USE OF MODAL REPRESENTATION FOR THE SUPPORTING STRUCTURE IN MODEL BASED FAULT IDENTIFICATION OF LARGE ROTATING MACHINERY: PART 1 – THEORETICAL REMARKS

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ABSTRACT

Fault identification by means of model based techniques, both in frequency and in time domain, is often employed in diagnostics of rotating machines, when the main task is to locate and to evaluate the severity of the malfunction. The model of the fully assembled machine is composed by the sub-models of the rotor, of the bearings and of the foundation, while the effect of the faults is modelled by means of equivalent force systems. Some identification techniques, such as the least square identification in frequency domain, proposed by the authors, have proven to be quite robust even if the sub-models are not fine tuned. Anyhow, the use of a reliable model can increase the accuracy of the identification.

Normally a supporting structure is represented by means of rigid foundation or by pedestals, i.e. 2 d.o.f. mass – spring - damper systems, but these kind of models are often not able to reproduce correctly the dynamical behaviour of the supporting structure, especially in large machines where coupled modes are present. Therefore, peculiar aspect of this paper is the use of a modal foundation to model the supporting structure of the machine and the method is discussed in detail in this first part. The modal representation of the foundation is then introduced in the least square identification technique in frequency domain.

1 INTRODUCTION

Many different topics interest the dynamic of the rotors and it is a difficult, and very subjective, task to determine a possible ranking among them. However, if we consider two possible criteria, being first one economic and the second one scientific, the fault identification is one of the most important. On one hand, due to the economic criteria, the early identification of possible faults affecting a machine and the evaluation of their severity can reduce off-line time, maintenance periods, forecast residual life, avoid accidents and dangerous break-down. On the other hand, many researchers have dealt with the fault identification problem and a huge literature is available, by considering only the most recent years.

Some papers give an overview of the possible diagnostics techniques applied on different types of rotating machines or plant types. Hellmann [1] analyzes the fault detection in pumps presenting symptoms and possible diagnostics methods. Südmersen et al. [2,3] give an overview of different problems related to power plants, starting from the condition monitoring techniques and then taking into account the vibrations during speed transients of the turbogenerator or different reactive power conditions. Orbit shape analysis and vibration spectra are used. Sawicki [4] presents a review on condition monitoring techniques in rotating machinery and discuss experimental cases about two particular faults: rotor rub and thermally induced vibrations, also known as spiral vibrations.

In rotor dynamics field different techniques are used for identification purposes. White and Jecmenica [5] describe a knowledge based system that implements a fault matrix and fuzzy logic: experimental cases are reported for a gas turbine generator and an induction motor. Also Kirk and Guo [6] propose an expert system that uses rule based logic to relate possible faulty conditions of a small size steam turbine to different symptoms. Bayesian networks are proposed in [7]. Nalinakash and Satishkumar [8] use an artificial neural network, with a back-propagation learning algorithm with three layers, to detect unbalance, misalignment and roller bearing looseness in a small scale test-rig. Lucifredi et al. [9,10] apply some chaos related indicators to a small scale test-rig and to a turbo-gas unit to detect faults related to non-linearities.

Anyhow, when the identification task is to locate along the rotor train and to evaluate the severity of the malfunction, i.e. the most important piece of information from an engineering point of view, and not to simply indicate a faulty condition, model based techniques, both in frequency and in time domain, are often employed in diagnostics. Actually model based methods are not used in fault detection in rotor dynamics only, but in general in all the technical process [11]. If a survey is limited to rotor dynamics field, both the complete process model [12] and the mechanical model of the rotating machinery only are used. The last case is commonly considered in literature. Mayes and Penny [13] point out that the use of model based diagnostics methods aims to improve the accuracy of a diagnostics process by using additional knowledge that is specific to the problem in question. They give also an overview on time and frequency domain related techniques. Markert et al. [14,15] use least squares fitting in time domain to identify malfunction in a test-rig; the faults are modelled by means of equivalent force systems. Kiciński [16] presents a study on model based diagnostics of a 200 MW turboset. The application of model based identification using least square fitting in frequency domain in rotating machinery is discussed by the authors in [17-20] with several application on real machines and test-rigs affected by different faults. Among the possible different model based approaches [21,22] that can be classified in:

i) *parameter estimation*, when the characteristic constant parameters of the process, or of the components are affected by the fault;

ii) *state estimation*, when the constant parameters are unaffected by possible faults and only the state of the system, which is represented by a set of generally unmeasurable state variables (function of time), is affected by the faults; in this case the model acts as a state observer;

iii) *parity equations*, when the faults affect some of the unmeasurable input variables, the parameters are constant, and only output variables are measured and compared with calculated model output variables;

the last approach is used and malfunction effects are modelled by means of a equivalent forcing systems.

In general, model based techniques in rotor dynamics consider the model of the fully assembled machine, which is composed by the sub-models of the rotor, of the bearings and of the foundation. Some identification techniques, such as the least square identification in frequency domain, proposed by the authors, have proven to be quite robust even if the sub-models are not fine tuned [17-20]. Anyhow, the use of a reliable model can increase the accuracy of the identification and in this paper the modelling of the supporting structure is concerned.

Even some papers are present in literature and also some text books presents rigid foundations, it is wide accepted that for the accurate modelling of the rotor system, the dynamic response of the supporting structure have to be considered [23,24]. Some authors have considered pedestals, i.e. 2 d.o.f. lumped mass–spring-damper systems in correspondence of the shaft supports. Feng and Hahn [25] present a study about pedestal parameter identification in a test-rig, in which the identified pedestal parameters are not depending on the rotating speed range. Rieger and Zhou [26] use a multi-level system for the rotor assembly in which pedestals and foundation supports are differentiated, but in any case their parameters are constant in the operating range. However constant pedestal coefficients cannot explain complex dynamical behaviour of supporting structures, so that in some cases the pedestal coefficients are considered function of the rotating speed, but in any case coupling effects cannot be taken into account. Another possible approach is to consider the transfer function, or equivalently the mechanical impedance, of the supporting structure. A similar approach is used by Vázquez et al. [27] to simulate the experimental response of a test-rig and the results are compared with those of a model with pedestals. Bonello and Brennan [28] present a mechanical impedance matrix and apply it to a test-rig to simulate experimental unbalance response.

