

Fuzzy evaluation of heterogeneous quantities: Measuring urban ecological efficiency[☆]

P. Giordano^{a,*}, P. Caputo^b, A. Vancheri^c

^a Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

^b Department of Architecture, Built Environment and Construction Engineering (A.B.C.), Politecnico di Milano, via Bonardi 9, 20133 Milano, Italy

^c Dipartimento tecnologie innovative, Scuola universitaria professionale della Svizzera italiana, Galleria 2, CH-6928 Manno, Switzerland

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1. Introduction

The wide and long-lasting debate related to the global environmental emergencies, population dynamics, energy demands trend and resources depletion was recently stressed again also in the framework of the European targets for year 2020 and 2050. To that end, it is very important to provide reliable models and applications in order to improve energy systems, limiting the risk of dangerous errors and difficultly predictable negative effects.

In the present article we introduce a fuzzy logic based methodology for the analysis of the urban metabolism and for the construction of an index of urban efficiency. We will firstly introduce the metabolic point of view used in this construction (Section 2), its conceptual background based on the use of fuzzy logic methods (Section 3) and the empirical meaning of this index as a simulator of decision processes of experts (Section 3.2). The

remaining parts are dedicated to the specific calculation of this index for the cities of Barcelona and Lugano (Section 4) and to the presentation of possible results and their practical uses (Section 5).

2. Metabolic point of view on cities

Cities are commonly considered as out of equilibrium open complex systems characterized by a high level of internal organization. Complex patterns of human activity and spatial morphological complexity are aspects of this high level of internal organization. A commonly accepted point of view about the functioning of cities is the metabolic one (see e.g. Acebillo et al., 2013; Zhang, 2013; Agudelo-Vera et al., 2012; Baynes and Wiedmann, 2012; Broto et al., 2012; Castán Broto et al., 2012; Hodson et al., 2012; Villarroel Walker and Beck, 2012; Elvidge et al., 2011; Kennedy et al., 2011, 2009; Minx et al., 2010; Rapoport, 2011; Weisz and Steinberger, 2010; Niza et al., 2009; Pulselli et al., 2005; Warren-Rhodes and Koenig, 2001; Baccini, 1997 and references therein). Like biological living systems, cities are characterized by massive inflows of energy and matter that sustain all the processes that make their lives possible. As a consequence of these processes, cities discard in the environment flows of matter and energy in form of pollutants, waste and heat.

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* Corresponding author at: Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria.

E-mail addresses: paolo.giordano@univie.ac.at (P. Giordano), paola.caputo@polimi.it (P. Caputo), alberto.vancheri@supsi.ch (A. Vancheri).

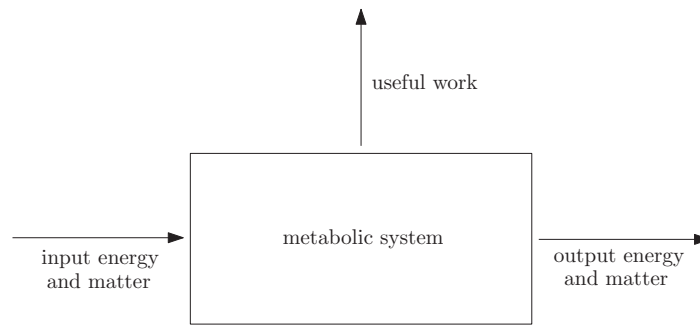


Fig. 1. Metabolic and ecological systems are chemical machines performing work at the expenses of free energy inputted by the environment in form of inflows and outflows of matter and energy.

Metabolic and ecological systems can be seen as chemical machines that perform some kind of useful work transforming substances and energy, inputted in the system from an external substrate, into energy and matter of some other forms. As a consequence of these activities, they discard in the environment output products (see Fig. 1 and Coscieme et al., 2013; Pulselli et al., 2011 for a similar description and representation of ecosystems). Useful work is spent inside the system to maintain its complex organization and functioning and to increase the biomass.

Many metabolic processes in living systems are well known from a chemical point of view, and thermodynamic quantities, like free energy of production, are tabulated. In the case of cities one would be faced with the problem of defining and measuring these quantities in a wide and heterogeneous range of phenomena including the functioning of the built environment and the functioning of the social system (see Pincetl et al., 2012; see Zhang et al., 2006 for a comprehensive list of these quantities). Of course, a thermodynamic approach could exclude some aspects that can be considered as relevant for our aims. Environmental pollution, depletion of resources and productivity cannot be assessed with pure thermodynamic tools because political, normative, and ethical aspects play a pivotal role. What is sustainable, efficient and desirable in terms of urban productivity (useful work) has to do with models of sustainable development, models of development of the human society and related values and norms. These aspects cannot be fully captured by a pure thermodynamic point of view: e.g. the impact on health of a given pollutant, the costs-benefit balances of a given urban policy or the quality of life in a city cannot be reduced to a measure of entropy or informational organization of the system.

Anyway, like metabolic and ecological systems, cities can sustain their activities and maintain themselves as complex organized and productive systems thanks to exchanges of matter and energy with the environment. So, there are also analogies. In case of cities, dissipation is meant in a broad sense as environmental impact with respect to a desired level of depletion of resources and environmental pollution. Also productivity is meant in a broad sense, including economical productivity, cultural and social productivity and quality of life and of the built environment. This productivity is sustained by the metabolic activity of cities. Like in metabolic systems, efficiency is the capability of the system to be productive with a low level of dissipation.

This point of view raises two main questions that will be faced in the following sections.

- How is it possible to measure the productivity and the environmental impact of a city?
- How should these measures be joined in an overall measure of urban efficiency?

Answering these questions is made especially difficult by the tremendously heterogeneous nature of the variables that describe the urban system (see Section 4).

3. Material and methods: fuzzy evaluation of hierarchical quantities

3.1. Hierarchical organization of the index

At the end of the previous section we have mentioned the problem of relating many heterogeneous variables in a unique measure of urban efficiency. A related difficulty is that, given its broad comprehensiveness, the index of urban efficiency cannot be a direct measure of some socio-economic or ecological quantity (for a different approach using indicators derived from Shannon's information entropy of measurable fluxes, see Zhang et al., 2006; Bodini et al., 2012; for indicators based on emergy related to urban systems, see e.g. Huang and Chen, 2005; Zhang et al., 2011; Yang et al., 2014 and references therein). For this reason, the index has to be thought as a highly plausible evaluation, in a scale conventionally ranging from 0 to 100, of the functioning of the urban system. The index is obtained aggregating many other evaluations of partial aspects of the system that in turn will depend on other evaluations. The whole construct has hence the form of a hierarchy conceptually relating many aspects of the urban system at different levels of analysis. At the top of the hierarchy one finds the overall evaluation of efficiency of the urban system, whereas the bottom is occupied by the variables listed in Section 4 and in Table 5 (see Fig. 2). We

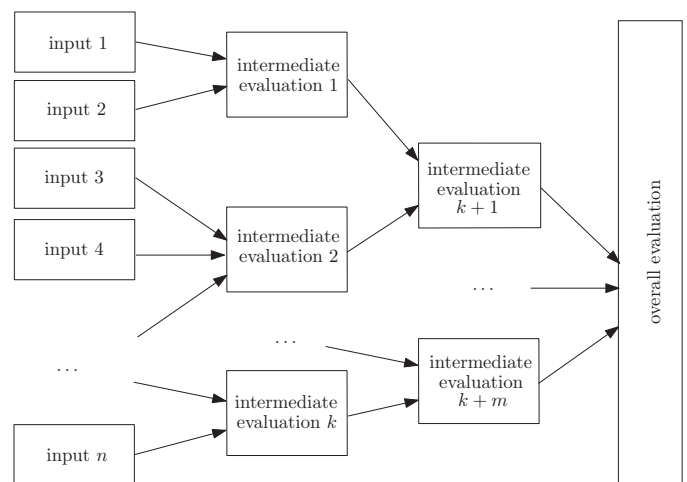


Fig. 2. The overall metabolic efficiency of the urban system is obtained through a hierarchical sequence of partial evaluations. The hierarchy is a possible logical analysis of the concept of urban efficiency in terms of lower level concepts. The analysis ends when only empirical data about urban metabolism are involved.

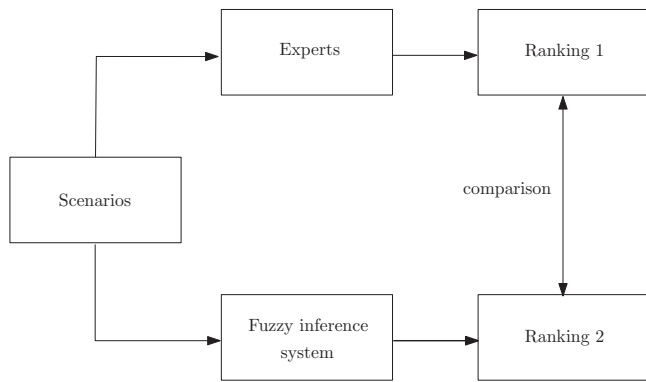


Fig. 3. The fuzzy inference system as a simulator of decision processes of experts. The same scenario is submitted to a pool of experts and to the fuzzy inference system worked out for computing the index of urban efficiency. The evaluations provided by the index are used for ranking the scenarios. A comparison between the ranking produced by the index and the one produced by the experts can be used for purposes of model calibration and validation.

can say that the whole hierarchy is a possible logical analysis of the concept of urban efficiency.

The full hierarchy among variables used to construct our index of urban efficiency is shown in Figs. 7–10.

This approach raises three main problems:

1. What is the conceptual meaning of the evaluation of urban systems obtained through the index?
2. How can the validity of this index be assessed?
3. What are the more suited mathematical tools to connect the hierarchy of Fig. 2 to the definition of a corresponding index?

We will answer the first two questions in Sections 3.2, whereas Section 3.3 will be devoted to the third one.

3.2. The index as a simulator of decision processes of experts

In this section we will address the first and second questions raised at the end of Section 3.1 concerning the method for validating the index of urban efficiency and, more in general, the empirical meaning of the evaluation provided by the index. The aim of this section is not to provide an effective method for model calibration and validation but to show that in principle such a method exists and is presumably feasible.

