Simple Lock-In Technique for Thickness Measurement of Metallic Plates

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I. Introduction

If a magnetizing coil, powered by an alternating current, is placed near a metallic object, the variable magnetic field induces into this latter a distribution of eddy currents, whose amplitude and phase depends on many parameters: object material and geometry, coil dimensions and position, signal amplitude, and frequency. These induced currents generate their own magnetic field, which depends on the object geometry and its electrical and magnetic properties. By detecting this field, these characteristics could in principle be extracted, giving important information about the object properties. However, in the general case, this problem is mathematically intractable, and it has been solved only for simple geometries [1]: a cylindrical coil above a metal plate, or wound around a cylindrical conductor. Even in this case, the exact solutions are quite complex and can be managed only by numerical methods or by suitable approximations. This complexity notwithstanding, the method of eddy-current testing has been considerably developed and became an important technological tool [2].

One of the most important applications of this method is the determination of conductance and thickness of metal plates and layers, where a considerable precision can be attained [3]. In this kind of measurements, a field coil is placed above the metal plate, it is excited with an alternating current, and its impedance is measured. This latter is then compared to the coil impedance as measured without the metal plate: the impedance difference contains information about the plate thickness and conductance. Depending on the measurement method, the real [3] or imaginary [5] part of the impedance is studied. The work of Yin *et al.* [4] demonstrated that the phase signature is of particular interest when measuring the plate thickness, as it is sensitive to this parameter and relatively insensitive to other measurement circumstances, as, e.g., the liftoff, i.e., the distance between the plate and the measuring coil [6]. However, the coil impedance technique has some drawbacks.

1) The phase dependence on the frequency demonstrated in [4] is relatively featureless, as it smoothly decreases for increasing frequencies. In order to simplify and possibly automate the measurement, it would be useful to have a prominent feature, like a peak at a given frequency related to the plate thickness. It has to be pointed out that such a peak exists in the imaginary part of the impedance [5].
2) Measuring the impedance with a good precision requires an impedance bridge or impedance meter, which can be complex and expensive. By concentrating the measurement on the phase only, it is possible to use simple phase comparators, already existing as inexpensive integrated circuits.[1]

In this paper, we show how it is possible to overcome these drawbacks, and obtain a simple instrument, which could in principle be automated.

II. Theory

Let us consider a field coil, having radius r_0, length h, and n turns per unit length, placed above a metal plate of thickness c (Fig. 1) and very large lateral extension. The metal is nonmagnetic and has conductivity σ. The distance between the field coil and the metal place (the liftoff) is l_1. A sinusoidal current $I_0 \times \sin(\omega t)$ in this coil produces a magnetic field that induces eddy currents into the metal plate. These currents, in turn, produce a magnetic field, which adds to the main field; both are detected by a pickup coil having radius r_p placed at the lower end of the field coil and concentric to it.

The voltage signal induced in one turn of the pick-up coil is given by [1]

$$V = j\omega \times 2\pi \times r_p \times A\left(r_p, l_1\right) \tag{1}$$

where $A(r_p, l_1)$ is the vector potential generated by both the magnetizing coil current and the eddy currents, evaluated at

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[1] An example, employed in this paper, is the CD4046 phase comparator, belonging to the 4000 series of digital building blocks.

