Development of a framework for the design of Autonomous Vehicle Storage and Retrieval Systems

Abstract: In today's competitive environment with increasingly faster deliveries and smaller order sizes, material handling providers are progressively developing new solutions. A more recent development in automated material-handling technology for unit load storage and retrieval is the autonomous vehicle storage and retrieval system (AVS/RS). The paper investigates the main design trade-offs for this new solution using simulation, and proposes a comprehensive design framework. Using data from a recently implemented AVS/RS, the application of the proposed framework is presented and the key design differences between the two types of AVS/RS configuration (i.e. "tier captive" versus "tier to tier") are identified.

Keywords: warehousing; autonomous vehicle storage and retrieval systems; simulation; automation; design

1. Introduction

The autonomous vehicle storage and retrieval system (AVS/RS) is a new automated material-handling technology for unit load storage and retrieval. In a traditional automated storage and retrieval system (AS/RS), unit loads are handled using aisle-captive storage cranes that move simultaneously vertically and horizontally. In an AVS/RS, unit loads are handled by vehicles that move horizontally along rails within the storage racks, while vertical movement is provided by lifts mounted along the rack periphery (Ekren and Heragu 2012).

Based on the format of vehicle assignment to storage tiers, there are two main configurations (Heragu *et al.* 2011):

- AVS/RS with a "tier to tier" configuration;
- AVS/RS with a "tier captive" configuration.

In the "tier to tier" configuration, vehicles may move from one tier of the storage racks to another using lifts. In the "tier captive" configuration, each vehicle is dedicated to a single tier and therefore cannot move to another one. Lifts are used only to move the unit loads to the destination tiers.

Evaluating AVS/RS performance is a complex process for warehouse designers due to the compound effect of the kinematic behaviour of vehicles and lift. This complexity may, in turn, be viewed as a function of the rack configuration (i.e. number and length of storage aisles and number of storage tiers), which makes it difficult to design these systems and evaluate their suitability. This complexity is important at the design conceptualisation stage where designers explore alternative configurations and material handling technologies, and estimate the performance of each, before settling on a final design (Heragu *et al.* 2011). In this initial warehouse design step it is very important to understand the impact of design variables on overall cost and system throughput. It is crucial that an AVS/RS be designed so that it can efficiently handle demand requirements, avoiding bottlenecks and overcapacity (Ekren 2011).

Based on the literature review, studies related to the design of the AVS/RS rack configuration are few and mainly focus on systems with a “tier to tier” configuration and a palletised unit load as the handling unit. This shortcoming is even more significant in light of the fact that several of the main solution providers have implemented numerous AVS/RSs with a “tier captive” configuration and the tote as the handling unit (Marchet *et al.* 2011b).

Therefore, the focus of this paper is on AVS/RS with a “tier captive” configuration and the tote as the handling unit. With reference to this type of system, the aim of this paper is twofold: first, to study the main design trade-offs; and second, to develop a comprehensive design framework to assist in the identification of the most suitable solution, i.e. that minimises costs, given the user requirements. In other words, given a warehouse problem, the framework should support the warehouse designer to identify the optimal design layout for an AVS/RS in terms of number of storage aisles, columns and tiers. According to other studies on AVS/RS, the benefit of a design framework is in the early technology selection, or "conceptualisation" phase of system development (Fukunari and Malmborg 2008, Dallari *et al.* 2009). Since the successful implementation of an AVS/RS system is highly dependent on an appropriate design, this is a key stage which needs to be better understood.

The remainder of the paper is organised as follows. The main studies of AVS/RS in the literature are presented in Section 2. The AVS/RS evaluated in this paper, the key performance indicators (KPIs) for AVS/RS and the cost modelling are described in Section 3. In Section 4 the simulation model used to evaluate AVS/RS performance is presented and the results of the analysis under different rack configurations and demand levels are discussed. A design framework is developed in Section 5 based on results presented in Sections 3 and 4. Finally, in Section 6 an application of the proposed framework and some design guidelines are presented. Conclusions and future developments are proposed in Section 7.

2. Literature review

Studies on AVS/RS may be classified into two main areas of research:

- AVS/RS performance analysis (i.e. throughput and cycle time);
- AVS/RS design criteria.

With regard to the first research area (i.e. AVS/RS performance analysis), the first study was conducted by Malmborg (2002). With reference to a "tier to tier" configuration, a model was proposed to estimate vehicle utilisation and cycle time as a function of the number of storage columns, tiers, vehicles and lifts.

Following this study, there were several papers on the analysis of AVS/RS performance. In most of these studies, the focus was on AVS/RS with a "tier to tier" configuration and a palletised unit load, and the performance evaluation was generally carried out using analytical models, which were then validated through simulation. For instance, Kuo *et al.* (2007) modelled the movement of autonomous devices as an M/G/V queue nested within an M/G/L queue to estimate the waiting times for vehicle and lift service, and Zhang *et al.* (2009) proposed a model that represents storage and retrieval transactions as customers and vehicle-lift pairs as parallel servers. In some other cases, simulation models were developed to evaluate the impact of rack configuration on performance (Ekren *et al.* 2010) or to identify which factors affect system cycle time by applying Design Of Experiment (Ekren and Heragu 2010).

The "tier captive" configuration has been studied in only two papers, by Heragu *et al.* (2011) and Marchet *et al.* (2011b), who presented analytical models based on an open queuing network approach.

The performance of an AVS/RS can be analysed by examining single command cycles only (Kuo *et al.* 2007, Roy *et al.* 2009), or both single and dual command cycles (Malmborg 2003, Fukunari and Malmborg 2009), where either a storage or retrieval transaction is completed in the same cycle. According to Zhang *et al.* (2009) the achievable benefit in terms of efficiency, which is dependent on an increase in the proportion of dual command cycles, is limited for several reasons: first, a high proportion of dual command cycles is difficult to attain and second, storage and retrieval transactions paired in the same cycle are usually associated with different storage tiers.

Papers that address the second research area (i.e. AVS/RS design criteria) can be divided in two categories: i) papers that examine the design issue by comparing the

AVS/RS with the AS/RS (Automated Storage and Retrieval System) and ii) papers that address the design issue by analysing the system performance according to different rack configurations. Table 1 shows that the majority of contributions in the second research area examined AVS/RS with a “tier to tier” configuration and a palletised unit load as the handling unit. These studies mainly involved simulations that were carried out using the Arena software. In terms of project parameters, the number of storage positions and demand rate are the two synthetic values that were most often used. The number of lifts and vehicles was also considered in some studies. Finally, the average number of experiments examined was 52.

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The first study to compare AVS/RS and AS/RS was performed by Malmborg (2002). The author compared the two technologies by varying the system configuration (i.e. storage rack shape, number of lifts and number of vehicles). Fukunari and Malmborg (2008) expanded on the work by Malmborg (2002) by performing an economic comparison between the AVS/RS and AS/RS solutions. Fifteen problem scenarios were considered, with storage capacities ranging from 10,000 to 30,000 storage positions, and transaction demand levels ranging from 100 requests/hour to 300 requests/hour (assuming the use of the Poisson distribution). For each scenario, a comparison was made between the lowest cost AVS/R and AS/R system configurations where vehicle or crane utilisation was below 90%. Equipment costs were assessed, namely for vehicles and lifts for AVS/RS and cranes and accumulation conveyors for AS/RS. Based on the optimal solutions identified, the authors provide some highlights on AVS/RS rack design with “tier to tier” configuration and a palletised unit load as the handling unit: while optimal AS/RS configurations tend to use few but long aisles to minimise the number of cranes, the authors found that optimal AVS/RS configurations tend to use more and shorter aisles to optimise the movement pattern of vehicles and shorter racks to avoid inefficiencies associated with vertical travel. Ekren and Heragu (2011) suggest that better operational performance is achieved with AVS/RS than with AS/RS, under a variety of conditions. Specifically, they found that the AVS/RS configuration that minimises cycle time has a large number of vehicles and aisles (i.e. use of short aisles).

With regard to papers that address the design issue by analysing system performance, Ekren and Heragu (2010) studied the effect of rack configuration (i.e. number of tiers, aisles and columns) on AVS/RS performance in six scenarios by means of simulation. Their analysis confirms that it is better to have many short aisles as opposed to fewer aisles with a larger number of storage columns. Ekren (2011) compared system performance (e.g. average cycle time, average utilisation of lifts and vehicles) and costs (vehicle, lifts and rack costs) for 55 different rack configurations. Depending on the performance required, the optimal configuration was found to vary. According to the authors, the choice of design profile should be based on company/designer priorities (e.g. configuration with average performance measures and low cost or configuration that maximises performance).

Therefore, although there are several studies that address the AVS/RS design issue, a general design framework is lacking. The availability of several tools (Heragu *et al.* 2011) that allow the designer to evaluate performance for configurations of interest only partially mitigates this need. The choice of which configurations to examine and in what order is, however, left to the warehouse designer. The lack of design frameworks is common both to warehouse design in general and design of specific automated solutions (Hassan 2002, Baker and Canessa 2009). In the preliminary phases of the design process for automated solutions such as AS/RS heuristic rules of thumb are often used, which are easy to apply and have an acceptable degree of accuracy (Malborg 2001). An example is Zollinger's rule, which is applicable to the process of identifying a cost-effective storage rack design for a given level of transaction demand (Zollinger 1996).

In summary, several studies have been performed on AVS/RS in the last decade. Those studies chiefly focussed on AVS/RS with a "tier to tier" configuration and a palletised unit load as the handling unit, and assumed that the vehicle moves vertically together with the lift and the number of lifts and vehicles is independent of the number of aisles and the number of tiers on the rack. As the literature review has shown, there is limited information and analysis related to the design criteria for AVS/RS with a "tier captive" configuration and the tote as the handling unit, notwithstanding its more frequent use in a number of industrial tote handling applications. Because of the different ways in which AVS/RS systems function, it is not possible to apply the results from the type of system studied most today (i.e. AVS/RS with a "tier to tier" configuration and the palletised unit load as the handling unit) to the type of system considered in the present study (i.e. AVS/RS with a "tier captive" configuration and the tote as the handling unit). For this reason, a specific study was required.

