Analysis of non-linear response of the human body to vertical whole-body vibration

Marco Tarabini*, Stefano Solbiati, Giovanni Moschioni, Bortolino Saggin and Diego Scaccabarozzi

Department of Mechanical Engineering, Politecnico di Milano, Lecco, Italy

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1. Introduction

People are normally exposed to whole-body vibration (WBV) either at workplace or during their daily life activities. Many studies evidenced that the continuous exposures to WBV have serious consequences on health, mainly because of the cyclic stress induced on the vertebral column, which may lead to physical and psychological fatigue and musculoskeletal disorders (Hulshof and van Zanten 1987; Bovenzi and Hulshof 1998; Tiemessen, Hulshof, and Frings-Dresen 2007). The biodynamic response of the human body has been extensively investigated in the sitting posture, analysing the effect of anthropometric angles and of the vibration magnitude. The number of studies focused on the characterisation of the response of standing subjects is more limited (Coermann 1962; Edwards and Lange 1964; Miwa 1975; Matsumoto and Griffin 1998, 2011; Subashi, Matsumoto, and Griffin 2006; Tarabini et al. 2013a), although it might be useful in structural dynamics, when assessing the interaction between human beings and structural elements in terms of both the discomfort due to structural-transmitted vibrations and the forces transmitted to the structure by the occupants. In addition, a better understanding of the physiological mechanisms at the basis of the measured response, in terms of resonances and contributions of the involved body parts (head, internal viscera and muscle activity), may improve current mathematical models for the response prediction (apparent masses and transmissibility).

The correct identification of the human body response to vibration is usually complex, both because of the variable nature of the human body (materials and structure) and because of the difficulties related with the measurement chain (Tarabini et al. 2012). In addition, tissues have thixotropic behaviour, which may cause the frequently observed softening phenomenon (Huang and Griffin 2009).

Non-linearity with respect to the vibration magnitude was found affecting the human response under either sinusoidal (Holmlund and Lundström 1998; Holmlund, Lundström, and Lindberg 2000) or random vibration (both single- and multi-axial). Results are commonly derived for seated subjects but their validity was confirmed by few studies also on standing persons (Matsumoto and Griffin 1998; Subashi, Matsumoto, and Griffin 2006). As the vibration magnitude increases, the apparent mass (APMS) and transmissibility change in shape: resonances shift to lower frequencies and the peak amplitudes decreases or increases (the first under sinusoidal stimuli while the second under random excitations). The response of seated subjects to vibration is also non-linear, regardless of the adopted posture; the importance of non-linearity is influenced by the degree of muscular activity and extension of the contact area on the seat (Nawayseh and Griffin 2003, 2005). A similar

*Corresponding author. Email: marco.tarabini@polimi.it

effect of the muscle tension was found also for the standing persons. This fact may suggest a key role of the muscular activity in the outspread of non-linearity.

Despite the non-linear behaviour, different studies reported that the ordinary coherence function was approximately one (although non-linearity causes the coherence function to decrease). Furthermore, linearity was always assumed by considering that both the power spectral density (PSD) and cross-spectral density (CSD) methods gave compatible results. Low coherence trials were mostly considered outliers frequency response functions (FRFs) and were sometimes rejected (Wang, Rakheja, and Boileau 2008; Stein et al. 2009), but the impact of this practice on the results has never been quantified.

The non-linearities of the APMS are probably caused by a complex combination of factors (tissue dynamics, active response of muscles, cross-axis response; Hinz et al. 2006), which were not quantified by the measuring technique adopted in most literature studies. Aim of this paper is to analyse the non-linear response of the human body to WBV with the techniques that are typically used for the analysis of non-linear systems (Bendat and Piersol 1993) to evaluate if a non-linear model can be effectively applied.

2. Method

This section describes the procedures for a complete characterisation of the non-linear behaviour of the human body and the experimental set-up for the method validation. The procedures for the analysis of non-linear system are described in Section 2.1. Section 2.2 summarises the anthropometric characteristics of subjects who underwent the tests for the validation of the proposed method. Section 2.3 describes the experimental set-up, while the statistical analyses performed to compare the linear and non-linear behaviour of the human body are presented in Section 2.4.

2.1. Data analysis

The response of the human body to vertical WBV is commonly expressed in terms of APMS; in this paper, non-linearities in the response have been analysed using the conditioned output spectra and the multiple coherence functions, described in the next sections.

2.1.1. Human body response to WBV

In the existing studies, the response of the human body to vibration is analysed using the driving-point APMS or impedance and the vibration transmissibility, i.e. linear estimators of the FRF. The techniques for the analysis of non-linear systems are described in many different textbooks (Bendat 1998; Bendat and Piersol 2000; D'Antona and Ferrero 2006); we report in the following a short summary in order to ease the method understanding. The response $y(t)$ of a generic system to an input $x(t)$ is:

$$y(t) = H\{x(t)\}. \tag{1}$$

H is the system operator, defined as a mapping of the possible outputs $y(t)$ to the possible inputs $x(t)$ (e.g. how the human body converts a vertical force stimulus into an acceleration). A mechanical system is linear if the operator H is linear, i.e. if H is homogeneous, additive, time invariant, causal and stable. The existing literature studies evidenced that the human body behaviour is:

- not homogeneous: it was shown that the APMS depends on the vibration level; hence, if the input is scaled by a factor Φ, the output is not scaled by a factor Φ;
- not additive: given that response to random excitation was found to be different from the response to swept sine;
- not time invariant: because involuntary muscle tension may change the body FRF;
- causal: because in absence of mechanical stimulus the body does not (unintentionally) generate forces in the frequency range typical of WBV; and
- stable: if the input vibration level is finite, the inertial forces or the vibration at different positions is also finite.

Nevertheless, the human body has always been analysed with FRF, which relies on the hypothesis of system linearity. In a linear system, the output $y(t)$ is the convolution between the input time history $x(t)$ and the time-domain response of the system to the Dirac function (impulse response) $h(t)$:

$$y(t) = x(t) \otimes h(t). \tag{2}$$

Thanks to the convolution theorem, the above equation can be written, in frequency domain, as:

$$Y(f) = X(f)H(f), \tag{3}$$

where $H(f)$ is the so-called frequency (f) response function. In the specific case of the APMS, for instance, $X(f)$ is the acceleration $a(f)$ and $Y(f)$ is the force $F(f)$, while for vibration transmissibility $X(f)$ and $Y(f)$ are the vibration spectra measured at different positions.