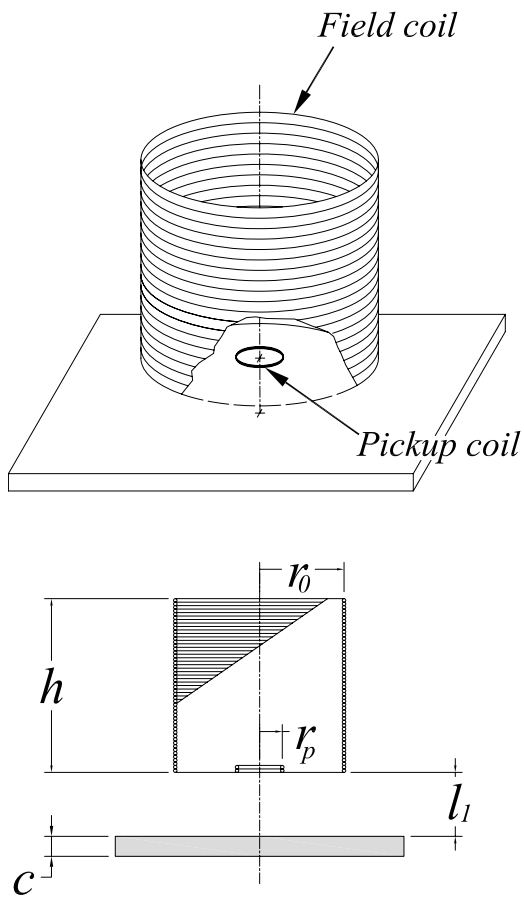

Fig. 1. Schematic representation of the experimental arrangement. Field coil produces a varying magnetic field that induces eddy currents into the metal plate. Pickup coil senses the total magnetic field.

the position of the pick-up coil. This potential is given by [1]

$$A\left(r_p, l_1\right) = \frac{1}{2}\mu_0 \times \overline{I} \times \int_0^\infty \frac{1}{\alpha} J_1\left(\alpha \cdot r_0\right) \times J_1\left(\alpha \cdot r_p\right) \times \left[1 - \exp(-\alpha h)\right] \times \left[1 - F\left(\alpha\right) \times \mathrm{e}^{-2\alpha l_1}\right] \times d\alpha \tag{2}$$

where $\overline{I}$ is the complex (magnitude and phase) magnetizing current, α is an integration parameter, J_1 is the Bessel functions of first kind and first order, and $F(\alpha)$ is the function

$$F\left(\alpha\right) = \frac{\left(\alpha_1^2 - \alpha^2\right) - \left(\alpha_1^2 - \alpha^2\right) \times \exp\left(2\alpha_1 c\right)}{\left(\alpha_1 - \alpha\right)^2 - \left(\alpha_1 + \alpha\right)^2 \times \exp\left(2\alpha_1 c\right)} \tag{3}$$

with

$$\alpha_1 = \sqrt{\alpha^2 + j\omega \times \mu_0 \times \sigma}. \tag{4}$$

The complexity of these expressions makes difficult the analytical calculation of the phase of the pickup signal $V(\omega)$. A great simplification is possible by using an approximation, already described in [5], which we will extend further. The Bessel functions can be approximated with a damped sine [8], and (2) becomes

$$A\left(r_p, l_1\right) = \frac{\mu_0}{2}\overline{I} \times \int_0^\infty \frac{2}{\pi\sqrt{r_p r_0}} \times \frac{1}{\alpha^2} \sin\left(\alpha r_p - \frac{\pi}{4}\right) \times \sin\left(\alpha r_0 - \frac{\pi}{4}\right) \times \left[1 - \exp(-\alpha h)\right] \times \left[1 - F\left(\alpha\right) \times \mathrm{e}^{-2\alpha l_1}\right] \times d\alpha.$$

This expression makes clear the oscillating behavior of the integrand; there is a shorter oscillation period of $2\pi/r_0$ and a longer one of $2\pi/r_p$ (in terms of the α variable). With the dimensions of our coil, the shorter period is ~500 and the longer one ~4000; with such numbers, the term $1/\alpha^2$ makes the integrand negligible after a few short periods or one long period at most. Instead, the function $F(\alpha)$ is a ratio between two quantities having the same exponential (and power) dependence on α, and therefore does not change very much in such a short interval.

This means that, as done in [5], the expression $1 - F(\alpha) \times \exp(-2\alpha l_1)$ can be evaluated at a suitable value α_0 of the integration variable, and taken out of the integral

$$A\left(r_p, l_1\right) \cong \frac{1}{2}\mu_0 \times \overline{I} \times \left[1 - F\left(\alpha_0\right) \times \mathrm{e}^{-2\alpha_0 l_1}\right] \times \int_0^\infty \frac{1}{\alpha} J_1\left(\alpha \cdot r_0\right) \times J_1\left(\alpha \cdot r_p\right) \times \left[1 - \exp\left(-\alpha h\right)\right] d\alpha. \tag{5}$$

The integral is now a purely real function, and therefore affects only the signal magnitude, while the phase is entirely contained in the term between square brackets before the integral. An important point is the value α_0 at which the function $F(\alpha)$ is evaluated. This value is related to the product of Bessel functions appearing into the integral; an argument based to their asymptotic expansion [8] suggests that it should be the reciprocal of the geometrical mean between the dimensions of field coil and pickup coil

$$\alpha_0 = \frac{1}{\sqrt{r_0 r_p}}. \tag{6}$$

Approximation of (5) in (1) yields the phase difference between the pickup signal V and the magnetizing current $\overline{I}$

$$\angle V - \angle \overline{I} = \frac{\pi}{2} + \Delta\varphi = \frac{\pi}{2} - \angle\left[1 - F\left(\alpha_0\right) \times \mathrm{e}^{-2\alpha_0 l_1}\right] \tag{7}$$

where $\pi/2$ is the fundamental phase shift from the Faraday's law and $\Delta\varphi$ is the additional phase shift due to the metal plate. This term contains the information about the plate thickness we want to extract.