Anyhow, some machines show a definite influence of foundation resonances over a large range of rotational speed. These reasons justify in many cases the need for a more appropriate representation of the supporting structure and a modal approach for the supporting structure is proposed by several authors. In this case the complex response of the supporting structure can be conveniently taken into account. Konishi et al. [29] propose the application on a large scale turbogenerator, but the modal analysis of the foundation is made by means of a 3D model and not on the basis of experimental results. Feng and Hahn [30] and Smart et al. [31] perform the experimental identification of the modal parameters of a test-rig. The two last papers point out the main disadvantage of this kind of method, i.e. the necessity of an effective experimental identification of the modal parameters of the foundation, but it should be emphasized that a modal model of the foundation provides in any case some more degrees of freedom for improving the tuning of the complete system model and the subsequent identification quality.

Performing the foundation modal identification is not easy on real machines installed in power plants, but a case is shown by Vania in [32].

Finally, the fault considered in the paper is not certainly the most fascinating among possible faults in rotating machinery, being the unbalance, but it is worthy to note that it is one of the most common not only as described in literature [33-40], but also in real large rotating machines, such as turbogenerators, in which the presence of many different components assembled together and blade erosion or loss can easily determine it, requiring field balancing.

The first part of the paper discusses the theoretical aspects of considering a modal foundation in the model used to identify a fault by means of the least square identification technique in frequency domain in a real machine. The experimental validation is performed in the second part by using experimental data coming from an in-situ balancing on a 320 MW steam turbogenerator of a power plant, in which the modal characterization of the foundation have been carried out experimentally.

2 THE FULLY ASSEMBLED MACHINE MODEL WITH THE MODAL REPRESENTATION OF THE FOUNDATION

As stated in the previous paragraph, the foundation of a rotating machine can be modelled by means of a modal representation. In this paragraph, the implementation of the modal foundation in the fully assembled model of a rotating machine, i.e. composed by the rotor, the bearings and the foundation, is presented. The rotor train, is modelled by means of finite beam elements, taking also into account the shear and the secondary effect of rotatory inertia. Due to the fact that the experimental data used in the fault identification are related to the lateral vibration of the machine, the axial and the torsional behaviour is neglected and only 4 d.o.f.s (two translational and two rotational) are considered per each node. The generalized displacement vector $\mathbf{x}_j^{(r)}$ of the rotor *j*-th node is:

$$\mathbf{x}_j^{(r)} = \left\{ x_j^{(r)} \quad \vartheta_{x_j}^{(r)} \quad y_j^{(r)} \quad \vartheta_{y_j}^{(r)} \right\}^{\mathrm{T}} \tag{1}$$

Two subsequent nodes, the *j*-th and the *j*+1-th, define the *j*-th element of the machine as shown in figure 1 along with the reference systems. If the rotor has n_r nodes, thus $n_r - 1$

elements, the vector $\mathbf{x}^{(r)}$ of the generalized displacements of all the rotor nodes is composed by all the ordered vectors $\mathbf{x}_j^{(r)}$ as shown in eq. (2):

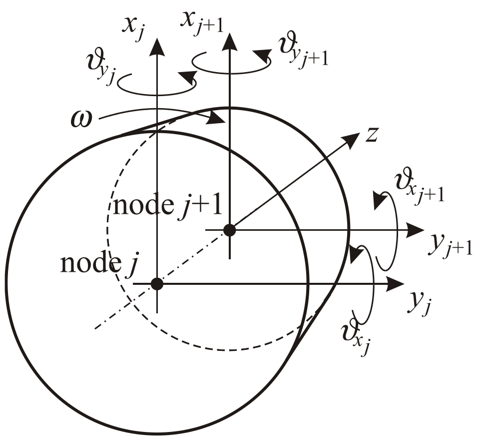

Figure 1 Reference systems of a general rotor element.

$$\mathbf{x}^{(r)} = \left\{ x_1^{(r)} \quad \vartheta_{x_1}^{(r)} \quad y_1^{(r)} \quad \vartheta_{y_1}^{(r)} \quad \cdots \quad x_{n_r}^{(r)} \quad \vartheta_{x_{n_r}}^{(r)} \quad y_{n_r}^{(r)} \quad \vartheta_{y_{n_r}}^{(r)} \right\}^{\mathrm{T}} \tag{2}$$

In a similar manner, also the d.o.f.s of the foundation, horizontal and vertical displacements, which are connected by the l bearings to the rotor, can be ordered in a vector:

$$\mathbf{x}^{(f)} = \left\{ x_1^{(f)} \quad y_1^{(f)} \quad \ldots \quad x_l^{(f)} \quad y_l^{(f)} \right\}^{\mathrm{T}} \tag{3}$$

Obviously, the actual foundation also has internal d.o.f.s. Finally, for the modal model of the foundation, a vector $\boldsymbol{\eta}^{(f)}$ is defined that contains the k ordered modes, that are taken into account in the modal representation:

$$\boldsymbol{\eta}^{(f)} = \left\{ \eta_1^{(f)} \quad \cdots \quad \eta_k^{(f)} \right\}^{\mathrm{T}} \tag{4}$$

