

Enabling Onboard Analytical Batch Filtering for Angles-Only Navigation to Non-Cooperative Space Targets

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Abstract. Angles-only relative navigation enables autonomous operations with non-cooperative targets such as defunct satellites and orbital debris, which lack the means of communicating with other satellites or ground stations. In fact, this navigation paradigm relies on gathering information about the target only by means of passive cameras mounted on the chaser satellite. The camera provides a pair of bearing angles which are used by the navigation filter to estimate the relative orbit. As of today, sequential filters are typically employed for on-board applications due to their computational efficiency, whereas batch filters are mostly confined to ground-based operations, where their higher computational cost poses no limitation. In this work we revisit the semi-analytic batch least-squares (LSQ) filtering framework with the aim of bringing its on-board implementation closer to practical feasibility. The proposed LSQ filter preserves the analytical propagation required for real-time use while improving accuracy and observability through three key upgrades: (i) a closed-form propagation model including first-order J_2 , J_4 , J_6 and second-order Keplerian and J_2 effects; (ii) an exact measurement model with improved mean-to-osculating transformation; (iii) an efficient circular-buffer configuration. The proposed filter is tested in a high-fidelity 750 km sun-synchronous orbit simulation with realistic perturbations and is shown to achieve accurate relative state estimation without manoeuvre-induced observability.

Introduction

Angles-only relative navigation is a key technology for autonomous operations with non-cooperative resident space objects such as defunct satellites and debris lacking GNSS receivers, inter-satellite links or cooperative beacons. Missions such as on-orbit inspection, servicing, end-of-life disposal and active debris removal [1, 2, 3] must therefore rely on navigation systems capable of estimating relative states solely from onboard passive optical sensors [4]. Cameras are ideal due to their availability, low power, light weight and wide dynamic range. While sequential filters (extended/unscented Kalman filter) are the standard choice for onboard implementation [5], batch least-squares (LSQ) filtering may offer a robust complementary approach for onboard state estimation. Although traditionally used for precise ground-based orbit determination, an analytical formulation with a sliding observation window might enable computationally efficient onboard use, as suggested (but not demonstrated) by Ardaens [6]. In fact, these filters could serve as: (i) Cross-check mechanisms to reduce/eliminate outliers before they are used by the sequential filter. (ii) Re-initialisation in the event of filter divergence. (iii) Entirely novel hybrid batch-sequential architectures. This work extends the analytical LSQ framework by Gaias et al. [7], which used bearing angles and relative orbital elements (ROE) as states. Their inclusion of J_2 yielded weak observability compared to the completely unobservable Keplerian case studied in [8]. However, their relatively benign scenario characterised by high altitude, negligible differential drag (DD) and simplified propagation and measurement models limits applicability and estimation accuracy.



Specifically, the J_2 model did not include effects induced by non-zero semimajor-axis. Moreover, the first-order mapping from relative orbital elements and line of sight (LoS) omitted the orbital curvature and mean-to-osculating transformations, reducing accuracy and observability. Nonetheless, the fully analytical nature proposed in [7] made the algorithm highly efficient. Here, the manoeuvre-free phase presented in [7] is revisited with the aim of improving estimation accuracy and observability while preserving analytical tractability. The proposed LSQ-based filter introduces the following improvements: (i) Enhanced propagation: analytical model including first-order J_2 , J_4 , J_6 and second-order Keplerian and J_2 effects [9]. (ii) Improved measurement: exact non-linear LoS transformation coupled with a refined mean-to-osculating transformation [10]. (iii) Circular-buffer implementation: the data batch is implemented as a sliding window, sized according to [11] and managed as a circular buffer to reduce memory operations. The LSQ filter is validated in a model-in-the-loop setup using a high-fidelity orbital simulator that accounts for higher-order gravity, third body effects, solar radiation pressure, and atmospheric drag. Camera noise, illumination, and shadowing are excluded in this first analysis and will be addressed in future work. The test scenario, consistent with [7], is inspired by the PRISMA mission and uses a 750 km sun-synchronous orbit. No manoeuvres is executed to avoid any observability enhancement. Preliminary results demonstrate good filter performance and a notable improvement in estimation accuracy compared to typical TLE-level initialization.