3. AVS/RS description

This section describes the AVS/RS examined, which has a "tier captive" configuration and the tote as the handling unit. The "tier captive" configuration was chosen as it is more frequently adopted in warehouses with small handling units that require a high throughput volume.

In the following sub-sections the layout that was assessed, the main notations used in the paper, AVS/RS performance, and the cost modelling process, which was developed based on interviews with material handling providers, are reported.

3.1 *Layout*

Figure 1 shows a representation of an AVS/RS for the handling of totes, while Figure 2 illustrates a single tier. As the figures show, the storage racks are single-deep and double-sided. Each storage position is the same size and can hold one tote. Lifts are mounted at fixed locations at one end of each storage aisle. The input/output (I/O) point is located at the first tier beside each lift. Vehicles move along one dimension only, each within a specific tier of a storage aisle. The number of lifts installed in the system is equal to the number of aisles (A), while the number of vehicles is equal to the product of the number of aisles times the number of tiers (T). The configuration is "tier captive", so the vehicles cannot move from one tier to another. To allow the mutual independence of lift and vehicle, the first position on either side of the storage aisle in all tiers serves as a buffer and is used to manage the transfer of totes between vehicles and lifts (Figure 2). One buffer (called *buffer out*) handles the totes which have been retrieved, the other one (called *buffer in*), located on the other side of the storage aisle, handles the totes to be stored. As such, totes (not vehicles) wait for the lift, so lift and vehicle can work independently of one another. It should be noted that the lift is needed in every cycle, even for totes located in storage positions located on the first tier, as the lift permits the transfer of the tote to/from the buffer.

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As the literature review has shown, performing dual command cycles in an AVS/RS is rather difficult. Therefore, only single command cycles have been considered and, more precisely, this study focuses on single retrieval cycles. This is due to the fact that the retrieval phase is the most critical activity from an organisational viewpoint, as it is directly related to service level and – in contrast to the storage phase – cannot be postponed to a period of low-workload. Furthermore, in some contexts (e.g. AVS/RS used as a dispatching buffer) storage takes place independently of retrieval.

3.2 *Main notations*

The notation used in the remainder of the paper is presented below:

λ_S = System retrieval demand rate [retrievals/hour]

λ_A = Storage aisle retrieval demand rate [retrievals/hour]

A = number of storage aisles

T = number of storage tiers

C = number of storage columns on each side of a storage aisle

n = total number of storage positions ($2*A*T*C$)

H_{max} = maximum height of the racks [m]

L_{max} = maximum length of the racks [m]

μ_w = unit width of clearance per storage position including allowances [m]

μ_h = unit height of clearance per storage position including allowances [m]

v_V = average horizontal velocity of vehicle [m/s]

v_L = average vertical velocity of lift [m/s]

γ_V = vehicle delay due to acceleration, deceleration and braking delay times [s/single command cycle]

γ_L = lift delay due to acceleration, deceleration and braking delay times [s/single command cycle]

ε_V= time allowance for charging and discharging load from vehicle [s/single command cycle]

ε_L = time allowance for charging and discharging load from lift [s/single command cycle]

3.3 *AVS/RS Key Performance Indicators*

As with other highly-automated systems, the main AVS/RS Key Performance Indicators (KPIs) are flow time and throughput.

Flow time is defined as the total time required by the system to retrieve a tote. It takes into account the time elapsed between the vehicle retrieval request and the moment at which the tote is released by the system, i.e. it reaches the I/O point. As Equation (1) shows, the flow time (FT) is the sum of two variables, namely the tote waiting time for the vehicle (W_v), i.e. the time elapsed between the vehicle retrieval request and the moment at which the vehicle starts to move to the retrieval address, and the cycle time (τ).

$$\mathrm{FT} = \mathrm{W_v} + \tau \tag{1}$$

In turn, τ is calculated by:

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \tau_7 + \tau_8 + \tau_9 \tag{2}$$

where:

τ_1 is the time required for the vehicle to travel from the *buffer out* to the retrieval address;

τ_2 is the time required for the vehicle to load the tote at the retrieval address;

τ_3 is the time required for the vehicle to move the tote from the retrieval address to the *buffer out*;

τ_4 is the time required for the vehicle to discharge the tote at the *buffer out*;

τ_5 is the tote waiting time for the lift, i.e. the time elapsed from the tote discharging at the *buffer out* to the moment in which the lift moves from its current position to the retrieval tier;

τ_6 is the time required for the lift to travel from the I/O point to the retrieval tier;

τ_7 is the time required for the lift to load the tote at the *buffer out*;

τ_8 is the time required for the lift to move the tote from the retrieval tier to the I/O point;

τ_9 is the time required for the lift to discharge the tote at the I/O point.

Throughput represents the number of totes that the system can retrieve per time unit. In conventional automated systems (i.e. AS/RS), throughput may be estimated as the inverse of the average cycle time (Tompkins *et al*. 2010). In "tier captive" AVS/RS this approach is not applicable due to the use of different resources (i.e. lifts and vehicles). Furthermore, throughput is a function of bottlenecks (i.e. of vehicles or lift). The creation of bottlenecks is a function of the rack configuration and kinematic features of vehicles and lifts.

3.4 *Modelling cost structure*

We propose a cost modelling procedure for an AVS/RS based on the sum of three cost items: equipment cost, rack cost and cost of the space. Uniquely, compared to previous studies, rack and space costs were included in this study, resulting in a more comprehensive economic evaluation. This may be useful both for the comparison with other types of automated tote S/R systems (e.g. mini-load) and in studying the most suitable rack configuration. Indeed, AVS/RS racks need rails to allow vehicle movement. As such, AVS/RS racks usually have a significant impact on the overall investment required (according to material handling providers, the rack cost is usually more than 25% of the overall investment). Moreover, the space required varies as a function of the rack configuration. The greater the vertical extent of the AVS/RS, the less space is required, and thus the cost is reduced.

Costs are expressed in terms of annualised costs. In particular, AC is calculated as follows:

$$AC = (AC_L + AC_V * T) * A + AC_R * (2 * T * A * C) + C_S * S \tag{3}$$

where:

AC_L = annualised cost of a lift [€/year]

AC_V = annualised cost of a vehicle [€/year]

AC_R = annualised cost of a storage rack position [€/storage position*year]

S = space required [m^2]

C_S = cost of the space [€/m^2*year]

The lift cost increases proportionally with the number of aisles, whereas the vehicle cost depends on the combination of the number of aisles (A) and tiers (T). According to Equation (3), if the storage capacity is constant, the number of storage aisles A is the most significant variable. To reduce AC, it is necessary to first minimise the number of aisles and then maximise the aisle length to reduce the number of vehicles.

Finally, the cost of energy consumption could also be included in Equation (3). The expected AVS/RS energy consumption is lower than in AS/RS, as horizontal movements are performed by vehicles, which are lighter than cranes. However, data provided by material handling providers yields only a rough estimate of the energy consumption per cycle, and it is therefore not possible to differentiate energy consumption costs as a function of rack configuration.

4. Evaluation of AVS/RS performances

The aim of this section is to study AVS/RS performance (e.g. throughput, flow time and cycle time) by varying the rack configuration.

4.1 *Simulation modelling*

To evaluate the AVS/RS performance, a simulation model was developed using Arena (version 13.0). Figure 3 illustrates the simulation flow chart for a retrieval cycle. A retrieval cycle requires the vehicle in the storage tier where the tote to be retrieved is located. The vehicle first retrieves the tote and then discharges it at *buffer out*. The lift moves the tote to the output point. During this cycle, it may happen that, for a pending retrieval, the vehicle is already busy or the vehicle has to wait for the *buffer out* to be empty before discharging the tote (i.e. limited buffer capacity), or the tote has to wait for the lift.

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The main assumptions of the simulation model are as follows:

- Distribution followed by the retrieval demand to the system: Poisson with parameter λ_S. Therefore the time between two subsequent arrivals follows an exponential distribution with parameter $1/\lambda_S$, in accordance with previous studies in the literature, i.e. Fukunari and Malmborg (2008) and Ekren *et al.* (2010);
- Storage policy: random. In most unit load SR systems, a space-conserving random storage policy is used because of capital cost considerations (Heragu 2008). According to this policy, the probability that a retrieval is required in a certain storage aisle is identical for each aisle and is equal to $1/A$; similarly, the probability that a retrieval is required in a certain tier is identical for each of them and is equal to $1/T$. Therefore, the system demand rate λ_S is evenly distributed among the storage aisles (A) and the storage tiers (T) and the demand rate at each storage aisle follows a Poisson distribution with parameter λ_A equal to λ_S/A;
- Modelling of lift and vehicle service time: the service times are composed of a variable part and a fixed part. In accordance with the previous literature (Malmborg 2003, Fukunary and Malmborg 2009), the first one (i.e. variable part) is calculated using an average velocity (v_V for the vehicle and v_L for the lift), and the second one (i.e. fixed part) is the time for charging and discharging the tote from lift and vehicle, assumed to be constant (ε_V for the vehicle and ε_L for the lift). In this study, the extra delay due to acceleration, deceleration and braking times is also included in the fixed component (γ_V for the vehicle and γ_L for the lift), in accordance with other simulation models used to study AVS/RS performance (Ekren *at al.* 2010);
- Number of totes handled per cycle by lifts and vehicles: 1;

- Vehicle dwell point policy: point-of-service-completion (POSC);
- Lift dwell point policy: point-of-service-completion (POSC);
- Lift and vehicle dispatching policy: first-come-first serve (FCFS);
- Maximum number of totes in queue at *buffer out*: 1.

The simulation model is assumed to be a non-terminating system, making it possible to conduct a steady state analysis (Ekren and Heragu 2010) and, similarly to Fukunary and Malmborg (2008), the specified length of each simulation is 48 hours. The model was run for 20 independent replications. The warm up period is calculated following the procedure proposed by Welch (Law and Kelton 2000), and observations belonging to the warm up period, (3 hours' length on average) have been omitted from the analysis.