Therefore, the vector $\mathbf{x}$ of all the d.o.f.s of the complete assembled model of the machine has order $(4n_r + 2l) \times 1$ and it is ordered by considering first the d.o.f.s of the rotor and then those of the foundation:

$$\mathbf{x} = \begin{Bmatrix} \mathbf{x}^{(r)} \\ \mathbf{x}^{(f)} \end{Bmatrix} \tag{5}$$

while the vector $\boldsymbol{\xi}$ of the d.o.f.s of the complete assembled model with modal foundation has order $(4n_r + 2l) \times 1$ and it is again ordered by considering first the d.o.f.s of the rotor and then the modes of the foundation:

$$\boldsymbol{\xi} = \begin{Bmatrix} \mathbf{x}^{(r)} \\ \boldsymbol{\eta}^{(r)} \end{Bmatrix} \tag{6}$$

Using Lagrange's method for the rotor and neglecting the effect of external forces, the d.o.f.s of the rotor and of the foundation can be considered separately:

$$\begin{gathered} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}^{(r)}}\right) - \frac{\partial T}{\partial x^{(r)}} + \frac{\partial U}{\partial x^{(r)}} = \frac{\delta W}{\delta x^{(r)}} \rightarrow \\ \left[\mathbf{M}^{(r)}\right]\ddot{\mathbf{x}}^{(r)} + \left(\left[\mathbf{C}^{(r)}\right] + \Omega\left[\mathbf{G}^{(r)}\right]\right)\dot{\mathbf{x}}^{(r)} + \left[\mathbf{K}^{(r)}\right]\mathbf{x}^{(r)} = \frac{\delta W}{\delta x^{(r)}} \end{gathered} \tag{7}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}^{(f)}}\right) - \frac{\partial T}{\partial x^{(f)}} + \frac{\partial U}{\partial x^{(f)}} = \frac{\delta W}{\delta x^{(f)}} \rightarrow \left[\mathbf{M}^{(f)}\right]\ddot{\mathbf{x}}^{(f)} + \left[\mathbf{C}^{(f)}\right]\dot{\mathbf{x}}^{(f)} + \left[\mathbf{K}^{(f)}\right]\mathbf{x}^{(f)} = \frac{\delta W}{\delta x^{(f)}} \tag{8}$$

In regards to the rotor train, the mass matrix $[\mathbf{M}^{(r)}]$, which takes also into account the secondary effect of the rotatory inertia, the internal damping matrix $[\mathbf{C}^{(r)}]$, the stiffness matrix $[\mathbf{K}^{(r)}]$, which takes also into account the shear effect, and the gyroscopic matrix $[\mathbf{G}^{(r)}]$, all of order $(4n_r \times 4n_r)$, can be defined by means of standard Lagrange's methods and beam elements as shown e.g. in [41], while the structure of $[\mathbf{M}^{(f)}]$, $[\mathbf{C}^{(f)}]$ and $[\mathbf{K}^{(f)}]$ is not relevant at this stage.

In order to consider the r.h.s of eqs. (7) and (8), large rotating machinery such as that considered later on in this paper, are normally supported by *l* oil-film bearings that realize a coupling between the rotor train and the supporting structure. Even if the exact calculation of the forces exchanged between the journal and the bearing case, due to the oil-film, requires appropriate methods that are far from the scope of present paper, a widely accepted simplification in rotor dynamics simulation (see again [41]) is the modelling of the oil-film force field by means of linearized stiffness and damping coefficients function of the rotating speed. Therefore, the expression of the linearized forces of the oil-film of the *i*-th bearing on the journal located in the *j*-th node is:

$$\mathbf{F}_i^{(br)}(\Omega) = -\begin{bmatrix} k_{xx_i}^{(b)}(\Omega) & 0 & k_{xy_i}^{(b)}(\Omega) & 0 \\ 0 & 0 & 0 & 0 \\ k_{yx_i}^{(b)}(\Omega) & 0 & k_{yy_i}^{(b)}(\Omega) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_j^{(r)} \\ \vartheta_{x_j}^{(r)} \\ y_j^{(r)} \\ \vartheta_{y_j}^{(r)} \end{Bmatrix} - \begin{bmatrix} c_{xx_i}^{(b)}(\Omega) & 0 & c_{xy_i}^{(b)}(\Omega) & 0 \\ 0 & 0 & 0 & 0 \\ c_{yx_i}^{(b)}(\Omega) & 0 & c_{yy_i}^{(b)}(\Omega) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_j^{(r)} \\ \dot{\vartheta}_{x_j}^{(r)} \\ \dot{y}_j^{(r)} \\ \dot{\vartheta}_{y_j}^{(r)} \end{Bmatrix} = \\ = -\left[\mathbf{K}_i^{(b)}(\Omega)\right]\mathbf{x}_j^{(r)} - \left[\mathbf{C}_i^{(b)}(\Omega)\right]\dot{\mathbf{x}}_j^{(r)} \tag{9}$$

while that of the forces on the supporting structure is:

$$\mathbf{F}_i^{(bf)}(\Omega) = -\begin{bmatrix} k_{xx_i}^{(b)}(\Omega) & k_{xy_i}^{(b)}(\Omega) \\ k_{yx_i}^{(b)}(\Omega) & k_{yy_i}^{(b)}(\Omega) \end{bmatrix} \begin{Bmatrix} x_j^{(f)} \\ y_j^{(f)} \end{Bmatrix} - \begin{bmatrix} c_{xx_i}^{(b)}(\Omega) & c_{xy_i}^{(b)}(\Omega) \\ c_{yx_i}^{(b)}(\Omega) & c_{yy_i}^{(b)}(\Omega) \end{bmatrix} \begin{Bmatrix} \dot{x}_j^{(f)} \\ \dot{y}_j^{(f)} \end{Bmatrix} = \\ = -\left[\widehat{\mathbf{K}}_i^{(b)}(\Omega)\right]\mathbf{x}_i^{(f)} - \left[\widehat{\mathbf{C}}_i^{(b)}(\Omega)\right]\dot{\mathbf{x}}_i^{(f)} \tag{10}$$