Equation (3) is meaningless in the presence of non-linearities; there are different ways to check the hypothesis of linearity, but the most common is the coherence function $\gamma(f)$ (Mansfield 2005):

$$\gamma^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f) \cdot S_{yy}(f)}, \tag{4}$$

where $S_{xx}(f)$and $S_{yy}(f)$ are the power spectra of the input and the output, respectively, and $S_{xy}(f)$ is the cross-spectrum between the input and the output signals. Equation (4) is always 1 if the spectral quantities are computed by Fourier transforming the input and output time histories. Assuming that $x(t)$ and $y(t)$ are random phenomena, their spectra can be estimated using the Welch method, i.e. by splitting the signals into N overlapped segments and computing the average of the N resulting spectra. If Equation (4) is computed using spectral density estimators, the coherence function drops below unity if there is contaminating noise on the measured signals, if there are leakage errors not reduced by windowing, if there are non-measured inputs affecting the output or if the system $H(f)$ is non-linear. In the investigation of the human body response to vibration, the measurement chain noise is usually limited and the only vibration input is the one measured by the acceleration sensor. The leakage is usually negligible and therefore low coherence values should be only endorsed to a non-linear response of the system.

The common practice in WBV analysis for the identification of system non-linearity is the comparison between the PSD and the CSD estimators. In the first case, the FRF is obtained as the ratio between the power spectral densities of the input and output signals ($S_{xx}(f)$ and $S_{yy}(f)$, respectively). In the specific case of the APMS:

$$\mathrm{APMS}_{\mathrm{PSD}}(f) = \sqrt{\frac{S_{yy}(f)}{S_{xx}(f)}}. \tag{5}$$

The CSD method involves the evaluation of the cross-spectrum between the input and output signals:

$$\mathrm{APMS}_{\mathrm{CSD}}(f) = \frac{S_{xy}(f)}{S_{xx}(f)}. \tag{6}$$

Equation (6) is known as H_1 linear estimator, which has the twofold advantage, as opposed to Equation (5), of preserving the information on the phase and removing the noise in the measurement system output (Bendat and Piersol 1993, 2000). The comparison between $\mathrm{APMS}_{\mathrm{PSD}}$ and $\mathrm{APMS}_{\mathrm{CSD}}$ is equivalent to the analysis of coherence, given that $\gamma^2(f)$ is the ratio between the CSD and the PSD estimators of the transfer function.

2.1.2. *Analysis of non-linear system*

If the coherence drops below unity the system is non-linear; the origin of non-linearity can be the lack of homogeneity or additivity, or the variation of the modal parameters. In the first case, the non-linear system can be investigated with the procedure developed by Bendat and Piersol (1993), while if the modal parameter are not constant the system can be modelled as a linear system with uncertainty.

The procedure described by Bendat consists in modelling a non-linear single-input, single-output (SISO) system as a multiple-input, single-output system with non-linear inputs. In other terms, if the human body response to vibration is non-linear, the response (inertial force) to the stimulus (vibration) is the sum of one linear function of the input and $M-1$ linear response to non-linear functions of the input:

$$Y(f) = H_{1y}(f) \cdot X(f) + \sum_{i=2}^{M} H_{iy}(f) \cdot \mathcal{F}_i(X(f)) + N(f). \tag{7}$$

$H_{1y}(f)$ is the linear FRF associated with the input $X(f)$. Similarly, $H_{iy}(f)$ is the linear FRF associated with a non-linear function $\mathcal{F}_i$ of the input, for instance the input squared or the input square-root. $N(f)$ is the noise, accounting for the non-modelled non-linearities and for the uncorrelated measurement noise. With this scheme (graphically summarised in Figure 1), a linear system with transfer function $H_{1y}(f)$ is in parallel with $M-1$ non-linear systems $g_2(x)\ldots g_n(x)$ followed by linear transfer functions $H_{2y}(f) \ldots H_{My}(f)$. The model fully describes the non-linear system behaviour (if the system is not homogeneous and additive), while the FRF estimator $H(f)$ would only point out the linear component of the model $H_{1y}(f)$.

Neither the linear FRFs nor the noise are known a priori or directly measurable; after the identification, all the model parameters are completely defined along with other useful conditioned quantities (the conditioned output spectra and the multiple coherence functions). The set of linear FRFs $\{H_i(f)\}$ relate mathematically each input to the total output (their physical meaning is often awkward). The identification procedure needs that inputs are stationary (ergodic) or transient random signals, even correlated but not perfectly. The four compulsory conditions for a correct system identification are (Bendat and Piersol 1993):

1. the ordinary coherence function between any pair of input must be different from unity, which means no redundant information;
2. the ordinary coherence function between any input and the output must not be equal to unity, otherwise the other inputs have no contributions to the output and the system is described by a SISO model;
3. the multiple coherence function between any input and the others should not equal unity (i.e. the input is redundant and then eliminated); and
4. the multiple coherence function between the output and the given inputs should be close to unity; conversely, some inputs are omitted or non-linearities have large effects.

As previously evidenced, the validity of the non-linear model is checked using the conditioned outputs, computed by modifying the generally correlated set of inputs $\{X_i(f)\}$ using the conditioned spectral density techniques (Golub and Reinsch 1970), in order to obtain an equivalent set of inputs $\{U_i(f)\}$ in which all the inputs are uncorrelated. With this new set of inputs, it is possible to identify a new set of frequency responses $\{L_i(f)\}$, generally different from $\{H_i(f)\}$, which relate each conditioned input to the total output. The product between the inputs $\{U_i(f)\}$ and the frequency responses $\{L_i(f)\}$ is a set of linearly independent outputs $\{Y_i^*(f)\}$, referred to as conditioned output. The best linear system estimator is $L_1(f)$ which differs from $H_1(f)$ if the non-linear effects are important (Bendat 1993).

