Warping influence on the resistance of uprights in steel storage pallet racks

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1. Introduction

The use of thin-walled cold-formed steel members is significantly increased in the last decades, especially in the field of logistic, where goods and products are frequently stored in pallet racks (mainly, adjustable and drive-in pallet racks), i.e. in structural framed systems made of components manufactured from cold forming steel coils [1–3]. As it can be noted from Fig. 1, adjustable pallet racks (in the following simply indicated as racks) are composed by a regular sequence of upright frames, i.e. built-up laced members. They are connected to each other in the down-aisle (longitudinal) direction by pairs of horizontal beams sustaining pallet units, which generally have boxed cross-section. The structural system is braced by the upright frames in the cross-aisle direction (transversal) but the need to optimize the rack performances in terms of stored pallet units should hamper to locate bracing systems in the down-aisle direction. In these cases, stability to lateral loads is hence provided by the sole degree of flexural continuity associated with the beam-to-column joints and the base-plate connections. Otherwise, if it is possible to locate longitudinal vertical bracings, the semi-continuous braced frame model has to be considered for design. Other key elements of the structural system are the columns (uprights), which have in many cases monosymmetric lipped channel cross-sections (Fig. 2), generally completed by additional lips located at the end of the rear flanges used to bolt, or to weld, lacings to uprights. Owing to the impossibility to develop the calculations by hand, a structural analysis, generally including non-linear geometrical effects, is required for routine design via suitable software based on the finite element (FE) method. Uprights are oriented to have their symmetry axis parallel to the cross-aisle direction and the shear center of the cross-section (point S in Fig. 3) is never coincident with its centroid (point O). From the computational point of view, a suitable beam FE formulation is hence required, which has to be necessary characterized by 7 degrees of freedom (*7DOFs*) for each node (Fig. 3): 3 displacements (u_0, v_s and w_s), 3 rotations (φ_x, φ_y and φ_z) and the warping function θ, which is defined as:

$$\theta = \theta(x) = -\frac{\mathrm{d}\varphi_x}{\mathrm{d}z}. \tag{1}$$

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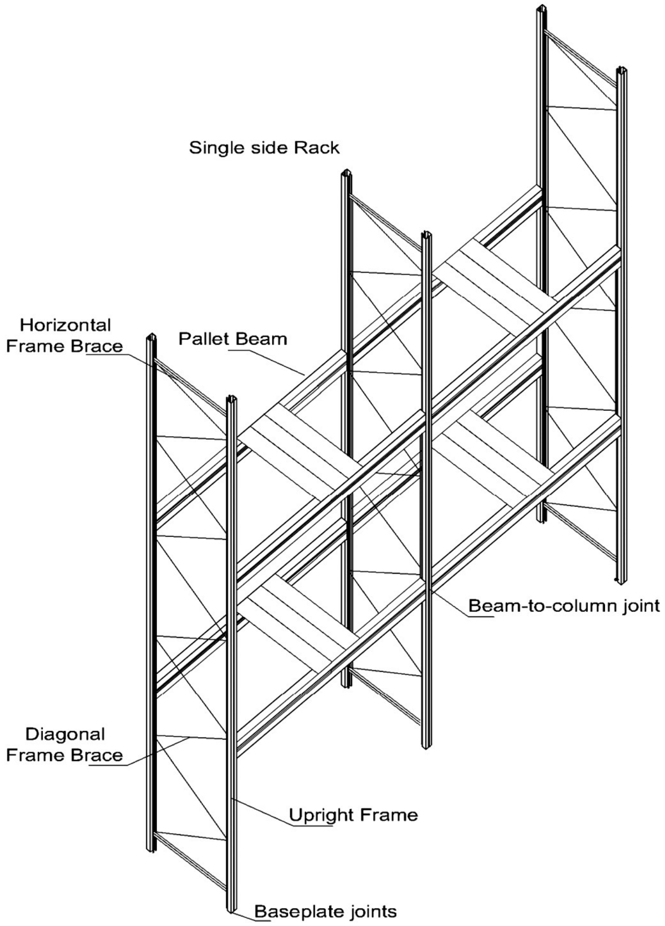


Fig. 1. Typical adjustable pallet rack configuration and key rack components.

Theoretical approaches have been developed in the past [4,5] mainly to predict directly the overall response and the behavior of isolated members. Despite suitable beam formulations have been already proposed in the last decades [6–8], a very limited number of FE analysis programs offer libraries with 7DOFs beam elements [9–12]. Some of them are however not able to represent efficiently the complex behavior of open thin-walled sections, or to capture their actual buckling modes, which are significantly influenced by the coupling between bending and torsion [13]. For members having monosymmetric cross-section, FE beam formulations are significantly different if compared with the ones adopted for bisymmetric cross-sections. Let j and k denote the two nodes of the beam, and the governing matrix displacement equations of the FE element can be written in a general form [14], valid with reference to both elastic $[K]^E$ and geometric $[K]^G$ stiffness matrices, such as:(2)

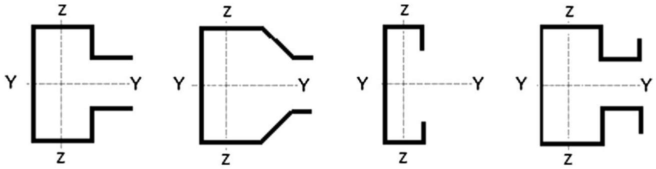


Fig. 2. Examples of cross-section for uprights in adjustable pallet racks.

$$\left[\begin{array}{c|c} [K]_{jj} & [K]_{jk} \\ \hline [K]_{kj} & [K]_{kk} \end{array}\right] \begin{bmatrix} \{u\}_j \\ \{u\}_k \end{bmatrix} = \begin{bmatrix} \{f\}_j \\ \{f\}_k \end{bmatrix} \tag{2}$$

With reference to the more general case of 7DOFs beam formulation, the nodal displacement, $\{u\}_j$ and $\{u\}_k$, and the associate force vectors, $\{f\}_j$ and $\{f\}_k$, can be defined (Fig. 3) respectively as:

$$\{u\}_j = \begin{bmatrix} u_o \\ v_s \\ w_s \\ \varphi_x \\ \varphi_y \\ \varphi_z \\ (\theta) \end{bmatrix} \tag{3a}$$

$$\{f\}_j = \begin{bmatrix} N \\ F_y \\ F_z \\ M_t \\ M_y \\ M_z \\ (B) \end{bmatrix}. \tag{3b}$$

The presence of terms θ and B is only in the beam formulations including also the 7th DOF. As it results also from ref. [15], these formulations are very complex, especially for what concerns the definition of the geometric stiffness matrix $[K]^G$. With reference to a beam element of length L_b, by considering its area (A), second moments of area (I_z and I_y) along principal axes, uniform and non-uniform torsional constants (I_t and I_w, respectively) and assuming E and G represent Young's modulus and tangential material modulus, respectively, the stiffness elastic sub-matrices $[K]^E_{jj}$ (or equivalently $[K]^E_{kk}$) and $[K]^E_{jk}$ (or $[K]^E_{kj}$) are defined as:(4a)

$$[K]^E_{jj} = \left[\begin{array}{cccccc|c} \frac{EA}{L_b} & 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L_b^3} & 0 & 0 & 0 & \frac{6EI_z}{L_b^2} & 0 \\ & & \frac{12EI_y}{L_b^3} & 0 & -\frac{6EI_y}{L_b^2} & 0 & 0 \\ & & & \frac{GI_t}{L_b} + \left(\frac{12EI_w}{L_b^3} + \frac{1}{5}\frac{GI_t}{L_b}\right) & 0 & 0 & \left(\frac{6EI_w}{L_b^3} + \frac{3}{30}\frac{GI_t}{L_b}\right) \\ & & Symmetric & & \frac{4EI_y}{L_b} & 0 & 0 \\ & & & & & \frac{4EI_z}{L_b} & 0 \\ \hline & & & & & & \left(\frac{4EI_w}{L_b^3} + \frac{4}{30}\frac{GI_t}{L_b}\right) \end{array}\right] \tag{4a}$$

(4b)

$$[K]^E_{jk} = \left[\begin{array}{cccccc|c} -\frac{EA}{L_b} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L_b^3} & 0 & 0 & 0 & \frac{6EI_z}{L_b^2} & 0 \\ 0 & 0 & -\frac{12EI_y}{L_b^3} & 0 & -\frac{6EI_y}{L_b^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{GI_t}{L_b} - \left(\frac{12EI_w}{L_b^3} + \frac{1}{5}\frac{GI_t}{L_b}\right) & 0 & 0 & \left(\frac{6EI_w}{L_b^3} + \frac{3}{30}\frac{GI_t}{L_b}\right) \\ 0 & 0 & \frac{6EI_y}{L_b^2} & 0 & \frac{2EI_y}{L_b} & 0 & 0 \\ 0 & -\frac{6EI_z}{L_b^2} & 0 & 0 & 0 & \frac{2EI_z}{L_b} & 0 \\ \hline 0 & 0 & 0 & \left(-\frac{6EI_w}{L_b^3} - \frac{3}{30}\frac{GI_t}{L_b}\right) & 0 & 0 & \left(\frac{2EI_w}{L_b^3} - \frac{1}{30}\frac{GI_t}{L_b}\right) \end{array}\right] \tag{4b}$$

Terms between brackets are related to the sole formulations including the 7th degree of freedom (warping), which influences directly also the terms associated with uniform torsion, i.e. term (4.4). It should be

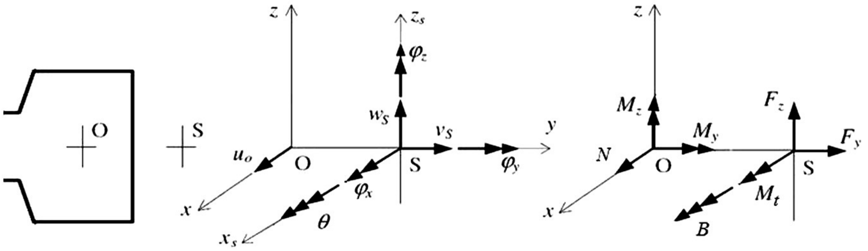


Fig. 3. Nodal displacements and internal forces and moments for a 7DOFs beam element.

noted that classical 6DOFs beam formulations are characterized, for what concerns the elastic stiffness matrix $[K]^E$, and in particular the uniform torsion contribution, by the presence of term $\frac{GI_t}{L_b}$, while in the 7DOFs formulation the contribution $\left(\frac{12EI_w}{L_b^3}+\frac{1}{5}\frac{GI_t}{L_b}\right)$ has to be added directly (for $[K]_{jj}^E$ in sub-matrix (4a)) or subtracted (for $[K]_{jj}^E$ in sub-matrix (4b)) to $\frac{GI_t}{L_b}$. Furthermore, with reference to the geometric stiffness matrix $[K]^G$, a traditional 6DOFs beam formulation requires the knowledge of the sole value of the internal axial load N. Otherwise, in the case of beam formulations including warping, also bending moments (M_y and M_z), torsional moment (M_t), bi-moment (B) and shear actions (F_y and F_z) contribute significantly to form the geometric stiffness $[K]^G$, whose terms depend strictly also from the distance between the load application point and shear center [15]. Only the presence of the 7th degree of freedom makes possible to estimate correctly both frame displacements and internal forces and moments, which influence significantly the local state of stress of upright cross-section. Furthermore, these formulations taking into account the coupling between flexure and torsion are the only ones capable to capture directly also the overall flexural–torsional buckling of the frame as well as of isolated columns, beams and beam-columns.