In order to consider the coupling effect of the oil-film forces their expressions in eqs. (9) and (10) are inserted into the l.h.s. of eqs. (7) and (8), taking into account the relative displacements of the nodes of the rotor and of the foundation in correspondence of the bearings, and the fully assembled system of equation is built up. This requires the definition of the stiffness coupling matrices $[\mathbf{K}^{(rr)}]$, $[\mathbf{K}^{(rf)}]$, $[\mathbf{K}^{(fr)}]$, $[\mathbf{K}^{(ff)}]$ and the corresponding damping matrices $[\mathbf{C}^{(rr)}]$, $[\mathbf{C}^{(rf)}]$, $[\mathbf{C}^{(fr)}]$, $[\mathbf{C}^{(ff)}]$, which are sparse and respectively of order $(4n_r \times 4n_r)$, $(4n_r \times 2l)$, $(2l \times 4n_r)$ and $(2l \times 2l)$. The structure for the stiffness matrices is:

$$[\mathbf{K}^{(rr)}] = \mathrm{diag}\left(\cdots \quad \left[\mathbf{K}_i^{(b)}(\Omega)\right] \quad \cdots\right) \tag{11}$$

$$[\mathbf{K}^{(fr)}] = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & k_{xx_i}^{(b)}(\Omega) & 0 & k_{yx_i}^{(b)}(\Omega) & 0 & \cdots \\ \cdots & k_{xy_i}^{(b)}(\Omega) & 0 & k_{yy_i}^{(b)}(\Omega) & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \quad [\mathbf{K}^{(rf)}] = [\mathbf{K}^{(fr)}]^{\mathrm{T}} \tag{12}$$

$$[\mathbf{K}^{(ff)}] = \mathrm{diag}\left(\cdots \quad \left[\widehat{\mathbf{K}}_i^{(b)}(\Omega)\right] \quad \cdots\right) \tag{13}$$

Damping matrices have similar structure, while the dependence of Ω is omitted for sake of brevity. This way, the fully assembled system of equations results:

$$[\mathbf{M}]\ddot{\mathbf{x}}+[\mathbf{C}]\dot{\mathbf{x}}+[\mathbf{K}]\mathbf{x}=0 \tag{14}$$

with:

$$[\mathbf{M}]=\begin{bmatrix}[\mathbf{M}^{(r)}] & 0\\ 0 & [\mathbf{M}^{(f)}]\end{bmatrix} \tag{15}$$

$$[\mathbf{C}]=\begin{bmatrix}[\mathbf{C}^{(r)}]+\Omega[\mathbf{G}^{(r)}]+[\mathbf{C}^{(rr)}] & -[\mathbf{C}^{(rf)}]\\ -[\mathbf{C}^{(fr)}] & [\mathbf{C}^{(f)}]+[\mathbf{C}^{(ff)}]\end{bmatrix} \tag{16}$$

$$[\mathbf{K}]=\begin{bmatrix}[\mathbf{K}^{(r)}]+[\mathbf{K}^{(rr)}] & -[\mathbf{K}^{(rf)}]\\ -[\mathbf{K}^{(fr)}] & [\mathbf{K}^{(f)}]+[\mathbf{K}^{(ff)}]\end{bmatrix} \tag{17}$$

Now, in order to consider the modal representation of the foundation, the modal mass matrix $[\tilde{\mathbf{M}}^{(f)}]$, the modal damping matrix $[\tilde{\mathbf{C}}^{(f)}]$, the modal stiffness matrix $[\tilde{\mathbf{K}}^{(f)}]$ and the eigenvector matrix $[\Phi]$, respectively of order $(k\times k)$, $(k\times k)$, $(k\times k)$ and $(2l\times k)$, are defined as:

$$[\tilde{\mathbf{M}}^{(f)}]=\mathrm{diag}(m_1^{(f)},\dots,m_k^{(f)}) \tag{18}$$

$$[\tilde{\mathbf{C}}^{(f)}]=\mathrm{diag}(c_1^{(f)},\dots,c_k^{(f)}) \tag{19}$$

$$[\tilde{\mathbf{K}}^{(f)}]=\mathrm{diag}(k_1^{(f)},\dots,k_k^{(f)}) \tag{20}$$

$$[\Phi]=\begin{bmatrix}X_{1,1}^{(f)} & \cdots & X_{1,k}^{(f)}\\ Y_{1,1}^{(f)} & \cdots & Y_{1,k}^{(f)}\\ \vdots & \vdots & \vdots\\ X_{l,1}^{(f)} & \cdots & X_{l,k}^{(f)}\\ Y_{l,1}^{(f)} & \cdots & Y_{l,k}^{(f)}\end{bmatrix} \tag{21}$$

and the modal transformation in eq.(22) is used.