The above expression is easier to evaluate than the exact result, but it can be further simplified and expressed in terms of a universal function. By series expansion of the exponential terms in (3) and after a lengthy algebra, it turns out that the phase difference can be expressed as

$$\Delta\varphi \propto -\frac{2z}{1 + z^2} \tag{8}$$

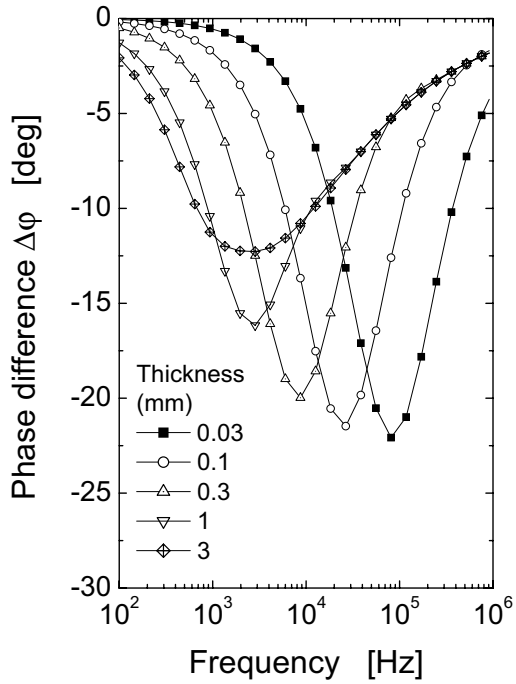

Fig. 2. Theoretical phase difference, calculated after (7), between the magnetizing current and pickup voltage as a function of the current frequency for copper plates of increasing thickness. Fundamental phase shift of $\pi/2$ of the Faraday's law has been subtracted from the phase.

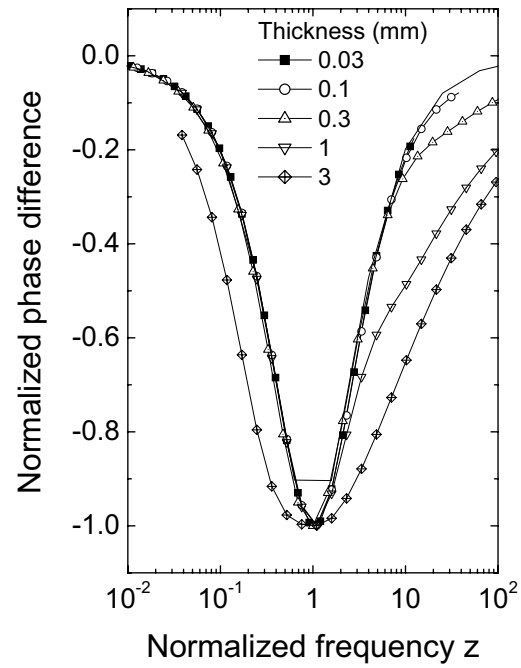

Fig. 3. Phase curves of Fig. 2 normalized to their peak value, and plotted against the normalized frequency of (9) and (10), collapse to a single curve.

where z is a normalized frequency

$$z = \frac{\omega}{\omega_N} \tag{9}$$

with

$$\omega_{\mathrm{N}} = \frac{2\alpha_0}{\sqrt[3]{\alpha_0 l_1}} \times \frac{1}{\mu_0 \sigma} \times \frac{1}{c}. \tag{10}$$

This approach yields a universal equation (8), which gives a function for the phase difference $\Delta\varphi(z)$ valid for any plate material and thickness, and measurement parameters. The fitting of the curve to the actual case is made by a single parameter, i.e., the normalization frequency ω_N.