Routine rack design, also in the case of complex warehouses, is currently developed neglecting all the aspects associated with non-uniform torsion, mainly because no practical indications arise from researchers on this topic. No significant studies have been up-to-now developed to investigate the influence of warping effects on pallet rack response, despite the fundamental need to guarantee a safe design. Design provisions have been very recently updated for Europe [16,17], for the United States [18] and for Australia and New Zealand [19], but practical indications on the key rules to adopt for the numerical analysis, as well as on the minimum requirements of the FE analysis software, are completely omitted in these codes. As a consequence, the verification checks actually adopted for serviceability and ultimate limit states are incorrect, being based on values of internal action and moments and

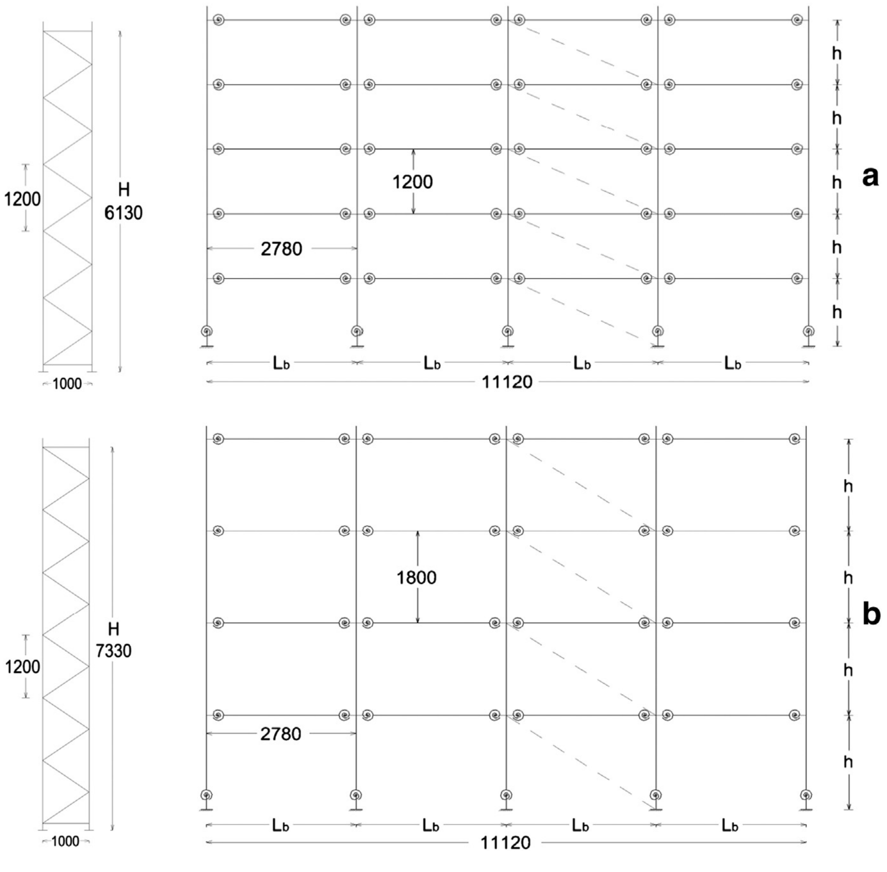


Fig. 4. Geometry of M_5 (a) and M_4 (b) racks (all dimensions are in millimeters): the D-brace upright frame (cross-aisle direction) and the semi-continuous frame (down-aisle direction) models.

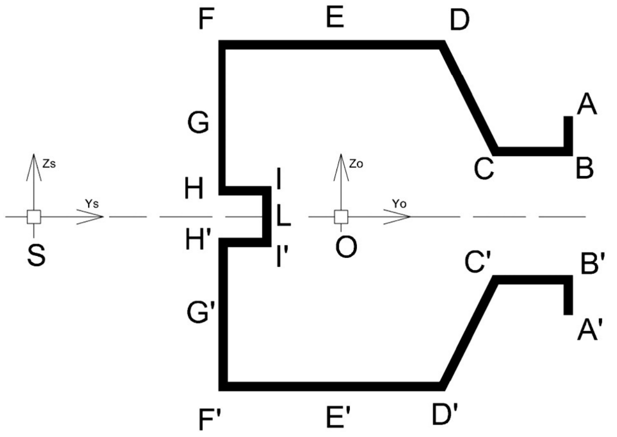


Fig. 5. The upright cross-section of the considered racks.

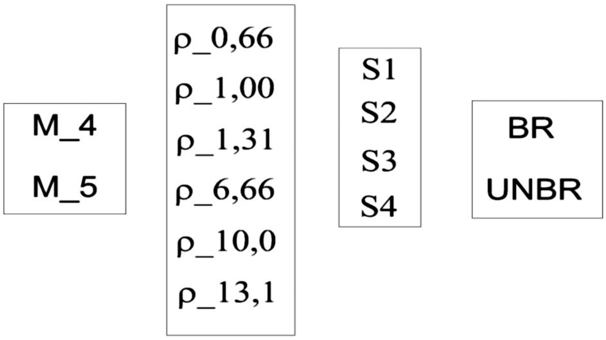


Fig. 8. Summary of the parameters of the numerical analysis.

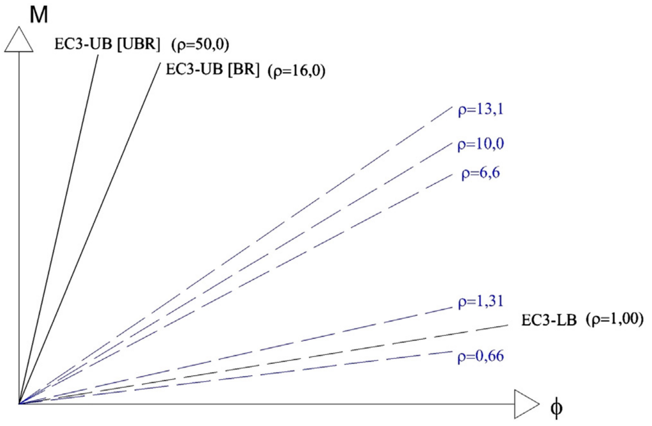


Fig. 6. Rotational stiffness of the beam-to-column joints considered in the analysis (dashed line).

displacements deriving from traditional FE analysis programs with 6DOFs beam element. This should lead to an unsafe design but no indications are available to quantify the effective degree of reliability of a design carried out assuming these incorrect assumptions for monosymmetric cross-section members.

A research project is currently in progress in conjunction between the Politecnico di Milano and the Università di Pavia on the warping influence on the pallet rack design. A suitable finite element beam formulation developed by authors [15] has been implemented in Śiva [20], the FE analysis software used to obtain the results herein summarized. In a previous research of one of the authors [21], warping influence on frame buckling and stability verification checks of beam-column has been already investigated by using a commercial FE analysis program [12]. This paper, which regards other rack configurations, is focused on the resistance checks and presents main outcomes of a numerical analysis on medium-rise racks. Two different beam formulations have been considered in order to appraise the differences in the internal forces and moments due to the presence of the 7th degree of freedom. Furthermore, design equations for both global and local resistance verification checks have been applied in order to evaluate the reduction of the design safety level due to neglecting of warping. Finally, practical indications are proposed to designers in terms of suitable warping safety factors γ_w to use when design is based on approaches neglecting warping torsion effects but an adequate safety level is however required.

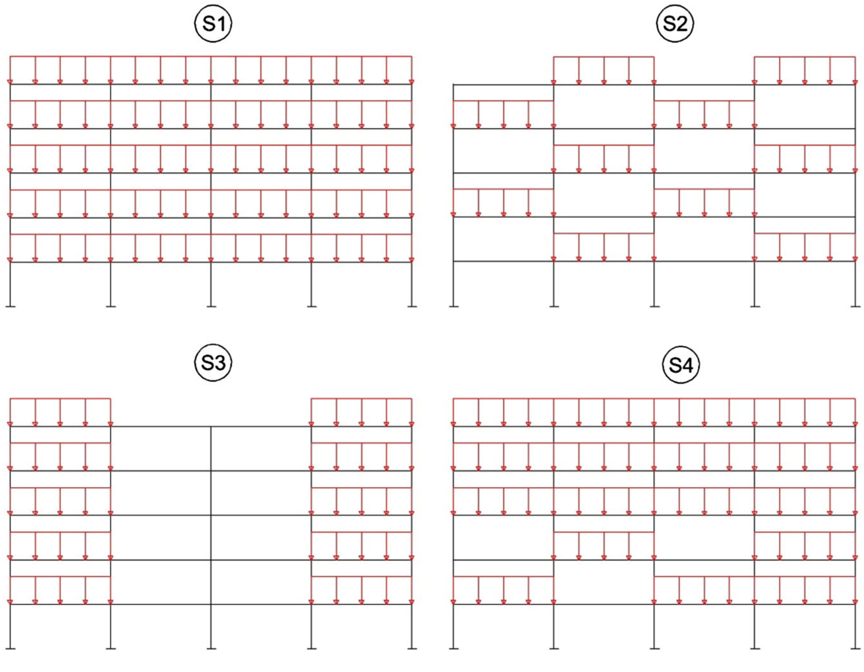


Fig. 7. The considered load conditions for the parametric analysis.

Table 1
Influence of the warping base restraint on M_y and M_z for M_5 racks (_a prevented, _b free warping).

Rack M_5		UNBR				BR			
		C.U.		E.U.		C.U.		E.U.	
L.C.		$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$
S1	Mean	1.08	0.77	1.05	1.00	1.13	0.87	1.02	1.25
	Dev	0.14	0.07	0.07	0.07	0.07	0.11	0.04	0.13
	Min	0.769	0.545	0.914	0.714	0.923	0.500	0.935	1.000
	Max	1.604	0.977	1.361	1.191	1.333	1.095	1.154	1.667
S2	Mean	0.95	1.13	0.97	1.14	0.99	1.00	0.93	1.15
	Dev	0.12	0.21	0.03	0.29	0.04	0.13	0.03	0.21
	Min	0.614	0.667	0.889	0.500	0.877	0.667	0.830	0.800
	Max	1.314	2.000	1.042	2.400	1.119	1.500	1.000	2.100
S3	Mean	0.97	1.15	0.93	1.11	0.97	1.16	1.14	0.88
	Dev	0.02	0.08	0.16	0.24	0.06	0.23	0.08	0.09
	Min	0.892	1.000	0.623	0.500	0.733	0.500	0.960	0.500
	Max	1.024	1.429	1.560	2.100	1.129	2.000	1.340	1.000
S4	Mean	0.98	1.13	0.98	0.97	0.92	1.08	1.00	1.09
	Dev	0.01	0.10	0.03	0.04	0.01	0.06	0.05	0.08
	Min	0.960	0.900	0.863	0.800	0.895	1.000	0.917	0.900
	Max	1.023	1.500	1.036	1.000	0.929	1.250	1.195	1.300

Table 2
Influence of the warping base restraint on M_y and M_z for M_4 racks (_a prevented, _b free warping).