Propagation Model

Quasi-non-singular ROE provide an intuitive parametrisation of the relative motion of the target (“ T ”) with respect to the chaser (“ C ”). Recalling the classical orbital elements (OE) vector $\alpha = [a, e, i, \Omega, \omega, M]^T$, the ROE are as a non-linear combination of the of α_C and α_T [9]. In particular, the dimensional (quasi-non-singular) ROE are defined as $a\delta\alpha = a_C[\delta a, \delta\lambda, \delta e_x, \delta e_y, \delta i_x, \delta i_y]^T$, where the components denote relative semi-major axis, mean relative longitude, eccentricity and inclination vectors. This representation gives a geometrically meaningful description of the motion in the Radial-Transversal-Normal frame (R) and separates the weakly observable range analogue $a\delta\lambda$ from the strongly observable components [7]. The GCL model [9] is selected for the LSQ filter since it accurately propagates $a\delta\alpha(t_0)$ at initial time t_0 to time t , accounting for J_2 , J_4 , J_6 , and second-order Keplerian and J_2 effects (DD is not modelled). The manoeuvre-free, analytical propagation model is:

$$a\delta\alpha(t) = \mathbf{F}_{\text{GCL}}(t_0, t, a\delta\alpha(t_0), \alpha(t_0)). \quad (1)$$

Measurement Model

In angles-only navigation, the chaser perceives the target solely through the bearing angles vector composed by a pair of azimuth (α) and elevation (ϵ) angles measured in the camera frame. In this article we follow the convention used in [5]. The camera frame (C) is oriented in anti-flight direction as in [7]. At time t , the navigation subsystem knows: the current knowledge of the ROE (from the filter), the chaser absolute OE (from external sources, e.g., GNSS) and the orientation of the inertial frame (I) relative to the camera ${}^C\mathbf{R}_I(t)$. The LoS vector is thus obtained using the exact fully non-linear transformation reported in [10]. By then the applying the definition of bearing angles, the resulting complete non-linear sequence of operations defines the modelled measurement:

$$\mathbf{y}(t) = [\alpha(t), \epsilon(t)]^T = \mathbf{h}_{\text{NL}}(a\delta\alpha(t), \alpha_C(t), {}^C\mathbf{R}_I(t)). \quad (2)$$

Circular Buffer Batch LSQ

In this work we implement an updated version of batch LSQ filter proposed in [7]. In fact, we use the more accurate [11] GCL analytical propagation model and, instead of relying on the

inaccurate ROE-to-LoS mapping presented in [7], we compute it numerically by taking the Jacobian of Eq. (2) with respect to $a\delta\alpha(t)$. This results in a slightly higher computational cost which is however offset by the much higher accuracy [11]. The LSQ filter operates on a data structure composed of B data-points. The “batch size” B corresponds to the number of data-points available to the filter where the latter are *synchronous* collections of measurement time t_{meas} and corresponding observed bearing angles pair $\mathbf{z}(t_{\text{meas}})$, chaser absolute orbit $\mathbf{a}_C(t_{\text{meas}})$ and camera attitude ${}^C\mathbf{R}_I(t_{\text{meas}})$. When a new measurement arrives, it is stored in a fixed, preallocated memory block together with the corresponding timestamp and the chaser absolute OE and camera attitude at that same instant. This operation effectively adds a new data point to the batch. This memory acts as a sliding window managed as a circular buffer, where the write position advances with each new entry and wraps around once the buffer is full, overwriting the oldest data to maintain a constant batch size (defined according to the guidelines of our previous work [11]). This strategy avoids physically shifting data in memory. Instead, only the read position is updated. When this index exceeds the buffer size, it is reset to the initial position. The process repeats, overwriting old data with a first-in, first-out logic. Once the buffer is fully updated, corresponding to a complete non-overlapping shift of the sliding window, the LSQ algorithm is executed using the current buffer contents as input. The selected algorithm also requires an *a priori* estimate of the $a\delta\alpha$ and its associated uncertainty. During the first execution, this information is provided by external sources (e.g., NORAD TLE). For subsequent executions, the *a priori* information is derived from the previous estimate, propagated to the new estimation epoch. Additionally, the LSQ requires specifying the time instant at which the estimate is computed. In this work, it corresponds to the earliest measurement time within the buffer.