Ordinary coherence functions $\gamma_i^*(f)$ can be computed between the conditioned output and conditioned inputs in order to assess, at each frequency, the percentage of the output due to each uncorrelated input. The multiple coherence function between the input and the output is computed by combining all partial coherence functions $\gamma_i^*(f)$:

$$\gamma_{y:M}^2(f) = 1 - \prod_{i=1}^{M} (1 - [\gamma_i^*(f)]^2). \tag{8}$$

The multiple coherence function can be used to assess the validity of the proposed non-linear model, while the comparison between the conditioned output and the output can be used to assess to which extent the output is correctly modelled by the non-linear system. Given that, in our case, the conditioned output is the force spectrum, the latter can be used to compute the conditioned APMS. With reference to the scheme of Figure 1, assuming that y is the measured force

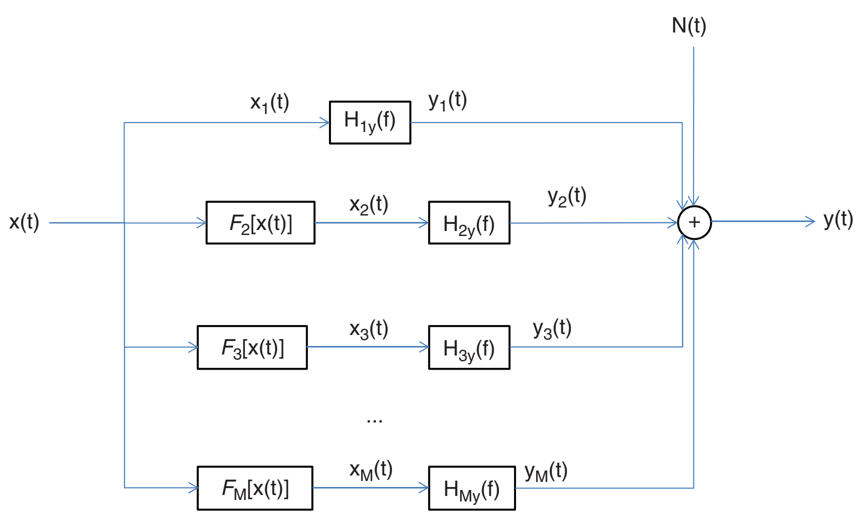

Figure 1. Single-input–single-output non-linear model of a linear system in parallel with n non-linear systems.

and x measured the acceleration:

$$\mathrm{APMS}_{\mathrm{COND}}(f) = \frac{S_y(f) - S_n(f)}{S_x(f)}. \tag{9}$$

In order to ease the comparison with the existing literature works, the apparent masses of Equations (5), (6) and (9) have been normalised with respect of the static mass.

If the system is not ‘fixed parameter’, the above-described procedure is useless and it is more convenient to describe the human body behaviour using a linear model with uncertainty. Uncertainty can be evaluated analysing the data dispersion in different postures and with different vibration levels, computing the coefficient of variation (COV) (the ratio between the standard deviation and the mean APMS):

$$u_{\mathrm{APMS}}(f) = \frac{\sigma_{\mathrm{APMS}}(f)}{\mu_{\mathrm{APMS}}(f)}. \tag{10}$$

This quantity is frequency dependent and has been summarised with the RMS between 2 and 20 Hz (hereinafter u_{APMS}). The computation of u_{APMS} on the non-normalised APMS of the entire data-set (eight subjects in upright and legs bent posture, 0.5, 1 and 1.5 m s^{-2}) using a linear model provides for an indication of the expected APMS variability in uncontrolled conditions. This parameter indicates the model uncertainty for the evaluation of the effect of people on structures, where the posture, the vibration level and the subjects’ masses are unknown. Conversely, the computation of u_{APMS} of the normalised APMS on a data-set that includes only a specific posture, a single vibration level using the conditioned response model provides for an indication of the APMS variability in controlled condition. The parameter is indicative of the modelling uncertainty for subjects with known body mass in a given posture.

2.2. *Subjects*

Eight male subjects (students and staff of the Politecnico di Milano) were involved in the experiments, which were carried out in compliance with both the EU legislation on workers’ vibration exposure and with the Politecnico di Milano ethical guidelines. Data of subjects’ ages, heights, weights and BMIs are summarised in Table 1.

2.3. *Experimental set-up*

The experimental set-up is similar to the one adopted in previous experiments (Tarabini et al. 2013a): vertical vibration was generated by an electrodynamic shaker (LDS V830, maximum stroke of ± 50 mm) and transmitted to standing subjects by a rigid metallic plate (600 × 600 mm in size, first natural frequency larger than 60 Hz). The force at the shaker–plate interface was measured by three piezoelectric load cells PCB 212B, whose sensitivity has been identified with a fit-to-purpose calibration procedure. The plate acceleration was measured by a piezoelectric accelerometer. Both the acceleration and force signals were filtered and amplified by two B&K Nexus conditioning units (band-pass filter between 0.1 and 100 Hz) and eventually sampled by two NI 9234 acquisition modules (24 bits A/D converter, sampling frequency of 2560 Hz). Signals were digitised and stored on a personal computer; the collected time histories were afterwards processed.

Table 1. Age, height, weight and body mass index of the eight subjects who underwent the experiments.

Subject	Age (years)	Height (cm)	Weight (kg)	BMI (cm^2/kg)
1	28	170	66	23
2	27	167	55	19
3	25	171	67	23
4	26	181	73	22
5	30	176	72	23
6	29	173	78	26
7	18	165	60	22
8	18	176	70	23
Average	25.1	172.4	67.6	22.6
SD	4.7	5.2	7.4	1.9
Median	26.5	172	68.5	23

Stimuli were white noises with bandwidth 2–20 Hz at three vibration magnitudes (RMS 0.5, 1 and 1.5 m s^{-2}). Input signals were generated each time, by changing the settings of the control software (LMS Virtual.Lab Rev 7). The method should provide for more information with respect to the commonly used procedure in which the same three acceleration waveforms are used in each session of trials (Subashi, Matsumoto, and Griffin 2006).

Subjects stood on the force plate in two different postures. In the upright posture, subjects stood with straight legs; in the legs bent posture the knees angle (150°) was verified with an artefact in a comfortable position to maintain the static equilibrium. In both postures, subjects kept their feet 25- cm apart, wearing their own shoes, given the weak effect of the feet/plate interface on the APMS (Tarabini et al. 2013a). In both postures, subjects kept their upper body in a comfortable and upright position, with both the arms resting on the hips. The order of presentation for the stimuli and the postures was randomised and the total exposure duration did not exceed 60 seconds for each configuration (total vibration exposure approximately 360 seconds). Subjects were advised to look straight to a fixed point during the trial and to maintain the same posture without any involuntary movement of the body.

2.4. *Hypothesis testing*

Parametric and non-parametric statistical analyses were performed on both the medians and the means of the apparent masses frequency by frequency. Non-parametric methods are distribution-free methods, which make no assumption on the population distribution; conversely, parametric methods are derived and applied for a particular parametric family of distributions (e.g. the normal distribution) (Montgomery and Runger 2002). The non-parametric paired Wilcoxon signed-rank test was used for testing the equality of the medians at different vibration levels; this test is widely adopted in literature to assess the differences between human responses under different conditions (e.g. the APMS magnitude against the excitation level) because the distribution of the experimental data is expected to be not normal (Morioka and Griffin 2006).