Rack M_4		UNBR				BR			
		C.U.		E.U.		C.U.		E.U.	
L.C.		$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$	$\frac{M_y^{7\_a}}{M_y^{7\_b}}$	$\frac{M_z^{7\_a}}{M_z^{7\_b}}$
S1	Mean	0.95	1.13	1.00	1.15	0.98	0.88	0.94	1.13
	Dev	0.04	0.19	0.01	0.12	0.04	0.14	0.06	0.09
	Min	0.765	0.800	0.964	0.909	0.867	0.500	0.722	1.000
	Max	1.000	2.000	1.040	1.500	1.100	1.250	1.120	1.500
S2	Mean	1.04	0.83	0.89	0.95	1.04	1.00	1.01	0.87
	Dev	0.02	0.18	0.06	0.13	0.03	0.00	0.02	0.07
	Min	1.000	0.500	0.648	0.722	0.946	1.000	0.952	0.667
	Max	1.132	1.500	1.030	1.500	1.168	1.002	1.089	1.000
S3	Mean	1.07	1.08	1.03	0.95	1.11	0.97	1.02	1.01
	Dev	0.05	0.22	0.04	0.12	0.06	0.08	0.02	0.06
	Min	1.000	0.500	1.000	0.667	1.000	0.667	0.971	0.833
	Max	1.253	2.000	1.206	1.364	1.385	1.250	1.077	1.250
S4	Mean	1.00	0.94	0.95	1.00	0.97	0.85	1.00	1.21
	Dev	0.01	0.03	0.08	0.00	0.04	0.123	0.03	0.15
	Min	0.985	0.875	0.600	1.000	0.800	0.500	0.878	1.000
	Max	1.003	1.000	1.100	1.001	1.015	1.111	1.109	1.739

2. Design rules for resistance of rack uprights

As previously mentioned, attention is herein focused on the sole resistance verification checks and all the proposed research outcomes maintain their validity independently by the code used for design. Furthermore, owing to the familiarity of the authors with the European design approaches, reference is made to the contents of the European design rules for steel structures (EC3), which have been prevalently developed with reference to the cases of members having cross-section with two axes of symmetry. In part 1-1 of EC3 [22], which regards the general rules and the rules for building, the non-coincidence between the shear center and the centroid of the cross-section is ignored and the verification checks of beam-columns are referred mainly to bisymmetric I-shaped and hollow cross-sections. Several research activities are currently in progress in Europe to improve these rules, in order to include also the case of I-shaped unequal flanges cross-section members [23] but no adequate attention seems up to now to be paid to the case of monosymmetric cross-sections having a generic geometry. As to resistance check, a very general yield criterion is proposed in European as well as in the other steel codes for the elastic verification. With reference to the critical point of the cross-section, the following condition has to fulfill:

$$\left(\frac{\sigma_{x,Ed}}{f_{y,\mathrm{EU1}}}\right)^2+\left(\frac{\sigma_{z,Ed}}{f_{y,\mathrm{EU1}}}\right)^2-\left(\frac{\sigma_{x,Ed}}{f_{y,\mathrm{EU1}}}\right)\cdot\left(\frac{\sigma_{z,Ed}}{f_{y,\mathrm{EU1}}}\right)+3\cdot\left(\frac{\tau_{Ed}}{f_{y,\mathrm{EU1}}}\right)^2\le 1 \tag{5}$$

where $\sigma_{x,Ed}$ and $\sigma_{z,Ed}$ are the design value of the local longitudinal and transverse stress respectively, τ_{Ed} is the design value of the local shear stress and $f_{y,\mathrm{EU1}}$ represents the design yielding stress (i.e. the value of the yielding stress divided by the material safety factor associated with the considered code).

It should be noted that it is clearly recommended in [22] to take into account the stresses due to torsion in Eq. (5) and, in particular:

- the shear stress τ_{Ed} has to include the contribution $\tau_{t,Ed}$ due to the St. Venant torsion $T_{t,Ed}$ and $\tau_{w,Ed}$ due to the warping torsion $T_{w,Ed}$;
- the normal stress $\sigma_{x,Ed}$ has to include $\sigma_{w,Ed}$ due to the bi-moment B_{Ed}.

No practical indications are provided to engineers for the correct evaluation of stresses $\tau_{t,Ed}$ and $\sigma_{w,Ed}$, which usually can require very complex computations due to the complex rack upright geometry (Fig. 2). Furthermore, a conservative approximation for all the cross-section classes is proposed in this code: in the cases of cross-sections subjected to axial load (N_{Ed}) and bending moments along principal axes ($M_{y,Ed}$ and $M_{z,Ed}$) it is required that:

$$\frac{N_{Ed}}{N_{Rd,\mathrm{EU1}}}+\frac{M_{y,Ed}}{M_{y,Rd,\mathrm{EU1}}}+\frac{M_{z,Ed}}{M_{z,Rd,\mathrm{EU1}}}\le 1 \tag{6}$$

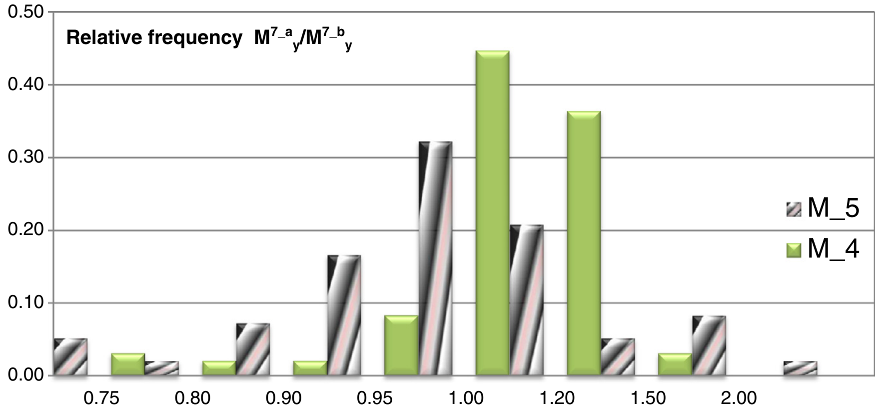


Fig. 9. Influence of the warping restrain at the base column on M_y (data related to both more stressed C.U. and E.U.).

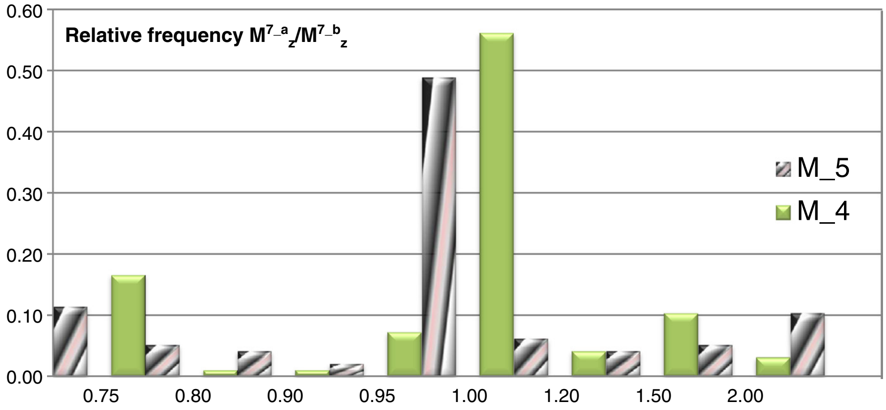


Fig. 10. Influence of the warping restrain at the base column on M_z (data related to both more stressed C.U. and E.U.).

where N_{Rd}, $M_{y,Rd}$ and $M_{z,Rd}$ are the design values of the resistance depending on the cross section classification and subscript EU1 indicates the accordance with design procedure of ref. [22].

As to cold formed members, which are considered in part 1-3 of EC3 [24], it should be noted that very general statements are provided with regard to the possible influence of torsional moments. The direct stresses ($\sigma_{N,Ed}$) due to the axial force N_{Ed}, and the ones ($\sigma_{My,Ed}$ and $\sigma_{Mz,Ed}$), associated with bending moments $M_{y,Ed}$ and $Mz_{,Ed}$, respectively, should be based on the relative effective cross-sections. Properties of the gross cross-section have to be considered to evaluate the shear stresses τ due to transverse shear forces, $\tau_{Fy,Ed}$ and $\tau_{Fz,Ed}$, the shear stresses due to uniform torsion, $\tau_{t,Ed}$, and both normal, $\sigma_{w,Ed}$, and shear stresses, $\tau_{w,Ed}$, due to warping. Owing to the need to reduce the parameters influencing the outcomes of this study, only class 3 profiles are herein considered, for which the effective and the gross cross-sections are coincident.

The total direct stress $\sigma_{tot,Ed}$ and the total shear stress $\tau_{tot,Ed}$ must be respectively obtained as:

$$\sigma_{tot,Ed} = \sigma_{N,Ed} + \sigma_{My,Ed} + \sigma_{Mz,Ed} + \sigma_{w,Ed} \tag{7a}$$

$$\tau_{tot,Ed} = \tau_{Fy,Ed} + \tau_{Fz,Ed} + \tau_{t,Ed} + \tau_{w,Ed}. \tag{7b}$$

In cross-sections subject to torsion, it is required that the following conditions have to be satisfied:

$$\sigma_{tot,Ed} \le f_{ya,\mathrm{EU3}} \tag{8}$$

$$\tau_{tot,Ed} \le {f_{ya,\mathrm{EU3}}} \Big/ {\sqrt{3}} \tag{9}$$

$$\sqrt{\sigma^2_{tot,Ed} + 3 \cdot \tau^2_{tot,Ed}} \le 1.1 \cdot f_{ya,\mathrm{EU3}} \tag{10}$$

where $f_{ya,\mathrm{EU3}}$ is the increased average yield strength due to the forming process and subscript EU3 indicates that reference has to be done to the design safety factor of ref. [24].

European engineers base the rack design on EN 15512 [16], which declares clearly that pallet racks are standard products for which design by calculation alone may not be appropriate. Test procedures are therefore specified where current analytical methods are not given, or are not appropriate, in its Annex A, but no attention is paid to the relevant effects of torsion on design, the philosophy of which has to be in accordance with EN 1990 [24], EN 1993-1-1 [21] and EN 1993-1-3 [23]. Only the expression to evaluate torsional and flexural–torsional buckling load of isolated members is directly presented [16]. Current design practice neglects hence warping for both analysis as well as verification checks and this could lead to a very non-conservative design. Only the very recent Australian standards [19] include a more adequate resistance check criteria for monosymmetric profiles. In particular, in the case of uprights, the section capacity requirement must include also the contribution due to bi-moment ($\mathrm{B_{Ed}}$) acting on the cross-section. It is required that:

$$\frac{N_{Ed}}{N_{Rd,AS}} + \frac{M_{y,Ed}}{M_{y,Rd,AS}} + \frac{M_{z,Ed}}{M_{z,Rd,AS}} + \frac{B_{Ed}}{B_{Rd,AS}} \le 1 \tag{11a}$$

Table 3
Influence of the warping on M_y and M_z for M_5 racks.

Rack M_5		UNBR				BR			
		C.U.		E.U.		C.U.		E.U.	
L.C.		$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$
S1	Mean	0.83	2.41	1.24	0.84	1.30	1.77	1.52	0.68
	Dev	0.06	0.58	0.07	0.11	0.15	0.69	0.12	0.07
	Min	0.555	0.700	0.938	0.293	0.339	0.154	1.040	0.338
	Max	1.217	7.051	1.526	1.482	1.858	6.750	2.083	1.103
S2	Mean	1.28	3.32	1.71	0.94	1.12	1.53	1.04	1.44
	Dev	0.12	0.71	0.11	0.33	0.04	0.28	0.04	0.31
	Min	0.669	0.621	1.291	0.302	0.872	0.639	0.892	0.518
	Max	2.073	6.534	2.182	4.161	1.367	3.150	1.317	3.615
S3	Mean	1.80	1.10	1.51	0.66	0.96	3.13	1.14	1.57
	Dev	0.27	0.12	0.12	0.07	0.05	0.80	0.07	0.15
	Min	0.823	0.413	0.937	0.291	0.695	0.432	0.891	0.519
	Max	2.816	1.666	2.232	1.096	1.220	6.986	1.732	2.274
S4	Mean	1.00	1.44	1.00	3.79	1.09	0.49	0.98	2.54
	Dev	0.02	0.06	0.02	0.61	0.04	0.04	0.03	0.27
	Min	0.921	1.144	0.916	1.774	0.865	0.344	0.804	1.636
	Max	1.079	1.717	1.169	8.364	1.249	0.587	1.133	3.418

Table 4
Influence of the warping on M_y and M_z for M_4 racks.