Preliminary Results

The LSQ filter undergoes model-in-the-loop validation by means of a high-fidelity simulation tool. The ground-truth simulation is obtained over 20 chaser orbital periods using a Dormand-Prince 5th-order integrator with a fixed step of 10 (s). The dynamical model includes the GGM03S gravity field up to degree and order 30, a cannonball drag model and atmospheric density computed from NRLMSISE-00. Camera noise, illumination and shadowing are excluded in this first analysis and will be addressed in future work. No manoeuvres are simulated to avoid artificial observability enhancement. Coherently with [7], the simulated case study is inspired by the PRISMA mission and uses a 750 km sun-synchronous orbit with LTAN = 9.5 hours for the chaser absolute orbit, whose spacecraft physical parameters can be found in [6]. The ground-truth initial conditions for the relative orbit are taken from Table 6 of [7] which also provide the initial guess for the batch LSQ filter with an error consistent with NORAD TLE accuracy, representing typical knowledge for debris or non-cooperative targets. The same level of uncertainty is the one used by the filter for the first call, in particular we assume that the *a priori* standard deviation vector $\sigma_{a\delta\alpha} = [20, 1000, 100, 100, 100, 100]^T$. Moreover, for the successive calls, a minimum uncertainty is always enforced on the propagated uncertainty as suggested by [7]: $\sigma_{a\delta\alpha}^{\text{min}} = [3, 50, 10, 10, 10, 10]^T$. Each time the circular buffer completes a full update, the LSQ algorithm receives the 120 data-points batch of data and performs the estimate. In this paper, the camera is assumed to output a valid measurement every 250 (s). Thus, with a circular buffer of 120 data-points, the measurements are uniformly spread over almost 5 full chaser orbit periods, as suggested in [11]. The results of the periodic estimation outcome are reported in Fig. 1 for the first run of the least square in Fig. 2 for the last run.

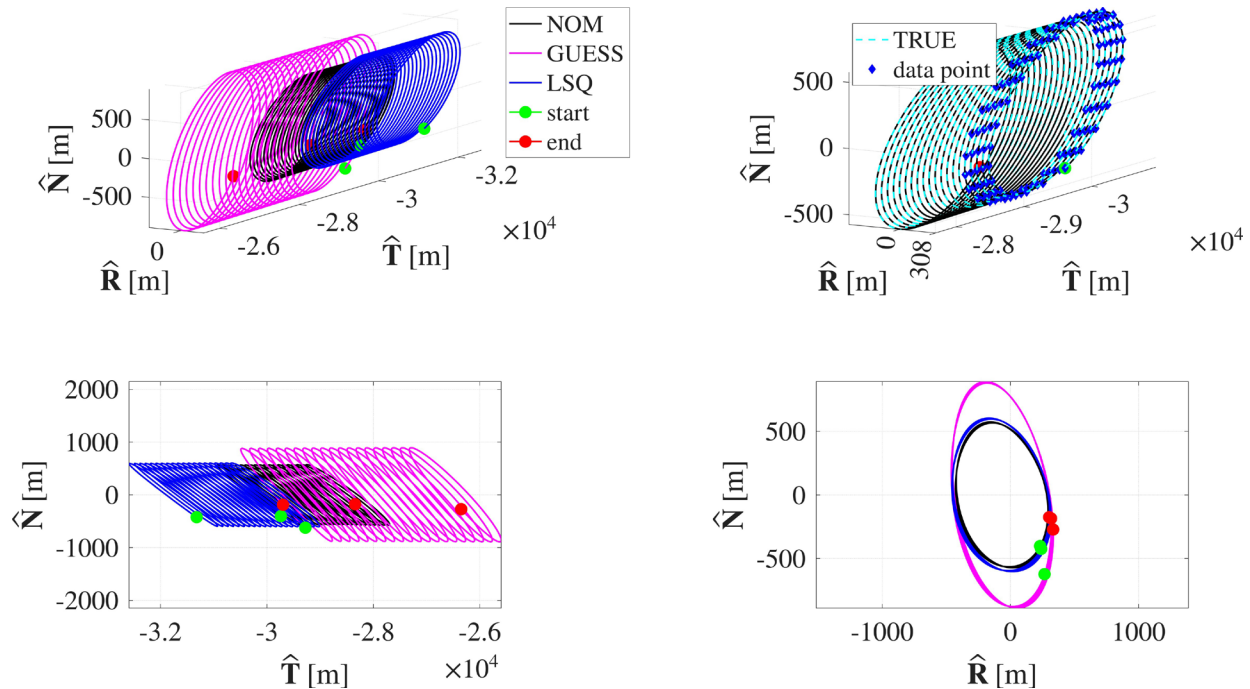


Figure 1: Results of the estimation procedure. Data collected over orbits 1 to 5.