The problem of the empirical meaning of the index concerns the possibility to relate the value of the index to some measurable quantity. Indeed, our index is not a simple function of some measurable quantities but is a complex construction connecting together many different aspects of the urban system in order to obtain an evaluation of the urban efficiency. In order to address this problem, we image to consider a pool of experts charged with the task of deciding what scenario of the future development of a city is more desirable among a few possible alternatives (a similar approach is used in Zhang et al., 2006 to implement analytic hierarchy process). The scenarios are represented through data like those contained in Table 5. Experts will analyze the economical and environmental consequences of the different scenarios using their own experience and knowledge; they will weight costs and benefits of the different alternatives and eventually they will produce a ranking of the different scenarios. As a black box, the pool of experts can be seen as a function receiving in input data about scenarios and producing in output a ranking (see Fig. 3). If experts are coherently applying some decision criteria, maybe complex and only partially explicit, we expect that the final result of their work will be at least partially predictable. Hence the interesting question arises whether it

is possible to model this decision processes. This task is equivalent to representing experts' knowledge through a computational tool.

If the index is able to statistically reproduce in a significant way the ranking of scenarios proposed by experts, then the whole construction behind the index can be validated and interpreted as a representation of the experts' knowledge.

This point of view has a few important consequences concerning the meaning and the use of the index. First, the index mixes together many relevant aspects concerning urban sustainability and productivity: economical theories about urban development, normative aspects concerning ecological policies, and ethical values about what should be considered sustainable and desirable from a human point of view. All these aspects are weighted and compared in the frame of a cost–benefit balance. For this reason, the index is not some sort of objective measure of the state and functioning of the urban system (for this reason is preferable to use the word “index” instead of “indicator”), but will include, encapsulated in a fuzzy inference system, also a prescriptive point of view. Second, the index will unavoidably mirror the particular point of view of the people that have contributed to its definition, calibration and validation. For this reason, the whole construction should be seen more as a methodological procedure for constructing indices. Indeed, the index should be improved by a progressive injection of new knowledge and updated with the evolution of values, knowledge and laws. Of course, on the contrary with respect to certain experts, an index is always reproducible.

3.3. From a hierarchy of subsystems to a cascade of Takagi–Sugeno fuzzy models

In evaluating different scenarios, a pool of experts can rank them but, of course, is not able to reliably quantify each scenario with a suitable score. On the other hand, these experts can try a fuzzy evaluation, e.g. with a judgment among “very bad, bad, medium, good, very good”. Fuzzy logic methods permit to have a continuous grade of judgment, each one corresponding to a number between 0 and 1. The idea is that, to evaluate the truth of sentences we are interested to, instead of using 0 for “false” and 1 for “true”, we can use any number between 0 and 1. So, e.g., the previous judgments from “very bad” to “very good” can be conventionally represented by truth values 0, 0.25, 0.5, 0.75 and 1. The present article uses only basic and intuitively clear methods of fuzzy logic (see e.g. Gottwald, 1993 and references therein for a deeper introduction). The first one of them is the notion of membership function. Let us assume that we want to consider the fuzzy truth value of a sentence $\mathcal{P}(x)$ depending on a quantitative parameter $x \in X$ (we will call x a *fuzzy variable*). In constructing a fuzzy model, the possible truth values of $\mathcal{P}(x)$ are represented by a function $\mu : X \rightarrow [0, 1]$, so that we can say that $\mathcal{P}(x)$ is true at the level $\mu(x) \in [0, 1]$. The actual form of μ is a modeling choice. In the present work, the membership function of a sentence of the form $\mathcal{P}_1(x) \text{ AND } \mathcal{P}_2(y)$ will be the product $\mu_1(x) \cdot \mu_2(y)$ (see Gottwald, 1993 for other ways to represent the logical operator AND).

We start by applying these ideas to the definition of our urban efficiency index. Indeed, we want to evaluate the input measured quantities x_1, \dots, x_n (leaves in the tree structure of Fig. 2) as fuzzy variables using three fuzzy membership functions $\mu_{k0}, \mu_{k1}, \mu_{k2}$ corresponding respectively to the fuzzy sentences

- the value of x_k is the worst
 - the value of x_k is medium
 - the value of x_k is the best.
- (1)

There are here two choices made by the interdisciplinary group working on the definition of the index. The first one is the choice of

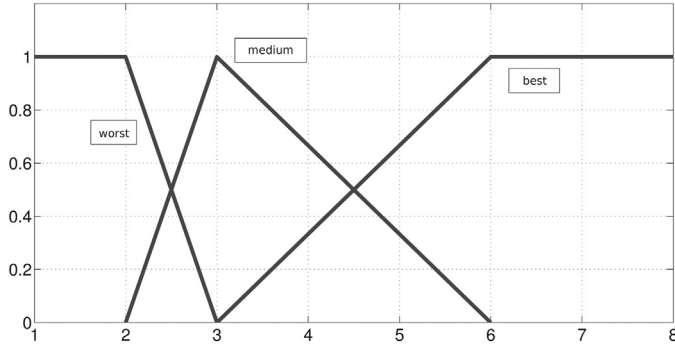


Fig. 4. An example of membership functions used in the definition of the urban efficiency index. In this case $w_k = 2$, $m_k = 3$ and $b_k = 6$.

crisp values w_k, m_k, b_k corresponding to crisp evaluations of worst, medium and best values of each variable x_k . Of course, this values depends on several factors, technical, legislative, and e.g. surely depending on the present level of sensibility regarding environmental protection. See Section 4 to have several examples of these crisp values. Depending on the meaning of the variable x_k , we may have $w_k < m_k < b_k$ (e.g. if x_k is the number of job positions, see Section 4) or $b_k < m_k < w_k$ (e.g. if x_k is CO₂ produced by private transportation). For example, in the former case we surely must have

$$\mu_{k0}(x) = 1 \quad \text{if } x \leq w_k$$

$$\mu_{k1}(m_i) = 1$$

$$\mu_{k2}(x) = 1 \quad \text{if } x \geq b_k.$$

The second choice made for defining the index is the form of these membership functions μ_{kj} . To reduce the number of parameters in the model, we opted for piecewise linear membership functions. More precisely, μ_{k0} (worst case) is decreasing exactly where μ_{k1} (medium case) is increasing; in the same way, μ_{k1} is decreasing exactly where μ_{k2} (best case) is increasing, see e.g. Fig. 4. This choice permits to have only w_k, m_k and b_k as parameters of these membership functions.

Let A_{kj} be the fuzzy sentence corresponding to the membership function μ_{kj} , e.g. A_{k1} represents “ x_k is medium”, and hence its truth is evaluated using μ_{k1} . We can hence say that the efficiency evaluation of our urban system necessarily passes through the partition of the input space of variables x_1, \dots, x_n into 3^n cases:

$$(x_1 \text{ IS } A_{1,j_1}) \text{ AND } (x_2 \text{ IS } A_{2,j_2}) \dots \text{AND} (x_n \text{ IS } A_{n,j_n}), \quad (2)$$

where $j_k = 0, 1, 2$. In our case, we hence have $3^{22} \simeq 3.14 \cdot 10^{10}$ possible cases. The idea to structure the analysis of the system into a cascade of subsystems permits to drastically reduce this huge number to 3^{n_s} , where n_s is the number of input leaves of each subsystem (in our case $1 \leq n_s \leq 5$, see Section 4). This is the most important implicit assumption in the definition of this index: it is coherently applicable only to urban systems whose efficiency can be evaluated by splitting the system into subsystems, i.e. where this global evaluation can be split into *independent* efficiency evaluations of each subsystem. The validation of this assumption is out of the scope of the present work, mainly because it needs a considerable amount of data from different urban systems and a suitable pool of experts, as explained above.

The next choice in defining the index is to observe that some of these input variables, from the technical point of view, must be considered more important than others in the final definition of the index. For example, in our case energy consumption for heating is considered, by policy makers, public utilities and urban designers of west countries, more important than water consumption for

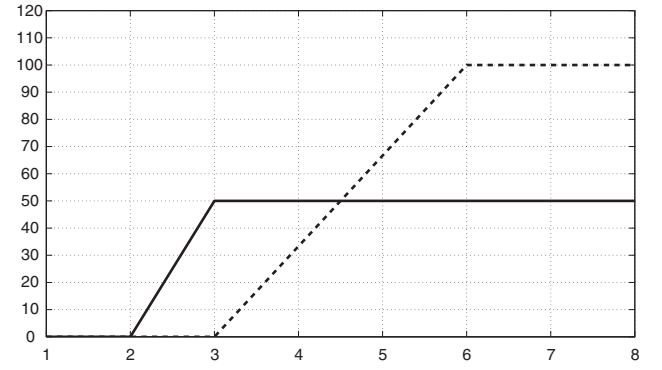


Fig. 5. An example of functions f_{kj} corresponding to membership functions of Fig. 4. In continuous line we have the function f_{k1} , and in dashed line the function f_{k2} . The function f_{k0} is always zero.

buildings. To consider this flexibility, we introduce an evaluation i_k of the *level of importance* of the input variable x_k in the final index. The greater the value of i_k the greater is the importance of the input variable x_k in the final index. In our urban efficiency index, we have $i_k = 1, 2, 3, 4, 5$.

