n')}\colon n \in N_1^{\mathcal{V}} \wedge n = v, n' \in N_2^{\mathcal{U}} \wedge t_{n'}^d = t_{u_1}^d\}$
7:     **if** $|\mathcal{Z}_v| \geq 2$ **then**
8:       **for all** $i\colon 2 \leq i \leq |\mathcal{Z}_v|$ **do**
9:         $\overline{E} \leftarrow \overline{E} \cup \{w_{e(n,n')}\colon n \in N_1^{\mathcal{U}} \wedge t_n^a = t_{u_{i-1}}^a, n' \in N_2^{\mathcal{U}} \wedge t_{n'}^d = t_{u_i}^d\}$
10:       **end for**
11:     **end if**
12:     $\overline{E} \leftarrow \overline{E} \cup \{w_{e(n,n')}\colon n \in N_1^{\mathcal{U}} \wedge t_n^a = t_{u_{|\mathcal{Z}_v|}}^a, n' \in N_2^{\mathcal{V}} \wedge n' = v\}$
13:   **else**
14:     $\overline{E} \leftarrow \overline{E} \cup \{w_{e(n,n')}\colon n \in N_1^{\mathcal{V}} \wedge n = v, n' \in N_2^{\mathcal{V}} \wedge n' = v\}$
15:   **end if**
16: **end for**

identifies the set of users that each vehicle v serves (line 3), sorts them in ascending order w.r.t. their respective recharge starting times (line 5), and includes in the set $\overline{E}$ the edges with weight equal to the corresponding recharge cost (line 6-15). We observe that different schedules can produce the same set of ordered sequences of users (these schedules only differ for the time epochs when the recharge periods start). The weights of the links in $\overline{E}$ correspond to the minimum recharge costs that can be obtained under the constraint that each vehicle is assigned that specific sequence of users. It follows that the scheduling $\overline{\mathcal{Z}}$ obtained from $\overline{E}$ has cost not larger than the cost of $\hat{\mathcal{Z}}$.

■

D. Polynomial-time Reduction of MWMP to SP

We now show the polynomial reducibility of MWMP to SP.

Theorem 2: The MWMP problem is polynomially reducible to the SP problem, i.e. $MWMP \leq_P SP$.

Proof: Analogously to Theorem 1, the proof follows the Cook reduction approach. To map a generic instance of MWMP with finite weights onto an instance of SP, we operate as follows. Let $G = (V, E)$ be a complete balanced bipartite graph as defined in Section III-A. The corresponding SP instance has $\frac{|V|}{2}$ vehicles ($\mathcal{V} = \{1, \ldots, \frac{|V|}{2}\}$) and $|V|$ users ($\mathcal{U} = \{1, \ldots, |V|\}$). Let $\mathcal{T} = \{-1, 0, \ldots, \frac{|V|^2}{4} + \frac{|V|}{2} + 1\}$ be the set of time slots within the optimization horizon. In the horizon we can distinguish the first and last timeslots and $\frac{|V|}{2}$ intermediate intervals $\mathcal{I}_h$ for $h \in \{1, \ldots, \frac{|V|}{2}\}$, each of length $\frac{|V|}{2} + 1$ slots. Figure 2 shows such instance for $|V| = 6$.

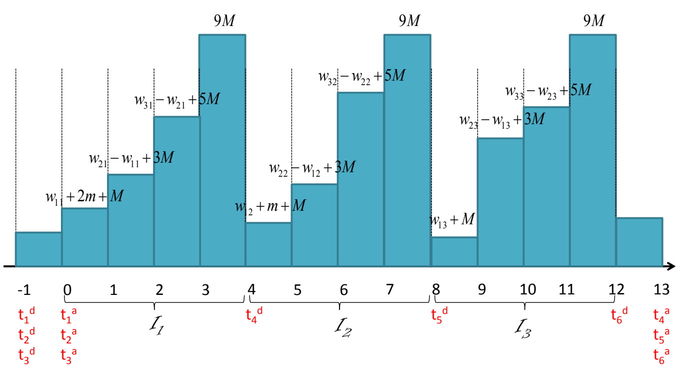

Fig. 2. Cost function obtained by means of Equation 7 from a bipartite complete balanced graph $G = (V, E)$ with $|V| = 6$.