In order to assess this result, we evaluated numerically the phase difference given by (3) and (7) for a series of copper plates, with thickness ranging from 30 μm to 3 mm, for the experimental arrangement described in the following sections; the results are shown in Fig. 2. Then, we scaled the frequency axis for each curve, as specified by (9) and (10) by using the appropriate thickness c for each plate. The resulting curves (normalized to the maximum phase) are shown in Fig. 3: it is evident that they collapse to this universal form of (8), also plotted in Fig. 3.

The phase difference shows a clear peak, which is shifted toward lower frequencies as the plate thickness increases. The presence of the peak can be broadly explained as follows. The pickup coil senses the sum of two magnetic fields: the main field (due to the field coil) and the induced field, generated by the eddy currents. These two fields are in time-quadrature, because the latter is induced by the former after the Faraday's law. Therefore, the phase angle of the resulting total field (with respect to the main field, taken as a reference) depends on their relative magnitudes (cfr. Fig. 4): if the main field dominates, the phase angle is small; if the induced field dominates, the phase angle tends to $\pi/2$. The main field remains constant as the frequency changes, if the latter is sufficiently low to ignore the coil self-inductance and the skin effect in its winding. Therefore, it is the magnitude of the induced fields, which establishes the total phase angle. When the frequency increases, two opposed effects are at play in establishing the magnitude of the induced field. First, by increasing the frequency the rate of change dB/dt of the main field increases, and so does the induced electrical field that generates the eddy currents in the plate. When these currents increase, the induced field grows and so does the phase difference sensed by the pickup coil. But, as the frequency increases further, the skin effect confines the eddy currents in a progressively thinner layer near the surface, whose resistance is higher. Thus, the eddy currents magnitude decreases, and so does the induced field. Therefore, the magnitude of the induced field (and thus the phase angle) benefits from higher frequencies, up to the point where the eddy currents do not fill any more the entire metal thickness, but begin to be confined to a progressively thinner region. Roughly speaking, the largest effect should happen at the maximum frequency at which the whole metal plate is still occupied by the eddy currents, just before the skin effect kicks in. The peak frequency thus increases for smaller plate thickness.

III. Experimental

The experimental arrangement is shown in Fig. 5. A magnetizing coil having diameter 2.5 cm, height 4 cm, and self-inductance of 140 μH is powered through a resistor $R = 120\ \Omega$. Parasitic inductance and capacitance of this resistor were <1 nH and 10 pF, respectively, and its frequency

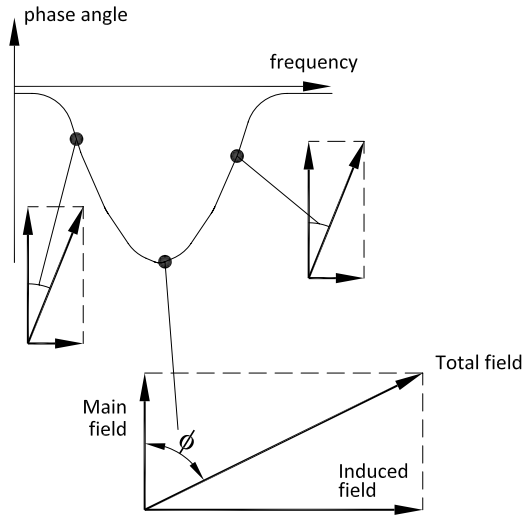

Fig. 4. Components of the total field sensed by the pickup coil at increasingly higher frequencies and resulting phase difference.

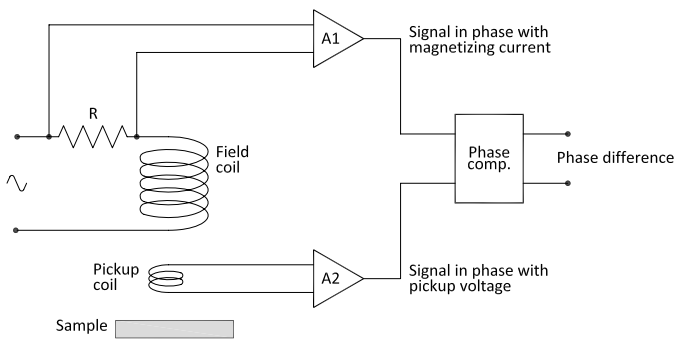

Fig. 5. Schematic experimental arrangement for measuring the phase difference between the main field and the pickup voltage.

behavior was flat up to hundreds of MHz, well above the maximum frequency of 100 kHz employed in our measurements. Therefore, by referring the pickup signal to the voltage drop across R (see. Fig. 5) we are properly referring it to the coil current and then to the main field. The cutoff frequency of the field coil in series with resistor R is about 850 kHz. At higher frequencies, the coil current (and then the main field) would decrease, but the phase reference taken across R would still be correct. The voltage signal from resistor R is amplified by amplifier A1. In our case, the signal is already relatively large and A1 is simply a voltage buffer with unit gain. Its purpose is to separate the input of the phase comparator from the driving voltage of the coil.