Each figure consists of four plots. The top-left panel shows a 3D view of three relative orbits in the R frame, centred on the chaser spacecraft. The nominal (NOM) orbit, in solid black, is obtained by propagating the *true* initial conditions with the analytical model $\mathbf{F}_{\text{GCL}}$. The initial guess (GUESS) orbit, in solid magenta, is propagated from the filter *a priori* guess, while the post-estimation (LSQ) orbit, in solid blue, uses the converged LSQ output. All orbits are propagated with the analytical model of Eq. 1, thus neglecting DD. The green and red markers indicate the start and end points, respectively. The second row presents the TN- and RN-plane projections (bottom-left and bottom-right plots). The top-right panel shows an additional 3D view, where the true (TRUE) orbit, obtained from the high-fidelity simulator, is superimposed on the nominal one with a dashed cyan line. The case study is inspired from PRISMA mission, where the high altitude orbit and almost equivalent ballistic coefficients of the two spacecraft [6] make DD effects negligible. Thus, given the high accuracy of the GCL model, NOM and TRUE orbits are practically equivalent. From the first figure, it is evident that the initial guess (derived from TLE) is significantly improved by the LSQ, despite the large uncertainty on $a\delta\lambda$. Recall that the authors of [7] had to impose an unrealistically small range uncertainty in the LSQ to ensure convergence, which in turn prevented meaningful updates of $a\delta\lambda$. The enhanced observability provided by the improved models allows this component to be accurately estimated.

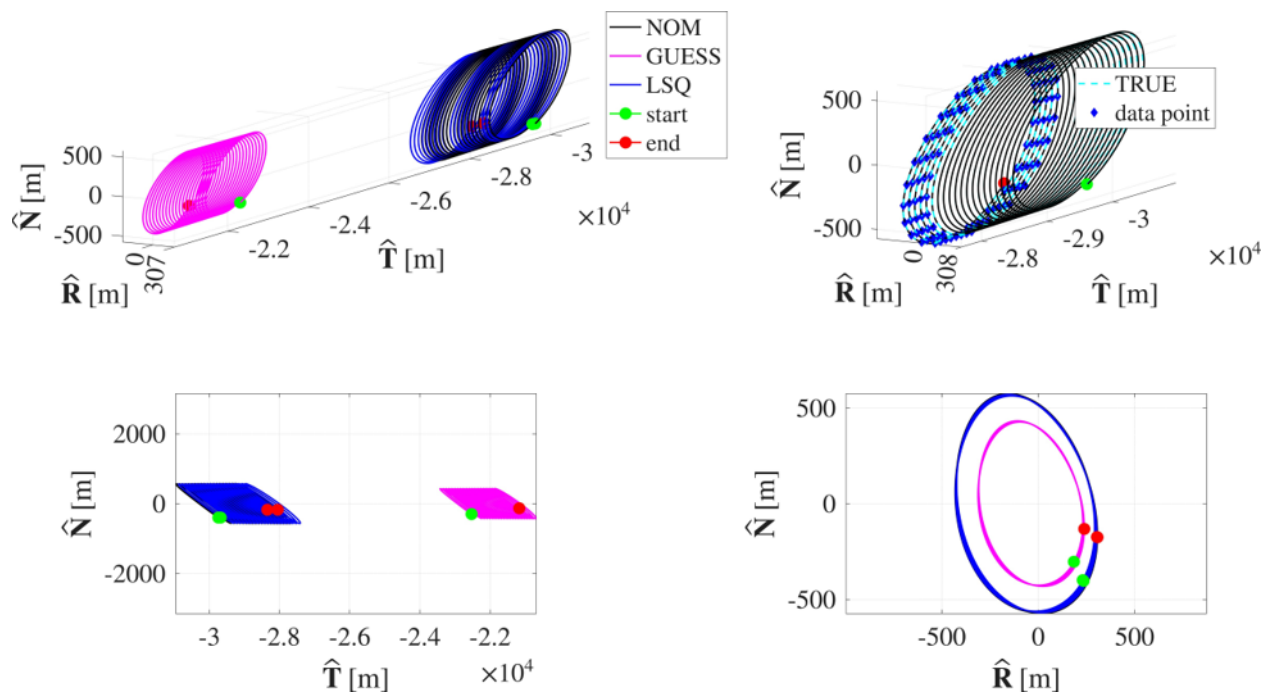


Figure 2: Results of the estimation procedure. Data collected over orbits 16 to 20.

The second figure illustrates two key aspects: first, that the previous LSQ runs degraded the estimate due to weak observability and the necessity to impose $\sigma_{a\delta a}^{\min}$ to prevent filter stagnation [7]. Secondly, the filter successfully recovers and achieves an even more accurate relative orbit estimate. In general, it is observed that progressively running the LSQ as described in this work results in estimates oscillating around the true orbit. This behaviour arises partly from the weak observability of the problem and partly from the need to impose a minimum uncertainty to prevent filter stagnation [7]. In conclusion, this work enhances the batch LSQ framework of [7]. The refined analytical propagation and exact non-linear ROE-to-LoS transformation improve accuracy and observability in manoeuvre-free scenarios, enabling estimation of the weakly observable $a\delta\lambda$. The circular-buffer sliding window further advances the framework toward onboard implementation.

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