Half of the users depart at the begin of timeslot -1 and return by the begin of timeslot 0 (for $u \in \mathcal{U}$: $1 \leq u \leq \frac{|V|}{2}$, $t_u^d = -1, t_u^a = 0$), the other half of users departs later on (one user at the end of every intermediate interval) and returns at the end of the scheduling period (for $u \in \mathcal{U}$: $\frac{|V|}{2} + 1 \leq u \leq |V|$, $t_u^d = (u - \frac{|V|}{2})(\frac{|V|}{2} + 1), t_u^a = \frac{|V|^2}{4} + \frac{|V|}{2} + 1$).

By construction, it follows that each vehicle will be used by two and only two users. Since all the vehicles are back at the begin of timeslot 0, this SP instance is equivalent to decide how to match the $\frac{|V|}{2}$ vehicles with the $\frac{|V|}{2}$ users departing later on. The rest of the proof will show that it is possible to determine energy costs so that the SP instance is able to capture all the variability of the $\frac{|V|^2}{4}$ weights of the MWMP instance.

In order to decouple the total energy recharge cost for the different vehicles, we can tune the required amount of energy to be recharged. In particular, the vehicle used by user $u \in \mathcal{U}$: $1 \leq u \leq \frac{|V|}{2}$ requires $e_u = ur$. For the second half of users we can consider arbitrary energy requests (e.g. $e_u = 0$ for $u \in \mathcal{U}$: $\frac{|V|}{2} + 1 \leq u \leq |V|$).

Let us denote by m and M two positive numbers such that $m > \max_{e(n,n') \in E} w_{e(n,n')}$, $M > \frac{m|V|}{2}$. We first observe that if all the weights of the links outgoing from a given node n (in V_1 or in V_2) are increased by the same amount x_n, the MWMP instance is transformed in an equivalent one with the same minimum weight matching (whose value will be increased by x_n). We can apply this transformation for all the nodes, thus obtaining a sequence of equivalent MWMP instances. This reasoning leads us to conclude that if we transform the weight of each link as follows:

$$w'_{e(n,n')} = w_{e(n,n')} + Mn^2 + m(\frac{|V|}{2} - n'), \tag{6}$$

we obtain an equivalent MWMP, because each weight has been increased by two quantities that depend only from the two terminating nodes.

We define the energy cost c_τ as follows:

$$c_\tau = \begin{cases} w'_{1,e(\lceil \frac{\tau+1}{\frac{|V|}{2}} \rceil)} \text{ if } \tau = 0 \vee \tau \bmod (\frac{|V|}{2} + 1) = 0 \\ M\frac{|V|^2}{4} \text{ if } \tau \bmod (\frac{|V|}{2} + 1) = \frac{|V|}{2} \\ w'_{e(\tau \bmod (\frac{|V|}{2}+1)+1, \lceil \frac{\tau+1}{\frac{|V|}{2}} \rceil)} - w'_{e(\lceil \frac{\tau+1}{\frac{|V|}{2}} \rceil, \tau \bmod (\frac{|V|}{2}+1))} \\ \qquad \text{otherwise} \end{cases} \tag{7}$$

We observe that the computation of c_t starting from the weights $w_{e_{(n,n')}}$ requires $O(|V|^2)$ operations.

