# Quantification of changes in modal parameters due to the presence of passive people on a slender structure

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## 1. Introduction

In the last few years, an increasing attention has been paid to human induced vibrations and Human–Structure Interaction (HSI). A review by Sachse et al. [1] proves this is a topic of interest, with its almost 150 quotations. The first key aspect of HSI is the influence of structural movement on the active forces exerted by humans to the structure itself; the second is the influence of humans on the dynamic properties of the structure they occupy. This work focuses on the latter topic, particularly on the influence of passive people on the modal properties of a structure. This influence has been widely studied for the last decades for structural engineering purposes. The first and simplest way to simulate people's presence on

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a structure is to model human occupants as added masses. As demonstrated by many works dealing with this topic [2–5], such an approach has been accepted for a long time. As for structural engineering, the first attempt to model humans using spring-mass systems dates back to 1976 [6]. These approaches could explain the reduction of structure natural frequencies due to people's presence, as reported by several authors in the past decades. Nevertheless, they could not explain the changes in structural damping due to passive people. The limitation of considering human occupants as mere added masses was first raised by Ohlson [4] and Rainer and Pernica [7] who suggested to use damped dynamic models of human occupants in order to obtain an accurate description of HSI. Foschi and Gupta [8] were among the firsts to adopt this approach. As for vertical vibrations, a significant increase of structural damping due to passive people has been widely evidenced in literature [9–12] and in the 1990s several attempts to quantify the effect of stationary occupants has been made experimentally [13–18]. Particularly, Brownjohn investigated people's effects under laboratory conditions pointing out that even a single subject could increase the damping ratio of a concrete plank from 0.8 to 9.2 percent of critical. In addition, he investigated the influence of human posture on the amount of added damping and proposed a 2 degrees of freedom (dof) model for the plank-human system. Besides laboratory conditions, other works such as [11] and [12] investigated such an influence on the dynamic properties of a football stadium and a sports hall respectively.

Along with experimental researches, some models to quantify the effect of human occupants on the dynamic properties of a slender structure have been proposed in the past decade, too. Particularly, Sachse [19–21] investigated the influence of human occupants on the dynamic properties of civil engineering structures, combining analytical studies and experiments. Sachse proposed to model the Human–Structure (H–S) system as a 2 dof system to explain the influence of human occupants on natural frequencies, damping ratios and Frequency Response Functions (FRFs). Her work proposed an extensive experimental investigation, deriving damped SDOF human models from experimental data and suggesting their use to predict the dynamic behaviour of the H–S system. According to the author, the properties of the human model to be adopted vary with the natural frequency of the empty structure, which is a non-physical assumption. As stated by the author herself, the structure modelling as a SDOF system is one of Sachse's work limitations. In addition, the effect of people's presence is evaluated in terms of mass ratio (mass of people over mass of structure). However, this does not explain the effect for people's distribution on the structure – e.g. a subject on a node of the structure would not modify the dynamic behaviour even though his mass is comparable to the mass of the structure. A few years later, Sim [22] and Sim et al. [23,24], proposed a dynamic model to represent a crowd as a system added to the main structure. In contrast to Sachse's work, Sim developed a crowd model employing the extensive research by Griffin et al. [25–28] carried out on seated and standing individuals. Using the results of the researches by Griffin et al., Sim derived an equivalent low-order model to represent the dynamic behaviour of the crowd. Particularly, the author demonstrated that no significant improvement was obtained using an order of more than 4 for the crowd's transfer function. As made by Sachse, Sim supposed the structure to be a SDOF system as well and the effect of people's distribution was not considered. In addition, a work by Alexander [29] proposed a theoretical analysis of the dynamics of the crowd-structure system in which the author investigated the role of higher vibration modes. Pavic and Reynolds [30] have recently proposed a 3 dof model to describe the interaction between a grandstand and the crowd which occupy it. The 3 dof represents the dynamics of the structure and of passive and active people respectively. In their work the authors suggest to scale the physical mass of people using mode shapes amplitude of the relevant mode of the empty structure. This was proved to be a reasonably correct assumption [31]. However, no mathematical justification to support such an assumption is proposed. In this case, too, the structure is modelled as a SDOF system. Therefore, the effect of closely spaced modes may not be considered. According to the above-mentioned researches, the reviewed models may explain only qualitatively the influence of human occupants on natural frequencies, damping ratios and FRFs and they may not provide an accurate estimation of the behaviour of the joint H–S system. In addition, despite the experimental evidence, the majority of international standards and codes [32–34] – developed both to design and evaluate structure dynamics under crowd action – neglect the influence of human occupants in terms of changes of structural modal parameters. This fact is explained with the lack of appropriate models to predict such a complex phenomenon. A recent guidance [35] (Joint Working Group, 2008) regarding dynamic performance requirements for permanent grandstands subjected to crowd action is an exception to this. Indeed, in this guidance recommendation to considered human–structure interaction is provided.

Despite the large amount of work on the topic, the influence of human occupants remains a complex issue and at present in literature there are no models being able to provide a reliable prediction of the experimental evidence starting from the properties of the empty structure. In this context, there is still ground for reliable methods that are able to account for people's presence.

This work focuses on the quantification of modal parameters changes of a structure occupied by passive people. The work takes advantage of theories from various backgrounds and matches them in order to develop a new method to predict the dynamic behaviour of the joint H–S system. The proposed approach requires the knowledge of the modal model of the empty structure and of the driving point FRF of each subject on the structure. No restriction on the number of degrees of freedom is required. A reliable prediction of the FRF of the joint H–S system may be achieved combining this information. This approach is able to provide a mathematical justification to the phenomenon observed. In fact, the effect of each human subject on the structure is evaluated locally, in contrast to other works available in literature. This approach managed to achieve unprecedented results with respect to other works in literature.