With respect to some input variables x_k , the final index must be non decreasing (this is the case of variables measuring the useful work of Fig. 1) and for some variables it must be non increasing (this is the case of variables measuring the inputs and outputs of Fig. 1). For simplicity, let us consider the former case and assume that $w_k < m_k < b_k$, analogously we can deal with other cases. The final choice in the construction of the index is the definition of three functions f_{k0}, f_{k1}, f_{k2} for every input variable x_k . The function f_{kj} corresponds to the contribution of the variable x_k to the final index assuming that we are evaluating the case corresponding only to “ x_k IS A_{kj} ” and $i_k \gg i_l$ for all $l \neq k$; in other words, we can say that f_{kj} gives the contribution of the variable x_k in the hypothetical situation where only the condition “ x_k IS A_{kj} ” holds, with truth value 1, and the variable x_k is considered the most important. These functions f_{kj} have to be chosen coherently with respect to the membership functions μ_{kj} , e.g. they surely have to verify general constraints as

$$f_{k0}(x) = 0 \quad \forall \quad x \leq w_k \quad (3)$$

$$f_{k1}(x) = 50 \quad \forall \quad x \geq m_k \quad (4)$$

$$f_{k2}(x) = 100 \quad \forall \quad x \geq b_k \quad (5)$$

$$f_{kj} \text{ non decreasing} \quad (6)$$

$$0 \leq f_{kj}(x) \leq 100, \quad (7)$$

(we recall that we want that our final index ranges between 0 and 100). Analogous general constraints can be stated when $b_k < m_k < w_k$ or when the final index is non increasing with x_k . Once again, in our case study we defined f_{kj} as piecewise linear functions and coherently with the membership functions μ_{kj} , so that they have only w_k, m_k and b_k as parameters. The function f_{k0} is always zero since it corresponds to the worst case. The function $f_{k1}(x)$ will be zero for $x \leq m_k$, it will be linearly increasing for $w_k < x < m_k$ and it will be constant to 50 for $x \geq m_k$. Analogously for f_{k2} , as in Fig. 5. In the case $b_k < m_k < w_k$ the functions f_{kj} are non increasing and can be defined analogously.

It is now natural to firstly consider as output of the j th rule (2) the weighted mean

$$\text{IF } (x_1 \text{ IS } A_{1,j_1}) \text{ AND } \dots \text{AND } (x_{n_s} \text{ IS } A_{n_s,j_{n_s}}) \text{ THEN } y_j = \sum_{k=1}^{n_s} \frac{i_k}{I} \cdot f_{k,j_k}(x_k), \quad (8)$$

where $I := \sum_{k=1}^n i_k$. We hence have that $0 \leq y_j \leq 100$ since (7) holds. It is also natural to put together all these y_j , for $j \in \{0, 1, 2\}^{n_s}$, as weighted mean with weights given by the fuzzy truth value of the antecedent in (8), i.e. by $\omega_j = \prod_{k=1}^{n_s} \mu_{k,j_k}(x_k)$, which is called *firing strength* of the j th rule (8). The final value of every intermediate evaluation (i.e. the first nested level of Fig. 2, just after the input variables) is therefore given by

$$y_\alpha = \frac{\sum_{j \in R_\alpha} \omega_j \cdot y_j}{\sum_{j \in R_\alpha} \omega_j} = \frac{\sum_{j \in R_\alpha} \prod_{k=1}^{n_s} \mu_{k,j_k}(x_k) \cdot \sum_{k=1}^{n_s} \frac{i_k}{I} \cdot f_{k,j_k}(x_k)}{\sum_{j \in R_\alpha} \prod_{k=1}^{n_s} \mu_{k,j_k}(x_k)}, \quad (9)$$

where $R_\alpha = \{0, 1, 2\}^n$ represents the set of all the rules for the considered α -intermediate evaluation. Of course, y_α depends on the input variables x_1, \dots, x_{n_s} of the considered α -subsystem. Let us note that, at the end, y_α is given by suitable sums and products of the functions μ_{kj} and f_{kj} , so that the fuzzy logic methods remain only a useful interpretation.

It may also worth noting that all the modeling data μ_{kj} , i_k and f_{kj} are the same for all the urban systems we want to evaluate using this index. Only the values of the input leaf variables x_k at the first level (see Fig. 2) depend on the particular system we are evaluating. We could thus say that μ_{kj} , i_k and f_{kj} represent the statistical mean of values and knowledge incorporated in the previously mentioned imaginary pool of experts.

If the condition “ x_k IS A_{kj} ” holds with truth value 1, i.e. $\mu_{kj}(x_k) = 1$ and $\mu_{kh}(x_k) = 0$ for all $h \neq j$, and if we consider the variable x_k as the most important, i.e. $i_k \gg i_l$ for all $l \neq k$ so that $i_k/I \simeq 1$ and $i_l/I \simeq 0$, then $y_\alpha \simeq f_{kj}(x_k)$. This gives more formally the previous interpretation of the function f_{kj} . Moreover, let X_k be the space of values for the input variable x_k , and let D_{pq} be the set where only the condition “ x_p IS A_{pq} ” holds, i.e.

$$D_{pq} := \{(x_1, \dots, x_n) \in X_1 \times \dots \times X_n \mid \mu_{pq}(x_p) = 1, \mu_{ph}(x_p) = 0 \forall h \neq p\}.$$

Assume that we can differentiate (9) with respect to x_p at a point $(x_1, \dots, x_n) \in D_{pq}$. Since $\mu_{ph}(x_p) = 0$ for all $h \neq q$, we can write

$$y_\alpha(x_1, \dots, x_n) = \frac{i_p}{I} \cdot f_{pq}(x_p) + \frac{\sum_{j \in R_\alpha} \prod_{k=1}^n \mu_{k,j_k}(x_k) \cdot \sum_{k=1}^n \frac{i_k}{I} \cdot f_{k,j_k}(x_k)}{\sum_{j \in R_\alpha} \prod_{k=1}^n \mu_{k,j_k}(x_k)}, \quad (10)$$

so that the second summand in (10) does not depend on x_p , and we get

$$\frac{\partial y_\alpha}{\partial x_p}(x_1, \dots, x_n) = \frac{i_p}{I} \cdot f'_{pq}(x_p). \quad (11)$$

This formula proves that, in the space D_{pq} , the index y_α will be non decreasing with respect to the input variable x_p if and only if $f'_{pq} \geq 0$, as we requested above in the construction of the functions f_{kj} .

We can continue recursively using the same method for the other levels in the hierarchy, until we arrive at the final index. The intermediate evaluations at level p will be the new input variables for the intermediate evaluations at level $p+1$, and whose worst, medium and best crisp values will be 10, 50, 90. We prefer to take 10 and 90 (instead of 0 and 100) as worst and best value for intermediate efficiencies, since otherwise it would be almost impossible to obtain a fully “worst” of “best” fuzzy evaluation in these efficiencies.

For example, if $x_k = w_k$, $\mu_{k1}(w_k) = \mu_{k2}(w_k) = 0$, for all $k = 1, \dots, n$, then $y_\alpha = y_j = 0$, where $j = (0, \dots, 0)$, because $f_{k0}(w_k) = 0$ by (3); if $x_k = m_k$, $\mu_{k0}(m_k) = \mu_{k2}(m_k) = 0$, for all $k = 1, \dots, n$, then $y_\alpha = y_j = 50$, where $j = (1, \dots, 1)$, because $f_{k1}(m_k) = 50$ by (4); finally, if $x_k = b_k$, $\mu_{k0}(b_k) = \mu_{k1}(b_k) = 0$ for all $k = 1, \dots, n$, then $y_\alpha = y_j = 100$, where $j = (2, \dots, 2)$, because $f_{k2}(b_k) = 100$ by (5). Of course, non trivial cases will depend on the firing strengths ω_j . All

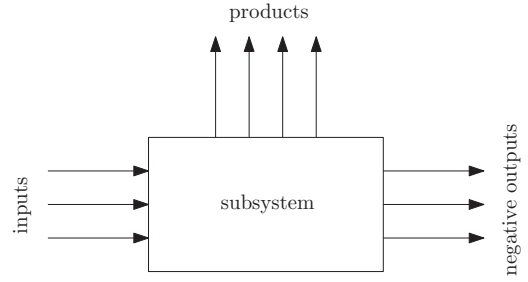


Fig. 6. The general metabolic schema for subsystems.

these general properties validate mathematically the definition (8) with respect to our intuition.

For the urban efficiency index, we have three main subsystems: the transportation subsystem, the built environment subsystem and the socioeconomic subsystem. Each one of these subsystems have three further subsystems called “inputs”, “products” and “negative outputs” (for the socioeconomic subsystem we have a simplified structure with only the “products” subsystem, see Section 4). At the end, the three efficiency evaluations are synthesized in the final index. We therefore have a hierarchy with $(3+1)+(3+1)+(1+1)+1 = 11$ subsystems and four levels of input/output fuzzy variables.

The fuzzy method we have just outlined is well know and used in several area of research. It is called Takagi–Sugeno fuzzy modeling, see Takagi and Sugeno (1985).

4. Construction of the urban efficiency index

In this section, we will describe in some details the data used for computing the index of urban efficiency and the structure of the hierarchy actually used for the implementation of the index.

In the present work, we will focus our attention more on the balance of energy or matter and less on the set of interactions causing this balance. Indeed, we will represent different part of a city using a black box approach (see Pulselli et al., 2011 for a general description of a black box approach to the study of complex systems; see Fig. 6; compare also with Fig. 1).

A city can be thought made of suitable subsystems (see e.g. Bodini, 2012; Bodini et al., 2012). From a functional point of view, a meaningful subdivision we can consider is the following:

- Transportation subsystem: how much movements there are in the city and using what modalities; how much energy they need and how much waste (e.g. CO₂ and other types of pollutants) they produce.
- Built environment subsystem: how much energy, water and surface, per head or per square meter, the city needs to guarantee its comfort and operative processes (excluding transportation); how much waste these processes produce.
- Cultural, sociological and economic subsystem: e.g. how much GDP the city produces, number of job positions, number of university students, energy consumption for industries, amount of taxes per head, healthy, welfare and services accessibility.