It can be checked that this definition of the costs leads to the following consequences (the reader may find useful to refer to Figure 2). First, because of the presence of the large constant M, energy costs are always increasing within each intermediate interval. Moreover, the presence of slots with cost $M\frac{|V|^2}{4}$ at the end of each intermediate interval makes inconvenient for the recharge of a vehicle to span two consecutive intervals. As a result, every vehicle will be charged by starting only at the begin of an interval. In particular, if the vehicle driven by user $n \in \{1, \ldots \frac{|V|}{2}\}$ is charged during the interval $\mathcal{I}_h$, the aggregate recharge cost is: $\sum_{k=(h-1)(\frac{|V|}{2}+1)}^{(h-1)(\frac{|V|}{2}+1)+n} c_k = w'_{e(n,h)}$, i.e. equal to the weight of link $e(n,h)$ in the equivalent MWMP we are considering. Second, the presence of m guarantees that the vehicle to be driven by user $u \in \{\frac{|V|}{2} + 1, \ldots |V|\}$ will not be charged before the interval $\mathcal{I}_{u - \frac{|V|}{2}}$. As a consequence, only one vehicle will be charged during each interval.

In conclusion this SP instance is equivalent to match each of the $\frac{|V|}{2}$ vehicles to a different recharging interval.

The optimality of the scheduling $\overline{\mathcal{Z}}$ obtained by conversion of the optimal matching $\overline{E}$ computed over graph G can be proved by showing that for every matching $\hat{E}$ there exists a schedule $\hat{\mathcal{Z}}$ which has cost equal to the cost of $\hat{E}$ plus a constant term depending only on the problem input size.

We start showing that if there exists a matching $\hat{E}$ over graph G such that $\sum_{e(n,n') \in \hat{E}} w_{e(n,n')} = \Xi$, then there exists a feasible schedule $\hat{\mathcal{Z}}$ defined as:

$$\hat{\mathcal{Z}} = \bigcup_{e(n,n') \in \hat{E}} \{(n, n, (\frac{|V|}{2} + 1)(n-1)), ((n' + \frac{|V|}{2}, n, |\mathcal{T}|)\} \tag{8}$$

Note that, since users $\frac{|V|}{2} + 1, \ldots, |V|$ do not consume any energy during their travel, the associated recharging periods last 0 epochs and can be scheduled at $t = |\mathcal{T}|$.

From Eqs. (6), (7) it follows that:

$$\sum_{(u,v,t) \in \hat{\mathcal{Z}}} \sum_{k=t}^{t + \lceil e_u / r \rceil - 1} c_k = \sum_{e(n,n') \in \hat{E}} w'_{e(n,n')} =$$

$$\sum_{e(n,n') \in \hat{E}} w_{e(n,n')} + \sum_{i=1}^{\frac{|V|}{2}} ((i-1)m + i^2 M) =$$

$$\Xi + m\frac{|V|}{4}\left(\frac{|V|}{2} - 1\right) + M\frac{|V|(|V|+1)(|V|+2)}{24}$$

i.e. the cost of the schedule $\hat{\mathcal{Z}}$ is obtained by adding to the cost of the corresponding matching $\hat{E}$ a positive quantity which depends only on the cardinality of V.

Moreover, by construction the following equality holds:

$$
\begin{aligned}
w'_{e(n,n')} &= \sum_{k=(n-1)(\frac{|V|}{2}+1)}^{k=(n'-1)(\frac{|V|}{2}+1)+n'-1} c_k \\
&= \min_{t\in\mathcal{T}:\ 0\le t<(\frac{|V|}{2}+1)n'} \sum_{k=t}^{t+n-1} c_k \quad \forall e(n,n')\in E
\end{aligned}
$$

Since the interval $[0, (\frac{|V|}{2}+1)n']$ covers the whole time span between the return of user n and the departure of user n', no recharge cheaper than $w'_{e(n,n')}$ can be scheduled for a vehicle serving both users. It follows that, for the set of associations between users and vehicles derived by the matching $\hat{E}$ over graph G, there is no feasible schedule ensuring lower overall cost than the cost of the matching $\hat{E}$ plus the constant term $m\frac{|V|}{4}\left(\frac{|V|}{2}-1\right)+M\frac{|V|(|V|+1)(|V|+2)}{24}$. Therefore, the solution $\overline{\mathcal{Z}}$ obtained from the matching of minimum weight $\overline{E}$ by means of Eq. (8) provides the schedule of minimum cost for the equivalent SP. ■

Theorem 3: The SP and MWMP problems are computationally equivalent, i.e. $SP \equiv_P MWMP$

Proof: The proof directly follows from Theorems 1 and 2. ■

E. Complexity Evaluation of SP

Let $\Gamma(|\mathcal{U}|,|\mathcal{V}|,|\mathcal{T}|)$ be the complexity of the SP problem. Let $\Psi(n)$ be the complexity of the MWMP problem. The variable n denotes the number of vertexes.