Under the assumption of symmetric and continuous distributions (samples with low skewness), the median is a valid estimate for the mean. Nevertheless, these two estimators have different definitions, which may lead to different results and interpretations in case of low sample size. Non-parametric methods may lose their distribution-free behaviour and become parametric when moderately large samples ($n > 20$) are submitted; in this case, the normal approximation is valid and the test statistic reduces to that of a normal distribution. The paired Student t-test was applied for testing the equality of the means, as an alternative for the paired Wilcoxon signed-rank test. This procedure is parametric (observations are sampled from a normal distribution) and evaluates whether there are differences between the means of paired dependent samples (i.e. one sample tested twice).

3. Results

3.1. *Non-linear model*

The biodynamic response of standing subjects to vertical WBV has been computed including in Equation (7) the input acceleration, the absolute value of the acceleration and the squared power of the acceleration time history. The complexity of the model might be increased with the addition of other non-linear functions, but the ones selected are the most common in typical non-linear model identification problems (Bendat 1998). Furthermore, our tests evidenced that the addition of cubic and high-order powers of the acceleration time history did not increase the modelling accuracy.

The constant-parameter FRFs in upright and legs bent postures are shown in Figures 2 and 3, respectively. The first (linear) FRF is very similar to that identified with the H_1 estimator used to derive the apparent masses. Subsequent linear FRFs (describing the response to the non-linear inputs acceleration modulus and squared acceleration) decreased in magnitude with the order of estimation (i.e. $\max(|H_{1y}|) > \ldots > \max(|H_{3y}|)$). The three FRFs are characterised by a dominant peak in the frequency range 3–5 Hz and by decreasing amplitudes above 5 Hz.

As evidenced in Section 2 and in Bendat (1998), the numerical values of the linear functions of Figures 2 and 3 are irrelevant, given that their physical meaning does not have a straightforward interpretation.

3.2. *Apparent mass*

The APMS of the eight subjects is compatible with the results of existing literature studies, with a dominant peak between 3 and 6 Hz. The biodynamic response in upright position differs from the one with legs bent, in terms of both resonance frequency and amplitude. Similarly, the coherence function depends on the posture and the subject. The coherence is almost one in upright position, while with legs bent varies from subject to subject: in three cases both the ordinary and the multiple coherence functions were close to one, thus evidencing the validity of the linear system approximation (as, for instance, in Figure 4b). In the remaining tests, the ordinary and the multiple coherence functions decreased to values below 0.5 at frequencies close to 6 Hz as in Figure 4a. The difference between the ordinary coherence and the multiple coherence

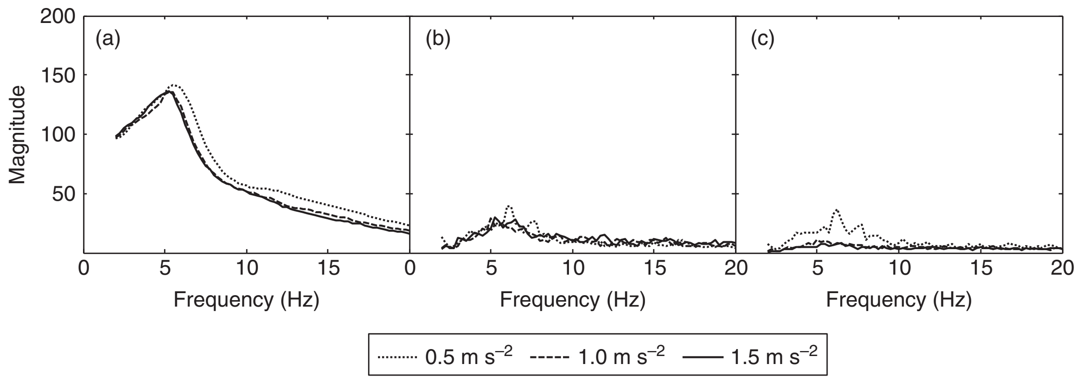

Figure 2. Mean constant-parameter linear functions (upright posture): (a) H_{1y} (kg), (b) H_{2y} (N/|m s^{-2}|), (c) H_{3y} (N/(m s^{-2})^2).

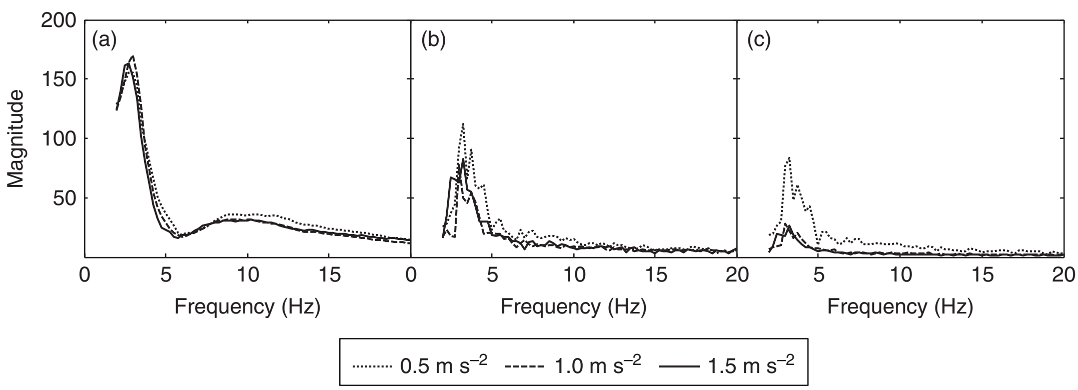

Figure 3. Mean constant-parameter linear functions (legs bent posture): (a) H_{1y} (kg), (b), H_{2y} (N/|m s^{-2}|), (c) H_{3y} (N/(m s^{-2})^2).

function, in each of the eight subjects who underwent our tests, is limited. The largest differences were noticed in subjects with bent legs: between 4 and 8 Hz.