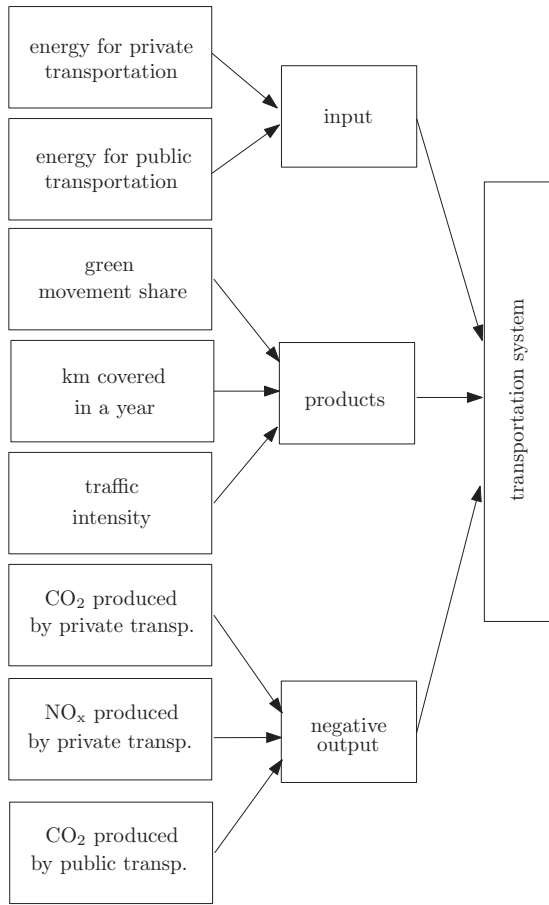


Fig. 7. The I/O metabolic schema for the transportation subsystem.

Each one of these subsystems can be abstractly represented using the schema of Fig. 6. In other words, each subsystem takes some quantities in input (usually, they are either masses or energies of some type) and produces two types of outputs. The first one has to be thought as all the “negative outputs”, e.g. every kind of waste produced by the subsystem. The other ones, are all the subsystem’s *products* (also called its *productivity*), analogous to *useful work*. We can hence state that we are thinking at a city as a network of engines producing some form of useful work (its products). These outputs are produced through exchanging of matter and energy with the external world or with other subsystems of the same city.

Each one of these subsystems has its own *efficiency*, depending on our evaluation of its inputs and its outputs. All together, these efficiencies contribute in our definition of the *urban efficiency index*.

Now, we will analyze each one of the previously listed subsystems to explicate a possible list of its inputs and outputs quantities. Of course, the subsystems we can really consider depend on the available data and their level of disaggregation, even if CO₂ produced by the considered urban systems has always been estimated (see Zhang, 2013, Section 5).

4.1. Transportation subsystem

The representation of the transportation subsystem is given in Fig. 7. Each input/products/negative output quantity is sufficiently explained by its own name. Anyway, for a more detailed explanation, see the next sections. The present work is strictly tied to the availability of a common data set between Barcelona and Lugano.

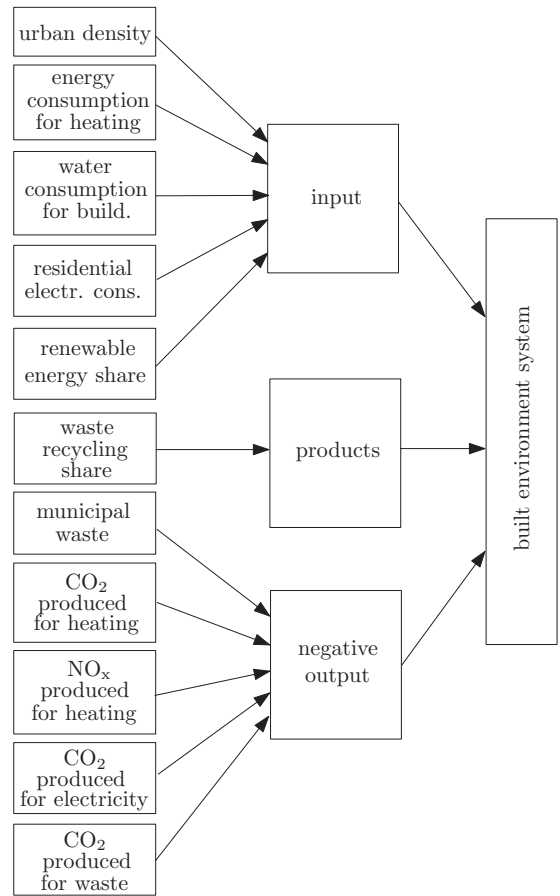


Fig. 8. The I/O metabolic schema for the built environment subsystem.

4.2. Built environment subsystem

The representation of the built environment subsystem is given in Fig. 8. For an explanation of each quantity, see the following sections.

4.3. Socioeconomic subsystem

The representation of the socioeconomic subsystem is given in Fig. 9. This subsystem has been represented in an extremely idealized way, with no input nor negative output. Of course, this is criticizable, but the main aim of the present work is to show the methodological approach we have used for the construction of the urban efficiency index and some of its possible uses. A more complete representation, including the industrial sector and the taxes as input, can be that of Fig. 10. It has not been considered in this

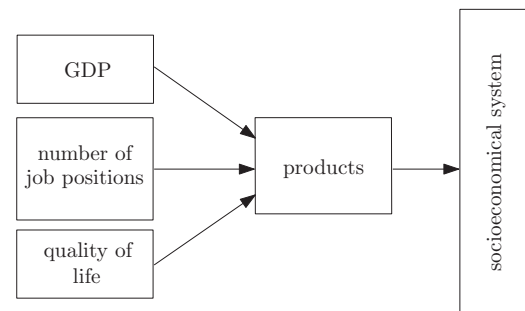


Fig. 9. The I/O metabolic schema for the socioeconomic subsystem.

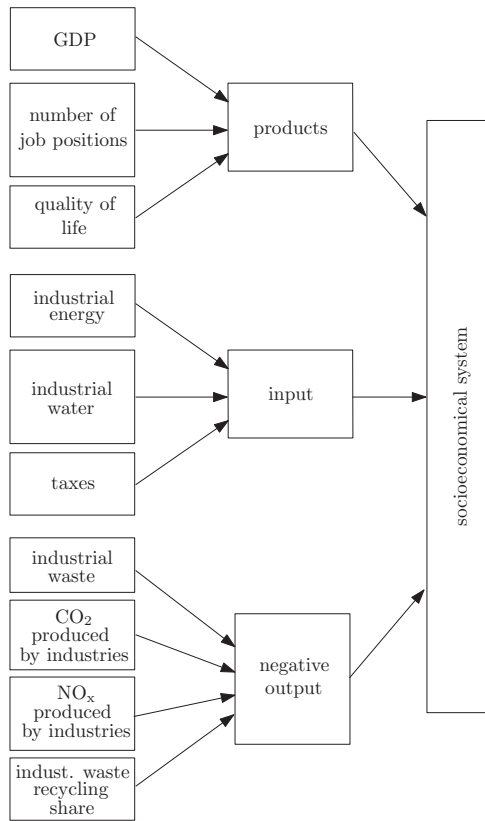


Fig. 10. A more complete I/O metabolic schema for the socioeconomic subsystem.

work because the corresponding data are not available for Lugano, but it will be considered in future works.

4.4. Fuzzy evaluations

Of course, as we underscored above, there is a certain degree of arbitrariness in the choice of the best/medium/worst evaluation of a given quantity or in the corresponding level of importance in the final index. In the present work we have tried to fix each value on the basis of the present state of the art in the environmental evaluation of this type of systems and with a certain eye toward a preservation of the environment. On the other hand, we have to see the methodology presented in this work as the construction of a decision support system to evaluate political decisions concerning the efficiency of a given city. Administrators aiming at using this approach will have the freedom to choose his/her own evaluations and levels of importance. Different choices of these parameters will correspond to different political and environmental decisions. For the sensitivity analysis of the model, see Section 5.

4.4.1. Transportation subsystem

The evaluations of the energy for private transportation (cars, vans and trucks), of the corresponding CO₂, and, finally, of the NO_x produced by private transportation, are all based on the km covered in a year in a urban cycle. For European countries, we started from the data gathered for 2008 by Emerdata (2013), obtaining an average value of 14,500 km/(y veh) for the total amount of km covered in a year in every driving cycle (urban plus extra-urban). We fixed this average value as the medium benchmark, i.e. medium = 14,500 km/(y veh), and set best = 7250 = 14,500/2 km/(y veh), and worst = 29,000 = 2 × 14,500 km/(y veh). The New European Driving Cycle (see e.g.

Wikipedia., 2013a) is supposed to represent the typical usage of a car in Europe. It consists of four repeated EVE-15 driving cycle and Extra-Urban driving cycle. In this typical usage, the Extra-Urban cycle correspond to 63% of the total covered km. For a urban driving cycle, we have hence fixed best = 2682 = 7250 × 37% km/(y veh), medium = 5365 = 14,500 × 37% km/(y veh) and worst = 10,730 = 29,000 × 37% km/(y veh).

Energy for private transportation:

In this case, for simplifying the methods, all evaluations refer only to cars. From Emerdata (2013) we obtain an average consumption of 0.0767 l/km for cars in European countries in 2008. The best evaluation corresponds to a total of 2682 km/(y veh) (urban cycle) and an average consumption of 0.897 kWh/km = (0.0767 l/km × 11.7 kWh/l), i.e. 2.41 MWh/(y veh) or less. The medium evaluation corresponds to a total of 5365 km/(y veh) and an average consumption of 0.897 kWh/km of gasoline, i.e. 4.81 MWh/(y veh). The worst evaluation corresponds to a total of 10,730 km/(y veh) and an average consumption of 0.897 kWh/km of gasoline, i.e. 9.62 MWh/(y veh) or more. Level of importance in the final index: 3.

Energy for public transportation:

It is not easy to evaluate the energy used for public transportation and the corresponding amount of CO₂ produced. Indeed, on the one hand we would like to say that a good public transportation system uses a small amount of energy and produces less CO₂ with respect to a worse public transportation system. On the other hand, a city characterized by a poor use of the public transportation would use small energy and produce small CO₂. Our idea is to compare, e.g., the energy per passenger of the public transportation with the energy per vehicle of the private transportation. We assume that the public system will be “best” if it is able to use less than 75% of the energy used by the “best” amount of energy per private vehicle. Of course, instead of 75% we can use another percentage, expressing the convenience of the public system with respect to the private one. Analogous ideas have been used to find the “medium” and the “worst” evaluations, and to evaluate the CO₂ produced by public transportation. An estimation of the number of passengers of the public transportation system has been obtained using the green movement share, see below. For these reasons, we assumed that the best evaluation corresponds to 1.48 = 1.97 × 75% MWh/(y inh) or less. The medium evaluation corresponds to 2.96 = 3.94 × 75% MWh/(y inh). The worst evaluation corresponds to 5.99 = 7.98 × 75% MWh/(y inh) or more. Level of importance in the final index: 3.