Corollary 1: The following upper bound to the complexity of the SP problem holds:

$$
\Gamma(|\mathcal{U}|,|\mathcal{V}|,|\mathcal{T}|) = O(\max\{(|\mathcal{U}|+|\mathcal{V}|)^2|\mathcal{T}|, \Psi(2(|\mathcal{U}|+|\mathcal{V}|))\}) \tag{9}
$$

Proof: The proof is a direct consequence of Theorem 1, which proves the polynomial-time reducibility of SP to MWMP. Hence, any instance of SP can be solved by mapping it onto an instance of MWMP through the construction of a bipartite graph $G=(V,E)$, finding the optimal solution of MWMP over G and converting it to the equivalent solution of SP. It follows that:

$$
\Gamma(n) = O((|\mathcal{U}|+|\mathcal{V}|)^2|\mathcal{T}| + \Psi(2(|\mathcal{U}|+|\mathcal{V}|)+|\mathcal{U}||\mathcal{T}|)
$$

■

The above bounds can be refined by defining $\Gamma(n)$ as the complexity of the SP problem versus the total size of input $n=|\mathcal{U}|+|\mathcal{V}|+|\mathcal{T}|$. Since it is known [16] that the complexity of MWMP is smaller than $O(n^3)$, the following bound can be easily derived: $\Gamma(n)=O(n^3)$. This bound is tight in the sense that it cannot be reduced without making assumptions on the relative size of the input variables. If, for example, the number of time epochs is assumed to be a fixed parameter, then the following tighter bound holds: $\Gamma(n)=O(\Psi(n))$.

Algorithm 3 Benchmark online algorithm for SP

1: $\delta_v \leftarrow 0\ \forall v\in\mathcal{V}$, $\overline{\mathcal{Z}} \leftarrow \emptyset$
2: Upon arrival of the traveling request (t_u^d, t_u^a, e_u) at epoch t_u^d
3: $v \leftarrow \text{rand}(\tilde{v}\in\mathcal{V}\colon \delta_{\tilde{v}} < t_u^d)$
4: $\overline{\mathcal{Z}} \leftarrow \overline{\mathcal{Z}} \cup (u, v, t_u^a)$, $\delta_v \leftarrow t_u^a + \lceil e_u/r \rceil$

Corollary 2: With $\Gamma(n)$ and $\Psi(n)$ defined above, the following holds:

$$
\Gamma(n) = \Omega(\Psi(\sqrt{n})) \tag{10}
$$

Proof: By virtue of Theorem 2, the following equality holds:

$$
\begin{aligned}
\Psi(n) &= O(n^2 + \Gamma\left(n + \frac{n}{2} + \frac{n(n+2)}{4} + 2\right) + n) \\
&= O(n^2 + \Gamma(n^2) + n).
\end{aligned}
$$

Thus, we have $\Gamma(n^2)+n^2 = \Omega(\Psi(n))$. Since, from an obvious lower bound, $\Gamma(n^2)=\Omega(n^2)$, we obtain

$$
\Gamma(n^2) = \Omega(\Psi(n))
$$

By performing a variable substitution, we obtain the thesis. ■

The exact complexity of MWMP is not known. The best lower bound in the literature for the MWMP is $\Psi(n)=\Omega(n^2)$. Applying this bound to Corollary 2 results in the obvious bound $\Gamma(n)=\Omega(n)$. However, if a larger lower bound for MWMP were found, then the lower bound for SP would be similarly increased.