Rack M_4		UNBR				BR			
		C.U.		E.U.		C.U.		E.U.	
L.C.		$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$	$\frac{M_y^7}{M_y^6}$	$\frac{M_z^7}{M_z^6}$
S1	Mean	2.03	2.93	0.98	7.15	1.36	0.80	1.02	1.71
	Dev	0.31	0.77	0.01	0.47	0.12	0.17	0.04	0.07
	Min	0.461	0.639	0.905	5.290	0.859	0.283	0.911	1.278
	Max	2.935	8.340	1.040	9.831	1.961	1.778	1.346	1.946
S2	Mean	1.10	3.53	2.90	7.01	1.00	1.75	3.39	5.17
	Dev	0.04	0.38	0.15	0.45	0.02	0.22	0.07	0.61
	Min	0.932	2.689	2.080	4.196	0.835	0.667	3.002	2.175
	Max	1.258	5.576	3.513	9.024	1.084	2.862	3.742	8.762
S3	Mean	0.99	3.23	1.03	6.41	1.07	1.64	1.25	1.63
	Dev	0.06	0.37	0.03	0.38	0.06	0.25	0.02	0.26
	Min	0.670	2.549	0.879	5.582	0.776	0.628	1.095	0.524
	Max	1.234	6.303	1.147	8.569	1.509	2.862	1.340	2.593
S4	Mean	1.01	3.11	1.02	7.58	1.24	0.64	1.01	1.27
	Dev	0.02	0.15	0.05	0.45	0.03	0.06	0.02	0.10
	Min	0.913	2.424	0.926	6.133	0.952	0.439	0.945	1.084
	Max	1.083	3.587	1.543	9.372	1.359	0.951	1.195	2.194

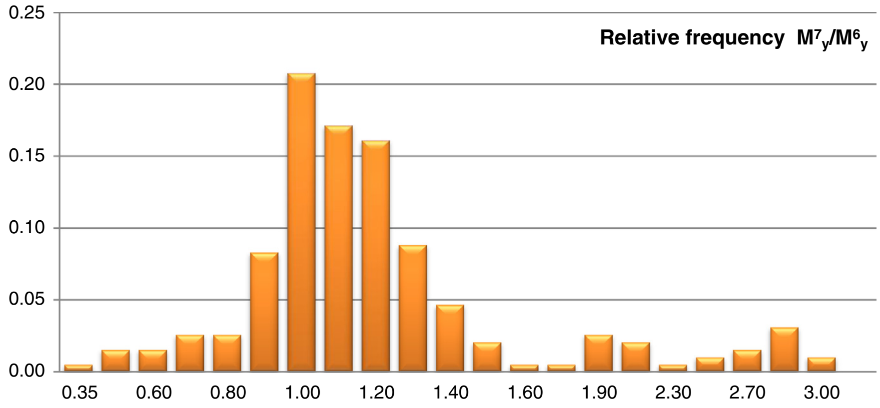


Fig. 11. Influence of warping on M_y for the more stressed C.U.

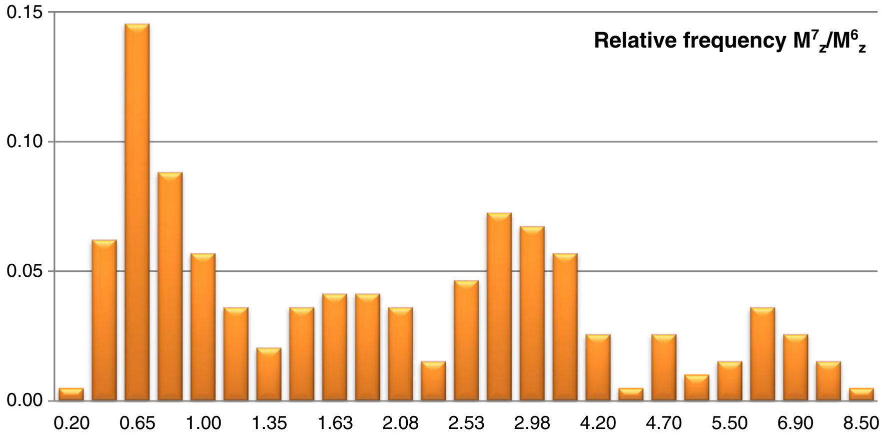


Fig. 12. Influence of warping on M_z for the more stressed C.U.

where subscript AS indicates that the cross-section resistance is evaluated in accordance with the design philosophy of ref. [19] and $B_{Rd,\mathrm{AS}}$ is the bi-moment section capacity defined as:

$$B_{Rd,\mathrm{AS}} = \frac{I_w}{\omega_{\max}} \cdot f_{y,\mathrm{AS}} \tag{11b}$$

where I_w is the warping constant and $\omega_{\max}$ is the maximum value of the static moment of the sectorial area.

3. The considered rack frames

In order to appraise the warping influence on the resistance check, attention has been focused on typical medium-rise rack configurations.

Table 5
Influence of warping on the global cross-section resistance checks for M_5 racks.

Rack M_5		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$
S1	Mean	1.06	1.43	1.14	1.58
	Dev	0.01	0.05	0.04	0.03
	Min	1.001	1.176	1.009	1.459
	Max	1.126	1.637	1.428	1.795
S2	Mean	1.35	1.75	1.19	1.35
	Dev	0.05	0.09	0.02	0.03
	Min	1.059	1.366	1.075	1.228
	Max	1.624	2.217	1.274	1.524
S3	Mean	1.48	1.53	1.23	1.48
	Dev	0.10	0.06	0.03	0.04
	Min	1.079	1.201	1.111	1.267
	Max	1.871	1.913	1.446	1.620
S4	Mean	1.16	1.32	1.07	1.33
	Dev	0.01	0.04	0.02	0.02
	Min	1.085	1.161	0.984	1.218
	Max	1.221	1.693	1.152	1.405

Table 6
Influence of warping on the global cross-section resistance checks for M_4 racks.

Rack M_4		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$	$\frac{SI^7_G}{SI^6_G}$
S1	Mean	1.13	1.32	1.07	1.31
	Dev	0.02	0.01	0.02	0.02
	Min	1.016	1.230	0.997	1.191
	Max	1.257	1.366	1.195	1.395
S2	Mean	1.23	2.26	1.13	2.41
	Dev	0.03	0.13	0.01	0.14
	Min	1.093	1.564	1.066	1.702
	Max	1.321	2.801	1.171	3.030
S3	Mean	1.21	1.34	1.14	1.57
	Dev	0.03	0.04	0.03	0.03
	Min	1.096	1.184	1.020	1.404
	Max	1.359	1.551	1.381	1.732
S4	Mean	1.14	1.41	1.18	1.27
	Dev	0.02	0.14	0.02	0.02
	Min	1.079	1.171	1.058	1.185
	Max	1.193	2.857	1.275	1.399

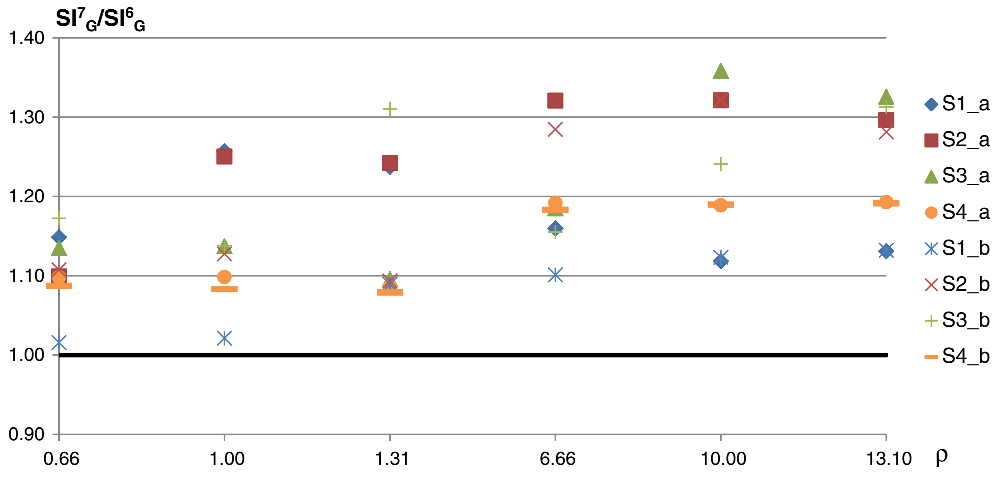


Fig. 13. Influence of warping on the global resistance check for the more stressed C.U. in M_4 UNBR racks.

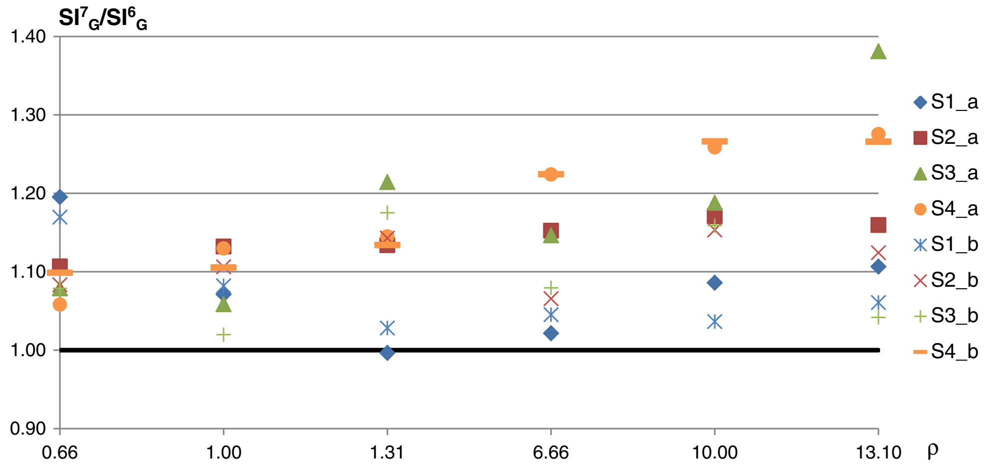


Fig. 14. Influence of warping on the global resistance check for the more stressed C.U. in M_4 BR racks.

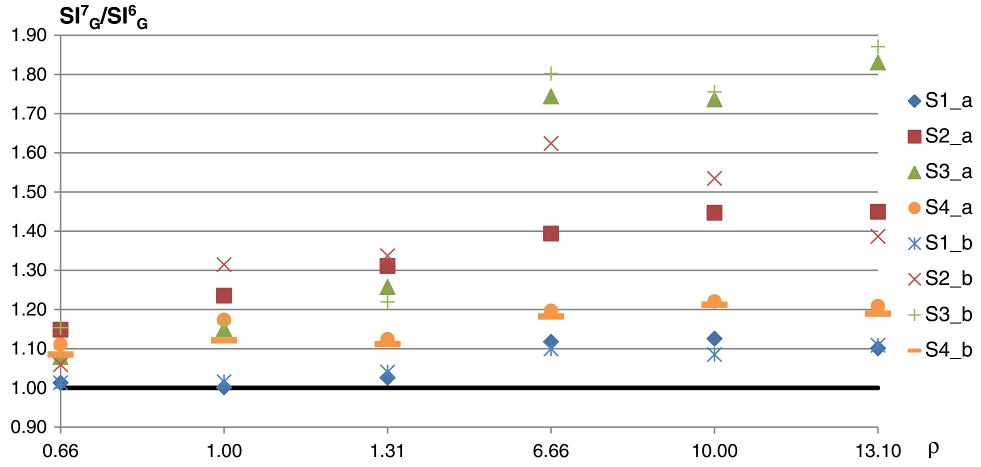


Fig. 15. Influence of warping on the global resistance check for the more stressed C.U. in M_5 UNBR racks.

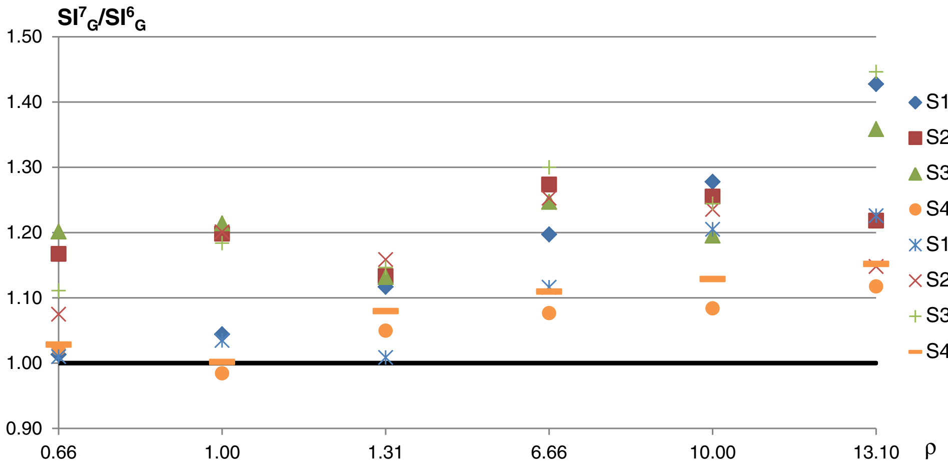


Fig. 16. Influence of warping on the global verification check for the more stressed C.U. in M_5 BR racks.