The (limited) benefits deriving from the adoption of a non-linear model are also evidenced by the analysis of the average apparent masses (see Figures 5 and 6). The plots show the linear and the conditioned APMS of standing subjects in upright

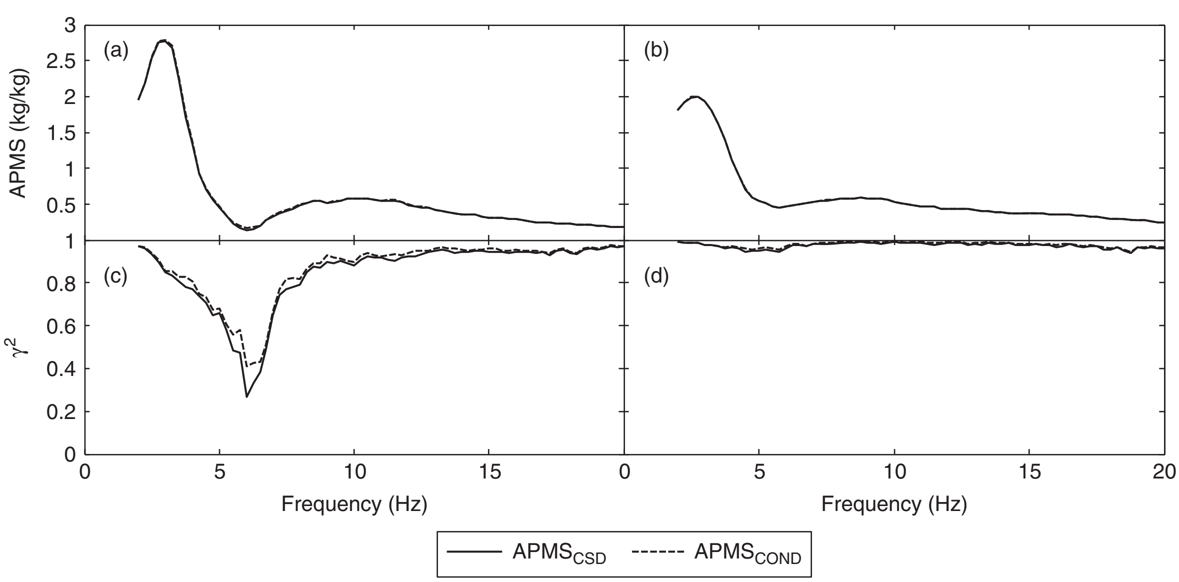

Figure 4. Subject 1: (a) normalised apparent masses, (c) coherence functions; subject 2: (b) normalised apparent masses, (d) coherence functions.

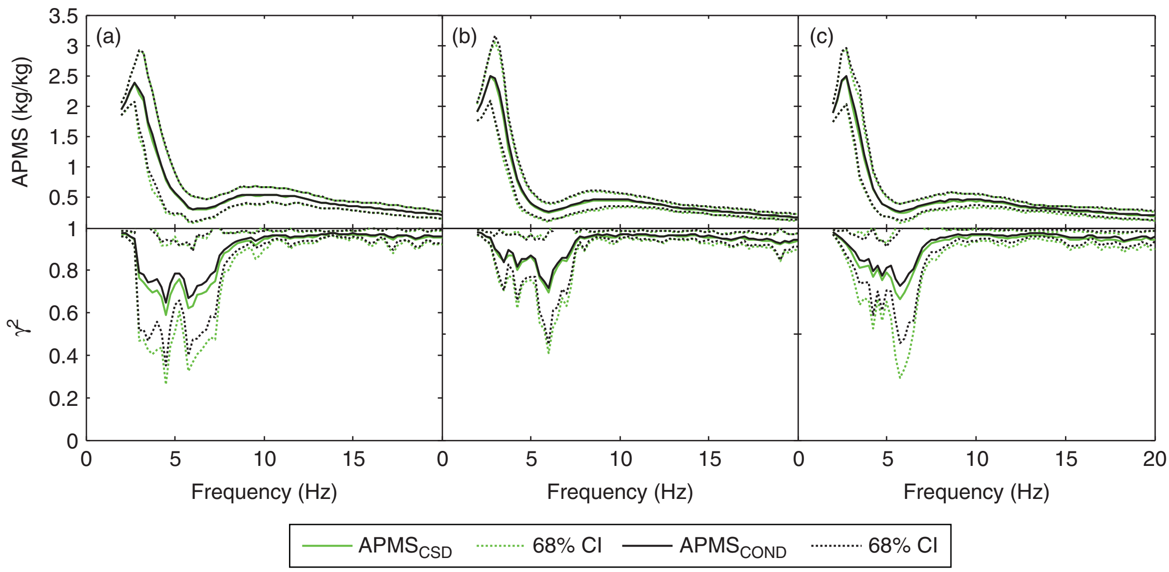

Figure 5. Normalised apparent masses and coherence functions (average and 68% confidence interval of eight subjects in legs bent posture): (a) $0.5\,\mathrm{m\,s^{-2}}$, (b) $1.0\,\mathrm{m\,s^{-2}}$, (c) $1.5\,\mathrm{m\,s^{-2}}$.

and legs bent postures with vibration levels between 0.5 and $1.5\,\mathrm{m\,s^{-2}}$. The differences between the apparent masses obtained using a linear and a non-linear model are low in comparison with the tests' repeatability. In the same way, the difference between the ordinary and the multiple coherence functions is negligible, independently from the posture and from the vibration level.

Uncertainties associated with the APMS of Figures 5 and 6 are summarised in Table 2. u_{APMS} of the entire data-set is approximately 40%, uncertainty in controlled condition ranges from 15% to 19% in the upright posture and from 28% to 30% when subjects stood with their legs bent. The effect of vibration magnitude on the measurement data dispersion is small (uncertainty changes smaller than 5%) and the differences between the linear and the conditioned models are negligible (less than 3%).

The relevance of the vibration level in the biodynamic response of the human body was investigated with parametric and non-parametric hypothesis tests. Both the paired Wilcoxon signed-rank test (see Figure 7) and the paired Student t-test (see Figure 8) evidenced that, either in the standing or legs bent postures, there is no difference between the biodynamic