Below is the work layout:

- Section 2 reports the mathematical basis of the proposed method.
- Section 3 describes the test structure (i.e. a lightly damped staircase) employed to validate the model, the experimental set-up and the tests carried out.

- Section 4 describes the tests performed to measure the driving point FRFs of the subjects involved in the tests and shows the results. These driving point FRFs and the modal model of the empty structure have been used to predict the dynamic behaviour of the joint H–S system. A satisfactory agreement has been found between the prediction and the experimental evidence.
- The results achieved are reported in Section 5.
- Finally, a generalisation of the presented model is proposed. This generalisation was achieved introducing people with average values of driving point FRFs, instead of employing the actual values. The results were in good agreement with the experimental evidence also in this case, as explained in Section 6.

## 2. Numerical model of the interaction between passive people and structure

A numerical model is presented to assess the changes in the structure modal parameters induced by people's presence. Two elements are required to this purpose:

1. a dynamical model of the empty structure;
2. a description of the dynamic behaviour of each person standing on the structure.

As for point 1, a modal model of the structure was employed. Such a model may be obtained from experimental data or from existing structural models (e.g. a finite element model).

As for point 2, the quantity used to represent the dynamic behaviour of the human body is the driving point FRF, i.e. the transfer function between the force $f^{\text{Human}}$ exerted by the person on the structure (Fig. 1) at the contact point (Ground Reaction Force) and the corresponding structure displacement ($x$), velocity ($\dot{x}$) or acceleration ($\ddot{x}$) in the frequency domain. In case of acceleration, this transfer function is commonly referred to as the "apparent mass" in literature [37]. Defined here as $M^{*}(\omega)=f^{\text{Human}}(\omega)/\ddot{x}(\omega)$, the apparent mass basically represents the relationship between the acceleration at the contact point and the Ground Reaction Force (GRF). $M^{*}(\omega)$ is a complex function in the frequency domain. Obviously, the GRF of each person on the structure depends on his/her dynamic properties (e.g. mass, stiffness, damping).

The quantity $M^{*}(\omega)$ may be measured experimentally with an appropriate measurement set-up. In the proposed approach each subject is then added to the model of the empty structure to obtain the response of the joint H–S system. General concepts have been introduced so far to explain the approach. Details about the model are given here below.

The approach proposed by Krenk [36] was employed to find a method that is able to account for people's presence on the structure. Krenk's model was originally developed for the introduction of dampers on discretised structural systems. The effect of each damper is introduced through the force exerted by the damper on the structure. In the frequency domain, this force may be expressed as the product between the FRF of the damper and the displacement of the point where the damper is located. This model was extended to the case of passive people by the authors of this paper. One of the advantages of this model is the possibility to introduce each subject and evaluate the corresponding effect individually.

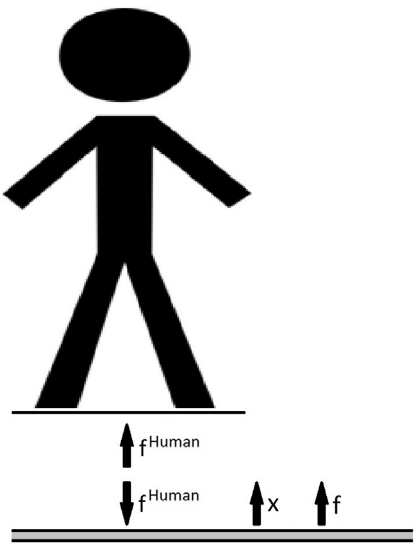


**Fig. 1.** Dynamic modelling of each subject through its driving point FRF.

The basic steps to obtain the transfer function of the joint H–S system are reported here below.

The FRFs (between a generic force vector f and structure displacement x) of the empty structure G($\omega$) may be expressed as [36]

$$\mathbf{G}(\omega) = \sum_{j=1}^{n} \frac{\boldsymbol{\phi}_j \boldsymbol{\phi}_j^{\mathrm{T}}}{\omega_j^2 - \omega^2 + 2i\zeta_j \omega \omega_j} \tag{1}$$

where $\boldsymbol{\phi}_j$ is the $j$th mode shape vector (scaled to the unit modal mass) measured at discrete points, $\omega_j$ is the natural frequency of the $j$th mode, $\zeta_j$ is the $j$th non-dimensional damping ratio and $n$ is the (arbitrary) number of modes taken into consideration.

The dynamic behaviour of the empty structure may be expressed as [36]

$$\mathbf{x}(\omega) = \mathbf{G}(\omega)\mathbf{f}(\omega) \tag{2}$$

Since the eigenvectors are measured at discrete points (named nodes in the following), $\mathbf{G}(\omega)$ is the matrix containing the FRFs associated to these points. Accordingly, $\mathbf{x}(\omega)$ is the vector that contains the responses in the points taken into consideration, while $\mathbf{f}(\omega)$ is a generic force vector containing forces applied in the nodes.

After defining the modal model of the empty structure $\mathbf{G}(\omega)$, people's contribution has to be added.