This study, which comprised of structural analyses and design verifications, has been carried out by considering the following key parameters:

1. the frame geometry (Fig. 4): two racks differing for their interstorey height (h) and overall height (H) were considered:
   - rack M_4 is characterized by 4 storeys with h = 1.80 m and H = 7.33 m;
   - rack M_5 is characterized by 5 storeys with h = 1.20 m and H = 6.13 m.

   For both frames, a 5 equal bay rack configuration (bay span of 2.78 m) was considered. Only the case of D-brace upright frame (Fig. 4), with alternate tension or compression diagonals, was included in this study with a panel height of 1.20 m, owing to the very limited influence of the type as well as of the height of the upright frame panel, as demonstrated in a previous research of one of the authors [20]. As to the rack components, they have been selected with reference to the most common adopted solutions. Upright cross-section is presented in Fig. 5, while rectangular hollow sections have been considered for the upright lacings (30 × 30 × 3 mm) and for the beams (100 × 50 × 3 mm). Owing to the need of limiting the number of variables of the analysis, all cross-section profiles have been selected in order to belong to

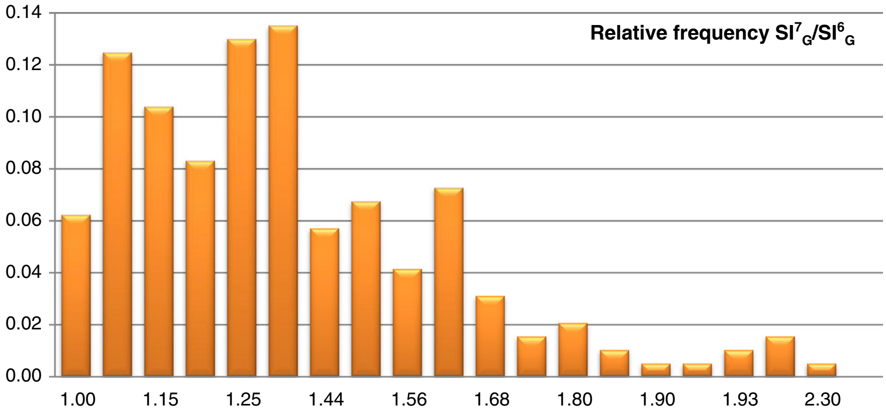


Fig. 17. Influence of warping in M_5 racks for global resistance check (more stressed C.U. and E.U.).

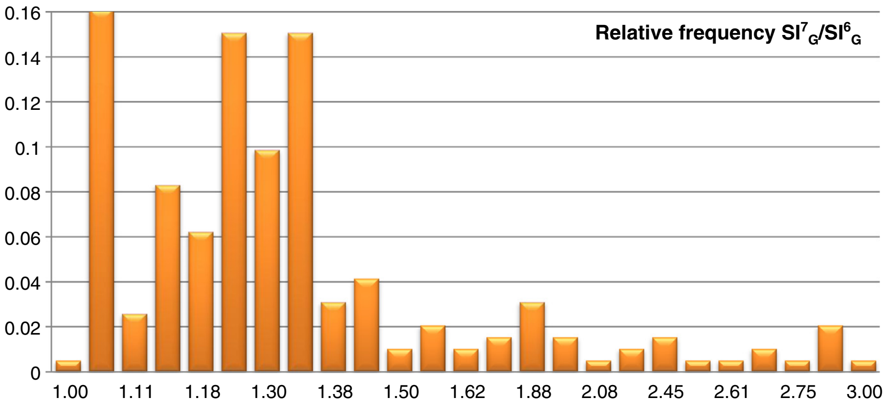


Fig. 18. Influence of warping in M_4 racks for global resistance check (more stressed C.U. and E.U.).

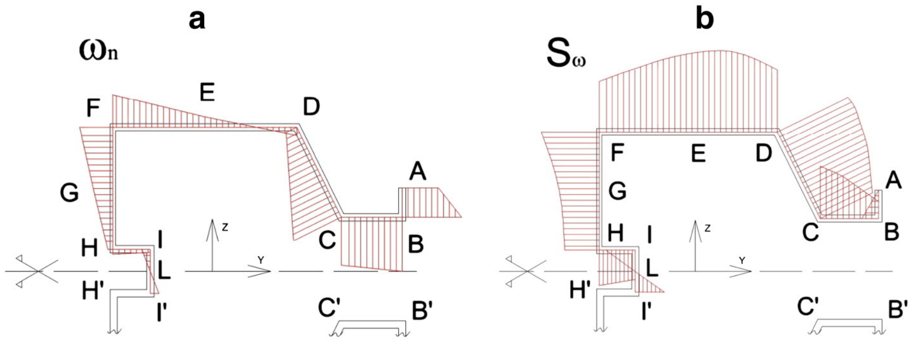


Fig. 19. Distribution of the sectorial area, ω_n (a) and the static moment, S_ω (b) for the considered upright cross-section.

class 3 [21], avoiding hence the influence of local and/or distortional buckling. All rack components and joints are commercial products and, for this reason, no more detailed performance data can be herein given. An overall frame imperfection equal to 3 mrad in terms of out-of-plumb of the uprights in both the cross-aisle and the down-aisle directions has been considered contemporaneously, which has been simulated via horizontal forces concentrated on each load level;

2. the frame typology: for each frame both cases of unbraced (UNBR) and braced (BR) frames in the down-aisle direction were considered. In the case of braced frames, in addition to the bracing in the vertical plane parallel to the main aisle of the rack (spine bracing), also a horizontal bracing has been located on each floor;
3. the degree of flexural stiffness of beam-to-column joints: attention was focused on semi-rigid beam-to-column joints of interest for practical application in rack routine. Considering the elastic rotational stiffness S_j of beam-to-column joints, reference was made to the classification criteria of EC3 1–8 [25]. In particular, the selected values of stiffness S_j have been defined as multiple (by means of term ρ) of a reference stiffness, $S_j^{\text{EC3 - LB}}$, as:

$$S_j = \rho \cdot S_j^{\text{EC3-LB}} \tag{12a}$$

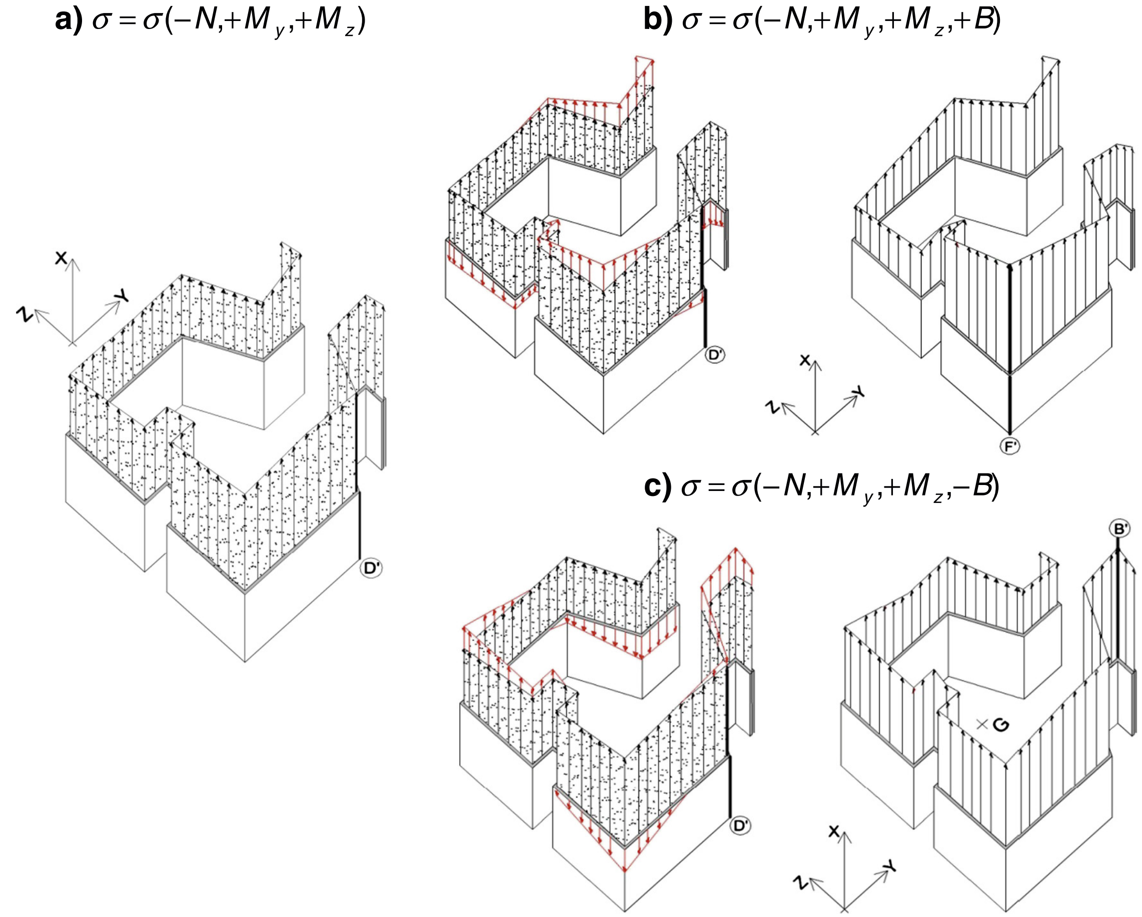
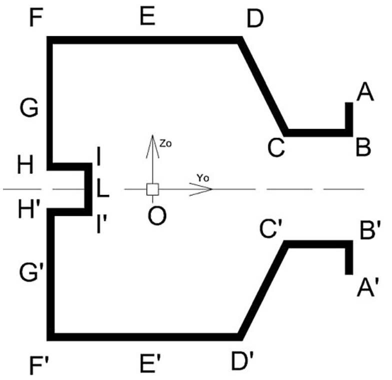


Fig. 20. Examples of influence of the bi-moment B on the location of the maximum normal stress in the cross-section upright of Fig. 19.

Table 7
Influence of the warping on the point where the maximum stress acts on the cross section.



N	M_z	M_y	$\sigma = \sigma(N, M_y, M_z)$ point	B	$\sigma = \sigma(N, M_y, M_z, B)$ point
−	+	+	D′	+	F′
				−	B′
−	+	−	D	+	B
				−	F
−	−	+	F′	+	F′
				−	B′
−	−	−	F	+	B
				−	F

where S_j^{EC3-LB} is the stiffness corresponding to the transition between flexible and semi-rigid joint domains, defined by the code as:

$$S_j^{EC3\text{-}LB} = 0.5\frac{E \cdot I_b}{L_b} \tag{12b}$$

where E is the Young's modulus, I_b is the second moment of area of beam section, L_b is the beam length and ρ is the stiffness parameter which has been considered, in the present study, ranging from 0.67 to 13.10, as it appears from Fig. 6 where they are plotted in the moment (M)–rotation (ϕ) reference system, together with the upper limit of the semi-rigid domain associated with both unbraced ($\rho = 50$) and braced ($\rho = 16$) frames. All the considered values of joint stiffness, which are typical of the possible configurations of beam-to-column joints associated with the considered upright (Fig. 5), have been deduced by test reports related to beam-to-column joint tests executed in accordance with ref. [16].

Table 8
Influence of the warping on the local stress resistance checks for M_5 racks.

Rack M_5		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$
S1	Mean	1.06	1.18	1.11	1.29
	Dev	0.01	0.03	0.03	0.03
	Max	1.125	1.383	1.362	1.508
	Min	0.991	1.001	0.989	1.150
S2	Mean	1.16	1.41	1.07	1.13
	Dev	0.04	0.07	0.01	0.02
	Max	1.395	1.838	1.148	1.259
	Min	0.968	1.104	0.981	1.004
S3	Mean	1.30	1.26	1.06	1.18
	Dev	0.09	0.04	0.02	0.03
	Max	1.638	1.489	1.225	1.343
	Min	0.949	1.013	0.922	1.060
S4	Mean	1.02	1.07	1.03	1.10
	Dev	0.01	0.05	0.01	0.01
	Max	1.056	1.541	1.107	1.153
	Min	0.990	0.982	0.957	1.053

Table 9
Influence of the warping on the local stress resistance checks for M_4 racks.