CO₂ produced by private transportation:

In this case, for simplifying the methods, all evaluations refer to cars. Many references could be taken into account for defining CO₂ emissions by cars. Information given at European level (European Commission, 2013) were considered in our research. The best evaluation corresponds to a total of 2682 km/(y veh) and an average production of 190 g/km of CO₂, i.e. 0.51 t/(y veh) or less. The medium evaluation corresponds to a total of 5365 km/(y veh) and an average production of 190 g/km of CO₂, i.e. 1.02 t/(y veh). The worst evaluation corresponds to a total of 10,730 km/(y veh) and an average production of 190 g/km of CO₂, i.e. 2.04 t/(y veh) or more. Level of importance in the final index: 4.

NO_x produced by private transportation:

Only gasoline passenger cars with direct injection engines are considered in the present work for the evaluation of nitrogen dioxides. Indeed, to evaluate the medium value of NO_x produced by transportation, we have used the following European (EU 27 plus Norway, Switzerland, Turkey) 2008 distribution of the vehicle fleet (see European Environment Agency, 2013):

Unlike the production of CO₂, depending on the average consumption of gasoline and not on the “Euro x” type of car, the

Table 1

Vehicle fleet (gasoline passenger cars) in 2008.

Pre Euro/conventional	9.1%
Euro 1	17.4%
Euro 2	22.4%
Euro 3	32.5%
Euro 4	18.5%
Euro 5	0.1%

nitrogen oxides depends on the “Euro” classification of the car. The data used in the present work (see [Official Journal of the European and Union, 2007](#); [Wikipedia., 2013b](#)) are summarized in [Table 2](#).

To estimate the NO_x emissions for Euro 1, Euro 2 and pre Euro/conventional cars, an exponential extrapolation ($R^2 = 0.96$) has been used. The best evaluation corresponds to a total of 2682 km/(y veh) corresponding to a vehicles distribution of 10% electrical and 90% Euro 5, i.e. 0.14 kg/(y veh) or less. The medium evaluation corresponds to a total of 5365 km/(y veh) with vehicles distribution equal to the 2008 European distribution of vehicle fleet (see [Table 1](#)), i.e. 1.21 kg/y veh. The worst evaluation corresponds to a total of 10,730 km/(y veh) with vehicles distribution of 100% Euro 3, i.e. 1.61 kg/y veh or more. Level of importance in the final index: 4. Our choice for the evaluation of the level of nitrogen dioxides produced by private transportation, can be really improved if we have data about the length of the average covered path in our city and the average speed used in this path.

CO₂ produced by public transportation:

Coherently to our threshold of 75% to evaluate a good public transportation system (see Energy for public transportation), and the benchmarks fixed for the CO₂ produced by private transportation, we can set the best evaluation for the CO₂ produced by public transportation to $0.32 = 0.42 \times 75\%$ t/y inh or less. The medium evaluation corresponds to $0.63 = 0.84 \times 75\%$ t/y inh. The worst evaluation corresponds to $1.27 = 1.69 \times 75\%$ t/y inh or more. Level of importance in the final index: 4. The evaluation of the nitrogen dioxides produced by public transportation has not been considered in the present work because the corresponding data are not available.

Green movement share:

The green movement share is the average percentage of citizens using slow or public transportation with respect to all the people moving in the city (i.e. using also private transportation), see e.g. [Siemens AG \(2009\)](#). The best evaluation corresponds to 50% or more. The medium evaluation corresponds to 40%. The worst evaluation corresponds to 30% or less. Level of importance in the final index: 3.

km covered in a year:

This quantity represents the average amount of km covered by a private car in a year in an urban driving cycle. The best evaluation corresponds to a total of 2682 km/(y veh) or less. The medium evaluation corresponds to a total of 5365 km/(y veh). The worst evaluation corresponds to a total of 10,730 km/(y veh) or more. Level of importance in the final index: 4.

Table 2

European emission standards for gasoline passenger cars (direct injection engines).

Tier	Year	NO _x (g/km)
Pre Euro/conventional	1988	0.56
Euro 1	1992	0.35
Euro 2	1996	0.22
Euro 3	2000	0.15
Euro 4	2005	0.08
Euro 5	2009	0.06

Transportation intensity:

The *transportation intensity* $I_r(t)$ of a given road r is defined as the ratio between the measured flux $q_r(t)$, at a given time t , of vehicles on the road and the maximum capacity q_{\max} of the road

$$I_r(t) := \frac{q_r(t)}{q_{\max}}.$$

(see, e.g., [Slinn et al., 2005](#)). To obtain a fuzzy evaluation of the transportation intensity, we propose the following procedure:

1. Let us suppose to have an estimation of the hourly average distribution of the transportation intensity for each link of the roads network (we obtained this data using a transportation simulation software, calibrated using a sufficient number of real flux measure points, see [Vancheri et al., 2014](#)).
2. Let us consider all the links passing the filter threshold of $I_r \geq 0.8$ for at least 1 h per day. They can be interpreted as the most used (and hence important) roads of our network. Let $l_{0.8}$ be the total length of these filtered links.
3. Let us consider all the links passing the threshold of $I_r \geq 0.9$ for at least 1 h per day (which, usually, is the hour of worst jamming, e.g. from 5 pm to 6 pm). Of course, they will be a subset of the previous set of filtered links. This set can hence be interpreted as the subset of jammed links among the most important link of the roads network. Finally, let $l_{0.9}$ be the total length of these jammed links.
4. Our evaluation is based on the fraction $l_{0.9}/l_{0.8}$. This fraction can be thought as an estimation of the probability to be in a jammed situation using one of the most important roads of the network.

The best evaluation corresponds to $l_{0.9}/l_{0.8} \leq 10\%$. The medium evaluation corresponds to $l_{0.9}/l_{0.8} = 30\%$. The worst evaluation corresponds to $l_{0.9}/l_{0.8} \geq 50\%$. Level of importance in the final index: 4.

4.4.2. Built environment subsystem

Urban density:

The urban density is defined as the ratio between the number of inhabitants and the urban surface area, which is the area of the residential built surface. The best evaluation corresponds to 9600 inh/km², i.e. the urban density of a dense and big city like Singapore. The medium evaluation corresponds to 5800 inh/km², equal to the mean value between the best and the worst values of urban density. The worst evaluation corresponds to 2000 inh/km², roughly indicating a high level of European urban sprawl (see [European Environment Agency, 2006](#)). Level of importance in the final index: 4.

Energy consumption for heating:

The technical literature referred to the European state of the art was considered, see [BPIE, 2011](#). The energy demand for heating takes into account for the worst and medium evaluation (set equal to 200 and 110 kWh/m² y respectively), the state of the art in Europe (referring, in particular, to residential buildings) and for the best evaluation, the Swiss Minergie standards (38 kWh/(m² y); see [www.minergie.ch](#)). Furthermore, in order to simplify the elaborations, the mean yearly efficiencies of natural gas boilers are taken into account as reference: 95% (best case), 90% (medium case) and 85% (worst case) have been taken into account as generation systems giving, at the end, energy consumptions for heating generation of 40, 122 and 235 kWh/(m² y) respectively. The figures just defined are in line with results reported in other studies, such as the recent ([BPIE, 2011](#)). Further, since the climate affects in a very important way the heating demand, these standards have been normalized with respect to the degree-days of Lugano (2638 K d), that could be taken as representative of a medium European climate. Therefore, the best evaluation corresponds to 15 Wh/(m² K d y) or

less. The medium evaluation corresponds to 46 Wh/(m² K d y). The worst evaluation corresponds to 89 Wh/(m² K d y) or more. Level of importance in the final index: 5.

Water consumption for buildings:

The present version of the index has been calibrated on European cities taking into account the available technical literature and, in particular ([Ambiente Italia Istituto di ricerche, 2007](#)). For non European cities the valuation of the water consumption for buildings must be changed, e.g. considering “worst” also values less than 100 l per inhabitant per day (in these cases, drinkable water availability becomes a sanitary problem). The best evaluation corresponds to 170 l/(inh d) or less. The medium evaluation corresponds to 220 l/(inh d). The worst evaluation corresponds to 280 l/(inh d) or more. Level of importance in the final index: 3.

Total electricity consumption:

As electricity consumption, only lighting and appliances have been taken into account (without air conditioning and ventilation). The values take into account the state of the art in Europe, considering as basis residential or commercial buildings equipped with high efficiency lighting and appliances and managed by an energy conscious approach. Therefore, the best evaluation corresponds to 15 kWh/m² y or less. The medium evaluation corresponds to 32.5 kWh/m² y. The worst evaluation corresponds to 50 kWh/m² y or more. The figures just defined are in line with results reported in other studies, such as ([Caputo et al., 2013](#)). Level of importance in the final index: 5.

Renewable energy share (renewable electricity/total electricity):

This share has been calculated as the ratio between the amount of avoided primary energy (i.e. primary energy I would have used in case I have not renewable sources) and the total amount of non renewable primary energy. The best evaluation corresponds to 20% or more (20% is the target defined by the EU 20–20–20 energy package for year 2020, considering all the final energy uses). The medium evaluation corresponds to 7.5%. The worst evaluation corresponds to 5% or less. Level of importance in the final index: 5.

Waste recycling share:

These benchmarks were defined taking into account the available technical literature and, in particular ([Ambiente Italia Istituto di ricerche, 2007](#)). The best evaluation corresponds to 45% or more. The medium evaluation corresponds to 35%. The worst evaluation corresponds to 15% or less. Level of importance in the final index: 4.

Municipal waste:

The best evaluation corresponds to 1 kg/(inh d) or less. The medium evaluation corresponds to 1.5 kg/(inh d). The worst evaluation corresponds to 2 kg/(inh d) or more. The figures just defined are in line with results reported in other studies, such as the recent ([ISPRA, 2012](#)). Level of importance in the final index: 4.