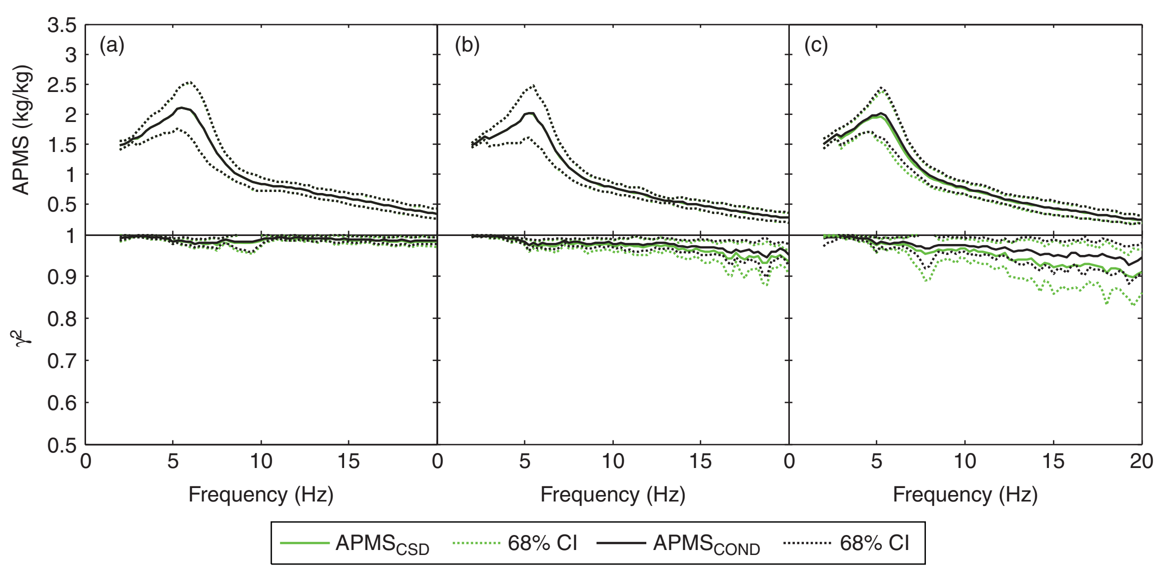

Figure 6. Normalised apparent masses and coherence functions (average and 68% confidence interval of eight subjects in upright posture): (a) $0.5\,\mathrm{m\,s^{-2}}$, (b) $1.0\,\mathrm{m\,s^{-2}}$, (c) $1.5\,\mathrm{m\,s^{-2}}$.

Table 2. Uncertainty associated with the experimental APMS.

Posture	Vibration level ($m\,s^{-2}$ RMS)	Mean COV (%) ($APMS_{COND}$)	Mean COV (%) ($APMS_{CSD}$)
All postures	All magnitudes	/	40.2
Upright	0.5	15.1	15.1
Upright	1.0	16.1	16.1
Upright	1.5	18.7	18.9
Legs bent	0.5	30.3	31.5
Legs bent	1.0	27.9	29.2
Legs bent	1.5	27.6	30.6

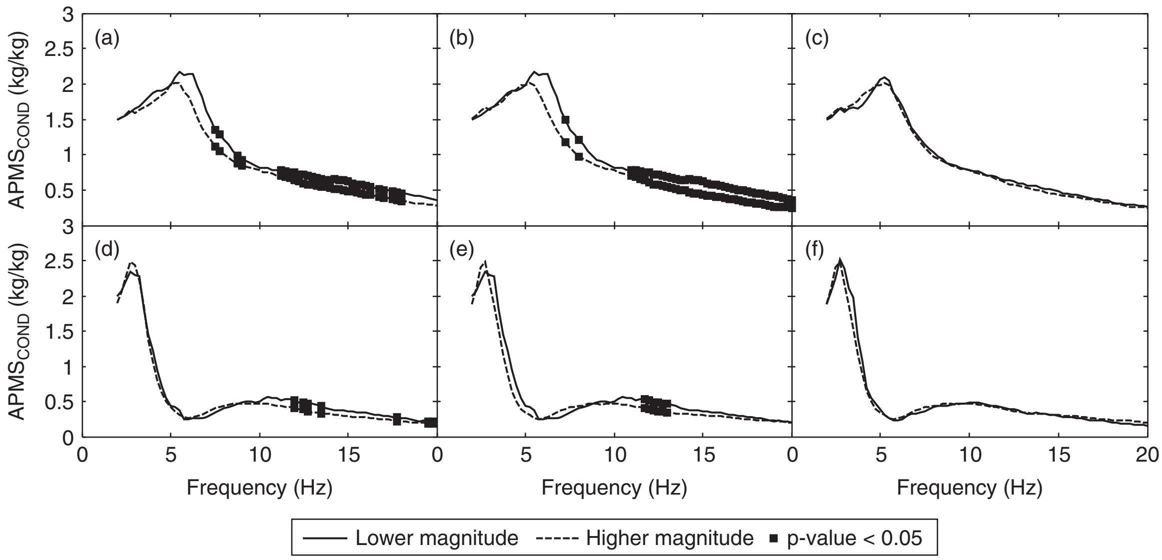

Figure 7. Results of the paired Wilcoxon signed-rank test; upright posture: (a) 0.5–1.0 $m\,s^{-2}$, (b) 0.5–1.5 $m\,s^{-2}$, (c) 1.0–1.5 $m\,s^{-2}$; legs bent posture: (d) 0.5–1.0 $m\,s^{-2}$, (e) 0.5–1.5 $m\,s^{-2}$, (f) 1.0–1.5 $m\,s^{-2}$.

response measured at 1 and 1.5 $m\,s^{-2}$. Conversely, the APMS measured with a stimulus of 0.5 $m\,s^{-2}$ is different from the ones measured at 1 and 1.5 $m\,s^{-2}$ at frequencies above 10 Hz, both in standing and legs bent postures.

A snapshot of the differences between the adoption of linear and non-linear models is provided by the conditioned force autospectra, i.e. the inertial forces generated by subjects estimated from the acceleration spectra. The conditioned force autospectra for vibration levels of 0.5, 1 and 1.5 $m\,s^{-2}$ are shown in Figure 9. The linear and the non-linear models are accurate at frequencies above 10 Hz (differences between the measured and the predicted response lower than 5%), while the modelling accuracy between 2 and 10 Hz is lower (maximum difference between the measured and the predicted response 20–40% at 5 Hz).

Nevertheless, the responses estimated using conditioned and linear models are compatible, and the multiple coherence is, at most, 10% larger than the ordinary coherence function (at 6 Hz with the stimulus of 1.5 $m\,s^{-2}$). Such a value is comparable to the intra-subject variability identified in similar conditions (Tarabini et al. 2013a).

4. Discussion

The contributions of the non-linear terms to the APMS are negligible in the whole range of frequencies. This means that the non-linear terms (quadratic, square root and absolute value) did not take part in the definition of the response, and other mechanisms were responsible for the low coherence at frequencies below 10 Hz. To our knowledge, it is the first time that the human body biodynamic response is modelled with a parallel non-linear system and, consequently, there is no material for comparisons. The noise term, which accounted for the non-modelled non-linearities and the uncorrelated noise on the measurement chain, decreased as the vibration magnitude rose and was always lower than 40% of the APMS. The largest error occurred in the anti-resonance region, where the APMS is approximately 0.5 times the (static) body mass.