$$\mathbf{x}^{(f)} = \left[\Phi\right]\boldsymbol{\eta}^{(f)} \tag{22}$$

The equation system of fully assembled rotating machine with modal foundation becomes:

$$\left[\bar{\mathbf{M}}\right]\ddot{\xi} + \left[\bar{\mathbf{C}}\right]\dot{\xi} + \left[\bar{\mathbf{K}}\right]\xi = 0 \tag{23}$$

where:

$$[\bar{\mathbf{M}}] = \begin{bmatrix} [\mathbf{M}^{(r)}] & 0 \\ 0 & [\tilde{\mathbf{M}}^{(f)}] \end{bmatrix} \tag{24}$$

$$[\bar{\mathbf{C}}] = \begin{bmatrix} [\mathbf{C}^{(r)}] + \Omega[\mathbf{G}^{(r)}] + [\mathbf{C}^{(rr)}] & -[\mathbf{C}^{(rf)}]\times[\Phi] \\ -[\Phi]^{\mathrm{T}}\times[\mathbf{C}^{(fr)}] & [\tilde{\mathbf{C}}^{(f)}] + [\Phi]^{\mathrm{T}}\times[\mathbf{C}^{(ff)}]\times[\Phi] \end{bmatrix} \tag{25}$$

$$[\bar{\mathbf{K}}] = \begin{bmatrix} [\mathbf{K}^{(r)}] + [\mathbf{K}^{(rr)}] & -[\mathbf{K}^{(rf)}]\times[\Phi] \\ -[\Phi]^{\mathrm{T}}\times[\mathbf{K}^{(fr)}] & [\tilde{\mathbf{K}}^{(f)}] + [\Phi]^{\mathrm{T}}\times[\mathbf{K}^{(ff)}]\times[\Phi] \end{bmatrix} \tag{26}$$

The matrices $\tilde{\mathbf{M}}^{(f)}$, $\tilde{\mathbf{C}}^{(f)}$ and $\tilde{\mathbf{K}}^{(f)}$ are diagonal under hypotheses largely acceptable in practice. They are only conceptually computed from a matrix representation of the whole foundation. In practice their value can be identified from experimental data or even assigned some what arbitrarily as a first approximation and then tuned using experimental information.

3 FAULT IDENTIFICATION

As it is was observed in the introduction, fault identification is performed by means of parity equations, by introducing external forces that are equivalent to the fault developed, rather than with parameter estimation approach, because the system parameters influence generally the complete mass, stiffness and damping matrices of the system. This way, few unknowns of the equivalent force have to be estimated. The estimation of external forces appears a simpler task than the estimation of system parameters, even if successful applications in rotor dynamics have been presented in literature [35,37], but limited to test-rigs o to small machines with few d.o.f.s. However, this approach requires that the model of machine, i.e. of the parts that compose it, is reliable and some tuning is usually done on it, but also the use of the nominal model of the machines, without costly fine tuning, allowed successful identification of faults in large real machines as shown in [20,42].

3.1 Definition of equivalent external forces

In real large rotating machines, the measuring points of the vibration along the shaft are few and the transducers are normally in correspondence of the bearings. The consideration of further measuring planes is practically impossible, therefore methods that reconstruct modal shapes of the rotor such those in [38] cannot be used. So, if the system of dynamic equations of a large rotating machine, with several d.o.f.s, is considered:

$$[\mathbf{M}]\ddot{\mathbf{x}}_t + [\mathbf{C}]\dot{\mathbf{x}}_t + [\mathbf{K}]\mathbf{x}_t = \mathbf{F}(t) \tag{27}$$

it seems difficult to identify the changes due to the developing fault in the matrices $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$, which are of high order, from measurement of vibration $\mathbf{x}_t$ in only few measuring planes along the shaft. The r.h.s. external forces $\mathbf{F}(t)$ are composed by the weight (which is known) and by the original unbalance and bow (which are unknown). The system parameter changes due to the fault are indicated as $[d\mathbf{M}]$, $[d\mathbf{C}]$ and $[d\mathbf{K}]$, and eq. (27) becomes:

$$([\mathbf{M}]+[d\mathbf{M}])\ddot{\mathbf{x}}_t + ([\mathbf{C}]+[d\mathbf{C}])\dot{\mathbf{x}}_t + ([\mathbf{K}]+[d\mathbf{K}])\mathbf{x}_t = \mathbf{W} + (U + M_u)\ e^{i\Omega t} \tag{28}$$

If the system behaviour is considered to be linear, which is acceptable for a wide class of faults as shown in [14,15,17,43], then the total vibration $\mathbf{x}_t$ can be considered as due to two superposed effects:

$$\mathbf{x}_t = \mathbf{x}_1 + \mathbf{x} \tag{29}$$

The first vibration vector $\mathbf{x}_1$ is the pre-fault vibration, which is due to the weight $\mathbf{W}$ and the unknown unbalance force $U\ e^{i\Omega t}$ and unbalance moment $M_u\ e^{i\Omega t}$. The second vibration $\mathbf{x}$ is due to the developing fault. The last is also called additional vibration. The vibration component $\mathbf{x}$ may be obtained by calculating the vector differences of the actual vibrations (due to weight, original unbalance, bow and fault) and the original vibrations measured, in the same operating conditions in a reference case (rotation speed, flow rate, power, temperature, etc.) before the fault was developing. A discussion about the possible errors introduced and their tracking in presented in [19]. Recalling the definition of the pre-fault vibration $\mathbf{x}_1$, the following equation holds:

$$[\mathbf{M}]\ddot{\mathbf{x}}_1 + [\mathbf{C}]\dot{\mathbf{x}}_1 + [\mathbf{K}]\mathbf{x}_1 = \mathbf{W} + (U + M_u)\ e^{i\Omega t} \tag{30}$$

which substituted in eq. (28) with eq. (29) gives:

$$[\mathbf{M}]\ddot{\mathbf{x}} + [\mathbf{C}]\dot{\mathbf{x}} + [\mathbf{K}]\mathbf{x} = -[d\mathbf{M}]\ddot{\mathbf{x}}_t - [d\mathbf{C}]\dot{\mathbf{x}}_t - [d\mathbf{K}]\mathbf{x}_t \tag{31}$$

The r.h.s. of eq. (31) can be considered as a system of equivalent external forces which force the fault-free system to have the change in the additional vibration $\mathbf{x}$ that is due to the developing fault only:

$$[\mathbf{M}]\ddot{\mathbf{x}} + [\mathbf{C}]\dot{\mathbf{x}} + [\mathbf{K}]\mathbf{x} = \mathbf{F}_f(t) \tag{32}$$

Note that in eq. (32) system parameters are time invariant and known, but depending from the operating speed as shown in the previous chapter. Moreover, using this last approach, the problem of fault identification is reduced to a force identification by means of parity equations. Keeping in mind that the final goal is the identification of faults, this approach is preferred since the unknown fault forcing vector is sparse and only few elements are actually different from zero, because the equivalent force is applied on few nodes of the rotor model. This fact reduces sensibly the number of unknowns to be identified. A rather complete overview of the equivalent forcing systems to the most common faults in rotating machinery is presented in [14,15,17,43].