CO₂ produced for heating:

This calculation takes into account the previously described energy consumption for heating. Despite of the very large alternatives relative to heating plants and systems, only gas fueled plants have been evaluated, considering a mean emission factor of 200 g/kWh of CO₂. Therefore, the best evaluation corresponds to 3 g/(m² K d y) or less. The medium evaluation corresponds to 9.2 g/(m² K d y). The worst evaluation corresponds to 17.8 g/(m² K d y) or more. It has to be stressed that also these values are normalized on the basis of the degree days. Level of importance in the final index: 5.

NO_x produced for heating:

From the point of view of production of NO_x, we considered only gas fueled systems for heating. From [Aste and Del Pero \(2012\)](#) we get that 45 mg/kWh, 125 mg/kWh and 230 mg/kWh can be considered as best, medium and worst evaluations resp. Therefore, as for the previously described CO₂ values, the best evaluation

corresponds to 0.7 mg/(m² K d y) or less. The medium evaluation corresponds to 5.8 mg/(m² K d y). The worst evaluation corresponds to 20.5 mg/(m² K d y) or more. It has to be stressed that also these values are normalized on the basis of the degree days. Level of importance in the final index: 5.

CO₂ produced for electricity:

This calculation takes into account the previously described electricity consumption. Despite of the very large alternatives relative to power generation in each country, the following emission factors have been taken into account for describing that wide panorama: 200 g/kWh of CO₂ for the best case (renewable, nuclear and partially gas based power generation), 450 g/kWh of CO₂ for the medium case (fossil, nuclear and renewable based power generation) and 700 g/kWh of CO₂ (totally fossil fuels based power generation, including a mix of gas, oil and coal) for the worst case. Combining these values with the electricity consumption values, we obtained that the best evaluation corresponds to 3 kg/m² y or less. The medium evaluation corresponds to 15 kg/m² y. The worst evaluation corresponds to 35 kg/m² y or more. Level of importance in the final index: 4.

CO₂ produced for waste:

These elaborations are based on [Kennedy et al. \(2009\)](#). Despite of the very large differences that can be founded in relation with the age and the performance of the incineration plant and with the characteristics of waste treated in these plants, an average emission factor of 468 g/kg of CO₂ treated waste has been taken. This value includes the fact that part (about the half) of waste is organic and its incineration can be considered as carbon neutral. Since, best, medium and worst evaluations depend only on the per capita amount of waste produced. Assuming 1, 1.5 and 2 kg/d per capita as waste generation, respectively, and considering about 33 m² per capita (mean value taken as constant in each evaluation), we obtained that the best evaluation corresponds to 5.2 kg/m² y or less. The medium evaluation corresponds to 7.8 kg/m² y. The worst evaluation corresponds to 10.4 kg/m² y or more. Level of importance in the final index: 4.

4.4.3. Socioeconomic subsystem

GDP:

The GDP has been evaluated using the Siemens data set (see [Siemens AG, 2009](#)). The best evaluation corresponds to the 80th percentile or more, the medium evaluation to the median, and the worst corresponds to less than the 20th percentile. The best evaluation corresponds to 40,000 euro/inh y or more. Let us note explicitly that using this fuzzy evaluation system, outlier values like that of Lugano (due to financial activities) are evaluated “best=100” exactly as any other value greater than 40,000 euro/inh y. This saturation effect permits to avoid unrealistic overestimation of the GDP. The medium evaluation corresponds to 26,200 euro/inh y. The worst evaluation corresponds to 19,000 euro/inh y or less. Level of importance in the final index: 3.

Number of job positions

The number of job positions has been evaluated in comparison with the employment rate of the EU-27 plus Switzerland. More precisely, let n_{jp} be the number of job positions, p_{15-64} the city's population with an age between 15 and 64 and $E_{27+CH} = 66.11\%$ this employment rate (source Eurostat, see [Eurostat, 2013](#)). We will evaluate the quantity

$$\Delta = \frac{n_{jp}}{p_{15-64}} - E_{27+CH}$$

using the following criteria: The best evaluation corresponds to $\Delta \geq +30\%$. The medium evaluation corresponds to $\Delta = 0$. The worst evaluation corresponds to $\Delta \leq -30\%$. This can be interpreted as how much a random citizen of our city feels better/the same/worse, from

Table 3Values of importance i_k for internal nodes.

Evaluation of the input of the transportation system	3
Evaluation of the products of the transportation system	4
Evaluation of the output of the transportation system	4
Evaluation of the input of the built environment system	5
Evaluation of the products of the built environment system	4
Evaluation of the output of the built environment system	5
Evaluation of the products of the socio-economic system	4
Efficiency of the transportation system	5
Efficiency of the built environment system	5
Efficiency of the socio-economic system	5

the employment point of view, than a random European person. Level of importance in the final index: 3.

Quality of life:

In the present work, the quality of life has been estimated using the Health-adjusted life expectancy at birth (HALE) in EU-27 plus Iceland, Norway and Switzerland in 2002. HALE is the average number of years that a person can expect to live in full health, and is calculated by subtracting from the life expectancy the average number of years in ill-health weighted for severity of the health problem (see [European Union Public Health Information System, 2013](#)). Using this index as an estimation of the quality of life, we are evaluating both the sanitary system of the city and its living conditions as a contribution to illness. The best evaluation corresponds to 80th percentile in HALE in the previously cited data set (total population): 72 years. The medium evaluation corresponds to the median value of HALE in the previously cited data set (total population): 71 years. The worst evaluation corresponds to 20th percentile in HALE in the previously cited data set (total population): 64.88 years. Level of importance in the final index: 4.

In the present work, the index of urban efficiency has been computed using data of two cities: Lugano (Switzerland) and Barcelona (Spain). The corresponding data we have collected and calculated are presented in [Table 4](#).

Importance of internal nodes:

Each one of the previous 22 variables is an input of a Takagi–Sugeno fuzzy model, but the outputs of these models are inputs of subsequent models in the hierarchy. In [Section 3.3](#) we already fixed $w_k = 10$, $m_k = 50$ and $b_k = 90$ as worst, medium and best values for these internal nodes. It remains hence to list the

Table 4

The data collected for Lugano and Barcelona and used for the evaluation of the urban efficiency index.

	Lugano	Barcelona	Units
Energy for private transportation	2.26	3.39	MWh/(y veh)
Energy for public transportation	1.21	1.63	MWh/(y inh)
CO ₂ produced by private transportation	0.50	0.83	t/(y veh)
NO _x produced by private transportation	2.61	5.08	kg/y veh
CO ₂ produced by public transportation	0.31	0.37	t/(y inh)
Green movement share	33.63	67.00	%
km covered in a year	2465.58	3476.16	km/(y veh)
Transportation intensity	55.76	30.00	%
Urban density	3467.32	18278.88	inh/km ²
Energy consumption for heating	39.44	34.16	kWh/(m ² K d y)
Water consumption for buildings	319.08	169.89	l/(inh d)
Total electricity consumption	45.10	60.55	kWh/(m ² y)
Renewable energy share	13.80	9.10	%
Waste recycling share	39.45	33.60	%
Municipal waste	1.47	1.46	kg/(inh d)
CO ₂ produced for heating	9.67	6.83	g/(m ² K d y)
NO _x produced for heating	0.43	5.80	mg/(m ² K d y)
CO ₂ produced for electricity	16.96	8.66	kg/(m ² y)
CO ₂ produced for waste	0.97	3.26	kg/(m ² y)
GDP	65260.62	47775.00	euro/inh
Number of job positions	49.78	58.75	%
Quality of life	73.02	72.06	y

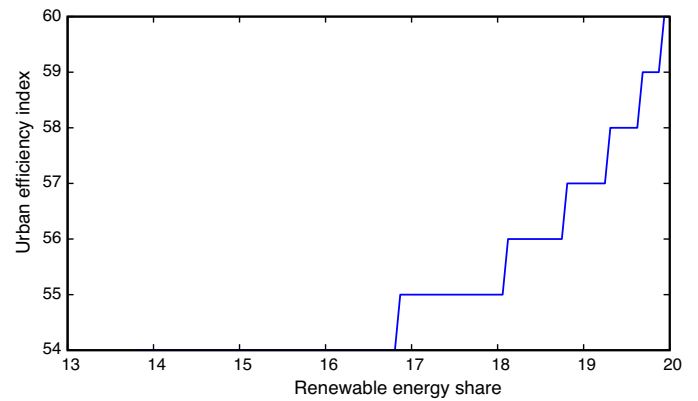


Fig. 11. Toy model representing the increasing of urban efficiency index for an increasing of renewable energy share from 13.8% to 20% (Lugano).

values of importance we chose for the definition of our index, and they are listed in [Table 3](#).

5. Results and sensitivity analysis

5.1. Results

The results are reported in [Table 5](#). The corresponding Matlab software is available for download at [Giordano et al. \(2014\)](#). We are perfectly aware of the fact that the benchmarks and the levels of importance assigned may appear as partly subjective. In the same way, the input data entered into the model does not have a uniform level of reliability and should be further verified. For these reasons, in the next section we present the results of a detailed sensitivity analysis. The index of urban efficiency defined above is based on 22 variables describing the urban structure of a city. It is not a model for estimating a quantity which is measurable, at least in principle, or a simple function of some available input data. As a consequence, the validity of the whole construction cannot be assessed easily, and it is especially important to find applications that shows the capability of the index to highlight interesting aspects and phenomena related to urban metabolism. The final aim is to obtain an index whose level of validation is strongly enough to enable its use for decision supporting purposes. This task can be accomplished

Table 5

Urban efficiency index for Lugano and Barcelona. The index is measured in a scale between 0 and 100. The table shows, beside the overall evaluation of urban efficiency, also the values of the eight intermediate indices that have been used for producing the final result in accordance with the hierarchical structure discussed in Section 4. Let us note that products and efficiency indices for the socio-economic systems are equal because of the particular structure represented in Fig. 9.