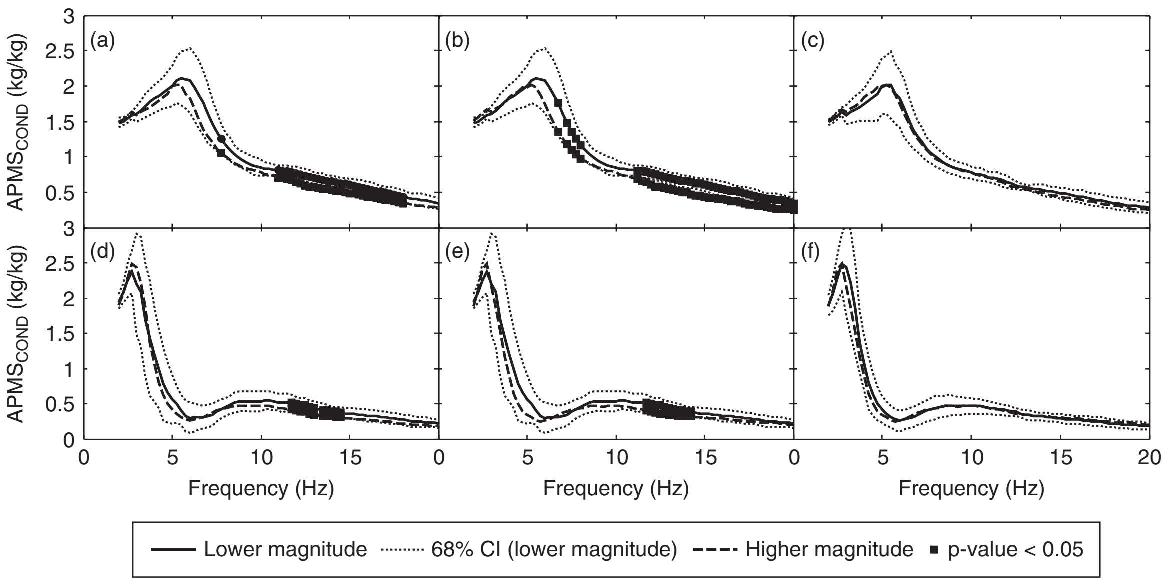

Figure 8. Results of the paired Student t-test; upright posture: (a) 0.5–1.0 m s^{-2}, (b) 0.5–1.5 m s^{-2}, (c) 1.0–1.5 m s^{-2}; legs bent posture: (d) 0.5–1.0 m s^{-2}, (e) 0.5–1.5 m s^{-2}, (f) 1.0–1.5 m s^{-2}.

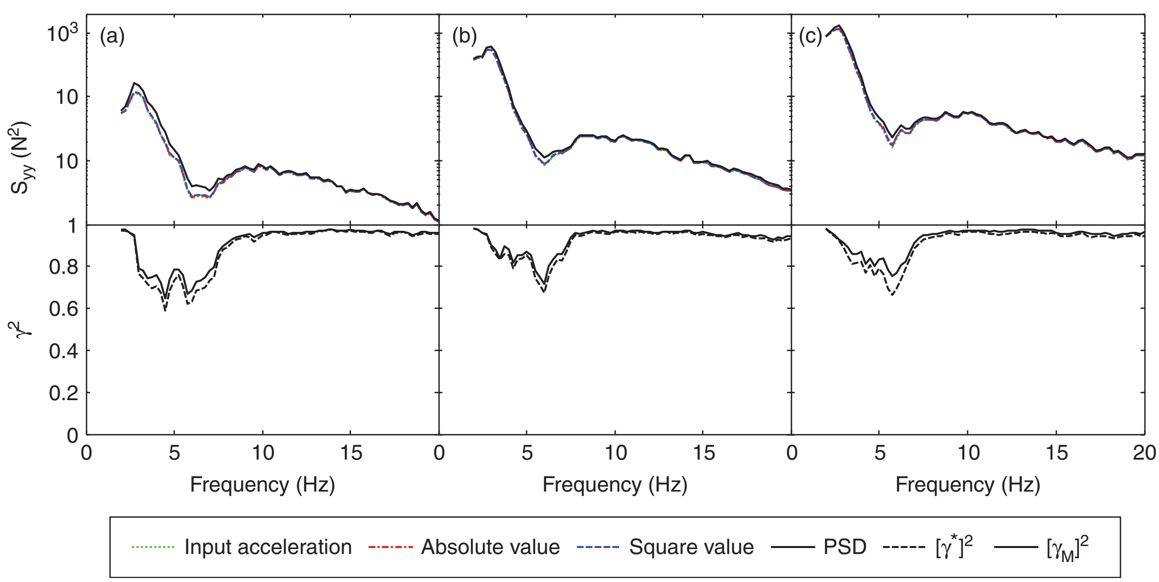

Figure 9. Conditioned autospectrum of the transmitted force (legs bent posture); (a) 0.5 m s^{-2}, (b) 1.0 m s^{-2}, (c) 1.5 m s^{-2}.

4.1. Coherence function

The ordinary coherence function has different behaviours in upright and legs bent postures. In the upright posture, coherence is close to unity for all subjects in the whole range of frequencies. When the lower limbs are bent, the average coherence function decreased between 2 and 8 Hz, with a minimum value around the anti-resonance frequency (approximately 6 Hz). In the existing studies, the information provided by the ordinary coherence function was mainly used for assessing whether a system was linear or not. High coherence values were interpreted as linear indicators and the adoption of linear identification techniques (i.e. CSD method) was fully justified. However, experimental results were in contrast with the assumption of linearity since the response of the human body exhibited a non-linearity with respect to the vibration magnitude. Besides linearity, the coherence function may indicate whether there is correlation between the output and the input signals: high coherence values are retrieved in case of good signal-to-noise ratio, as clearly exposed by Hinz et al. (2006). However, this aspect is considered of marginal importance because under controlled conditions and laboratory instrumentation (i.e. conditioning units, shielded cables) the signal-to-noise ratio is always adequate.