The performance assessment was based on the KPIs highlighted in Section 3.3:

- Average throughput;
- Average flow time, considering both the overall performance and its two components (i.e. tote average waiting time for the vehicle at the retrieval address, and average cycle time), and identifying the contribution of the average waiting time for the lift at the *buffer out*.

4.2 *Main simulation data*

Similarly to previous studies (e.g. Kuo *et al.* 2007), the analysis was performed by considering different combinations of storage capacity and retrieval demand level. A storage capacity value *n* equal to 10,000 storage positions and five retrieval demand rates at each storage aisle λ_A (i.e. 100, 250, 500, 750, 1000 retrievals/hour) were considered, in order to evaluate performance based on different resource utilisation levels.

In the AVS/RS design there are usually some physical constraints (Ekren and Heragu 2010). Specifically, in AVS/RS for the handling of totes the maximum height corresponds to an industrial building height. Therefore, a maximum system height of 10 m was assumed. The maximum length assigned to the storage aisles was 80 m, which corresponds to the maximum length for AVS/RSs currently in place.

Nine rack configurations were analysed, as a combination of three values of *T* (i.e. 8, 10 and 12) and three values of *A* (i.e. 4, 8 and 16). Here, the level of the *C* variable varies according to *T* and *A* values, and λ_S is equal to λ_A*A. Table 2 reports the three rack dimensions, the expected throughput of the vehicles in one aisle (ETH_V) and the expected throughput of the lift (ETH_L). ETH_V and ETH_L may be computed as follows:

$$ETH_V = \frac{3600}{\tau_V} * T \tag{4}$$

$$ETH_L = \frac{3600}{\tau_L} \tag{5}$$

According to the random storage policy hypothesis, τ_V is equal to the length of a retrieval cycle performed by a vehicle at the midpoint of the aisle:

$$\tau_V = \frac{\mu_W * C}{2 * v_V} * 2 + \gamma_V + \varepsilon_V \tag{6}$$

Similarly, τ_L is equal to the length of a retrieval cycle performed by a lift at the midpoint of its maximum distance travelled:

$$\tau_L = \frac{\mu_h * (T-1)}{2 * v_L} * 2 + \gamma_L + \varepsilon_L \tag{7}$$

Table 2 also reports the type of resource (i.e. lift or vehicles) that is expected to be the bottleneck for each rack configuration, based on the ETH_V and ETH_L results. The expected throughput of each storage aisle (ETH_A) and the expected throughput of the system (ETH_S) are defined as:

$$ETH_A = \min\ (ETH_V, ETH_L) \quad (8)$$

$$ETH_S = A * \ ETH_A \quad (9)$$

The combination of the five values of retrieval demand rate at each storage aisle (λ_A) and the nine rack configurations lead to the analysis of 45 types of experiments. Additional data used in all simulation runs are reported in Table 3.

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4.3 *Simulation results*

Simulation results are reported in Table 4. To enhance readability, they refer to a single aisle. Under the assumption of random storage policy, it should be noted that the behaviour of each aisle is identical.

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The following observations on the functioning of the AVS/RS system can be made based on these results:

- The throughput of one aisle may be estimated analytically by identifying the system bottleneck (i.e. lift or vehicles), according to Equation (8). In fact, looking at Table 2 and Table 4, the simulated throughput performance corresponds to the expected throughput of the vehicle in one aisle (ETH_V) where ETH_V is lower than ETH_L (e.g. experiments 28, 38 and 40). Conversely, the simulated throughput performance corresponds to the expected throughput of the lift ETH_L where ETH_L is lower than ETH_V (e.g. experiments 39, 41 and 42). The average cycle time and average flow time are hardly predictable *a priori*. These may be obtained by using simulation or analytical modelling (i.e. modelling the creation of queues in the system);
- The throughput performance corresponds to λ_A where the expected throughput ETH_A is higher than the demand rate λ_A, (e.g. experiments 10-18);
- For experiments 19, 28, 31 and 37-45, it may be observed that where the demand level λ_A is similar to the expected throughput ETH_A, the high utilisation of lifts and vehicles leads to the creation of queues and, therefore, to long average cycle times and average flow times. In general, the increase in the average cycle time is related to the waiting time for the lift at *buffer out*, whereas the increase in average flow time may be correlated to both a long waiting time for the lift at *buffer out*, and a long waiting time for the vehicle at the retrieval address. For instance, the average waiting time for the vehicle ranges from 1.28 s to 45.50 s where there is a low demand rate (e.g. experiments 10-18), and from 8.10 s to 17,534 s where there is a high demand rate (e.g. experiments 37-45). For those cases with vehicle or lift utilisation level below 0.9 the related waiting times are reasonably low. For instance, in the experiment 22 the average flow time is 133.27 s, which is the sum of the

average waiting time for the vehicle (i.e. 74.61 s) and the average cycle time (i.e. 58.66 s, including the average waiting time for the lift of 2.26 s). These results confirm previous studies that use a resource utilisation level of 0.9 during the design phase of AVS/RS systems (Fukunary and Malmborg 2008, Kuo *et al.* 2007);

- The rack configuration, and specifically the relationship between the number of tiers (*T*) and number of storage columns (*C*), impacts on the creation of the bottleneck. In the presence of high racks the bottleneck is usually caused by the lift, which has to travel longer vertical distances and serves a greater number of tiers (e.g. please refer to the throughput performance in experiments 43, 44 and 45). When the rack height decreases, the storage aisle tends to be longer, and the bottleneck may be caused by the vehicles, as they have to travel longer distances (e.g. please refer to the throughput performance in experiments 37, 38 and 40). However, as shown in Table 2, this does not occur when A = 16. In this case, even with the number of tiers equal to 8, the bottleneck is still caused by the lift: indeed, in this case, the aisle length (C = 40) allows vehicles to quickly move the totes to the *buffer out*;
- The maximum throughput of a single aisle does not necessarily correspond to the configuration with the maximum possible rack height. Such a result is shown in Figure 4, which illustrates the expected throughput per aisle (ETH_A), varying T and A. It can be noted that, when the number of aisles is 4, the throughput performance of a single aisle tends to increase as T increases. In fact, as shown in Table 2, for a number of tiers ranging from T = 8 (i.e. rack configuration 1) to T = 10 (i.e. rack configuration 4), ETH_V increases as a result of shorter travel distances, although the bottleneck is still caused by the

vehicle. Moving from T = 10 to T = 12 (i.e. rack configuration 7), a further reduction in C leads to a change in the creation of the bottleneck, which is now caused by the lift. A different result is obtained when A = 16: the throughput performance of the single aisle tends to decrease as the number of levels increases. This is due to the fact that the lift causes the bottleneck (i.e. rack configuration 3, 6 and 9), and therefore the increase in rack height worsens lift performance.

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5. A framework for the AVS/RS design

As previously illustrated, the rack configuration affects both cost structure and system performance. In Figure 5 an AVS/RS design framework is presented, the purpose of which is to facilitate the identification of the suitable rack configuration, i.e. the rack configuration that meets user requirements (i.e. storage capacity and throughput capacity) at a minimum cost, given the required service level (i.e. average flow time) and the physical constraints. The framework aims to facilitate the task of designers, by outlining important design guidelines and helping designers to make informed decisions. The framework is derived from a literature review on warehouse design conceptualisation, the analysis of system performance (Section 4) and the modelling of the cost structure (Section 3.4).

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In step 1 the size of the storage position is defined as a function of the product features (e.g. in the pharmaceutical industry product totes are smaller in size than in the food industry), and the technology used by the vehicle to charge and discharge the tote.

Once the size of the storage position has been defined, an initial solution (i.e. system configuration) is identified (step 2). As the purpose of this design framework is cost minimisation, this solution corresponds to the rack configuration which presents the minimum cost among those meeting the storage capacity requirements. Beginning with the maximum number of levels T_{max} and storage columns C_{max}, the minimum number of aisles A_{min} that satisfy the physical constraints and the required storage capacity is calculated in step 2:

$$T_{max} = \text{lower integer part } [H_{max} / \mu_h] \qquad (10)$$

$$C_{max} = \text{lower integer part } [L_{max} / \mu_w] \qquad (11)$$

$$A_{min} = \text{upper integer part } [n / (2 * T_{max} * C_{max})] \qquad (12)$$

The initial rack configuration determined by T_{max}, C_{max} and A_{min} might give rise to an overcapacity in terms of the number of storage positions and vehicle fleet size. Therefore, the rack configuration is determined in step 2 as follows:

$$A_0 = A_{min} \qquad (13)$$

$$T_0 = \text{upper integer part } [n / (2 * A_{min} * C_{max})] \qquad (14)$$

$$C_0 = \text{upper integer part } [n / (2 * A_{min} * T_0)] \qquad (15)$$

In step 3, the expected system throughput ETH_S is calculated in order to verify whether the current solution satisfies the constraint on the throughput target. Based on the results of Section 4.3, the expected system throughput may be estimated according to the Equation (9). In general, during the system design phase there is usually the need to set a constraint on the resource utilisation level (i.e. lower than 1). Therefore, the throughput requirement should be appropriately increased, to obtain a resource utilisation level that is reasonable. Based on the results of Section 4.3, a throughput target that is the ratio between the throughput requirement and a resource utilisation level of 0.9 is proposed. This will limit the creation of queues and, as a consequence, both cycle time and flow time will be relatively low, thus satisfying the service level requirement.

If the solution meets the throughput target, then the procedure ends with the calculation of the annualised cost of the proposed configuration, and the average flow time by means of simulation (step 4). If the throughput of the proposed rack configuration is lower than the target value, the design procedure is not yet complete. The increase of ETH_S is obtained as a function of the bottleneck position. According to Section 4.3, if the bottleneck is caused by the lift, then the number of aisles should be increased (i.e. $A = A + 1$) and T is calculated, given C_{max} and the current value of A (step 5) by:

$$T = \text{upper integer part } [n / (2 * A * C_{max})] \qquad (16)$$

Subsequently, the value of C is calculated, using T calculated above and the current value of A:

$$C = \text{upper integer part } [n / (2 * A * T)] \qquad (17)$$

Following step 5, the procedure continues with a return to step 3.