IV. Performance Assessment

In this Section we quantify the savings of the optimal scheduling with day-ahead knowledge over simpler approaches. As discussed in the previous Sections, the optimal solution to the SP can be obtained by solving the corresponding MWMP and converting the optimal matching to the optimal schedule. As a benchmark, we consider Alg. 3, which is an on-line heuristic approach that does not assume any future knowledge about the traveling requests of the users in terms of departure/arrival times and amount of energy usage during the travel. Under assumption that all the vehicles are fully charged at the beginning of the scheduling horizon, whenever a travel request by user u comes, Alg. 3 assigns to u a randomly chosen vehicle among the ones parked at the recharging station and currently not under recharge (i.e. with full battery level). Then, as soon as v returns at time t_u^a after serving user u, it is immediately recharged during the next $\lceil e_u/r \rceil$ consecutive epochs. Therefore, the complexity of the algorithm is $O(|\mathcal{U}|)$, i.e. shows linear dependency on the number of users. Note that, since each recharge is scheduled immediately after the vehicle's return without introducing any waiting time, Alg. 3 always provides a feasible solution, given that the considered instance of SP admits at least one feasible scheduling.

Numerical results are obtained assuming a scenario of a fleet ranging from 200 to 500 vehicles and a set of 500 users. The

TABLE I

PERFORMANCE COMPARISON OF THE OPTIMAL SCHEDULING VS. THE ONLINE BENCHMARK

$\lvert\mathcal{U}\rvert$	$\lvert\mathcal{V}\rvert$	Aver. Gap [%]	Max. Gap [%]	Min. Gap [%]
500	500	24.4	167.8	2.8
500	400	23.8	177.4	3.6
500	300	21.9	168.8	3.1
500	200	17.6	110.5	2.75

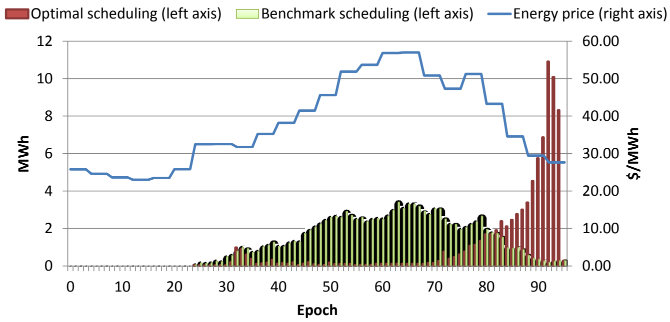

Fig. 3. Daily trend of overall scheduled recharge (optimal and benchmark approaches) vs. energy cost, assuming $|\mathcal{V}| = 400$, $|\mathcal{U}| = 500$.

departure time, arrival time and amount of energy required for the travel have been computed based on the TripChaining dataset [17], whereas the trend of the hourly energy cost on the day-ahead energy market has been obtained from [18]. Results have been averaged over 365 days (each 24-hours period from 00:00 to 23:59 is divided in 96 epochs of 15 mins duration).

Table I compares the performance of the optimal day-ahead scheduling, which assumes full knowledge of the departure and arrival times of the users, to the on-line benchmark, which does not rely on any information about the future traveling partterns of the users. Results show that the on-line strategy increments the total recharge cost by on average 17-25%, with peaks above 110%. The savings allowed by the optimal approach decrease when the fleet size decreases, since a small fleet imposes that each EV must sequentially serve multiple users, which reduces the periods in which the EV is plugged at the recharging station and limits the recharge cost minimization, which must be performed over shorter time spans. An example of the daily recharge schedule is plotted in Fig. 3 for both optimal and benchmark approaches, from which it results that the optimal scheduling delays most of the recharges to the end of the day, when the energy cost is significantly lower.

V. CONCLUSIONS

This paper evaluates the computational complexity of an algorithm for optimal day-ahead scheduling of the battery recharge of a fleet of electric vehicles. We provide analytical upper and lower bounds to the problem complexity and compare the performance of the proposed algorithm to an on-line benchmark approach, showing that it leads to considerable cost savings in presence of time-variable electricity tariffs.

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