According to the definition of apparent mass, each person fixed to the $i$th node of the structure introduces a force depending on the apparent mass and on the structure acceleration $\ddot{x}$ of the point itself (Fig. 1). Therefore, the GRF of each passive subject connected to the $k$th point of the structure is

$$\mathbf{f}_{\mathbf{k}}^{\mathrm{Human}}(\omega) = M^*(\omega)\ddot{x}_k(\omega) = -M^*(\omega)\omega^2 x_k(\omega) = H(\omega)x_k(\omega) \tag{3}$$

where $M^*(\omega)$ is the apparent mass of the subject.

In terms of the full displacement vector $\mathbf{x}(\omega)$, Eq. (3) may be expressed in the following matrix form:

$$\mathbf{f}_{\mathbf{k}}^{\mathrm{Human}}(\omega) = H(\omega)\mathbf{w}_k \mathbf{w}_k^{\mathrm{T}} \mathbf{x}(\omega) \tag{4}$$

where $\mathbf{w}_k$ identifies the connection of the person to the structure, as exemplified in Fig. 2.

The total force vector in the case of m people on the structure may be expressed as

$$\mathbf{f}^{\mathrm{Human}}(\omega) = \mathbf{WHW}^{\mathrm{T}}\mathbf{x}(\omega) \tag{5}$$

where $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_m]$ represents the connection of m subjects and $\mathbf{H}$ is the (diagonal) transfer function matrix, containing the $H(\omega)$ functions of all the subjects.

$$\mathbf{G}^{-1}(\omega)\mathbf{x}(\omega) = \mathbf{f}(\omega) - \mathbf{f}^{\mathrm{Human}}(\omega) = \mathbf{f}(\omega) - \mathbf{WHW}^{\mathrm{T}}\mathbf{x}(\omega) \tag{6}$$

Including the force vector expressed by Eq. (5) in Eq. (2), Eq. (6) is obtained (refer to Fig. 1):

The modified equation of motion becomes

$$[\mathbf{G}^{-1}(\omega) + \mathbf{WHW}^{\mathrm{T}}]\mathbf{x}(\omega) = \mathbf{G}_{\mathrm{H}}^{-1}(\omega)\mathbf{x}(\omega) = \mathbf{f}(\omega) \tag{7}$$

Thus, in Eq. (7) $\mathbf{G}_{\mathrm{H}}(\omega)$ represents the new transfer function of the H–S system. The new frequency response function may be expressed explicitly in terms of the frequency response function $\mathbf{G}(\omega)$ by the Woodbury matrix identity [38]:

$$(\mathbf{A} + \mathbf{UCV})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}(\mathbf{C}^{-1} + \mathbf{VA}^{-1}\mathbf{U})^{-1}\mathbf{VA}^{-1} \tag{8}$$

as

$$\mathbf{G}_{\mathrm{H}} = [\mathbf{G}^{-1} + \mathbf{WHW}^{\mathrm{T}}]^{-1} = \mathbf{G} - \mathbf{GW}(\mathbf{H}^{-1} + \mathbf{W}^{\mathrm{T}}\mathbf{GW})^{-1}\mathbf{W}^{\mathrm{T}}\mathbf{G} \tag{9}$$

This simple equation allows to calculate the transfer function of the joint H–S system. This approach allows to evaluate the effect due to the presence of each subject separately. This effect is a function of the subject's characteristics and posture (e.g. standing, one leg). The driving point FRFs contained in the matrix $\mathbf{H}$ depends upon these properties. In addition, the effect of each subject is a function of the point where the subject is located (i.e. a function of the mode shape components). The matrix $\mathbf{W}$ allows taking into account the position of each subject.

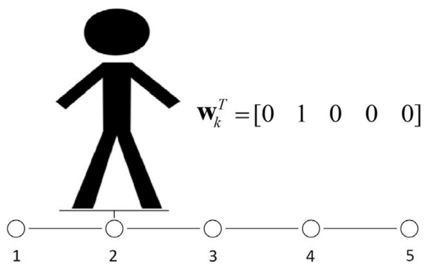


**Fig. 2.** Connection of a subject at the node 2 of the structure.

The proposed method has been validated by means of experimental tests. All tests have been carried out in accordance with The Code of Ethics of the World Medical Association (Declaration of Helsinki) for experiments involving humans. The layout for such tests and the structure used to this purpose are presented in the next section.

## 3. Test structure and experimental data collection

After choosing an appropriate test structure, a slender staircase, several tests were carried out to verify the effectiveness of the proposed approach. The structure was first tested without people to attain its FRFs. Then, it was tested with people standing still in defined points. This allowed to determine the FRFs of the joint H–S system. All the FRFs were processed to obtain the corresponding modal parameters by means of experimental modal analysis techniques [39].

This section describes the experiments carried out on the staircase and the corresponding results in terms of FRFs and modal parameters. Then, these results will be used to validate those predicted by the model (Sections 5 and 6).

The staircase taken into consideration (Fig. 3) connects the ground and the first floors in the main building of the Politecnico di Milano Bovisa Campus. The structure under test is very flexible: even a few people on it are able to change the damping considerably.

The first step was a modal characterisation of the structure with and without people. Therefore, the structure was instrumented with 19 accelerometers measuring in the vertical direction and forced by accelerating a known mass with an electro-dynamic shaker. Fig. 4 shows the position of the accelerometers.

The force exerted on the structure by the moving mass could be estimated by multiplying the value of the mass and its acceleration measured with an accelerometer placed on the mass itself.

The preliminary tests showed that the first natural frequency of the structure is approximately at 8 Hz. Therefore, the structure was forced with band-limited random noise between 5 and 20 Hz. In this frequency band the force Root Mean Square (RMS) value was 16.8 N. At first, some experimental tests on the empty structure were performed in order to extract its modal properties. Then, additional tests with respectively 3, 6 and 9 people standing in different points of the structure were carried out. In all the cases, the structure was forced to determine the corresponding FRFs with the shaker.