Conversely, if the bottleneck is due to the vehicles, then the procedure moves to step 6. In this latter case, the suggested procedure is to increase T, thus increasing ETH_V. This reduces the aisle length and, at the same time, increases the number of vehicles per aisle. Before performing step 6, T is checked to see if it is equal to T_{max}. If T is equal to T_{max} the procedure is to return to step 5 and then to step 3, without completing step 6. If not, the number of levels is increased (i.e. $T = T + 1$) and C is then calculated, using the current values of A and T according to Equation (17). After step 6, the procedure returns to step 3.

As suggested in literature, once the most appropriate rack configuration has been identified, fine tuning of the solution through simulation can take place, in order to improve the system performance, for example by considering alternative operating policies.

6. Framework application

Using the data from an implementation of an AVS/RS with a “tier captive” configuration in the United Kingdom by Knapp, a material handling provider, we show an application of the proposed design framework. The aim of presenting this application is twofold: to provide an experimental validation of the framework and to derive some guidelines for AVS/RS design.

The warehouse used in this analysis is a distribution centre that serves all the stores in the UK for a retailer in the apparel sector. The warehouse is made up of various storage areas. Operationally, once a customer order has been received, item picking takes place in the storage areas. Picked items are packaged in customer boxes. When they are ready, the customer boxes are stored in the AVS/RS to await shipment. More detailed data about the company have been withheld for confidentiality reasons.

The framework was first applied to the specific case. Then, several constraints related to the building housing the AVS/RS were removed in order to apply the model to evaluate 9 scenarios. Sixty four rack configurations were assessed for each scenario.

6.1 *Case study*

The user requirements for this case study were 9500 storage positions (i.e. customer boxes) and a system throughput of 4000 retrievals/hour (based on the demand peak, an average system throughput of 3000 retrievals/hour is sufficient). In order to respect shipping windows, the average flow time for a box must not be high (i.e. less than 5 minutes). Based on the product features, the width and the height of each storage position are 0.6 and 0.7 metres, respectively. Due to the storage area's height limitation, the maximum rack height is 11 m (i.e. T_{max} is equal to 15). Similarly, the maximum rack length is 40 m (i.e. C_{max} equal to 66).

As shown in Table 5, sixty-four rack configurations were assessed, as a result of eight different aisle numbers (*A*) and eight different tier numbers (*T*). For each configuration, the expected system throughput, the average flow time and the annualised cost were calculated. ETH_S values were estimated using Equation (9). The average flow time was obtained through simulation. The cost of each solution was calculated as shown in Section 3.4, assuming 10 years of service and a 10% interest rate, In accordance with Marchet *et al.* (2011a). Table 6 reports the unit costs obtained from interviews with material handling providers.

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Take in Tables 5 and 6

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The minimum cost configuration that satisfies all the requirements has 6 storage aisles and 12 tiers, with a calculated *AC* of 237,595 €. The results of the simulation confirm that the existing system meets the throughput requirement, without adversely affecting the service level. In fact, the average cycle time is 0.81 min, and the average flow time is 1.28 min. As expected, resource utilisation was found to be less than 0.9, i.e. 0.57 for the vehicles and 0.82 for the lifts. The minimum cost configuration can be obtained by following the proposed design framework. In fact, application of the framework involves an initial configuration (step 2) with A_{min} = 5, T_0 = 15 and C_0 = 64. This configuration, in which the average cycle time and average flow time are both 1.95 hours, is not acceptable as it does not satisfy the throughput target (i.e. 3678 retrievals/hour compared to a throughput target of 4000/0.9 which is equal to 4445 retrievals/hour). Because the bottleneck in this configuration is caused by the lift, according to the framework, the number of aisles should be increased and a configuration with A = 6, T = 12 e C = 66 should be evaluated, which results in the minimum cost solution. It should be noted that the minimum cost rack configuration identified by applying the framework corresponds with that implemented in the studied case.

6.2 *Scenario analysis*

Additional analyses were performed by specifying different combinations of maximum rack lengths (i.e. maximum number of storage columns) and throughput requirements. More specifically, three values for the maximum number of storage columns (i.e. 55, 75 and 95) and throughput requirements (i.e. 3000, 4000 and 5000 retrievals/hour) were considered. Therefore, 9 scenarios were evaluated (see Table 7). All the other values remained the same as those used in the case study (see Section

6.1). The storage capacity value *n* is equal to 9500 storage positions and maximum number of tiers is 15. The estimate of performance (i.e. throughput and flow time) was conducted in the same manner as in Section 6.1.

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Take in Table 7

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Table 8 shows the results of applying the framework to scenarios where C_{max} is 55 (i.e. scenarios 1, 2 and 3). The first solution identified by the framework always produces the minimum cost from among those solutions that satisfy the physical constraints (rack configuration 24 in Table 8). This rack configuration is already the final solution for scenario 1 in that ETH_S is greater than TH_{target} (3,334 retrievals/hour). For scenarios 2 and 3, in which the throughput capacity had to be increased to satisfy the service requirements, the framework suggests to increase the number of aisles since the bottleneck occurs at the lift. This means to evaluate only rack configuration 30 in scenario 2 and rack configurations 30 and 36 in the scenario 3. In all three scenarios, the optimal solution identified using the framework is the minimum cost solution (among those that respect the layout and service constraints).

In terms of the results for the other six scenarios (Tables 9 and 10) a similar search process was observed. Furthermore, as C_{max} increases, the framework's starting solution becomes focussed on a smaller number of aisles. This implies that the initial solution has a lower throughput capacity and a greater number of alternatives are considered prior to identifying the optimal solution (from 3 rack configurations assessed in scenario 3 up to 8 assessed in scenario 9).

Generalising the results of these 576 experiments (64 rack configurations * 9 scenarios), it can be concluded that the proposed design framework always identifies the optimal solution from among the potential solutions based on the physical and service constraints. The framework application involves the computation and assessment for only a small number of alternative rack configurations (e.g. 8 for scenario 9 and only 1 configuration for scenario 1), such that simulation was needed only to further evaluate the performance of identified configuration and to fine-tune it. This derives from the fact that the framework allows the warehouse designer to assess alternative rack configurations in an intelligent manner. It should be noted that the framework is dependent on the assumptions made with respect to cost structure, particularly the relationship between vehicle cost and lift cost. However, any significant change in this respect is, at the moment, considered to be highly improbable by material handling providers.

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Take in Tables 8, 9 and 10

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Based on the analysis of the scenarios presented above, the following design guidelines were identified. First, the most appropriate configuration differs from that for AVS/RS with a “tier to tier” configuration, which has been presented in the literature. While it is better to have a larger number of short aisles in systems with a “tier to tier” configuration, this study shows that in systems with a “tier captive” configuration a smaller number of longer aisles is more appropriate.

However, for both configurations (i.e. “tier to tier” and “tier captive”), the optimal solution may not be one that maximises rack height, making full use of the height of an industrial building.

7. Conclusions and further research

This study focused on the analysis of AVS/RS performance for product totes with a "tier captive" configuration. Performance assessment is highly complex due to the compound effect of the kinematic behaviour of the vehicles and the lift. In turn, this complexity is a function of the rack configuration (i.e. number and length of storage aisles and number of tiers).

First, examination of the AVS/SR cost structure showed that cost minimisation may be achieved by first minimising the number of aisles, given the physical building constraints, and then maximising aisle length to reduce the number of vehicles required.

Second, the effect of rack configuration on AVS/RS performance was studied through simulation using the Arena software. Simulation results confirm that throughput may be estimated analytically by identifying the system bottleneck (i.e. lift or vehicles). Analytical formulas for estimating throughput were then defined. However the average cycle time and average flow time are more hardly predictable a priori. Simulation results show that when the resource utilisation level (i.e. lifts or vehicles) is below 0.9 the impact of waiting times on flow time is minimal, and therefore flow time is not much greater than cycle time. When resource utilisation exceeds 0.9, system performance decreases appreciably. Therefore, in the design phase, a target value of 0.9 for resource utilisation should be used.

Finally, the simulation results illustrate the effect of rack configuration on throughput performance. The lift tends to be the cause of the bottleneck where the configuration has high racks. Conversely, for a given storage capacity, vehicles tend to create the bottleneck as the number of storage tiers is reduced. As a consequence, the maximum throughput for an aisle does not necessarily correspond to the configuration with the maximum rack height. Furthermore, the results confirm the relationship between system throughput and the two main design variables, i.e. number of aisles (*A)* and number of tiers (*T*): to increase the system throughput, the number of tiers *(T)* needs to be adjusted when vehicles create the bottleneck; conversely the number of aisles (*A)* should be adjusted if the bottleneck is caused by the lift. Recognising this pattern makes it possible to increase the system throughput performance while avoiding overcapacity (i.e. at a minimal incremental cost).

The in-depth examination of system performance with varying rack configurations, together with the cost structure modelling, were the basis for the development of a design framework for AVS/RS with a "tier captive" configuration and the tote as the handling unit. This framework is a useful tool for warehouse designers, as it permits a rapid identification of the most appropriate rack configuration. The primary purpose of the framework is to assist designers in the conceptualisation phase of system development. It makes it possible to design AVS/RS that satisfy user requirements while avoiding overcapacity and then decreasing the overall system cost.