The pickup coil (having a diameter of 3 mm) detects the induced voltage V, and its signal is amplified by A2. Amplifier A2 has a gain of 1000, given the small value of the pickup signal. The outputs of the two amplifiers are fed to a simple phase-comparator integrated circuit,[1] which gives an output signal proportional to their phase difference.

In order to have a precise phase measurement, it would be advisable that the amplifiers A1 and A2 have a large bandwidth and therefore a flat frequency response with negligible phase lag in the frequency range of interest. However, if these additional phases are small, they can be accounted for simply

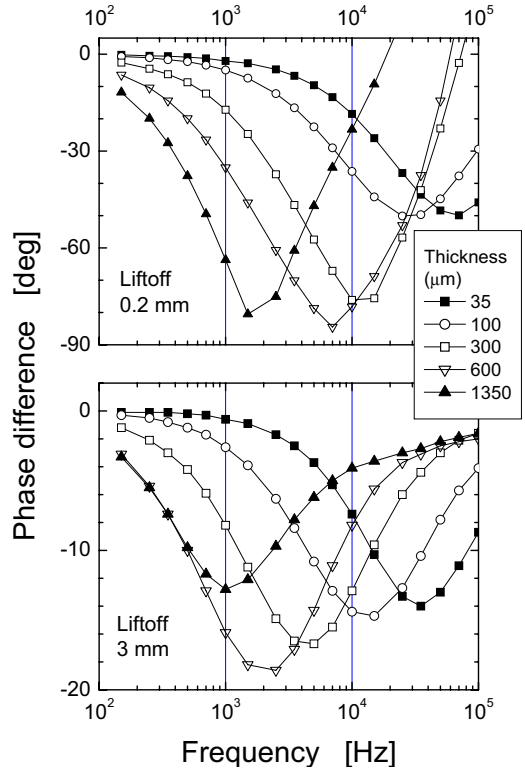

Fig. 6. Experimental phase difference measured for five different copper plate thickness and for two values of the coil liftoff. The fundamental phase shift of $\pi/2$ of the Faraday's law has been subtracted from the phase data.

by performing a reference measurement with field and pickup coil in position, but without any metal plate: the pickup coil detects only the main field and its phase contains all the spurious contributions from the experimental setup. Then, the metal plates are placed in position, and the reference measurement is subtracted from the measured one.

As metal sample, we used copper plates with thickness ranging from 35 μm to 1.35 mm; for the calculations a copper conductivity value of 5.8×10^7 S·m was used. For each sample, the skin depth was larger than the plate thickness in the relevant frequency range: therefore, the eddy currents probe the sample in its entire thickness, and over an area of about 5 cm^2. Any surface or localized conductivity change is averaged out.

A second set of measurements have been made with an aluminum plate 3 mm in thickness, and a 304 stainless steel plate, 2 mm in thickness. The frequency of the magnetizing current was swept from 100 Hz to 100 kHz, and the phase difference was measured as a function of the frequency. The plates of various thickness have been measured with a liftoff ranging from 0.1 to 10 mm, in order to check the sensitivity of the method to the liftoff predicted by (10).

All measurements have been made at room temperature (20 °C), and no appreciable heating of the samples occurred during the measurements.

IV. Results and Discussion

The phase differences thus obtained are shown for the copper plates in Fig. 6, where we subtracted the fundamental $\pi/2$ phase shift between magnetic field and induced voltage.