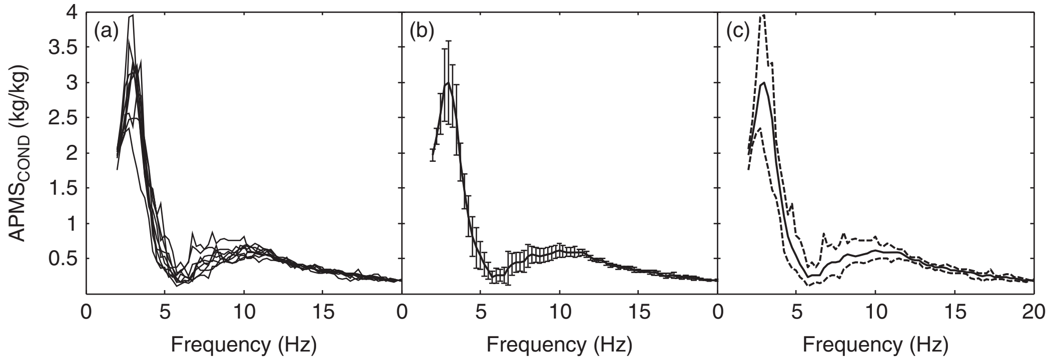

Figure 10. (a) normalised conditioned apparent mass for subject 1 (intra-subject variability), (b) normalised apparent mass for subject 1 (mean and standard deviation), (c) normalised apparent mass for subject 1 (upper and lower envelopes).

The low coherence therefore indicates the presence of non-measured inputs or that the mechanical system is non-linear (i.e. that the system is not additive, homogeneous, time invariant, fixed parameter or causal). Since the only input in the frequency range of interest is the one provided by the stimulus and that a non-linear model does not increase the coherence value, the non-linearity is reasonably caused by the variation of the modal parameters in time. This is consistent with what was evidenced in Mansfield and Maeda (2005), where the authors investigated the APMS of seated subjects in case of repeated movements of the upper body: subjects twisted their torso to the left and right with accompanied arm movements. In this situation, the coherence was lower than that derived for the static posture; it was hypothesised that the twisting motion was responsible of the whole drop-off in the coherence function at low frequencies.

In some of the existing studies, authors rejected trials with low coherence (Wang, Rakheja, and Boileau 2008; Stein et al. 2009). Results presented in this paper evidenced that the low coherence may be associated with low frequency motion during the tests and involuntary muscular actions, given that the non-linearity in the response and the measurement noise were proven trivial. The stationarity of the response has been verified dividing the time history into sub-records using the Welch approach and computing the APMS on each of them (see Figure 10). The analysis returned the state of the apparent masses as a function of time and the FRFs were then statistically analysed. Coherence is low where the APMS exhibits large COVs and both the lower and the upper envelopes showed significant misalignments in the shape (peaks and drops occurred at different frequencies).

4.2. Effect of vibration magnitude

Both the parametric and the non-parametric tests evidenced that the biodynamic response to vertical WBV is affected by the vibration magnitude only at frequencies larger than 10 Hz. In this frequency range, the APMS is lower than the static mass and the effect of modelling inaccuracy is less critical than the one deriving from an error in the resonance region.

In the existing studies (Matsumoto and Griffin 1998; Subashi, Matsumoto, and Griffin 2006), the effect of vibration magnitude on the APMS resonance frequency and amplitude have been investigated using the paired Wilcoxon signed-rank test; results evidenced that both the resonance frequency and amplitude are magnitude dependent. The Wilcoxon signed-rank test is a non-parametric method, which uses the plus and minus signs of the differences between paired observations. Since the apparent masses were normally distributed, the differences between the Wilcoxon test and the paired *t*-test outline that the effect of vibration magnitude is comparable to the intrinsic phenomenon variability; also in this case, it is convenient to adopt a linear system with uncertainty rather than to use frequency-dependent models.

4.3. Model accuracy

Experimental results confirmed that the normalised APMS of standing subjects is influenced by the posture and, in specific frequency ranges, by the vibration level; consequently, the adoption of posture, vibration level and body mass-dependent APMS models provides for the best description of the biodynamic response to WBV. Results evidenced that the effect of the vibration magnitude on the normalised APMS is definitely smaller than that of the posture and that the effect of vibration magnitude on the non-normalised APMS is negligible in comparison with the effect of the body mass. The benefits deriving from the adoption of the non-linear models (parallel of linear systems) are negligible. After the correction of all the non-linear effects (i.e. using non-linear, vibration magnitude-dependent models), the modelling uncertainty (15–30%) is governed by the subjects' posture and body mass. The uncertainty augmentation deriving from the adoption of a unique linear model (3%; see Table 2) is tolerable, similar to what happens in the case of hand–arm vibration (Tarabini et al. 2013b). This consideration is particularly useful in the identification of the response of mechanical structures in the presence of crowds. From this perspective, the body can be modelled with a linear system with uncertainty, thus easing the numerical modelling of man interaction with lightly damped structures (metallic bridges, slender staircases and similar structures).

4.4. *Suggestions for future studies*

The method described in this paper can be applied to any study focused on the analysis of the non-linear behaviour of the human body (for instance, study of the biodynamic response of sitting and recumbent subjects or of the hand–arm response to vibration). In our case, the computational effort required for the adoption of the parallel linear system model was not justified by a reduction of the modelling uncertainty. Nevertheless, the non-linear model allowed understanding the origin of the non-linearity (the human body is not a fixed-parameter system) and it seems reasonable to apply the proposed approach to other postures, in order to identify the modelling accuracy in different conditions and the influencing factors.

5. Conclusions

Non-linearities in the human body response to WBV were analysed using the conditioned output spectra and the multiple coherence functions. The contributions of the non-linear terms to the APMS are negligible (i.e. the modelled non-linear terms did not take part in the definition of the response) and the non-linearity is associated with the variation of the modal parameters in time, due to low frequency motion during the tests and involuntary muscular actions.

Both in the standing and legs bent postures, the biodynamic response to vertical WBV is affected by the vibration magnitude for frequencies above 10 Hz, where the effect of modelling inaccuracy is less critical. The responses modelled with conditioned and linear models are very similar and differences between the ordinary and the multiple coherence functions are comparable to the intra-subject variability. According to our experience, the procedure for the identification of the origins of non-linear effects consists of the computation of the APMS using the Welch approach and the H_1 estimator of the transfer function. The ordinary coherence function has to be analysed without any procedure for the outliers' rejection; if the coherence is close to unity the linear model is valid, otherwise low coherence indicates that the system is non-linear. The causes of the non-linearity may be the variation of modal parameters during the test or the presence of non-linear terms in the response. The variation of modal parameters is evidenced by the analysis of FRF COV using the Welch approach while the presence of non-linear terms is outlined by a difference between the conditioned and the conventional APMS.

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