Finally, an application of the framework was presented which used data from an AVS/RS in the United Kingdom. The results of over 576 experiments show that the proposed design framework always identifies the optimal solution from among the potential solutions, given the physical and service constraints, and involves the analysis and assessment of a small number of alternative rack configurations. Finally, the results have highlighted the main differences in design approach between the "tier to tier" configuration studied more frequently in the literature and the "tier captive" configuration examined in this paper. In "tier to tier" systems, where vehicles may access locations on different storage tiers and a lift is not dedicated to a single aisle, it is more appropriate to have a high number of short aisles. However, the optimal configuration for a “tier captive” AVS/RS (i.e. one lift for each aisle and as many vehicles as the number of tiers) tends to have a smaller number of longer aisles.

This first step in the research involved an assessment of the performance of AVS/RS under various design scenarios. Using the proposed design framework, it will be possible, through future research, to perform an economic comparison of AVS/RS with other automated systems, such as miniload. In addition, it will be possible to compare the results with empirical results obtained from an analysis of the industrial applications of these automated solutions and, if needed, extend the analysis to other aspects of warehousing such as picking activity.

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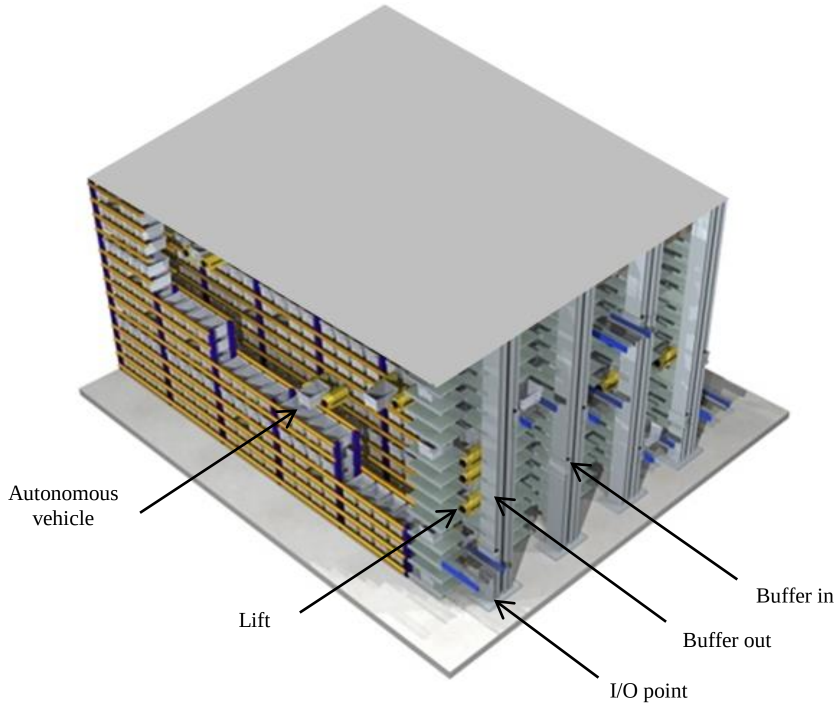

Figure 1. AVS/RS with a “tier captive” configuration and the tote as the handling unit.

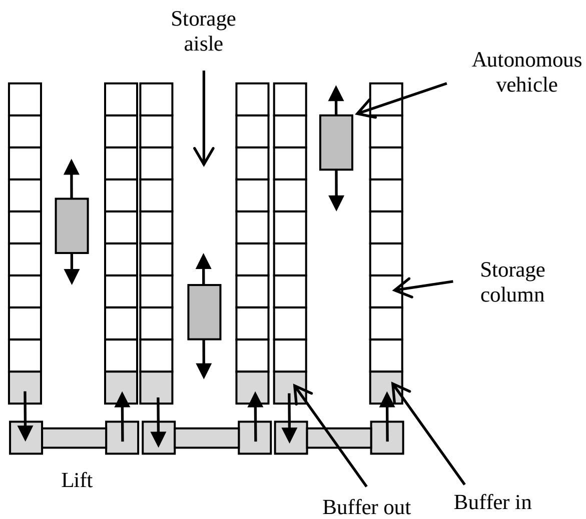

Figure 2. Depiction of a tier of an AVS/RS with a “tier captive” configuration and the tote as the handling unit.

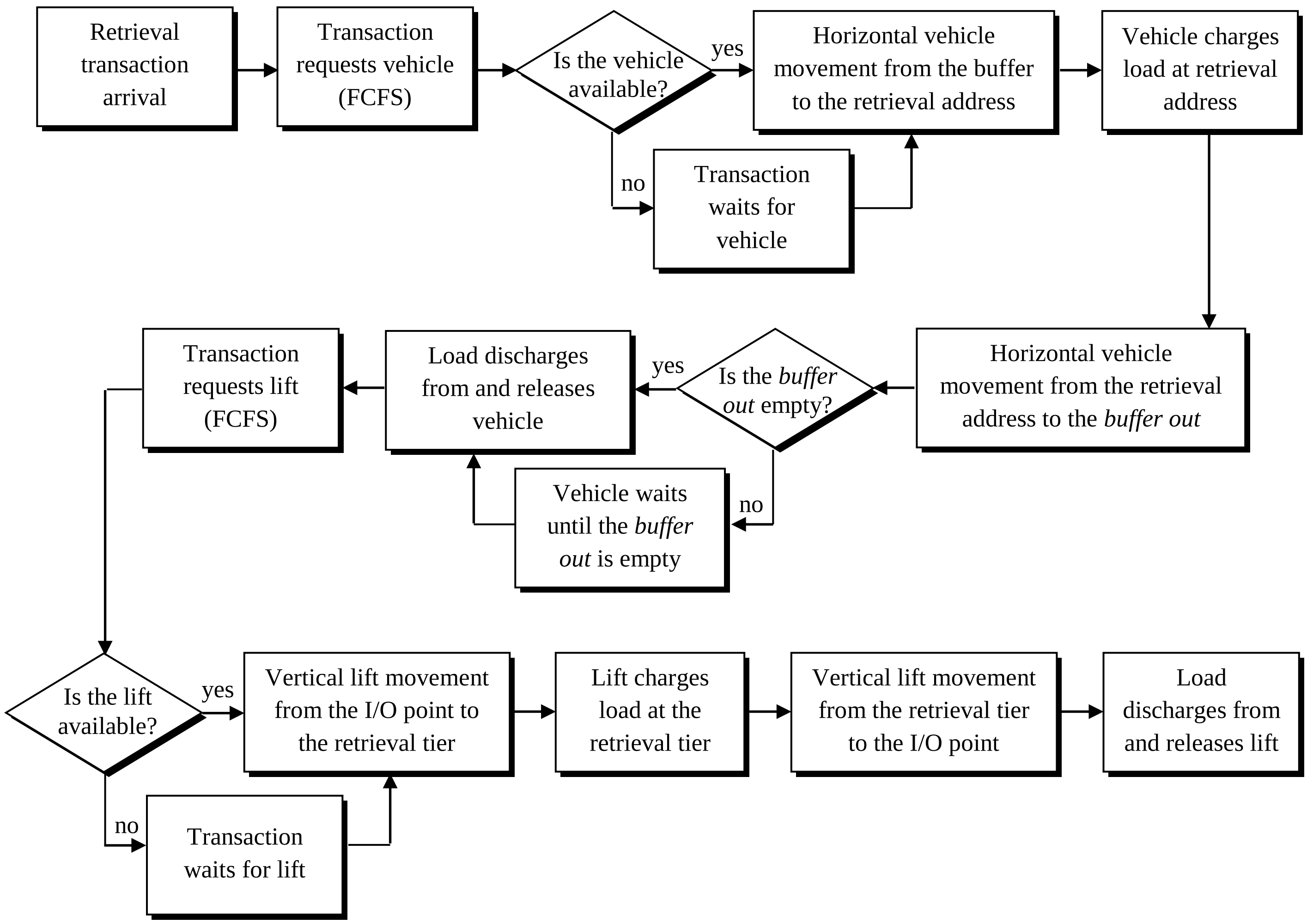

Figure 3. Simulation flow chart for retrieval cycle.

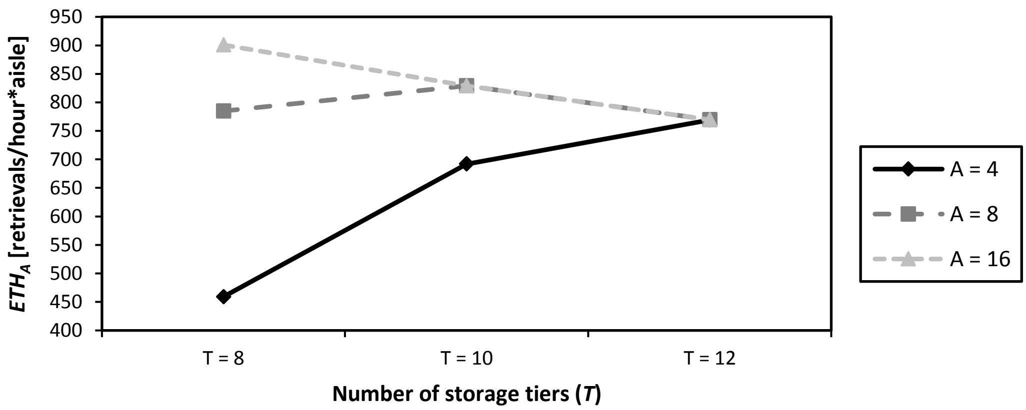

Figure 4. Expected throughput per aisle (ETH_A) varying the number of storage tiers (T) and number of aisles (A), assuming a number of storage locations per aisle equal to 2500 when A = 4, 1250 when A = 8, and 625 when A = 16.