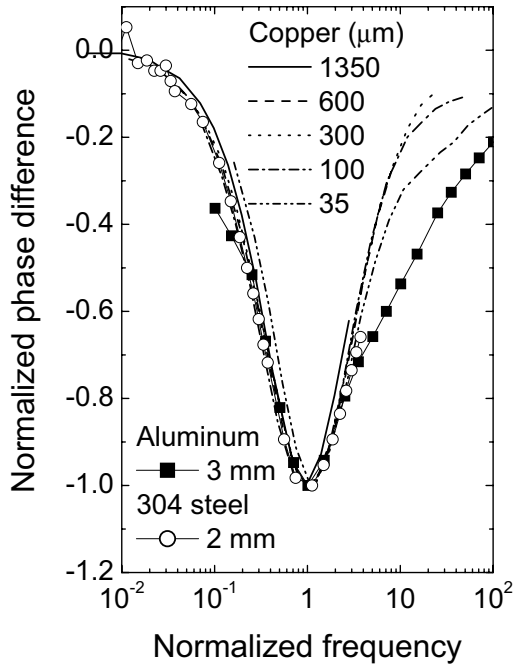

Fig. 7. Experimental curves of Fig. 6 (copper plates) and additional curves for a 304 steel plate and an aluminium one, normalized to their peak value, and plotted against the normalized frequency of (9) and (10).

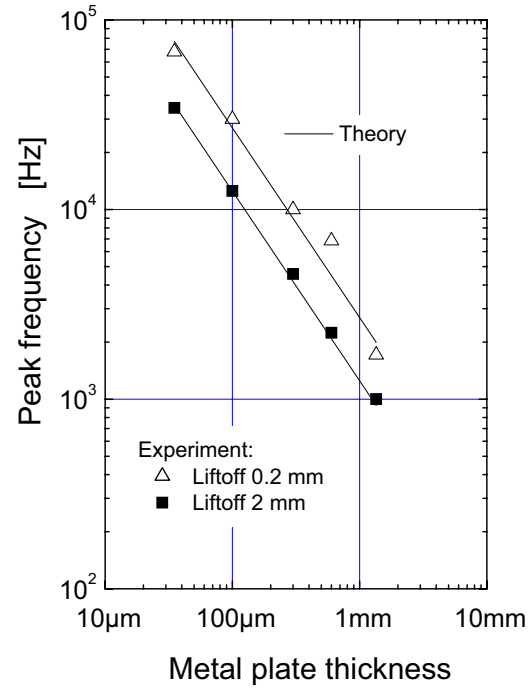

Fig. 8. Comparison between the experimental peak frequencies of the phase (symbols) and the theoretical values expected from (10), for copper plates at two different liftoffs.

The behavior found in our experiment confirms the theoretical background, and generally agrees with the expected one shown in Fig, 2.

In order to check the universal curve predicted by (8), the phase curves obtained for the copper, aluminum, and steel plates have been normalized to their peak, and plotted as a function of the normalized frequency of (9) and (10). The result is shown in Fig. 7, where all curves collapse to the universal form, notwithstanding the large difference in conductivity and thickness of the individual plates.

The frequencies of the phase peak have been compared with the theoretical predictions of (10), and the results are shown in Fig. 8, where the peak frequency is shown as a function of the plate thickness for two different values of liftoff; the agreement is reasonably good for a liftoff of 0.2 mm, and very good for a liftoff of 2 mm.

A second set of measurements have been made in order to study the effect of the liftoff: the phase measurements have been repeated for the same copper plate (thickness 300 μm) with different liftoffs. The results are shown in Fig. 9, where the peak frequency shows the dependence on the liftoff predicted by (10). This explains also the different qualities of the results shown in Fig. 8: at small values of liftoff, it was difficult to be precise enough in maintaining the same liftoff when changing the sample plate. A variation of ± 0.1 mm, e.g., is small in absolute value, but large if compared to the nominal 0.2 mm value. Fig. 8 shows that in this range a small change in the liftoff results in relatively large changes of the peak frequency. Instead, at a liftoff of 2 mm, a variation of ± 0.1 mm has a negligible effect. Therefore, a higher liftoff is less sensitive to positioning errors; the trade-off is a smaller phase difference (compare the two panels of Fig. 6) and then a reduced sensitivity. It is at present unclear why this technique leads to a liftoff dependence of the frequency peak, whereas no such dependence of the phase signature has been found for the measurements described in [6]. However, it has to be pointed out that the geometry and the configuration of our system are quite different from the double-coil one employed in [6]. This point requires further investigation.

V. Analysis of Errors

Measurement errors arise mainly from three sources:

1) error in locating the peak frequency;
2) error in the assumed conductance;
3) error in estimating the liftoff.