Rack M_4		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$	$\frac{SI^7_\sigma}{SI^6_\sigma}$
S1	Mean	1.12	1.07	1.06	1.08
	Dev	0.02	0.02	0.02	0.02
	Max	1.227	1.187	1.146	1.207
	Min	1.007	1.006	0.984	1.013
S2	Mean	1.07	1.81	1.00	1.95
	Dev	0.02	0.09	0.01	0.09
	Max	1.143	2.145	1.029	2.351
	Min	0.987	1.286	0.932	1.513
S3	Mean	1.04	1.08	1.05	1.21
	Dev	0.02	0.02	0.03	0.02
	Max	1.174	1.316	1.258	1.374
	Min	0.953	1.035	0.936	1.129
S4	Mean	1.02	1.03	1.11	1.03
	Dev	0.01	0.01	0.02	0.01
	Max	1.052	1.123	1.176	1.092
	Min	0.979	1.005	0.992	0.986

4. the load condition (Fig. 7): rack bays have been considered directly loaded by pallets and a uniform distributed load acting on each beam was assumed. Four different load conditions have been identified as representative for rack design:
   a) fully loaded condition, i.e. each bay is loaded (in the following indicated as S1);
   b) alternate loaded condition giving rise to single curvature on uprights when the rack is braced (S2);
   c) external bays only loaded on each load levels (S3);
   d) full load on the rack with the exception of few lowest beam level, near the middle of the racks (S4), as indicated in Fig. 7.
   It should be noted that S1 and S4 load conditions are recommended also by rack standard codes, while the other ones have been identified on the basis of the expertise of the authors in rack design.

The sole case of semi-rigid joints has been considered, owing to the need of limiting the number of variables influencing warping effects. The value of the base-plate joint stiffness corresponds to 0.11 and to 0.17 times the flexural upright stiffness $\left(\frac{EI_u}{h}\right)$ for M_5 and M_4 frames, respectively.

Finite element analysis has been executed by Śiva software [26]: in addition to the beam element formulation including warping, i.e., the beam formulation with 7DOFs [15], Śiva's library offers also the more traditional beam element based on the classical 6DOFs beam formulation [14]. Both types of analysis, i.e. with 6DOFs and 7DOFs beam element formulations have been executed for all the considered rack frames. Warping restraint has been considered free for the upright top as well as for the bracing upright members, also in correspondence of the intersection with upright. As to the end of beams, due to the available forms of end connectors, warping has been considered blocked. Owing to the different possibilities to connect the upright end to the industrial floor, i.e. due to the different types of available connections, both cases of column base with warping totally prevented (_a) or free (_b) have been considered. Fig. 8 presents the analysis layout explaining symbols used to present main research outcomes.

4. Influence of warping on internal forces and moments

At first attention has been focused on the influence of the warping restraint at the column bases related to the 7DOFs beam analyses. Tables 1 and 2 present the ratio of $\frac{M_y^{7-a}}{M_y^{7-b}}$ and $\frac{M_z^{7-a}}{M_z^{7-b}}$ between the bending moment (M_y^7 or M_z^7) obtained by considering prevented (_a) or free (_b) the column base warping. Mean value and standard deviation for all

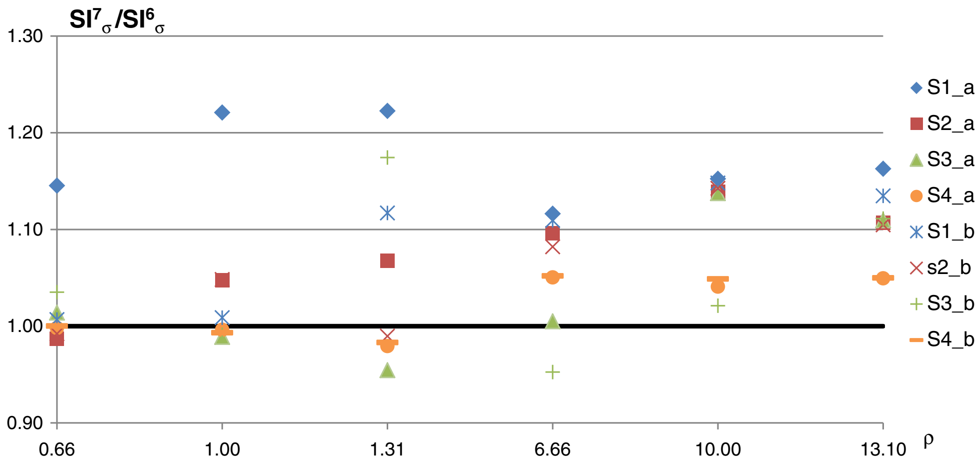


Fig. 21. Influence of warping on the local resistance check for M_4 UNBR racks (more stressed C.U.).

the racks under the same load conditions are reported in the tables together with the maximum (max) and the minimum (min) values of the ratio. These values are practically independent from the beam-to-column joint stiffness as well as from the location of the considered upright, i.e., internal (C.U.) or external (E.U.). As it can be noted from Figs. 9 to 10, related respectively to the distribution of M_y and M_z bending moment ratios, the greatest values of the relative frequency are in correspondence of unity. Furthermore, the great dispersion for bending moment ratios confirms the relevant influence of the warping base restraints. No general conclusions seem possible on the basis of reanalysis of these ratios but the importance of a correct base restraint modeling should be underlined, owing to the non-negligible influence on the values of internal forces and moments and, as a consequence, on design verifications.

Due to the differences in the stiffness matrices of 6DOFs and 7DOFs beam formulations, upright warping is expected to influence significantly the values of internal forces and moments and rack displacements. As to axial load (N) no significant differences have been observed while influence of warping is non-negligible for shear forces (F_y and F_z), despite the fact that generally shear doesn't represent a parameter governing rack design, as shown in the following. As to bending moments M_y and M_z, a summary of the differences can be directly appraised via Tables 3 and 4 related to $\frac{M_y^7}{M_y^6}$ and $\frac{M_z^7}{M_z^6}$ ratio, where data are grouped for each load condition. The mean value and the standard deviation of the ratio are reported, together with the maximum and minimum values of these ratios. Data related to free and prevented base warping have been treated together in M_y^7 and M_z^7 and these ratios are presented for the more stressed both internal (C.U.) and external (E.U.) upright. In a very limited number of cases the mean value of $\frac{M_y^7}{M_y^6}$ is lower than unity for M_5 (S1_UNBR_C.U., S3_BR_C.U. and S4_BR_E.U.) and M_4 (S1_UNBR_E.U. and S3_UNBR_C.U.) racks. If ratio $\frac{M_z^7}{M_z^6}$ is considered, the number of cases with mean value lower than unity increases slightly for M_5 (S1_UNBR_E.U., S2_UNBR_E.U., S3_UNBR_E.U., S1_BR_E.U. and S4_BR_C.U.) and for M_4 (S1_BR_C.U. and S4_BR_C.U.) racks. Owing to the great dispersion of these ratios, which is independent from the load conditions, as it can be directly appraised by the values of the standard deviation, it can be concluded that no prediction can be a priori made associated with the considered parameters on the influence of warping and hence the sole 7DOFs beam formulation appears adequate to evaluate correctly design internal actions. This very general remark is confirmed also by Figs. 11 and 12 where relative frequency of $\frac{M_y^7}{M_y^6}$ and $\frac{M_z^7}{M_z^6}$ ratios for both M_4 and M_5 racks is plotted. It can be noted in fact that with reference to M_y a quite limited number of racks presents a ratio lower than unity and a great concentration of data is in the range 1.0–1.15. Otherwise, if M_z is considered, a significant number of cases presents ratios lower than unity and several racks are characterized by bending ratios associated with a relative frequency significantly greater than 1.0 up to 8.5.

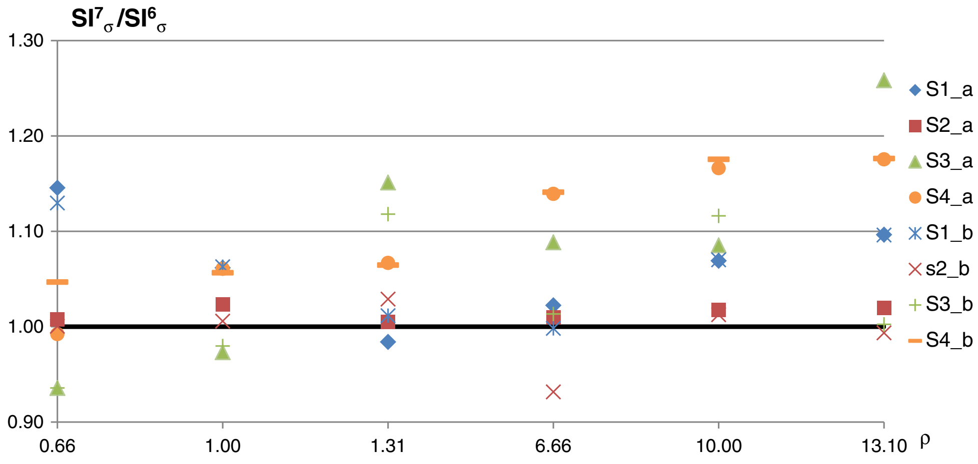


Fig. 22. Influence of warping on the local resistance check for M_4 BR racks (more stressed C.U.).

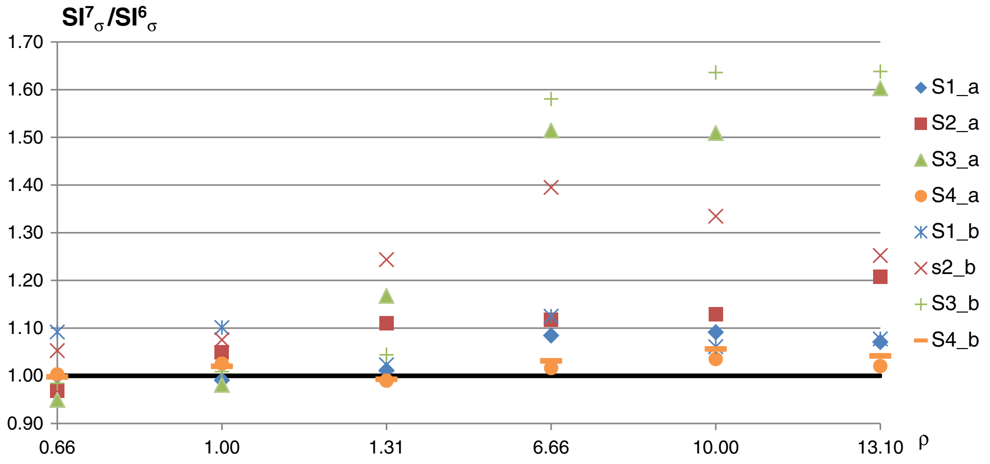


Fig. 23. Influence of warping on the local resistance check for M_5 UNBR racks (more stressed C.U.).