3.2 Weighted least square identification in frequency domain

Often in real machines the condition monitoring system allows to measure the vibrations of the machine during speed transients. Run-downs are generally preferred since it is assumed that the machine begins its speed transient starting from nearly equal initial conditions of operating. Additional vibrations are then obtained by means of the difference between a reference condition and the faulty condition. Due to the big inertia of the machines, these speed transient are normally long time lasting, of the order of several minutes. Therefore the transient operating condition is considered as a succession of steady-states, so much so that commercial condition monitoring systems make a synchronous sampling and an order analysis of the vibration.

If the harmonic balance criteria is applied to eq. (32), the following equations are obtained for each harmonic component, in which the force vector $\mathbf{F}_{f_n}$, has to be identified:

$$\left[-(n\Omega)^2\mathbf{M} + in\Omega\mathbf{C} + \mathbf{K}\right]\mathbf{X}_n = \mathbf{F}_{f_n} \tag{33}$$

A further advantage of this approach is that many types of faults of rotating machinery has effect, and thus related symptoms are acting, on few harmonic components as well known in literature starting from [44]. Moreover, since linearity in the system is considered, the effect of *m* faults developing simultaneously can be considered by means of the superposition of the effects for each harmonic component:

$$\mathbf{F}_{f_n} = \sum_{i=1}^{m}\mathbf{F}_{f_n}^{(i)} \tag{34}$$

Moreover, the *k*-th fault acts on few d.o.f. of the system, therefore the vector $\mathbf{F}_{f_n}^{(k)}$ is not a full-element vector which is convenient to be represented by means of:

$$\mathbf{F}_{f_n}^{(k)} = \left\{\mathbf{F}_L^{(k)}\right\}\bar{A}^{(k)}(\Omega) \tag{35}$$

where $\left\{\mathbf{F}_L^{(k)}\right\}$ is the localisation vector which has all null-elements except for the d.o.f. to which the forcing system is applied, and $\bar{A}^{(k)}(\Omega)$ is a complex number representing the amplitude and the phase of the fault.

In the case considered in the paper, in which an unbalance is going to be identified, it is sufficient to consider the 1x rev. component of the vibration. Taking into account that the foundation has a modal representation, the *k*-th unbalance can be expressed as:

$$\mathbf{F}_{f_1}^{(k)} = \left\{0 \quad \vdots \quad \underbrace{1 \quad 0 \quad i \quad 0}_{j\text{-th rotor node}} \quad \vdots \quad 0 \quad \underbrace{0 \quad \cdots \quad 0}_{\text{foundation modes}}\right\}^{\mathrm{T}} \cdot \left(mr\right)^{(k)}\Omega^2 e^{i\varphi^{(k)}} = \left\{\mathbf{F}_L^{(k)}\right\}\bar{A}^{(k)}(\Omega) \tag{36}$$

where the only elements different from zero are those relative to the horizontal and vertical d.o.f.s of the node *j*, where the unbalance is supposed to be applied.

Eq. (33) can be now rewritten considering that condition monitoring systems collects data for many rotating speeds, so the additional vibrations are available for several rotating speeds and a set of p rotating speeds is considered:

$$\mathbf{\Omega} = \left\{ \Omega_1 \;\; \Omega_2 \;\; \cdots \;\; \Omega_p \right\}^{\mathrm{T}} \tag{37}$$

Then, introducing the admittance matrix of the system, eq. (33) becomes:

$$\left[\mathbf{E}\left(n\mathbf{\Omega}\right) \right] \; \mathbf{X}_n = \begin{bmatrix} \mathbf{E}\left(n\Omega_1\right) & 0 & 0 & 0 \\ 0 & \mathbf{E}\left(n\Omega_2\right) & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \mathbf{E}\left(n\Omega_p\right) \end{bmatrix} \begin{Bmatrix} \mathbf{X}_n \\ \mathbf{X}_n \\ \vdots \\ \mathbf{X}_n \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{m} \mathbf{F}_{f_n}^{(i)}(\Omega_1) \\ \sum_{i=1}^{m} \mathbf{F}_{f_n}^{(i)}(\Omega_2) \\ \vdots \\ \sum_{i=1}^{m} \mathbf{F}_{f_n}^{(i)}(\Omega_p) \end{Bmatrix} = \mathbf{F}_{f_n}(\mathbf{\Omega}) \tag{38}$$

Normally, only a developing fault is going to be identified, since in actual machines the occurrence of multiple simultaneous faults is an uncommon occurrence and possibly avoided by safety procedures, indeed the presented method is general and have been applied in test-rigs and in real machines also in case of two simultaneous faults [17,20,42]. Anyhow the equivalent force fault identification problem in eq. (38) is overdetermined since the number of the observation (the measured vibrations at different rotating speeds) are grater than the number of the parameter of the fault that have to be identified. The procedure used to solve the problem is the following.

Let the machine model have n_r nodes and l bearings in which the vibrations $\mathbf{X}_{\mathrm{B}mn}$ are measured at p rotating speeds in two orthogonal directions, and the support structure is represented by means of a modal foundation with k modes. The model d.o.f.s are $(4n_r + k)$ while per each rotating speed are measured only $2l$ d.o.f.s.