Fig. 5 shows an example of experimental FRFs for:

- the empty structure;
- the structure with 2 groups of 3 different people placed in correspondence of the accelerometers 5, 13, 16;
- the structure with 6 people placed in correspondence of the accelerometers 5, 7, 13, 14, 16, 19;
- the structure with 9 people placed in correspondence of the accelerometers 5, 6, 7, 13, 14, 16, 17, 18, 19.

Table 1 shows the modal properties (i.e. eigenfrequencies fn and non-dimensional damping ratios $\zeta$) associated to these configurations, identified through the Polyreference Least Square Frequency Domain method [39].

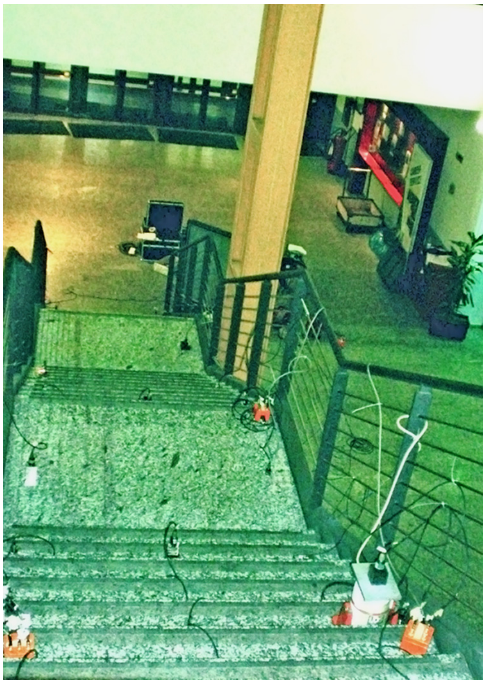

**Fig. 3.** Tested staircase.

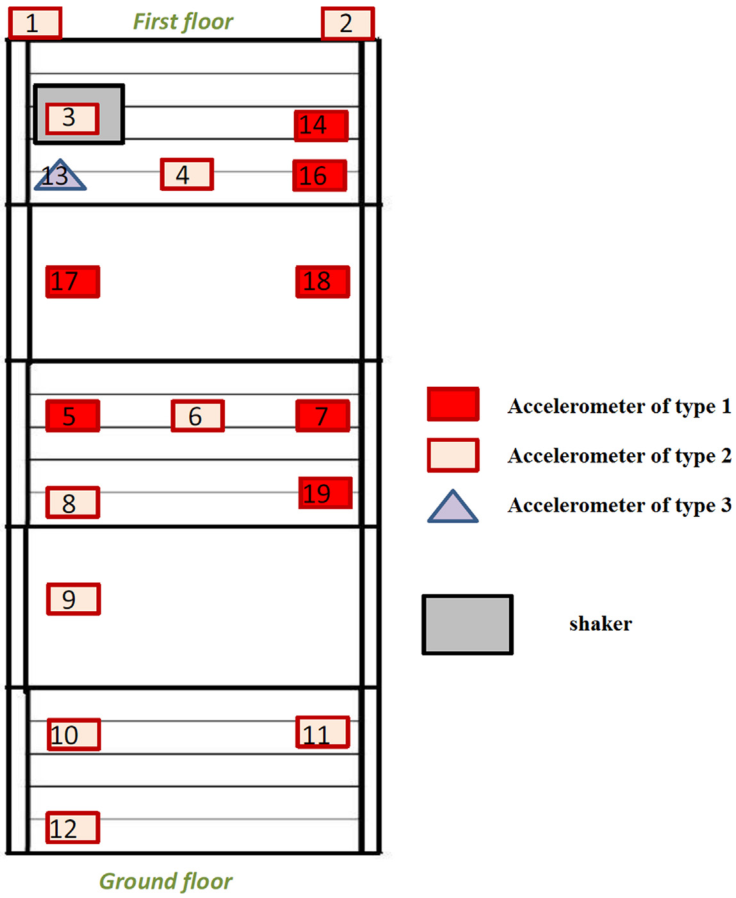


**Fig. 4.** Position of the accelerometers and the shaker.

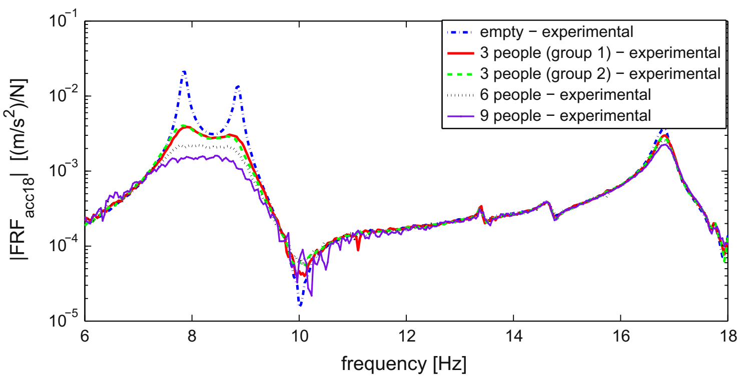


**Fig. 5.** Experimental FRFs.

**Table 1**
Modal properties of the staircase in different configurations.

| Empty | | 3 People, group 1 | | 3 People, group 2 | | 6 People | | 9 People | |
|---|---|---|---|---|---|---|---|---|---|
| $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] |
| 7.85 | 0.48 | 7.85 | 3.14 | 7.79 | 2.92 | 7.76 | 5.31 | 7.67 | 7.83 |
| 8.86 | 0.50 | 8.85 | 2.60 | 8.80 | 2.84 | 8.81 | 3.60 | 8.78 | 5.22 |
| 16.80 | 0.61 | 16.84 | 0.78 | 16.87 | 0.90 | 16.85 | 0.91 | 16.89 | 0.99 |