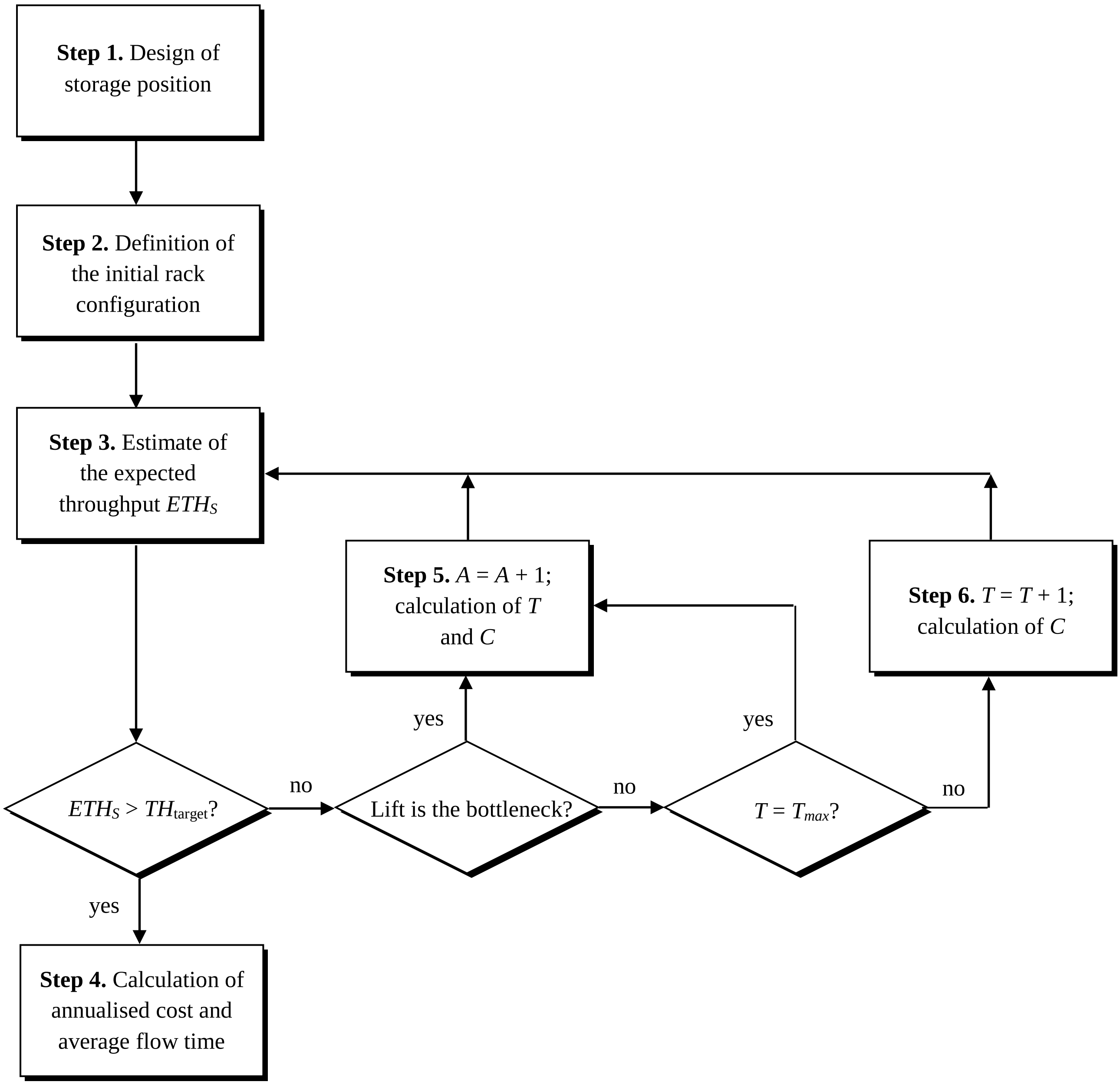

Figure 5. Design framework for the design of an AVS/RS with a "tier captive" configuration.

Table 1. Overview of the main studies on AVS/RS design criteria.

Paper	*Category of the paper*	*Configuration*	*Methodology*	*Project parameters*	*Number of experiments*	*Unit load*
Malmborg (2002)	A	"tier to tier"	Analytical model validated through simulation	Number of storage positions (10,000) and demand rate (150 transactions/hour)	25	Palletised unit load
Kuo *et al.* (2007)	A	"tier to tier"	Analytical model validated through simulation using AutoMod software	5 number of storage positions (10,000 to 30,000 with steps of 5,000) and 3 demand rates (100; 200; 300 transactions/hour)	15	Palletised unit load
Fukunari and Malmborg (2008)	A	"tier to tier"	Analytical model validated through simulation using AutoMod software	5 number of storage positions (10,000 to 30,000 with steps of 5,000) and 3 demand rates (100; 200; 300 transactions/hour)	15	Palletised unit load
Ekren and Heragu (2010)	B	"tier to tier"	Simulation model using Arena software	Number of storage positions (42,000) and 1 demand rate (450 transactions/hour)	32	Palletised unit load
Ekren *et al.* (2010)	B	"tier to tier"	Simulation model using Arena software	Number of storage positions (47,628)	160	Palletised unit load
Ekren (2011)	B	"tier to tier"	Simulation model using Arena software	Number of storage positions (47,628)	55	Palletised unit load
Ekren and Heragu (2011)	A	"tier to tier"	Simulation model using Arena software	Number of storage positions (47,628) and 2 demand rates (450; 500 transactions/hour)	110	Palletised unit load
Heragu *et al.* (2011)	B	"tier captive"	Analytical model validated through simulation	Number of storage positions (20,000) and 1 demand rate (500; transactions/hour)	4	Palletised unit load

A: papers that compare AVS/RS with AS/RS
B: papers that analyse system performance according to different rack configurations

Table 2. Rack configuration data.

Rack configuration (RC)	*Number of tiers(T)*	*Number of aisles (A)*	*Number of columns (C)*	*ETH_V [retrievals/ hour*aisle]*	*ETH_L [retrievals/ hour*aisle]*	*Expected bottleneck*	*ETH_A [retrievals/ hour*aisle]*	*ETH_S [retrievals/ hour]*
1	8	4	157	459	901	Vehicles	459	1836
2	8	8	79	785	901	Vehicles	785	6280
3	8	16	40	1217	901	Lift	901	14,416
4	10	4	125	692	829	Vehicles	692	2768
5	10	8	63	1149	829	Lift	829	6632
6	10	16	32	1714	829	Lift	829	13,264
7	12	4	105	953	769	Lift	769	3076
8	12	8	53	1543	769	Lift	769	6152
9	12	16	27	2234	769	Lift	769	12,304

Table 3. Data used in simulation experiments.

Variable	*Unit of measure*	*Data*
μ_w	m	0.5
μ_h	m	0.8
v_V	m/s	1.5
v_L	m/s	5
γ_V	s	7
γ_L	s	1
ε_V	s	3
ε_L	s	2
H_{max}	m	10
L_{max}	m	80

Table 4. Simulation results for a single aisle.

Ex.	*RC*	*λ_A [retrievals/ hours*aisle]*	*ETH_A [retrievals/ hours*aisle]*	*Throughput performance [retrievals /hours*aisle]*	*Average waiting time for the vehicle [s]*	*Average waiting time for the lift [s]*	*Average cycle time [s]*	*Average flow time [s]*	*Vehicle utilisation*	*Lift utilisation*
1	1	100	459	100	9.91	0.23	66.92	76.83	0.22	0.11
2	2	100	785	100	3.00	0.23	40.41	43.41	0.13	0.11
3	3	100	901	100	1.16	0.22	27.90	29.06	0.08	0.11
4	4	100	692	100	5.00	0.28	56.65	61.65	0.14	0.12
5	5	100	829	100	1.67	0.28	35.97	37.64	0.09	0.12
6	6	100	829	100	0.72	0.27	25.63	26.35	0.06	0.12
7	7	100	769	100	2.98	0.33	50.36	53.34	0.11	0.13
8	8	100	769	100	1.08	0.33	33.03	34.11	0.07	0.13
9	9	100	769	100	0.49	0.34	24.37	24.86	0.04	0.13
10	1	250	459	250	45.50	0.64	67.29	112.80	0.54	0.28
11	2	250	785	250	9.96	0.65	41.32	51.28	0.31	0.28
12	3	250	901	250	3.36	0.65	28.33	31.69	0.20	0.28
13	4	250	692	250	17.78	0.82	57.17	74.95	0.36	0.30
14	5	250	829	250	5.00	0.82	36.52	41.52	0.22	0.30
15	6	250	829	250	1.47	0.83	23.86	25.33	0.13	0.30
16	7	250	769	250	9.54	1.03	51.02	60.56	0.26	0.32
17	8	250	769	250	3.03	1.03	33.71	36.74	0.16	0.32
18	9	250	769	250	1.28	1.04	25.05	26.33	0.11	0.32
19	1	500	459	459	2954	1.44	68.17	3022	1	0.51
20	2	500	785	500	34.09	1.66	42.36	76.45	0.64	0.55
21	3	500	901	500	8.57	1.76	29.45	38.02	0.41	0.55
22	4	500	692	500	74.61	2.26	58.66	133.27	0.72	0.60
23	5	500	829	500	12.94	2.39	38.11	51.05	0.44	0.60
24	6	500	829	500	4.51	2.64	28.02	32.53	0.29	0.60
25	7	500	769	500	27.38	3.14	53.18	80.56	0.52	0.65
26	8	500	769	500	7.10	3.56	36.27	43.37	0.32	0.65
27	9	500	769	500	2.86	3.89	27.91	30.77	0.22	0.65
28	1	750	459	459	35,264	1.49	68.11	35,445	1	0.51
29	2	750	785	750	389.41	4.17	44.83	434.27	0.95	0.83
30	3	750	901	750	20.67	6.18	33.86	54.53	0.61	0.83
31	4	750	692	692	7398	5.29	61.64	7460	1	0.83
32	5	750	829	750	33.82	15.32	51.01	84.83	0.65	0.91
33	6	750	829	750	6.34	19.52	42.54	48.86	0.39	0.91
34	7	750	769	750	99.50	67.36	117.35	216.85	0.78	0.97
35	8	750	769	750	14.89	81.71	114.39	129.28	0.48	0.97
36	9	750	769	750	5.21	83.96	107.97	113.18	0.33	0.97
37	1	1000	459	459	17,534	1.44	68.16	17,602	1	0.51
38	2	1000	785	785	7033	4.10	44.79	7078	1	0.87
39	3	1000	901	901	57.58	3192	3219	3277	0.82	1
40	4	1000	692	692	10,030	4.48	60.87	10,091	1	0.83
41	5	1000	829	829	115.11	5453	5488	5603	0.87	1
42	6	1000	829	829	15.49	5554	5578	5593	0.58	1
43	7	1000	769	769	1939	5861	5910	7849	1	1
44	8	1000	769	769	27.85	7494	7526	7554	0.65	1
45	9	1000	769	769	8.10	7520	7542	7550	0.45	1

Table 5. Results for the 64 rack configurations considered in the application of the design framework to a real case. The alternatives explored by the framework are highlighted in grey.