Since the fault have to be identified not only in its severity but also in its position, in general the procedure have to be repeated per each node of the rotor, unless the research of the fault is limited in a specified interval of the nodes.

Eq. (38) represents the general system of equations for all the d.o.f.s of the considered fully assembled machine. The admittance matrix $\left[\mathbf{E}(n\mathbf{\Omega})\right]$ has order $\left((4n_r + k)p \times (4n_r + k)p\right)$.

Now, weighted least square identification is used in order to evaluate the module, the phase and a residual of a particular fault type, for instance an unbalance, starting from the first node.

The weighted least square technique is employed in order to increase the robustness of the identification. It is known that least square estimators assume that the noise corrupting the experimental data is of zero mean [45], i.e. an unbiased parameter estimate is obtained. On the field, sensor malfunction or the exceeding of the sensibility range crossing critical speeds can introduce bias. The use of weights, which allow to reduce the relevance of some measures or to discard them at all, increases the robustness of the identification. Similarly, the discarding of all experimental data corresponding to speeds close to critical ones and the variance analysis of experimental data of the same machine in similar speed-transient condition allow to increase the robustness of the estimate. These issues are analyzed in detail in [17,40] on real machines and test-rigs. Other techniques that improve the robustness of the least square method, that can be classified as regression diagnostics [45] or residual analysis, are discussed and applied on numerical models and on real machines in [19,46].

The equivalent force system, for each one of the m forces considered, is applied in each node of the rotor model, so for all the rotating speed the fault vector is of order $\left((4n_r+k)p\right)\times m$; taking into account that in this case the unbalance is considered, in order to eliminate the dependence on the rotating speed square that appear in the complex amplitude of eq. (36), it is convenient to insert Ω_j^2 in the localization vector of each rotating speed. Therefore, in the first node, the localization vector of fault i-th, for the j-th rotating speed is:

$$\left\{\mathbf{F}_L^{(i),(j)}\right\} = \Omega_j^2 \left\{ \underbrace{1 \quad 0 \quad i \quad 0}_{1^{st}\text{ node}} \quad \underbrace{0 \quad 0 \quad 0 \quad 0}_{2^{nd}\text{ node}} \quad \vdots \quad \underbrace{0 \quad 0 \quad 0 \quad 0}_{n_r^{th}\text{ node}} \quad \underbrace{0 \quad \vdots \quad 0}_{k\text{ modes}} \right\}^{\mathrm{T}} \tag{39}$$

and for all the p rotating speeds:

$$\mathbf{F}_L^{(i)} = \begin{Bmatrix} \left\{\mathbf{F}_L^{(i),(1)}\right\} \\ \vdots \\ \left\{\mathbf{F}_L^{(i),(p)}\right\} \end{Bmatrix} \tag{40}$$

The effect on the measured d.o.f.s $\left[\hat{\mathbf{X}}_{\mathrm{B}_n}\right]$, which vector is of order $(2lp\times m)$, due to unitary force systems applied in the first node on the model is now calculated. This is done by first substituting eq. (39) in the right hand side of eq. (38), and inverting matrix $\left[\mathbf{E}(n\boldsymbol{\Omega})\right]$, obtaining the transfer matrix $\left[\mathbf{H}(n\boldsymbol{\Omega})\right]$.

$$\mathbf{X}_n = \left[\mathbf{E}\left(n\boldsymbol{\Omega}\right)\right]^{-1}\mathbf{F}_{f_n}(\boldsymbol{\Omega}) = \left[\mathbf{H}\left(n\boldsymbol{\Omega}\right)\right]\mathbf{F}_{f_n}(\boldsymbol{\Omega}) \tag{41}$$

Then, the vibrations of the d.o.f.s, which are measured, are separated from the all the d.o.f.s of the system, by considering only the rows of $\left[\mathbf{H}(n\boldsymbol{\Omega})\right]$ corresponding to the measured d.o.f.s. The partitioned matrix is of order $\left(2lp\times(4n_r+k)p\right)$. Also the weight matrix of size $\left(2lp\times 2lp\right)$, which is for the *j*-th rotating speed:

$$\left[\mathbf{W}^{(j)}\right] = \mathrm{diag}\left(w_1^{(j)},\ldots,w_{2l}^{(j)}\right) \tag{42}$$

and where $w_i^{(j)}=1$ indicates that the corresponding measure is fully considered at the *j*-th rotating speed, is introduced and results:

$$\left[\hat{\mathbf{X}}_{\mathrm{B}n}\right] = \left[\mathbf{W}\right]\left[\mathbf{H}\left(n\Omega\right)\right]\Big|_{\substack{mesured\\ d.o.f.s}}\left[\mathbf{F}_L^{(1)} \;\; \vdots \;\; \mathbf{F}_L^{(m)}\right] \tag{43}$$

Now the array, of order $\left(m\times 1\right)$, of the complex values $\overline{A}^{(i)}$ (i.e. the modules and phases) of the equivalent force systems applied in the first node that fits best the experimental data $\mathbf{X}_{\mathrm{B}m_n}$, of order $\left(2lp\times 1\right)$, have to be estimated. The fitting is done in least square sense, since the number of the unknown (the modules and the phases) is less than the equations (recalling that data are corresponding to several rotating speeds and each of the sets is composed by several measuring planes, while the number of the faults in practical applications is one or two). The problem is equivalent to:

$$\min\left\|\left[\hat{\mathbf{X}}_{\mathrm{B}_n}\right]\begin{Bmatrix}\overline{A}^{(1)}\\ \vdots\\ \overline{A}^{(m)}\end{Bmatrix} - \mathbf{X}_{\mathrm{B}m_n}\right\| \tag{44}$$

whose general solution is given by means of the pseudo-inverse calculation:

$$\mathbf{A}^{(1)} = \left(\left[\hat{\mathbf{X}}_{\mathrm{B}n} \right]^{\mathrm{T}} \left[\hat{\mathbf{X}}_{\mathrm{B}n} \right] \right)^{-1} \left[\hat{\mathbf{X}}_{\mathrm{B}n} \right]^{\mathrm{T}} \left[\mathbf{W} \right] \mathbf{X}_{\mathrm{B}mn} \tag{45}$$