As it is formulated, the model proposed in this paper requires the FRFs of the empty structure (Section 2). No constrains are placed on the number of vibration modes used to reconstruct these FRFs, as in Eq. (1). All the modes in the frequency range of interest may and should be considered. The vibration modes reported in Table 1 are those having an appreciable amplitude in the range of frequencies of interest for the structure taken into consideration (i.e. the frequency range where the effect of passive people is considerable). These modes were used to reconstruct the FRFs of the empty structure and perform the subsequent analysis.

The values reported in Table 1 show that people might cause a high increase of damping in correspondence of the first two eigenmodes. A slight increase may be noticed in correspondence of the third mode as well (Fig. 5 and Table 1). In addition, the results of the tests performed with different groups of people showed that different subjects do not behave in the same way and the effect on the modal parameters of the structure is not merely proportional to the mass of the subjects standing on the structure. In the two tests with three people (groups 1 and 2) the mass of each subject in group 1 is lower than that of the corresponding subject in group 2 and the total mass is about 15 percent higher in this second case. Despite this, as for group 1, the damping associated to the first mode is higher. This highlights how the characteristics of the subject may play a crucial role for the final effect on the dynamic behaviour of the joint H–S system.

The above-mentioned experimental data were used to validate the approach proposed in Section 2. Such an issue requires to add people's effect. This is possible by measuring the actual apparent mass $M^*(\omega)$ of the subjects standing on the structure. Then, the measured $M^*(\omega)$ curves are used in the model, as explained by Eqs. (3)–(9). The next section of this paper shows the experimental tests carried out to measure $M^*(\omega)$, while Section 5 discusses model results and compares them to the experimental evidence.

## 4. Apparent mass estimation

This section describes the experimental set-up and procedure used to measure $M^*(\omega)$ for the three people composing group 1 in the experimental tests on the staircase (Section 3, Fig. 5 and Table 1).

The apparent mass was measured imposing a vertical vibration with a large electro-dynamic shaker (maximum displacement of $\pm 50$ mm). The subjects were standing over a rigid surface (0.6 m $\times$ 0.6 m in size), whose natural frequency was higher than 60 Hz. According to the characteristics of the tested staircase, the stimulus was white noise in a frequency range between 4 and 25 Hz with a vibration amplitude set to 0.5 m/s² RMS. The plate acceleration was measured with an accelerometer, while the force was measured by three piezoelectric load cells interposed between the shaker head and the plate. Fig. 6 shows the experimental set-up. The apparent masses of the three subjects taken into consideration were estimated as explained in [25] and are shown in Fig. 7.

The next section deals with the results of the proposed model using the values of the apparent masses shown in Fig. 7.

## 5. Prediction of people effects with measured apparent masses

The modal model presented in Section 2 was used to predict the behaviour of the structure occupied by people. The results were compared with those obtained experimentally. To do this, each subject was added to the model using its measured apparent mass, as described in the previous sections (Eqs. (3)–(9)).