Rack configuration (RC)	*Number of aisles (A)*	*Number of tiers (T)*	*Number of columns (C)*	*Number of storage positions (n)*	*ETH_S [retrievals/ hour]*	*Average flow time [s]*	*Annualised cost [€]*
1	4	8	149	9536	n.a.	n.a.	n.a.
2	4	9	132	9504	n.a.	n.a.	n.a.
3	4	10	119	9520	n.a.	n.a.	n.a.
4	4	11	108	9504	n.a.	n.a.	n.a.
5	4	12	99	9504	n.a.	n.a.	n.a.
6	4	13	92	9568	n.a.	n.a.	n.a.
7	4	14	85	9520	n.a.	n.a.	n.a.
8	4	15	80	9600	n.a.	n.a.	n.a.
9	5	8	119	9520	n.a.	n.a.	n.a.
10	5	9	106	9540	n.a.	n.a.	n.a.
11	5	10	95	9500	n.a.	n.a.	n.a.
12	5	11	87	9570	n.a.	n.a.	n.a.
13	5	12	80	9600	n.a.	n.a.	n.a.
14	5	13	74	9620	n.a.	n.a.	n.a.
15	5	14	68	9520	n.a.	n.a.	n.a.
16	5	15	64	9600	3678	7050	230,040
17	6	8	99	9504	n.a.	n.a.	n.a.
18	6	9	88	9504	n.a.	n.a.	n.a.
19	6	10	80	9600	n.a.	n.a.	n.a.
20	6	11	72	9504	n.a.	n.a.	n.a.
21	6	12	66	9504	4847	76.88	237,595
22	6	13	61	9516	4693	69.71	245,708
23	6	14	57	9576	4549	67.53	254,398
24	6	15	53	9540	4414	69.42	262,619
25	7	8	85	9520	n.a.	n.a.	n.a.
26	7	9	76	9576	n.a.	n.a.	n.a.
27	7	10	68	9520	n.a.	n.a.	n.a.
28	7	11	62	9548	5848	63.64	256,687
29	7	12	57	9576	5654	56.77	266,221
30	7	13	53	9646	5475	52.58	276,359
31	7	14	49	9604	5307	49.72	285,950
32	7	15	46	9660	5150	48.28	296,419
33	8	8	75	9600	n.a.	n.a.	n.a.
34	8	9	66	9504	7043	70.48	262,268
35	8	10	60	9600	6923	58.81	273,020
36	8	11	54	9504	6684	51.09	282,835
37	8	12	50	9600	6462	47.16	294,499
38	8	13	46	9568	6257	44.05	305,538
39	8	14	43	9632	6065	42.15	317,502
40	8	15	40	9600	5886	40.68	328,998
41	9	8	66	9504	7043	69.88	274,604
42	9	9	59	9558	8082	56.45	285,923
43	9	10	53	9540	7788	48.99	297,405
44	9	11	48	9504	7519	44.17	309,311
45	9	12	44	9504	7280	41.06	321,906
46	9	13	41	9594	7039	39.12	335,453
47	9	14	38	9576	6823	37.42	348,473
48	9	15	36	9720	6621	36.51	362,797
49	10	8	60	9600	8372	57.34	297,008
50	10	9	53	9540	8980	48.32	309,000
51	10	10	48	9600	8654	43.43	322,717
52	10	11	44	9680	8354	40.12	337,102
53	10	12	40	9600	8078	37.48	350,706
54	10	13	37	9620	7821	35.74	365,368
55	10	14	34	9520	7581	34.22	379,444
56	10	15	32	9600	7357	33.37	394,969
57	11	8	54	9504	9900	49.12	317,791
58	11	9	48	9504	9878	43.00	331,931
59	11	10	44	9680	9519	39.62	348,184
60	11	11	40	9680	9190	36.81	363,578
61	11	12	36	9504	8886	34.45	378,113
62	11	13	34	9724	8603	33.40	395,835
63	11	14	31	9548	8339	31.99	410,997
64	11	15	29	9570	8093	31.13	427,752

Legend: n.a.: the rack configuration does not satisfy the physical constraint

Table 6. Unit costs.

Cost item	*Unit of measure*	*Value*
Vehicle	€/unit	10,000
Lift	€/unit	50,000
Storage position	€/unit	30
Area	€/m^2*year	50

Table 7. Data for the scenarios examined in the framework application.

Scenario	*Maximum number of storage columns (C_{max})*	*λ [retrievals/ hours]*	*TH_{target} [retrievals/hours]*
1	55	3,000	3334
2	55	4,000	4445
3	55	5,000	5556
4	75	3,000	3334
5	75	4,000	4445
6	75	5,000	5556
7	95	3,000	3334
8	95	4,000	4445
9	95	5,000	5556

Table 8. Results of the design framework application for scenarios 1, 2 and 3 (i.e. C_{max} = 55). For each scenario the sequence of rack configurations evaluated is highlighted.

RC	*A*	*T*	*C*	*n*	*ETH*ETH_S *[retrievals/hour]*	*Average flow time [s]*			*Annualised cost [€]*	*Rack configurations evaluated*		
						SC1	*SC2*	*SC3*		*SC1*	*SC2*	*SC3*
1	4	8	149	9536	n.a.	n.a.	n.a.	n.a.	n.a.			
2	4	9	132	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
3	4	10	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
4	4	11	108	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
5	4	12	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
6	4	13	92	9568	n.a.	n.a.	n.a.	n.a.	n.a.			
7	4	14	85	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
8	4	15	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
9	5	8	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
10	5	9	106	9540	n.a.	n.a.	n.a.	n.a.	n.a.			
11	5	10	95	9500	n.a.	n.a.	n.a.	n.a.	n.a.			
12	5	11	87	9570	n.a.	n.a.	n.a.	n.a.	n.a.			
13	5	12	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
14	5	13	74	9620	n.a.	n.a.	n.a.	n.a.	n.a.			
15	5	14	68	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
16	5	15	64	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
17	6	8	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
18	6	9	88	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
19	6	10	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
20	6	11	72	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
21	6	12	66	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
22	6	13	61	9516	n.a.	n.a.	n.a.	n.a.	n.a.			
23	6	14	57	9576	n.a.	n.a.	n.a.	n.a.	n.a.			
24	6	15	53	9540	4414	48.55	69.42	10,648	262,619	1	1	1
25	7	8	85	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
26	7	9	76	9576	n.a.	n.a.	n.a.	n.a.	n.a.			
27	7	10	68	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
28	7	11	62	9548	n.a.	n.a.	n.a.	n.a.	n.a.			
29	7	12	57	9576	n.a.	n.a.	n.a.	n.a.	n.a.			
30	7	13	53	9646	5475	45.95	52.58	74.15	276,359		2	2
31	7	14	49	9604	5307	43.26	49.72	81.86	285,950			
32	7	15	46	9660	5150	48.93	48.28	119.28	296,419			
33	8	8	75	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
34	8	9	66	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
35	8	10	60	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
36	8	11	54	9504	6684	45.69	51.09	59.65	282,835			3
37	8	12	50	9600	6462	42.69	47.16	54.36	294,499			
38	8	13	46	9568	6257	40.11	44.05	51.28	305,538			
39	8	14	43	9632	6065	38.43	42.15	50.18	317,502			
40	8	15	40	9600	5886	36.88	40.68	50.55	328,998			
41	9	8	66	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
42	9	9	59	9558	n.a.	n.a.	n.a.	n.a.	n.a.			
43	9	10	53	9540	7788	44.32	48.99	55.63	297,405			
44	9	11	48	9504	7519	40.70	44.17	49.06	309,311			
45	9	12	44	9504	7280	38.15	41.06	45.30	321,906			
46	9	13	41	9594	7039	36.47	39.12	43.23	335,453			
47	9	14	38	9576	6823	34.91	37.42	41.64	348,473			
48	9	15	36	9720	6621	34.00	36.51	41.27	362,797			
49	10	8	60	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
50	10	9	53	9540	8980	43.87	48.32	54.45	309,000			
51	10	10	48	9600	8654	40.24	43.43	47.63	322,717			
52	10	11	44	9680	8354	37.57	40.12	43.40	337,102			
53	10	12	40	9600	8078	35.34	37.48	40.33	350,706			
54	10	13	37	9620	7821	33.78	35.74	38.45	365,368			
55	10	14	34	9520	7581	32.37	34.22	39.92	379,444			
56	10	15	32	9600	7357	31.53	33.37	36.27	394,969			
57	11	8	54	9504	9900	44.48	49.12	55.65	317,791			
58	11	9	48	9504	9878	39.94	43.00	47.03	331,931			
59	11	10	44	9680	9519	37.25	39.62	42.54	348,184			
60	11	11	40	9680	9190	34.93	36.81	39.19	363,578			
61	11	12	36	9504	8886	32.81	34.45	36.48	378,113			
62	11	13	34	9724	8603	31.88	33.40	35.40	395,835			
63	11	14	31	9548	8339	30.55	31.99	33.94	410,997			
64	11	15	29	9570	8093	29.71	31.13	33.17	427,752			

Legend: n.a.: the rack configuration does not satisfy the physical constraint

Table 9. Results of the design framework application for scenarios 4, 5 and 6 (i.e. C_{max} = 75). For each scenario the sequence of rack configurations evaluated is highlighted.