Their impact on the thickness estimation is quite different. Let us analyze first the error in locating the frequency of the phase peak, arising mainly from measurement made at discrete frequencies too spaced apart. From Eq. (10), it can be readily shown that a (small) uncertainty $\Delta\omega$ in locating the peak frequency translates in an uncertainty Δc on the thickness value given by

$$\Delta c\,(\Delta\omega) \approx c_{\text{true}} \times \frac{\Delta\omega}{\omega_{\text{peak}}}. \tag{11}$$

Therefore, if the measurement system has a fixed frequency uncertainty, the peaks at lower frequencies (corresponding to thicker plates) will suffer the highest relative uncertainty.

The error in the assumed metal conductance has the same effect

$$\Delta c\,(\Delta\sigma) \approx c_{\text{true}} \times \frac{\Delta\sigma}{\sigma}. \tag{12}$$

The metal conductance can change considerably, e.g., for temperature variation, mechanical stress, presence of impurities, change in metal composition, this is the most serious source

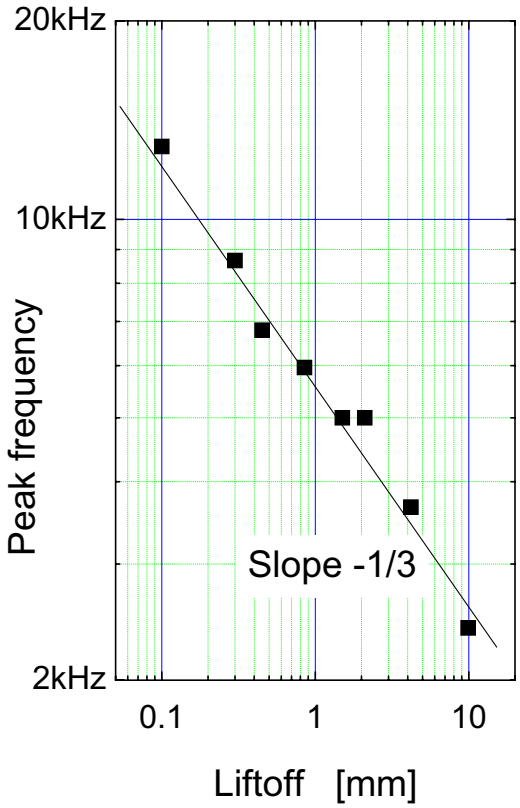

Fig. 9. Comparison between the experimental peak frequencies of the phase (symbols) and the values expected from (10), for a copper plate of fixed thickness (300 m) and variable liftoff. The solid line shows the dependence on reciprocal of the cubic root of liftoff, predicted by (10).

of error. If a high precision is needed, it would be advisable to measure the effective conductance of the sample.

Instead, an error in estimating the liftoff has a lesser impact

$$\Delta c\left(\Delta l_1\right) \approx c_{\text{true}} \times \frac{1}{3} \times \frac{\Delta l_1}{l_1}. \tag{13}$$

This error source can be made small, because in most cases the liftoff l_1 can be made much larger than its uncertainty Δl_1. For example, if the uncertainty is of the order of 0.1 mm (mechanically feasible in most cases), its effect can be greatly reduced by keeping the liftoff to the reasonable value of 1 mm.

VI. Conclusion

In this paper, we present a method for measuring the thickness of metal plates, based on the phase signature of the eddy currents induced by a field coil. The prominent feature of this method is the presence, in the phase signature, of a peak whose position is related to the plate thickness and conductance, and to the liftoff of the measuring coil. This peak has an universal shape, which scales for different samples by a factor containing the above-mentioned quantities. The presence of the phase peak allows a simple automatization of the method, as it can be translated to a voltage peak by means of an inexpensive phase comparator IC, in connection with a variable frequency oscillator, also available in various low-cost integrated circuits.[2]

[2]Very simple voltage controlled oscillators can be built using the classical 555 IC timer.

The results have a certain dependence on the liftoff and its variations, which is rather large for small liftoffs (<0.5 mm). This drawback can be overcome by increasing the liftoff at larger values (>1 mm), where this dependence becomes rapidly negligible. On the other side, this effect can be exploited if the plate thickness and conductance are already known, because in this case it can be used for measuring the thickness of an insulating layer deposited on the plate (e.g., a paint).

Acknowledgment

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