Index	Lugano	Barcelona
Transportation system (input)	100	71
Transportation system (output)	67	41
Transportation system (products)	38	69
Transportation system (efficiency)	53	42
Built environment (input)	24	52
Built environment (output)	67	58
Built environment (products)	48	43
Built environment (efficiency)	31	43
Socio-economic system (products)	100	100
Socio-economic system (efficiency)	100	100
Urban efficiency index	54	55

with a semi-quantitative study of the phenomena related to urban metabolism, where the index prove to be able to deepen our comprehension of the phenomena. A first example of how such type of semi-quantitative study could look like is given in Fig. 11.

In this figure, we can see the result of a toy model obtained by increasing the renewable energy share of Lugano from the present value of 13.8% to the best value of 20%. As a consequence, we linearly decrease the amount of CO₂ produced for electricity by the present value of 16.96 kg/m² y to the best value of 3 kg/m² y. Therefore, two correlated input variables are changed and the resulting values of efficiency, approximated to its nearest integer value, are represented in Fig. 11, where we can see that we have a gain of six points in efficiency, even if the increasing phase starts only around a renewable energy share of 17%.

Finally, the values of the intermediate efficiency indices in Table 5 are useful for three main related purposes. First of all, they enable us to understand how the overall evaluation has been obtained. This information would be further enriched if also the firing strengths of all the intermediate fuzzy systems in the hierarchy would be taken into account. Second, the values of the intermediate indices could be used as data for clustering and classifying cities based on their metabolic efficiency. Further, the modular structure of the model, permits also to analyze the singular performances of the different subsystems; this can be very useful for the individuation of the strengths and weaknesses of a city and also to redirect economic-financial efforts in the most effective way in order to improve quality of life, saving natural resources and limiting environmental impacts.

5.2. Sensitivity analysis

It is clear that in the definition of our urban efficiency index there are several parameters. For each one of the 22 input leaf variable x_k we have w_k, m_k, b_k , the worst, medium and best benchmarks related to x_k , a value of importance i_k and, of course, the concrete value of x_k for the urban system we are evaluating. Moreover, we have further 10 internal nodes with $w_k = 10, m_k = 50$ and $b_k = 90$, and whose values of importance are listed in Table 3. We recall that, in our choice for both the membership functions μ_{kj} and the output functions f_{kj} , we have only w_k, m_k, b_k as parameters. We therefore have a total of $22 \times 5 + 10 \times 4 = 150$ parameters. Therefore, it is natural to investigate the dependence of the index by a suitable variation of these parameters. On the one hand, this is important because some of the values of the 22 input variables may undergo a non-negligible estimate or measurement error. On the other hand, the whole method is subject to a considerable amount of choices in the benchmarks w_k, m_k, b_k and in the importance i_k . It is hence

Table 6

Variations considered for the parameters w_k, m_k, b_k (only if they are greater than 20%).

	Δw_k	Δm_k	Δb_k
Green movement share	20%	30%	40%
Urban density	20%	20%	20%
Renewable energy share	20%	26.7%	20%
Waste recycling share	20%	20%	31.1%
Quality of life (HALE)	20%	36.6%	55.8%
	Δb_k	Δm_k	Δw_k
NO _x produced by private transportation	20%	20%	30.1%
Water consumption for buildings	20%	30.9%	40%
Municipal waste	20%	26.7%	35%
CO ₂ produced for waste	20%	26.7%	35%

natural to ask whether “playing” with these parameters we can have a strong change of the urban efficiency index, and what changes can be considered non meaningful.

In the present analysis, we consider a variation of *at least* $\pm 20\%$ for w_k, m_k, b_k , of 20% for x_k , and $\pm 40\%$ for i_k . The latter is greater because it guarantees an integer change of class in the importance of the variable, but, at the same time, i_k is sampled with a Sobol quasi-random sequence so that it can assume also non-integer values. We also have to consider that this analysis must always respect the constraints $w_k < m_k < b_k$ (or $b_k < m_k < w_k$, depending on the type of variable x_k). It is for this constraints that we need to consider a variation of at least $\pm 20\%$ for w_k, m_k, b_k . Indeed, set $p=0.2$ and assume that $0 < w_k < m_k < b_k$ (analogously, we can deal with the case $w_k \leq 0$; see the variable “number of job positions” in Section 4.4.3); we want to sample w from the interval $[w_k - pw_k, w_k + pw_k]$, m from $[m_k - pm_k, m_k + pm_k]$ and b from $[b_k - pb_k, b_k + pb_k]$. But if these intervals are not pairwise disjoint, it can happen e.g. that $m < w$ or $b < m$ and we would have a violation of the constraints $w < m < b$. Assume, e.g., that

$$\begin{aligned} [w_k - pw_k, w_k + pw_k] \cap [m_k - pm_k, m_k + pm_k] &\neq \emptyset \\ [m_k - pm_k, m_k + pm_k] \cap [b_k - pb_k, b_k + pb_k] &\neq \emptyset. \end{aligned} \quad (12)$$

In this case, we sample the variations w of w_k with a Sobol sequence with values in the interval $[w_k - pw_k, w_k + pw_k]$, and we set the samples of m_k, b_k respectively as $m = w + d_1, b = m + d_2$; here d_1 is sampled, with the same type of sequence, in $[\max(0, m_k - w_k - pw_k), m_k - w_k + pw_k]$ and d_2 in $[\max(0, b_k - m_k - pm_k), b_k - m_k + pm_k]$. In this way, we have $d_1, d_2 > 0$, and hence $w < m < b$. However, the samples m, b verify $m_k + 2pw_k < m < m_k + 2pw_k$ and $b_k - 2pw_k - pm_k < b < b_k + 2pw_k + pm_k$. In our cases, if (12) holds, then $2pw_k > pm_k$ and $2pw_k + pm_k > pb_k$, so that the variations w of w_k are at most of $\pm p = \pm 20\%$, but those of m_k and b_k are greater. If only one of the intersections of (12) is not empty, we apply this method using only one among d_1 or d_2 . Table 6 shows the final variations for the parameters w_k, m_k, b_k ; all the non indicated parameters have a variation of $\pm 20\%$.

The hierarchical structure has been reflected also in the sensitivity analysis: we do not study the changes of the final index in a space of 150 variables (which is computationally unfeasible), but we consider changes in the parameters of each subsystems, keeping fixed the parameters of all the other subsystems. This assumption reflects that we are considering urban systems whose efficiency can be evaluated focusing on each single subsystem, one at a time. We recall that we have 11 cascaded Takagi–Sugeno subsystems, each one with a number of parameters between 5 and 25; therefore, like in the construction of the model (see Section 3.3), also in the sensitivity analysis, the hierarchical structure permits to deal with problems which, otherwise, would be unfeasible.

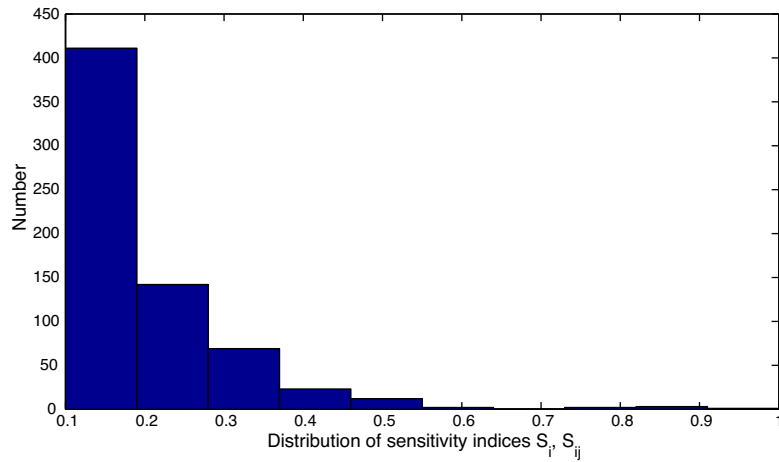


Fig. 12. Distribution of sensitivity indices S_i, S_{ij} .

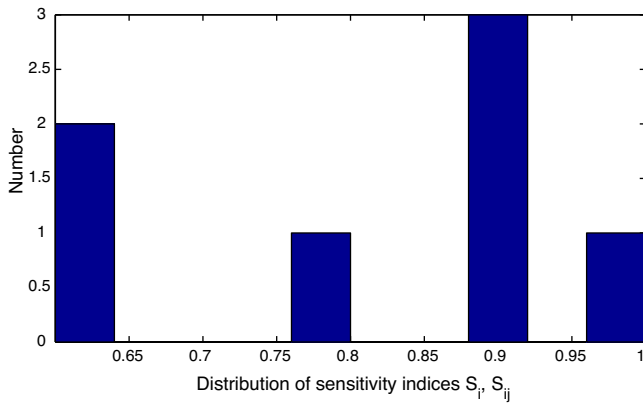


Fig. 13. Distribution of sensitivity indices $S_i, S_{ij} > 0.5$.

To perform this sensitivity analysis, we used the GSAT software (see Cannavò, 2012) to estimate Sobol' first S_i and second order S_{ij} global sensitivity indices (see Sobol', 1993) for all the parameters i, j . This permits to decompose the output variance into parts attributable to single parameters or interactions between two of them. Since for many models, a second order approximation already provides satisfactory results in the sensitivity analysis (Li et al., 2001), we did not consider interactions of more than two parameters. A verification of this assumption can be performed e.g. using the GUI-HDMR software (Ziehn and Tomlin, 2009) to estimate the error in approximating the model of our efficiency index with a second order polynomial HDMR (high dimensional model representation) expansion. In Fig. 12, the distribution of these sensitivities indices is shown. If we use the common distinction shown in Table 7 (see e.g. Cannavò, 2012), we can select only important and very important parameters and pairs of parameters. Their distribution is shown in Fig. 13, and they are listed by name in Table 8. All these results refer to Lugano, but very similar results can be obtained for Barcelona.

It seems reasonable to think that all the parameters of the internal node "Evaluation of the products of the socio-economic System"

Table 7
Relevance of parameters from their global sensitivity index.