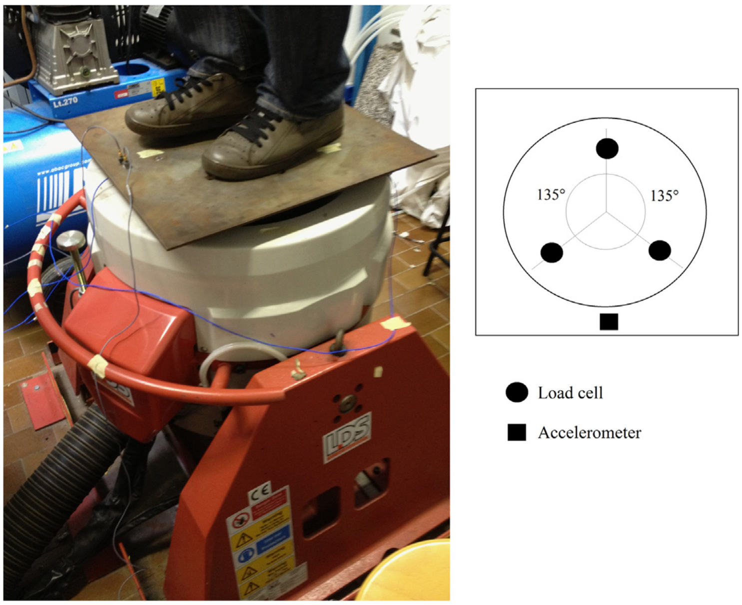


**Fig. 6.** Set-up to measure the apparent mass.

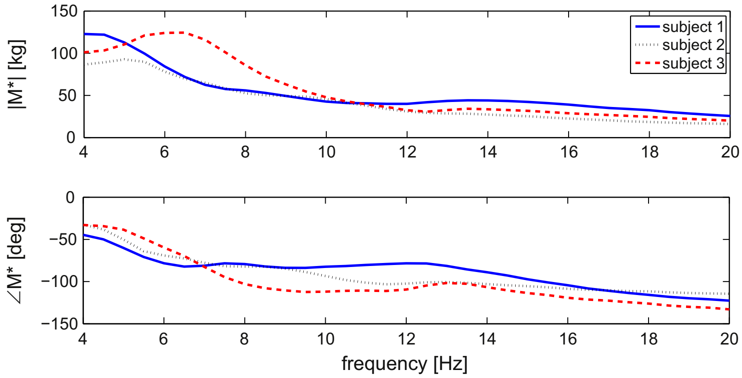


**Fig. 7.** Measured apparent masses.

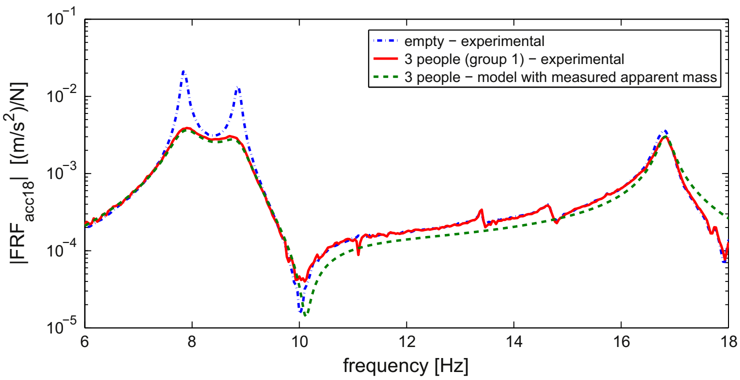


**Fig. 8.** Experimental and predicted FRF (3 people, group 1) using people's actual apparent mass. Response measured by accelerometer 18.

**Table 2**
Modal properties of the staircase with 3 people (experimental vs. model with measured apparent masses).

| Empty | | 3 People, group 1, Experimental | | 3 People, group 1, model with measured apparent masses | |
|---|---|---|---|---|---|
| $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] | $f_n$ [Hz] | $\zeta$ [%] |
| 7.85 | 0.48 | 7.85 | 3.14 | 7.85 | 3.26 |
| 8.86 | 0.50 | 8.85 | 2. 60 | 8.85 | 2.73 |
| 16.80 | 0.61 | 16.84 | 0.78 | 16.83 | 0.76 |

Fig. 8 shows the comparison between a measured FRF and a predicted FRF for the test with 3 people (group 1); while Table 2 shows the corresponding identified modal parameters.

The results of the model fit well with the experimental ones, both in terms of natural frequencies and damping ratios, proving the reliability of the model proposed.

In this case, people were added in the model through the measured apparent masses. Basically, this is not feasible because in the design phase it is not possible to know the exact characteristics of all the people that are using the structure then. Furthermore, the authors aimed to comprehend whether the proposed model is able to predict the dynamic behaviour of a structure when occupied by passive people: is there any way to predict H–S system dynamics, although no information about people who will occupy the structure are available? This is made possible by using average values of apparent masses instead of measured values. Such average values are already available in literature [25]. Therefore, the next section will show the results obtained with the same model, but using the average apparent mass proposed by Matsumoto and Griffin [25] to model people.

## 6. Prediction of people effects with average apparent masses

The previous section showed the reliability of the results obtained with the model using the actual apparent masses of the subjects on the structure. However, the actual apparent mass of the people occupying the structure is unknown in usual applications.

Thus, in such cases, only an average estimate may be obtained. The average models available in literature were also employed to verify the applicability of the proposed model, instead of using the actual apparent mass of each subject. This allowed to verify the robustness of the model. The authors chose to this purpose to use the work by Mastumoto and Griffin [25], who proposed mathematical models to represent the dynamic behaviour of standing subjects. In their work, Matsumoto and Griffin proposed these models to describe the response of human subjects – in different postures – exposed to vertical whole-body vibration in terms of apparent masses. Although the availability of a standard [40] and of several researches concerning the dynamic behaviour of humans, most of these deal with vertical vibration of sitting people [26,27,41,42]. Fewer works deal with standing humans exposed to vertical vibrations [25,28,43]. The model proposed by Matsumoto and Griffin [25] may be suitably employed to the purpose of this work because it represents people's dynamic behaviour through apparent mass curves, as made by this model.

Some of the problems occurring when modelling the dynamic characteristics of the human body deal with the variation of the following properties:

- the posture;
- the subject (inter-subject variability, which means that two subjects do not have the same dynamic behaviour);
- the subject (intra-subject variability, which means that there is a spread in results when a subject undergoes the same dynamic test several times);
- the vibration amplitude (for the same subject);

Matsumoto and Griffin investigated all these aspects. Particularly, they analysed the cases of subjects in normal standing posture, in legs bent posture and in one-leg posture. As for the case of normal standing posture, they also investigated the effect of the vibration amplitude on the results.

In all the cases taken into consideration in [25], Matsumoto and Griffin measured the apparent mass curve of 12 male subjects and then computed an average apparent mass curve for each case. Afterwards, different linear lumped parameter models were proposed to model the dynamic behaviour of humans. The values of such parameters were chosen in order to achieve numerical apparent mass curves as close as possible to the above-mentioned average apparent mass curves obtained through experimental measurements. Matsumoto and Griffin proposed different optimised models to represent the apparent mass for people in normal standing posture and some of them have been employed in this paper: the corresponding optimised analytical expressions of the apparent mass have been used in the model proposed in this paper, particularly in Eq. (3). Then, simulations have been carried out for the same staircase described in Section 3. These simulations differ from those analysed in Section 5 for the use of the above-mentioned modelled apparent masses in place of the measured ones.