RC	*A*	*T*	*C*	*n*	*ETH*	*Average flow time [s]*			*Annualised cost [€]*	*Rack configurations evaluated*		
						SC4	*SC5*	*SC6*		*SC4*	*SC5*	*SC6*
1	4	8	149	9536	n.a.	n.a.	n.a.	n.a.	n.a.			
2	4	9	132	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
3	4	10	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
4	4	11	108	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
5	4	12	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
6	4	13	92	9568	n.a.	n.a.	n.a.	n.a.	n.a.			
7	4	14	85	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
8	4	15	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
9	5	8	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
10	5	9	106	9540	n.a.	n.a.	n.a.	n.a.	n.a.			
11	5	10	95	9500	n.a.	n.a.	n.a.	n.a.	n.a.			
12	5	11	87	9570	n.a.	n.a.	n.a.	n.a.	n.a.			
13	5	12	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
14	5	13	74	9620	3910	74.87	1830	19,974	216,713	1	1	1
15	5	14	68	9520	3791	67.45	4697	22,098	222,652			
16	5	15	64	9600	3678	63.95	7050	24,172	230,040			
17	6	8	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
18	6	9	88	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
19	6	10	80	9600	n.a.	n.a.	n.a.	n.a.	n.a.			
20	6	11	72	9504	5013	68.20	93.02	499.68	229,882		2	2
21	6	12	66	9504	4847	59.96	76.88	2845	237,595			
22	6	13	61	9516	4693	54.64	69.71	5611	245,708			
23	6	14	57	9576	4549	59.19	67.53	8172	254,398			
24	6	15	53	9540	4414	48.55	69.42	10,648	262,619			
25	7	8	85	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
26	7	9	76	9576	n.a.	n.a.	n.a.	n.a.	n.a.			
27	7	10	68	9520	4903	61.14	76.99	111.34	247,553			3
28	7	11	62	9548	5848	53.74	63.64	83.22	256,687			
29	7	12	57	9576	5654	49.08	56.77	74.27	266,221			
30	7	13	53	9646	5475	45.95	52.58	74.15	276,359			
31	7	14	49	9604	5307	43.26	49.72	81.86	285,950			
32	7	15	46	9660	5150	48.93	48.28	119.28	296,419			
33	8	8	75	9600	5703	71.57	101.32	210.95	253,821			
34	8	9	66	9504	7043	57.80	70.48	95.32	262,268			
35	8	10	60	9600	6923	50.95	58.81	71.45	273,020			
36	8	11	54	9504	6684	45.69	51.09	59.65	282,835			
37	8	12	50	9600	6462	42.69	47.16	54.36	294,499			
38	8	13	46	9568	6257	40.11	44.05	51.28	305,538			
39	8	14	43	9632	6065	38.43	42.15	50.18	317,502			
40	8	15	40	9600	5886	36.88	40.68	50.55	328,998			
41	9	8	66	9504	7043	69.88	69.88	94.99	274,604			
42	9	9	59	9558	8082	56.45	56.45	67.41	285,923			
43	9	10	53	9540	7788	44.32	48.99	55.63	297,405			
44	9	11	48	9504	7519	40.70	44.17	49.06	309,311			
45	9	12	44	9504	7280	38.15	41.06	45.30	321,906			
46	9	13	41	9594	7039	36.47	39.12	43.23	335,453			
47	9	14	38	9576	6823	34.91	37.42	41.64	348,473			
48	9	15	36	9720	6621	34.00	36.51	41.27	362,797			
49	10	8	60	9600	8372	57.34	57.34	69.05	297,008			
50	10	9	53	9540	8980	43.87	48.32	54.45	309,000			
51	10	10	48	9600	8654	40.24	43.43	47.63	322,717			
52	10	11	44	9680	8354	37.57	40.12	43.40	337,102			
53	10	12	40	9600	8078	35.34	37.48	40.33	350,706			
54	10	13	37	9620	7821	33.78	35.74	38.45	365,368			
55	10	14	34	9520	7581	32.37	34.22	39.92	379,444			
56	10	15	32	9600	7357	31.53	33.37	36.27	394,969			
57	11	8	54	9504	9900	44.48	49.12	55.65	317,791			
58	11	9	48	9504	9878	39.94	43.00	47.03	331,931			
59	11	10	44	9680	9519	37.25	39.62	42.54	348,184			
60	11	11	40	9680	9190	34.93	36.81	39.19	363,578			
61	11	12	36	9504	8886	32.81	34.45	36.48	378,113			
62	11	13	34	9724	8603	31.88	33.40	35.40	395,835			
63	11	14	31	9548	8339	30.55	31.99	33.94	410,997			
64	11	15	29	9570	8093	29.71	31.13	33.17	427,752			

Legend: n.a.: the rack configuration does not satisfy the physical constraint

Table 10. Results of the design framework application for scenarios 7, 8 and 9 (i.e. C_{max} = 95). For each scenario the sequence of rack configurations evaluated is highlighted.

RC	*A*	*T*	*C*	*n*	*ETH*	*Average flow time [s]*			*Annualised cost [€]*	*Rack configurations evaluated*		
					[retrievals/hour]	*SC7*	*SC8*	*SC9*		*SC7*	*SC8*	*SC9*
1	4	8	149	9536	n.a.	n.a.	n.a.	n.a.	n.a.			
2	4	9	132	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
3	4	10	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
4	4	11	108	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
5	4	12	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
6	4	13	92	9568	3128	180.85	19,963	34,313	186,614	1	1	1
7	4	14	85	9520	3032	75,766	79,807	82,216	191,294			
8	4	15	80	9600	2943	1741	24,174	37693	197,054			
9	5	8	119	9520	n.a.	n.a.	n.a.	n.a.	n.a.			
10	5	9	106	9540	n.a.	n.a.	n.a.	n.a.	n.a.			
11	5	10	95	9500	3719	174.25	6649	23,471	197,701	2	2	2
12	5	11	87	9570	4177	111.32	335.52	15,172	203,900		3	3
13	5	12	80	9600	4039	87.08	317.62	17,625	210,188			
14	5	13	74	9620	3910	74.87	1830	19,974	216,713			
15	5	14	68	9520	3791	67.45	4697	22,098	222,652			
16	5	15	64	9600	3678	63.95	7050	24,172	230,040			
17	6	8	99	9504	n.a.	n.a.	n.a.	n.a.	n.a.			
18	6	9	88	9504	4263	114.96	457.20	12,860	215,825		4	4
19	6	10	80	9600	5094	84.38	141.00	1000	223,322		5	5
20	6	11	72	9504	5013	68.21	93.02	499.68	229,882			6
21	6	12	66	9504	4847	59.96	76.88	2845	237,595			
22	6	13	61	9516	4693	54.64	69.71		245,708			
23	6	14	57	9576	4549	59.19	67.53	8172	254,398			
24	6	15	53	9540	4414	48.55	69.42	10,648	262,619			
25	7	8	85	9520	4541	100.30	241.68	8533	231,551			7
26	7	9	76	9576	5559	74.16	107.34	263.78	239,626			8
27	7	10	68	9520	4903	61.14	76.99	111.34	247,553			
28	7	11	62	9548	5848	53.74	63.64	83.22	256,687			
29	7	12	57	9576	5654	49.08	56.77	74.27	266,221			
30	7	13	53	9646	5475	45.95	52.58	74.15	276,359			
31	7	14	49	9604	5307	43.26	49.72	81.86	285,950			
32	7	15	46	9660	5150	48.93	48.28	119.28	296,419			
33	8	8	75	9600	5703	71.57	101.32	210.95	253,821			
34	8	9	66	9504	7043	57.80	70.48	95.32	262,268			
35	8	10	60	9600	6923	50.95	58.81	71.45	273,020			
36	8	11	54	9504	6684	45.69	51.09	59.65	282,835			
37	8	12	50	9600	6462	42.69	47.16	54.36	294,499			
38	8	13	46	9568	6257	40.11	44.05	51.28	305,538			
39	8	14	43	9632	6065	38.43	42.15	50.18	317,502			
40	8	15	40	9600	5886	36.88	40.68	50.55	328,998			
41	9	8	66	9504	7043	69.88	69.88	94.99	274,604			
42	9	9	59	9558	8082	56.45	56.45	67.41	285,923			
43	9	10	53	9540	7788	44.32	48.99	55.63	297,405			
44	9	11	48	9504	7519	40.70	44.17	49.06	309,311			
45	9	12	44	9504	7280	38.15	41.06	45.30	321,906			
46	9	13	41	9594	7039	36.47	39.12	43.23	335,453			
47	9	14	38	9576	6823	34.91	37.42	41.64	348,473			
48	9	15	36	9720	6621	34.00	36.51	41.27	362,797			
49	10	8	60	9600	8372	57.34	57.34	69.05	297,008			
50	10	9	53	9540	8980	43.87	48.32	54.45	309,000			
51	10	10	48	9600	8654	40.24	43.43	47.63	322,717			
52	10	11	44	9680	8354	37.57	40.12	43.40	337,102			
53	10	12	40	9600	8078	35.34	37.48	40.33	350,706			
54	10	13	37	9620	7821	33.78	35.74	38.45	365,368			
55	10	14	34	9520	7581	32.37	34.22	39.92	379,444			
56	10	15	32	9600	7357	31.53	33.37	36.27	394,969			
57	11	8	54	9504	9900	44.48	49.12	55.65	317,791			
58	11	9	48	9504	9878	39.94	43.00	47.03	331,931			
59	11	10	44	9680	9519	37.25	39.62	42.54	348,184			
60	11	11	40	9680	9190	34.93	36.81	39.19	363,578			
61	11	12	36	9504	8886	32.81	34.45	36.48	378,113			
62	11	13	34	9724	8603	31.88	33.40	35.40	395,835			
63	11	14	31	9548	8339	30.55	31.99	33.94	410,997			
64	11	15	29	9570	8093	29.71	31.13	33.17	427,752			

Legend: n.a.: the rack configuration does not satisfy the physical constraint