Very important	$0.8 < S_i, S_{ij} \leq 1$
Important	$0.5 < S_i, S_{ij} \leq 0.8$
Unimportant	$0.3 < S_i, S_{ij} \leq 0.5$
Irrelevant	$0 < S_i, S_{ij} \leq 0.3$

appear as very important in Table 8 only because this node is the unique input for the second level sub-system of the socio-economic system (see Fig. 9). Since our metabolic schema for this system is only idealized and preliminary (see Section 4 and Fig. 10), we do not consider this result of sensitivity analysis as meaningful.

In order to evaluate the variation of the urban efficiency index when we vary the important parameters of Table 8 (excluding those concerning "prod. soc. econ. sys."), we can now realize an uncertainty analysis. We therefore consider a variation of $\pm 20\%$ (see Table 6), with respect to the value used in the model, of each pair of parameters in the first, second and last row of Table 8. Figs. 14–16 show the results, where we can see that:

- Only to have a better graphical result, and on the contrary with respect to what we did above, in these graphs the urban efficiency index is not approximated to the nearest integer.
- The variation of the urban efficiency index shown in these graphs is, in general, lower than the real total variation. Indeed, in these graphs only two parameters of a sub-system are changed whereas the others are kept constant, and these varying parameters explain only a percentage S_{ij} (see Table 8) of the global variation of the final index.
- Only a very small amount of parameters, among a total number of 150, result to have an important sensitivity. This reveals a surprising stability of the index.
- A variation of $\pm 20\%$ of these important parameters causes a variation of the final index between -9.2% and 16.7% in Fig. 14, between -40.7% and 0% in Fig. 15 and between -22.2% and 20.4% in Fig. 16.
- For this reason, the small variations we can see on these graphs are non meaningful. Even if one would naturally expect an increasing of the efficiency with respect to an increasing of the value of "waste recycling share" or of "quality of life", these small changes are probably due to situation which are evaluated, at a certain fuzzy degree, e.g. both as medium and good.

Table 8
Parameters with more than important sensitivities S_i, S_{ij} .

i	j	S_{ij}
m_k of waste rec. share	x_k of waste rec. share	0.6
w_k of qual. life	x_k of qual. life	0.6
w_k of prod. soc. econ. sys.	b_k of prod. soc. econ. sys.	0.9
m_k of prod. soc. econ. sys.	b_k of prod. soc. econ. sys.	1
b_k of prod. soc. econ. sys.	b_k of prod. soc. econ. sys.	0.9
b_k of prod. soc. econ. sys.	i_k of prod. soc. econ. sys.	0.9
i_k of eff. built env. sys.	i_k of eff. soc. econ. sys.	0.8

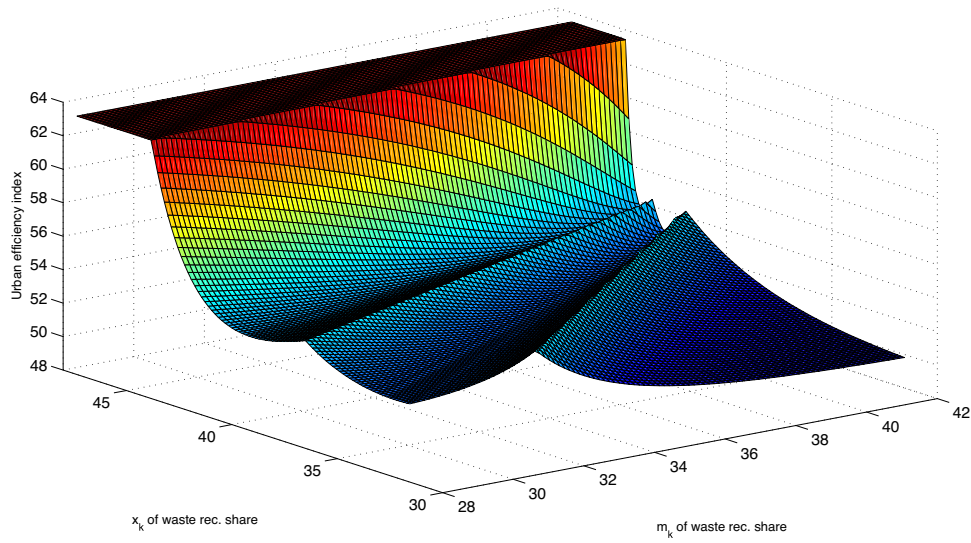


Fig. 14. Uncertainty analysis of the pair of parameters m_k and x_k of “waste of recycling share”.

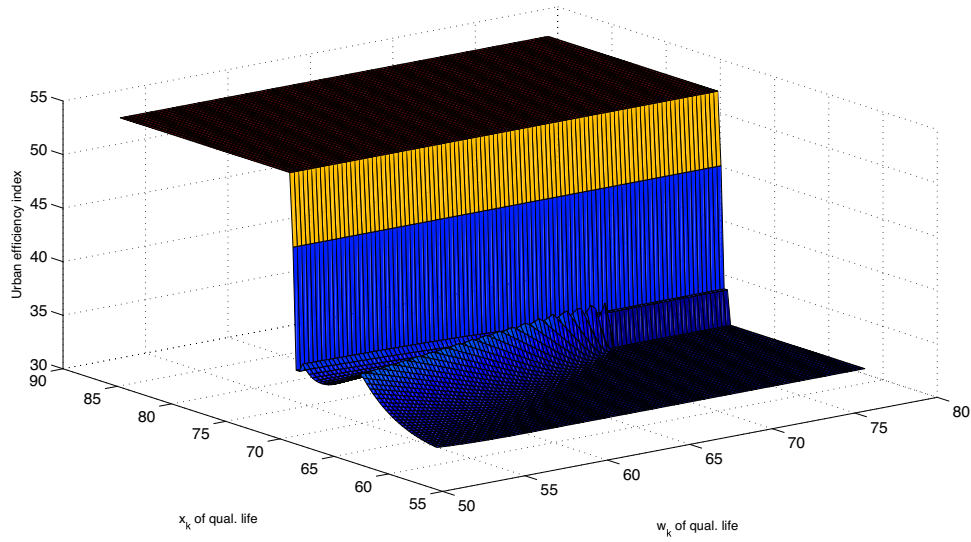


Fig. 15. Uncertainty analysis of the pair of parameters w_k and x_k of “quality of life”.

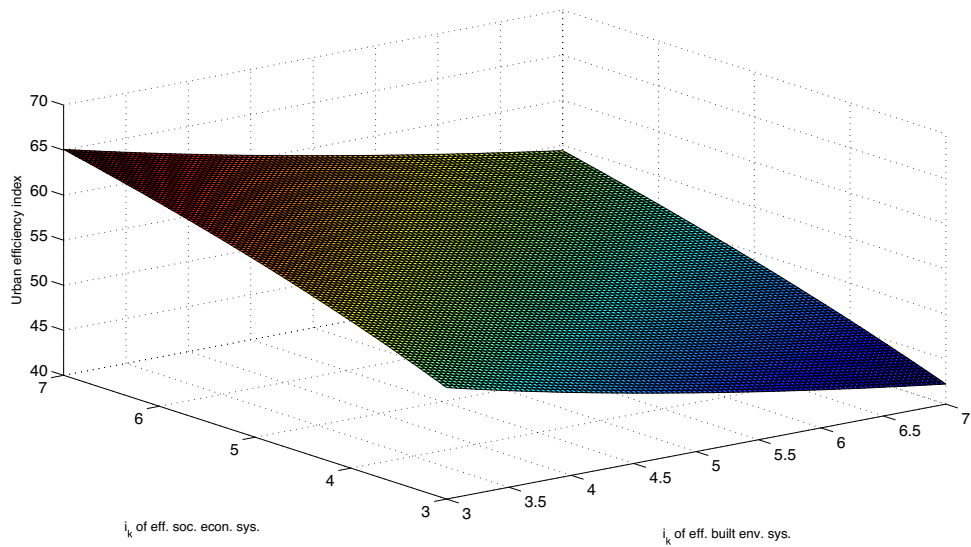


Fig. 16. Uncertainty analysis of the pair of parameters i_k of “efficiency of built environment system” and i_k of “efficiency of socio-economic system”.

- This gives us strong motivations to consider as more meaningful a suitable approximation of our urban efficiency index, e.g. by 21 or 11 integer scores in the range [0, 100] obtained by approximating the urban efficiency index with multiples of 5 or 10. In this case, we would have an index with a similar meaning, but with the previously mentioned increasing or decreasing properties, since small oscillations would be negligible.
- Rapid changes in the final index, for “quality of life” case, seem related to the short range between w_k and b_k for that variable.

6. Conclusions and further developments

We hope that our efforts could represent a contribution in developing new approaches and models related to the application of metabolism theories to urban areas. To that end, the overall structure of the model, the definition of the subsystems, the set of the 22 indicators, the adopted benchmarks and the levels of importance should be better verified and calibrated. In particular, in the present article we tried to highlight the choices, which can be considered as partly subjective, made during the construction of this index. We recall that a similar approach, based on the partial subjectivity of analytic hierarchy process, has been already used in Zhang et al. (2006) to implement interesting environmental indices related to complex urban ecosystems.

From the methodological point of view, we think that after managing a significant sample of cases of study, the capabilities of our models could help in representing results; for example, clustering cities on the basis of selected parameters could help in defining groups of cities with analogous characteristics or performances and in comparing the different groups (i.e. energy consumption in relation to urban density, or renewable energies percentage in relation to GDP, or services accessibility in relation to urban population, etc.). From the comparison and taking into account actual policies, economic and technological conditions, it is possible also to define different scenarios for improving the metabolism of a city in the future or, on the other hand, for accounting combinations of actions whose effects permit to conserve the present performance (in other words, how do we have to reallocate our resources in order to perform defined changes, but without further economic or environmental or social costs?). As conclusion, we know very well that it is very difficult to develop a model able to take into account urban complexity and urban people behavior, but we hope that the simplified, transparent and human-friendly model presented here could be useful to that end. Future developments will be aimed to test the model with other cases of study and to compare results obtained by other kind of tools and methodologies.

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