Fig. 9 gives an example of the results concerning the case of three people (group 1). The figure shows a comparison among the predicted FRFs using the measured apparent masses (the same as Fig. 8) and those achieved by means of the optimised numerical apparent mass curves coming from three different models proposed by Matsumoto and Griffin. These models are named 2a, 2c and 2d in the following (2a, 2c and 2d is the nomenclature given in [25]; all the three models are 2 DOF m-c-k oscillators but with different layouts). The optimised parameters for vibration amplitude of 1 m/s$^2$ were used to achieve the FRFs shown in Fig. 9.

Fig. 9 shows that the choice of the apparent mass model has little influence on the results. In addition, all the FRFs are in good agreement with the experimental ones and with that obtained by using measured apparent masses. Since the choice of the model has little influence on the result, 2a was used in the following as it is the model that best fits the measured apparent mass data, according to Matsumoto and Griffin. In addition, the optimised coefficients for vibration amplitudes of 0.25 m/s$^2$, 0.5 m/s$^2$ and 2 m/s$^2$ were also available for this model. Indeed, the apparent mass depends on the amplitude of vibration which the subject is exposed to. As for actual applications, it is not possible to know a priori these amplitudes of

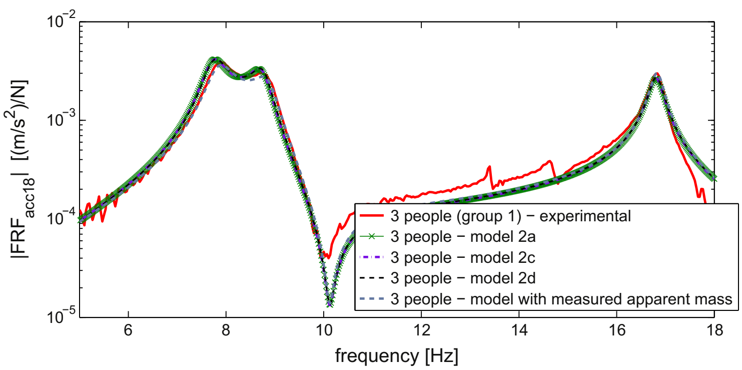


**Fig. 9.** Experimental and predicted FRF (3 people) using Mastumoto and Griffin's models 2a, 2c and 2d (2a, 2c and 2d is the nomenclature given in [25]). Response measured by accelerometer 18.

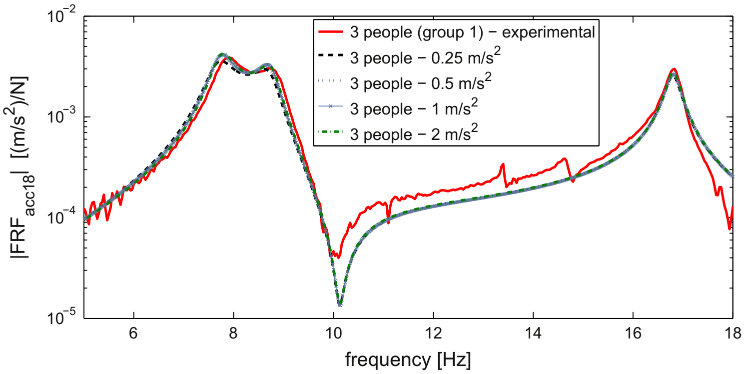


**Fig. 10.** Experimental and predicted FRF (3 people) using Matsumoto and Griffin's model 2a and different vibration amplitudes. Response measured by accelerometer 18.

**Table 3**
Comparison between experimental and predicted modal parameters – 3 people, group 1.

| $f_n$ [Hz], Empty experimental | $f_n$ [Hz], 3 People experimental | $f_n$ [Hz], 3 People (Matsumoto and Griffin's apparent mass – structure vibration of 1 m/s$^2$) | $\zeta$ [%], Empty experimental | $\zeta$ [%], 3 People experimental | $\zeta$ [%], 3 People (Matsumoto and Griffin's apparent mass– structure vibration of 1 m/s$^2$) |
|---|---|---|---|---|---|
| 7.85 | 7.85 | 7.74 | 0.48 | 3.14 | 2.89 |
| 8.86 | 8.85 | 8.74 | 0.50 | 2.60 | 2.37 |
| 16.80 | 16.84 | 16.81 | 0.61 | 0.78 | 0.85 |

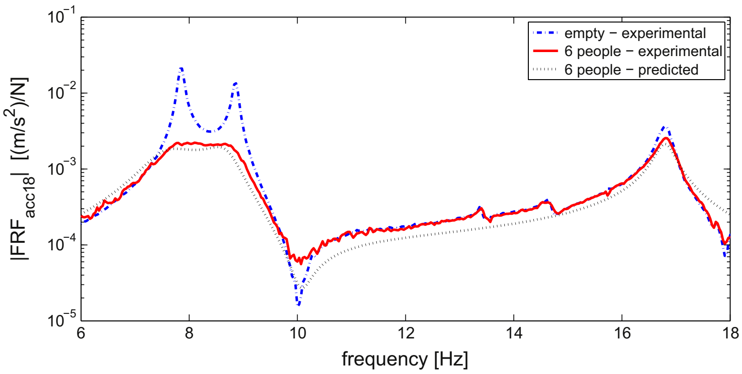


**Fig. 11.** Experimental and predicted FRF (6 people on the structure). Response measured by accelerometer 18.

vibration. Therefore, the effect of employing apparent masses yielded with different vibration amplitudes was investigated as well (Fig. 10). Table 3 reports the modal parameters identified using the FRFs computed for the case of 1 m/s$^2$. The difference among the FRFs in Fig. 10 is hardly noticeable and all the results are in good agreement with the experimental evidence. Such a result suggests that no strong assumptions are required on the amount of the vibration amplitudes of the structure in order to yield reliable information regarding the dynamic behaviour of the joint H–S system. It is useful to point out that all the vibration amplitudes taken into consideration may take place in actual cases on light civil structures.