5. Warping influence on the global resistance checks

Modern design codes base verification checks on the evaluation of a safety index (*SI*), which are fulfilled if $SI \leq 1$, and for routine rack design are associated with the use of beam formulations with 6DOFs per node. Owing to the fact that the considered upright belongs to class 3, the corresponding safety index (SI_G^6) is referred to the global properties of the cross-section in terms of axial (N_{Rd}) and bending resistance ($M_{y,Rd}$ and $M_{z,Rd}$), and it is defined from Eq. (6) as:

$$SI_G^6 = \frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} = \frac{N_{Ed}}{f_y A} + \frac{M_{y,Ed}}{f_y I_y} z_{\max} + \frac{M_{z,Ed}}{f_y I_z} y_{\max}. \tag{13a}$$

In the case of beam formulations including the influence of warping, the global safety index (SI_G^7) has to be taken into account necessarily also for the bi-moment contribution, as very recently recommended by Australian standards [19]. In accordance with the criteria associated with the Eq. (11), SI_G^7 can be defined as:

$$SI_G^7 = SI_G^6 + \frac{B_{Ed}}{f_y I_w} \omega_{\max}. \tag{13b}$$

Tables 5 and 6, related respectively to M_5 and M_4 frames, present the mean values of the ratio $\frac{SI_G^7}{SI_G^6}$, for the more stressed internal (C.U.) and external (E.U.) upright of each set of racks having the same load conditions. Standard deviation, maximum and minimum values of this ratio are reported in the tables, too. Also in these cases, data related to free and fixed warping of the base-plate connection have been grouped in SI_G^7. As to the dependency of this ratio from the joint stiffness, reference can be made to Figs. 13 and 14 for M_4 unbraced and braced frames, respectively, and to Figs. 15 and 16 for M_5 unbraced and braced frames where both $\frac{SI_G^{7-a}}{SI_G^6}$ and $\frac{SI_G^{7-b}}{SI_G^6}$ are plotted versus ρ. It can be noted that $\frac{SI_G^7}{SI_G^6}$ is always greater than unity, with greatest values associated with external uprights. This ratio is practically independent from the stiffness parameter ρ and the greatest values are generally for the S2 and S3 load conditions. No significant differences can be appraised with reference to the warping base restraints. Furthermore, the distribution of $\frac{SI_G^7}{SI_G^6}$ is plotted in Figs. 17 and 18 considering data related to both internal and external upright of M_5 and M_4 frames, respectively. It can be noted that several values fall in the range 1.0–1.4 but in several cases the bi-moment contribution in the resistance verification checks is very important. Its absence could lead to a very non-conservative and

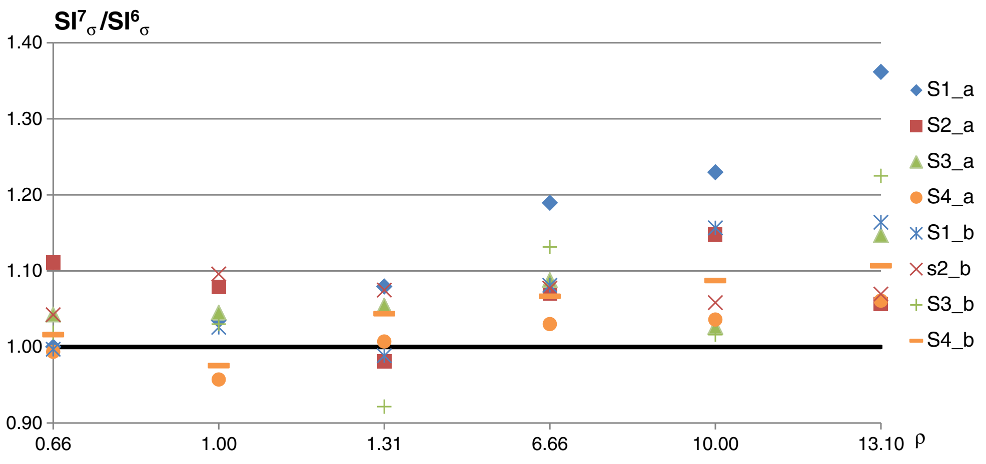


Fig. 24. Influence of warping on the local resistance check for M_4 BR racks (more stressed C.U.).

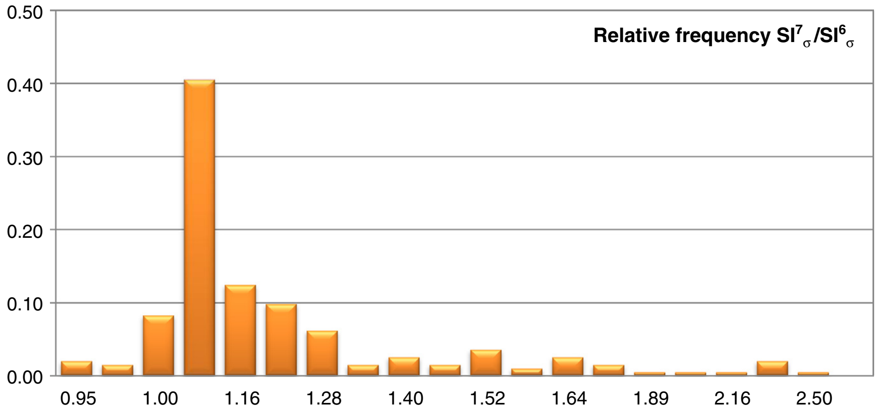


Fig. 25. Influence of warping on the local resistance check for M_5 racks (more stressed C.U. and E.U.).

dangerous design, being the safety index overestimated up to approximately 3 times.

6. Warping influence on the local resistance checks

Normal $\sigma_{w,Ed}(y, z)$ and shear $\tau_{w,Ed}(y, z)$ stresses due to the bi-moment B_{Ed} in a general point P of co-ordinate (y,z) defined with reference to the cross-section centroid (Fig. 3), can be expressed as:

$$\sigma_{w,Ed}(y,z) = \frac{B_{Ed}}{I_w} \cdot \omega(y,z) \tag{14a}$$

$$\tau_{w,Ed}(y,z) = \frac{T_w}{I_w} \cdot \frac{S_\omega(y,z)}{t} \tag{14b}$$

where T_w represents the non-uniform torsional moment, t is the thickness of the cross-section and all the other symbols have been previously defined.

As already mentioned, the use of Eq. (13b) in resistance checks could lead to a slightly conservative design, owing to the fact that the maximum of the sectorial area ($\omega_{\max}$), as well as of its first moment of area ($S_{\omega,\max}$), is not at the same location where stresses due to bending moments reach the maximum values. As a consequence, it should appear more appropriate, in order to guarantee an optimal use of the material, to evaluate the local distribution of the normal stresses summing the values of the stresses occurring at the same point of the cross-section. The distributions of the sectorial area $\omega(y, z)$ and of its first moment $S_\omega(y, z)$ are presented in Fig. 19 for the cross-section geometry of the considered upright. With reference to the sole normal stresses, owing to the influence of warping, the non-coincidence between the points where normal stress is maximum if a 6DOFs or a 7DOFs beam formulation is used as can be noted in Fig. 20. If reference is made to the sign conventions of Fig. 3, maximum normal stress is in point D′ if the sole axial load and positive bending moments are considered. Otherwise, if bi-moment B_{Ed} acts on cross-section, maximum stress is in correspondence of point F′ ($B_{Ed} > 0$) or point B′ if ($B_{Ed} < 0$). More in general, Table 7 indicates the point where normal stress is maximum when axial load is negative (compression) and moments are positive or negative. It appears that if the normal stresses due to warping are neglected, $\sigma = \sigma(N, M_y, M_z)$, or considered, $\sigma = \sigma(N, M_y, M_z, B_w)$, the point with the maximum stress coincides only in the following two cases: all the moments negative or when M_z is negative and M_y and B are positive. Otherwise a moderate member over sizing is possible when Eq. (13b)) is used.

Neglecting the presence of material safety factors γ_m, which depends on the considered code, resistance safety index based on the local stress value SI_σ can be defined, in accordance with Eq. (10), as:

$$SI_\sigma = \frac{\sqrt{[\sigma_{tot}(y,z)]^2 + 3 \cdot [\tau_{tot}(y,z)]^2}}{f_y}. \tag{15}$$

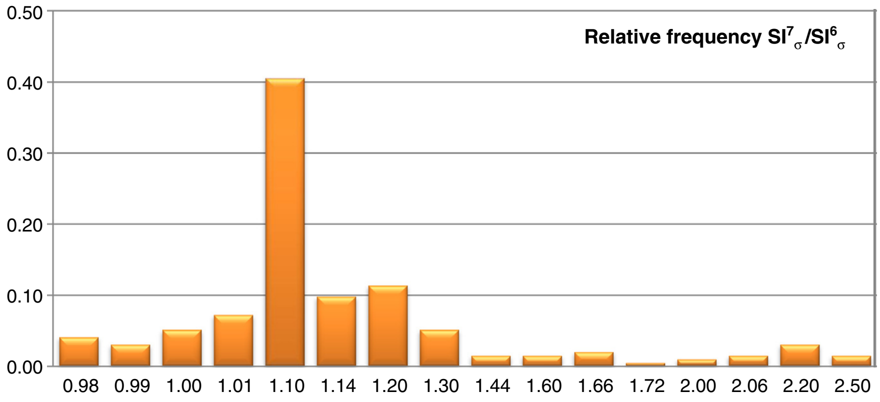


Fig. 26. Influence of warping on the local resistance check for M_4 racks (more stressed C.U. and E.U.).

Table 10
Influence of the resistance check criteria on the safety index for M_5 racks.

Rack M_5		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$
S1	Mean	1.01	1.20	1.03	1.24
	Dev	0.01	0.02	0.00	0.02
	Max	1.037	1.297	1.059	1.338
	Min	1.002	1.064	1.016	1.129
S2	Mean	1.17	1.25	1.11	1.19
	Dev	0.02	0.03	0.02	0.03
	Max	1.287	1.378	1.194	1.332
	Min	1.006	1.096	1.000	1.083
S3	Mean	1.14	1.22	1.13	1.25
	Dev	0.01	0.02	0.03	0.01
	Max	1.179	1.305	1.246	1.309
	Min	1.069	1.114	1.000	1.192
S4	Mean	1.15	1.23	1.04	1.24
	Dev	0.01	0.02	0.00	0.03
	Max	1.188	1.295	1.057	1.402
	Min	1.105	1.097	1.022	1.124

In the following, for the generic point P of co-ordinates (y,z), the verification checks associated with the use of both 6DOFs and 7DOFs beam elements can be expressed in terms of safety index, as:

$$SI_\sigma^6 = \frac{\sqrt{\left[\sigma_{tot}^6(y,z)\right]^2 + 3\cdot\left[\tau_{tot}^6(y,z)\right]^2}}{f_y} = \frac{\sqrt{\left[\sigma_{tot}^6(y,z)\right]^2 + 3\cdot\left[\tau_{Fy}(y,z)+\tau_{Fz}(y,z)\right]^2}}{f_y} \tag{16a}$$

$$SI_\sigma^7 = \frac{\sqrt{\left[\sigma_{tot}^7(y,z)\right]^2 + 3\cdot\left[\tau_{tot}^7(y,z)\right]^2}}{f_y} = = \frac{\sqrt{\left[\sigma_{tot}^6(y,z)+\sigma_{w,Ed}(y,z)\right]^2 + 3\cdot\left[\tau_{tot}^6(y,z)+\sigma_{w,Ed}(y,z)\right]^2}}{f_y} \tag{16b}$$

where terms $\tau_{Fy}(y,z)$ and $\tau_{Fz}(y,z)$ represent the tangential stresses due to the shear forces F_y and F_z, respectively.

As previously mentioned, the contribution of F_y and F_z to the resistance verification check is very modest and the influence of the term $\tau_{tot}^6(y,z)$ on the evaluation of SI_σ^6 is very limited, not greater than 0.1% of the maximum normal stress. If the bi-moment contribution is considered to evaluate $\tau_{tot}^7(y,z)$, shear stress influence on SI_σ^7 is greater than in the previous case, but, however, remains negligible, not greater than 0.5%. For each of the points of the upright cross-section indicated in Fig. 7 the local state of stress has been evaluated. As in the previous cases, by considering both SI_σ^{7-a} and SI_σ^{7-b} grouped in term SI_σ^7, Tables 8 and 9 present for M_5 and M_4 racks the mean value of $\frac{SI_\sigma^7}{SI_\sigma^6}$ ratio for each set of similar frames having the same load condition. Standard deviation with the maximum and minimum values of the ratio is reported in these tables, too. As for the global safety index SI_G, also in the case of index SI_σ reference has been made to the more stressed internal (C.U.) and external (E.U.) column. In Figs. 21 and 22, the ratio $\frac{SI_\sigma^7}{SI_\sigma^6}$ for the more stressed uprights is plotted versus ρ for M_4 unbraced and braced frames, respectively. Corresponding Figs. 23 and 24 present the data for M_5 frames. Previous comments related to the global safety index SI_σ are confirmed also in the case of local safety index SI_σ. Ratio $\frac{SI_\sigma^7}{SI_\sigma^6}$ is independent on the joint stiffness and the more relevant warping influence is for external uprights.