A full discussion on the possible numerical errors in calculating eq. (45) is reported in [47]. The modules and the phases of the complex values in the m rows of $\mathbf{A}^{(1)}$ are the identified faults in the first rotor node. Finally the residual in the first rotor node is determined, first obtaining the calculated response due to the identified fault in the first node:

$$\mathbf{X}_{\mathrm{B}n} = \left[\hat{\mathbf{X}}_{\mathrm{B}n} \right] \mathbf{A}^{(1)} \tag{46}$$

and then normalizing it:

$$\delta_{r_n}^{(1)} = \left(\frac{\left[\mathbf{X}_{\mathrm{B}_n} - \mathbf{X}_{\mathrm{B}m_n} \right]^{*\mathrm{T}} \left[\mathbf{X}_{\mathrm{B}_n} - \mathbf{X}_{\mathrm{B}m_n} \right]}{\mathbf{X}_{\mathrm{B}m_n}^{*\mathrm{T}} \; \mathbf{X}_{\mathrm{B}m_n}} \right)^{1/2} \tag{47}$$

The procedure is then iterated for all the n_r nodes of the rotor. If a fault only is considered, a set in $\mathbb{R}$ of relative residuals given by eq. (47), ordered by the node number, is obtained:

$$\boldsymbol{\delta}_{r_n} = \left(\delta_{r_n}^{(1)}, \ldots, \delta_{r_n}^{(n_r)} \right) \tag{48}$$

The s-th node location that corresponds to the minimum value of eq. (48) indicates the most probable location of the fault, whose estimation is given by the corresponding value of eq. (45). If m faults are taken into account, all the combination of the faults and the nodes have to be considered and the set of the relative residuals is in $\mathbb{R}^m$ space (see figure 2). The minimum of the set indicates in each one of the m dimensions the location of the corresponding fault and their estimation is given by the corresponding value of eq. (47). The closer to zero the minimum value of eq. (48) is, the better the estimation of the faults is. An effective way to graphically represent the relative residual as function of the rotor nodes up to two faults is presented in [17].

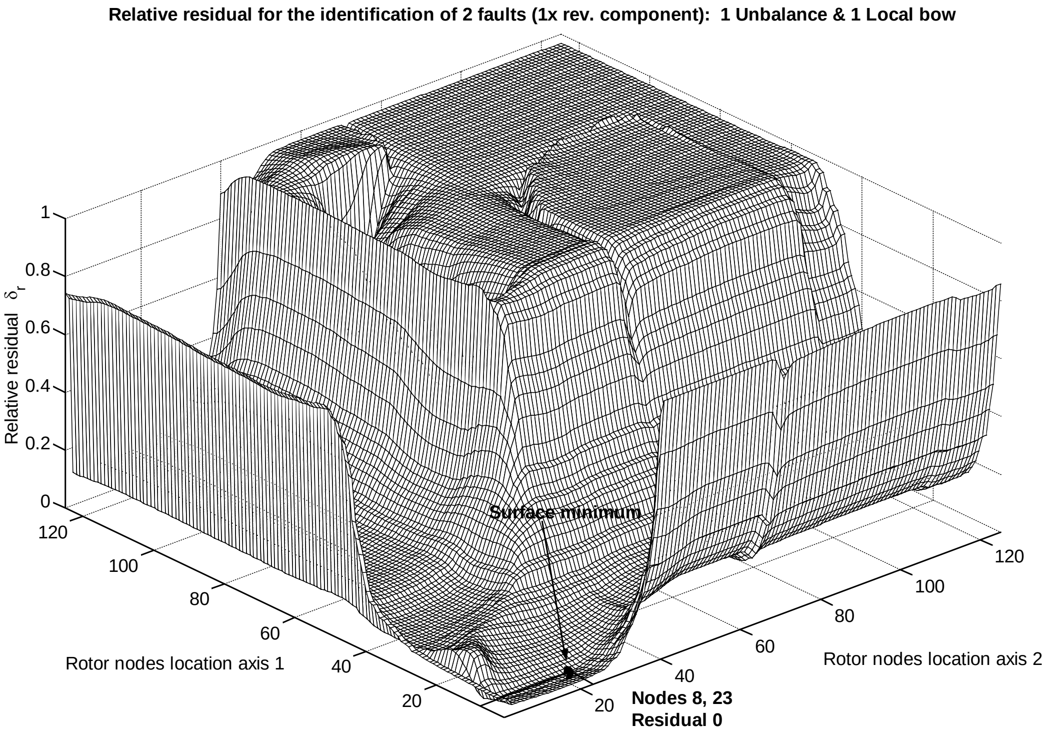

Figure 2 Relative residual minimum in case of two faults.

4 CONCLUSIONS

The first part of the paper showed how the modal model of the supporting structure can be easily included in the whole equation system of a rotating machine. The algorithmic developments of the method demonstrate that there are no particular constraints in the modal model of the foundation, such as completeness or problems of singular matrices. A modal model of the foundation can therefore be integrated in the frequency domain identification algorithm, illustrated in some detail in the paper, thus offering an effective alternative to simpler representations of the foundation and providing the specialist with some more degrees of freedom to get a better tuning of the whole system model to the measured frequency response curves. The modal parameters do not have to be identified by expensive modal tests, but can be obtained by pre-processing of data collected in normal operation or even judiciously assigned and, in case, further tuned.

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