Again, this result proves the effectiveness and robustness of the proposed approach.

The effect of a greater number of people on the structure was also investigated. Figs. 11 and 12 show the comparison between the experimental and predicted FRFs for two other tests, in the presence of 6 and 9 people on the structure respectively. Tables 4 and 5 show the comparison between experimental and predicted modal parameters. In these cases, too, the results show that the model predictions were satisfactory.

The experimental and predicted results were in very good agreement in all the tests performed, especially as regards the damping. Therefore, the model proposed to describe the dynamics of H–S systems is able to predict changes of modal parameters due to people's presence, even employing the average analytical apparent mass determined by Matsumoto and Griffin. Thus, these results prove the effectiveness of the proposed model and pave the way for many future applications.

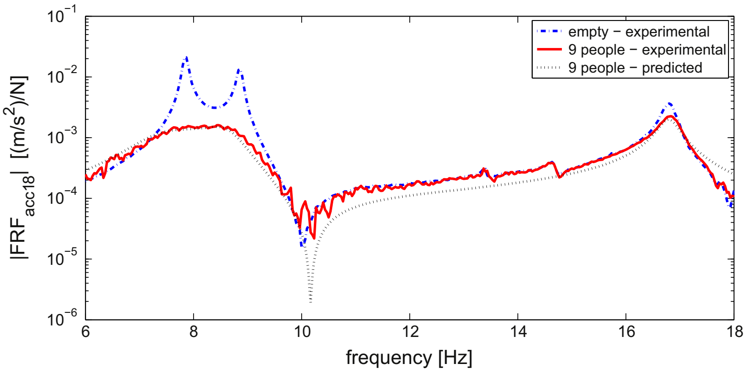


**Fig. 12.** Experimental and predicted FRF (9 people on the structure). Response measured by accelerometer 18.

**Table 4**
Comparison between experimental and predicted modal parameters – 6 people.

| $f_n$ [Hz], Empty experimental | $f_n$ [Hz], 6 People experimental | $f_n$ [Hz], 6 People (Matsumoto and Griffin's apparent mass) | $\zeta$ [%], Empty experimental | $\zeta$ [%], 6 People experimental | $\zeta$ [%], 6 People (Matsumoto and Griffin's apparent mass) |
|---|---|---|---|---|---|
| 7.85 | 7.76 | 7.57 | 0.48 | 5.31 | 6.21 |
| 8.86 | 8.81 | 7.68 | 0.50 | 3.60 | 3.50 |
| 16.80 | 16.85 | 16.80 | 0.61 | 0.91 | 1.05 |

**Table 5**
Comparison between experimental and predicted modal parameters – 9 people.

| $f_n$ [Hz], Empty experimental | $f_n$ [Hz], 9 People experimental | $f_n$ [Hz], 9 People (Matsumoto and Griffin's apparent mass) | $\zeta$ [%], Empty experimental | $\zeta$ [%], 9 People experimental | $\zeta$ [%], 9 People (Matsumoto and Griffin's apparent mass) |
|---|---|---|---|---|---|
| 7.85 | 7.67 | 7.46 | 0.48 | 7.83 | 8.93 |
| 8.86 | 8.78 | 0.64 | 0.50 | 5.22 | 4.42 |
| 16.80 | 16.89 | 16.80 | 0.61 | 0.99 | 1.11 |

## 7. Conclusions

This paper focuses on the analysis of the effects of passive people on the modal parameters of a lowly damped civil structure. Particularly, the work proposes a method that is able to quantify the effect of passive people on the modal parameters of a generic structure. The method only requires the modal parameters of the empty structure as an input. Each subject is then added locally on the structure by means of his/her apparent mass. With respect to other methods currently available in literature, the proposed approach places no constraints on the number of structural degrees of freedom taken into consideration. In addition, the proposed method considers the effect produced on the structure by each subject locally. Therefore, such a procedure allows a reliable quantification of people's effect.

A slender staircase was used to validate this approach. The structure was first tested without people in order to attain its modal model, so that the modal parameters of the first three vibration modes of the empty structure could be extracted. The structure was then tested experimentally with people standing in different points of the structure. The new properties of the joint human–structure system were estimated. The experimental evidence showed a high increase of damping ratios and a slight decrease of natural frequencies.

The experimental results of the tests carried out with people standing on the staircase were used to verify the approach proposed to predict the effect of passive people. People were first added in the modal model of the empty structure using their measured apparent mass to verify the method. In this case, the results obtained were very close to the experimental data. This provided actual evidence of the effectiveness of the proposed approach.

Then, the above-mentioned approach was used again to simulate people's effect. In this second case people were added employing average values of the apparent mass found in literature, instead of using the actual apparent mass of each human subject. The effect of using different models of the apparent mass was investigated as well. In addition, the apparent masses obtained with different levels of vibration were also used. This was performed to extend the results to real-life applications. Satisfactory results were obtained also in this case in terms of identification of changes of modal parameters due to people's presence.

To conclude, the approach proposed in this paper is able to provide a quantitative assessment of the effect of people and is able to predict the modal behaviour of a structure occupied by passive people.

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