Table 11
Influence of the resistance check criteria on the safety index for M_4 racks.

Rack M_4		UNBR		BR	
		C.U.	E.U.	C.U.	E.U.
L.C.		$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$	$\frac{SI_G^7}{SI_\sigma^7}$
S1	Mean	1.01	1.23	1.01	1.22
	Dev	0.01	0.02	0.01	0.02
	Max	1.059	1.316	1.053	1.325
	Min	1.001	1.119	0.999	1.130
S2	Mean	1.15	1.25	1.12	1.18
	Dev	0.01	0.01	0.01	0.02
	Max	1.197	1.307	1.145	1.266
	Min	1.075	1.197	1.080	0.996
S3	Mean	1.17	1.24	1.08	1.30
	Dev	0.01	0.02	0.01	0.03
	Max	1.215	1.313	1.153	1.391
	Min	1.117	1.132	1.038	1.114
S4	Mean	1.11	1.24	1.07	1.23
	Dev	0.01	0.02	0.00	0.02
	Max	1.140	1.300	1.086	1.293
	Min	1.086	1.132	1.051	1.146

The maximum values of the relative frequency plotted in Figs. 25 (M_4 racks) and 26 (M_5 racks) are in the range 1.08–1.12 despite the fact that a great dispersion can however be observed. In a very limited number of cases, the ratio $\frac{SI_\sigma^7}{SI_\sigma^6}$ is slightly lower than unity. This is due to the greater flexibility of the rack modeled via a 7DOFs beam formulation to which correspond slightly lower values of the bending moments acting on the cross-section; only in these very few cases a moderate conservative design could hence be obtained via a 6DOFs beam formulation. In all the other cases it appears fundamental to take adequately into account warping, owing to great influence also on the local verification safety index SI_σ^7, which is in some cases up to 2.5 times SI_σ^6.

7. A proposal for routine design

Results previously presented underline the non-negligible influence of warping effects in resistance verification checks, confirming the need to analyze racks as well as every frame with monosymmetric cross-section members via suitable analysis software programs having adequate 7DOFs beam element formulations. Two different criteria for the resistance verification of the cross-section have been previously considered to evaluate the global (SI_G) and the local (SI_σ) safety index. Tables 10 and 11 summarize the data related to the distribution of the ratio $\frac{SI_G^7}{SI_\sigma^7}$, in order to appraise the concrete influence of the verifications criteria on design. As expected SI_σ^7 is slightly lower than SI_G^7 owing to the more accurate evaluation of the state of stresses on cross-section.

Figs. 27 and 28 plot the distribution of $\frac{SI_G^7}{SI_\sigma^7}$ ratio for M_5 and M_4 racks, respectively. For internal uprights (C.U.) the global safety index for resistance check is slightly greater than the local one. With reference to the external uprights (E.U.) these differences are significantly greater, approximately up to 30% and appear to be independent from the load condition.

Correct design of these types of structures should result very complex for many engineers and practitioners: in addition to the availability of an appropriate software analysis, it is in fact also required to have an adequate knowledge on the theory of thin-walled member behavior [4–6] and the ability to determinate non-uniform torsional stiffness coefficients as well as the Wagner coefficients for very complex cross-section geometries (Fig. 2). As alternative to a refined design using 7DOFs beam formulation, traditional procedures based on 6DOFs beam structural analysis should however still be used by increasing the safety index (SI^6) via a suitable safety factor γ_w accounting for the neglected warping. An evaluation of (SI^7) should be obtained as:

$$SI^7 = \gamma_w \cdot SI^6. \tag{17}$$

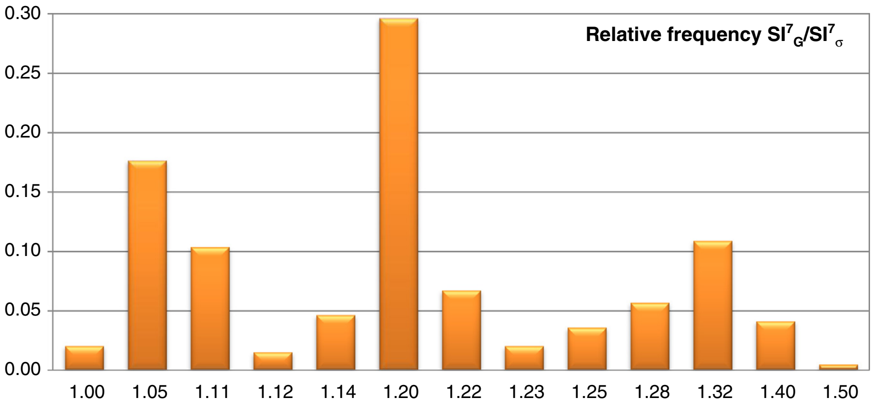


Fig. 27. Influence of the resistance check criteria on safety index of M_5 racks (more stressed C.U. and E.U.).

To this purpose, extensive parametric analysis similar to the one described in the present paper should allow to evaluate statically the value of γ_w. Nevertheless, as an example of practical application of this methodology, the data of this research have been statically considered to evaluate safety warping factors γ_w.

With reference to the limit state design philosophy [25] the distribution of all data governing design can to be considered of a Gaussian type and the upper characteristic value of safety factor γ_w can be defined as:

$$\gamma_w = \gamma_{w,95} = \widetilde{\gamma}_w + k_{st} \cdot s_w \tag{18}$$

where $\widetilde{\gamma}_w$ is the mean value of the distribution of the ratio $\frac{SI^7}{SI^6}$, s_w is its standard deviation and k_{st} is the coefficient to evaluate the 95% fractile value at a confidence level of 75% [16,25].

Tables 12 and 13 contain the values of γ_w^G and γ_w^σ respectively. Both indices have been evaluated for central (C.U.) and external (E.U.) uprights as well as for all the uprights by considering each frame typology. As to general remarks, it can be noted that:

- safety factors for C.U. are significantly lower than the ones of E.U.;
- if all the uprights are treated together, γ_w for unbraced frames is greater than the one for braced frames;
- factors γ_w associated with local cross-section verifications check (γ_w^σ) are slightly lower than the one associated with (γ_w^G).

It can be concluded that the range of variation of these γ_w factors is significantly wide. A proposal of the authors for warping safety factors γ_w^G and γ_w^σ to be used in routine design is summarized in Table 14 considering the cases of a unique value for all the uprights or values separated for external (E.U.) and internal (C.U.) uprights.

8. Conclusions

This paper deals with medium-rise racks, which are semi-continuous frames realized by monosymmetric cross-section members used as uprights, and attention has been focused on the influence of the warping on the resistance checks. The response of several geometric configurations of interest for rack practice has been considered; a parametric study has been carried out by using two FE beam formulations differing in the number of nodal degrees of freedom considered, both implemented in an analysis software for academic use developed by authors [15,20]. With reference to the resistance check approaches used worldwide, design results of a traditional 6DOFs analysis have been compared with those from a more refined formulation considering warping effects, i.e. characterized by 7DOFs per node. It has been demonstrated that warping plays a very important rule on the rack response and this reflects directly on the safety level of design, which is significantly overestimated if warping is neglected, especially for external uprights.

In order to reduce the overestimation of the safety due to the use of inadequate beam formulations, the values of safety factor accounting for warping γ_w have been evaluated, and Table 14 presents a proposal of the authors for an immediate use in resistance design verifications.

Furthermore, it should be noted that these research outcomes, which have been obtained with reference to racks, have a more general

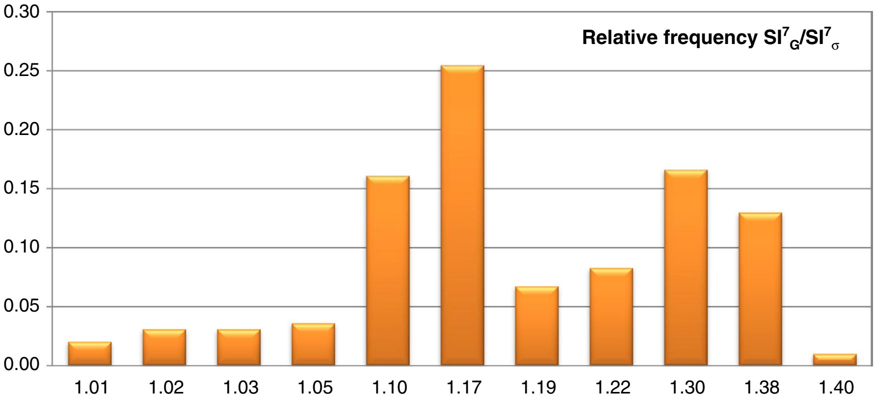


Fig. 28. Influence of the resistance check criteria on safety index of M_4 racks (more stressed C.U. and E.U.).

Table 12
Values of the safety factors γ_w^G.

γ_w^G	M_4		M_5		UNBR	BR	All
	UNBR	BR	UNBR	BR	M_4 & M_5	M_4 & M_5	
C.U.	1.20	1.15	1.33	1.19	1.26	1.16	1.20
E.U.	1.66	1.74	1.57	1.39	1.59	1.54	1.54
All	1.60	1.45	1.43	1.28	1.42	1.35	1.37

Table 13
Value of the safety factors γ_w^σ.

γ_w^σ	M_4		M_5		UNBR	BR	All
	UNBR	BR	UNBR	BR	M_4 & M_5	M_4 & M_5	
C.U.	1.08	1.08	1.18	1.09	1.13	1.08	1.10
E.U.	1.34	1.42	1.28	1.20	1.29	1.30	1.28
All	1.36	1.24	1.22	1.14	1.20	1.18	1.18

Table 14
Recommended value for warping safety factor γ_w^G and γ_w^σ.

	All	C.U.	E.U.
γ_w^G	1.40	1.20	1.50
γ_w^σ	1.20	1.10	1.30

validity, also for all the frames or sub-frames realized with members having the centroid not coincident with the shear center.

Appendix A. List of symbols

Latin lower case letters

d	differential.
f	limit theoretical stress of material.
h	inter-storey height
k	coefficient to evaluate the 95% fractile.
s	standard deviation.
t	thickness.
u	displacement along the x axis.
v	displacement along the y axis.
w	displacement along the z axis.
x	longitudinal axis of the beam.
y	symmetry axis of the cross-section.
z	non-symmetry axis of the cross-section.

Latin upper case letters

A	area of the cross-section.
B	bi-moment
BR	braced.
C.U.	central upright.
DOF	degrees of freedom.
E	Young's modulus.
E.U.	external upright.
E_d	design value.
F	shear force.
FE	finite element.
G	tangential modulus.
H	height.
I	second moment of area.
K	matrix stiffness.
L	length.
M	moment.
N	axial force.
R_d	resistance value.
S	beam-to-column joint stiffness, first moment of area.
UNBR	unbraced.

Greek lower case letters

γ	safety factor.
φ	rotation.
σ	normal stress.
θ	warping function.
ω	sectorial area.
τ	tangential stress.

Subscripts

AS	code [19]
b	beam.
Ed	design value.
EU1	code [22].
EU3	code [24].
Fy	shear force along y axis.
Fz	shear force along z axis.
G	global check.
j	initial node of the beam element, joint.
k	final node of the beam element.
m	material.
max	maximum.
min	minimum.
M_y	bending moment along y axis.
M_z	bending moment along z axis.
N	axial force.
o	position of the centroid.
Rd	resistance value.
s	position of the shear center.
St	standard.
t	uniform torsion.
tot	total.
w	warping.
x	longitudinal axis of beam element.
y	symmetry axis of the cross-section, yielding of the material.
ya	average yield strength.
z	non-symmetry axis of the cross-section.
σ	local resistance check.
ω	sectorial area.

Superscripts

_a	base warping prevented.
_b	base warping free.
6	analysis with a beam element formulation having 6DOFs per node.
7	analysis with a beam element formulation having 7DOFs per node.
E	elastic stiffness matrix.
EC3-LB	Eurocode [23] lower bound of the semi-rigid domain value.
EC3-UB	Eurocode [23] upper bound of the semi-rigid domain value.
G	geometric stiffness matrix, global resistance check.
σ	local resistance check.

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