Engineering Structures 304 (2024) 117572

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Damage detection of prestressed concrete beams affected by shear and flexure cracks through vibration monitoring

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ARTICLE INFO

Keywords:
Prestressed concrete beam
Damage detection
Shear cracks
Ambient vibrations
Operational modal analysis
Frequency domain decomposition

ABSTRACT

This paper reports the results of an experimental campaign carried out on prestressed concrete beams (PCB), equipped both with bonded and unbonded strands, with the aim to employ Operation Modal Analysis (OMA) techniques to detect early warning signs (i.e., variation of natural frequencies and/or relevant mode shapes) associated to the onset of structural damages. With respect to similar past experimental studies, the main novelties presented hereafter concern the application of OMA to the following situations: (1) a PCB equipped with bonded strands and affected by a “almost pure” shear-cracks pattern; (2) a PCB equipped with unbonded strands and affected by an almost negligible (i.e., reversible) flexural damage. Among the outcomes of this study, it was found that, especially in the second case, the assessment of variations in the PCB natural fundamental frequencies, is not always an effective index of damage. On the contrary, a qualitative comparison between the undamaged and damaged identified mode shapes can be helpful even for the spatial location of the damage. Eventually, despite the diverging opinions in the reference literature, it is found that OMA technique is not a suitable tool to assess the potential influence of variations of the precompression load on the PCB fundamental frequencies, unless this variation is so high to cause cracking.

1. Introduction

Among Operational Modal Analysis (OMA) techniques [1], the “Frequency Domain Decomposition” (FDD) [2] allows the identification of the natural frequencies of structural systems processing the time-histories of ambient vibrations’ records (i.e., “white noise”). In the last two decades, this algorithm has been successfully applied for the dynamic characterization of different civil artworks such as bridges [3,4], ancient cathedrals [5] and historic masonry towers [6]. OMA procedures allow to detect even small variations of both natural frequencies and relevant mode shapes that could highlights the onset of potential structural damages. In recent studies [7,8], OMA has been applied for the detection of flexural cracks in prestressed r.c. beams.

However, most studies [9–11] evaluated how the fundamental frequency and modal shape change in presence of significant damage. Pisani et al. [8] in fact asserted that for prestressed beams, in contrast to r.c. beams, changes in the modal parameters are recorded only when the structure is close to ultimate conditions, which makes this information hardly useful for monitoring and for the purpose of construction management choices. Limongelli et al. [9] also used OMA procedures to investigate how the fundamental frequency changes because of significant flexural damage, as evidenced by a residual beam deflection of about 1.5 mm after removal of the vertical load. In this regard, the novelty of the study presented hereafter in this paper is represented by the application of OMA to detect widely minor cracks width.

Zhao et al. [12] also argued that traditional modal parameters, such as natural frequencies, mode shapes, and modal damping ratios, are not sufficiently sensitive to reflect different damage levels.

Several studies evaluated whether through dynamic acquisitions the presence of flexural cracking damage could be captured; there are no studies investigating a shear cracking pattern. Zhao et al. [12] applied “vibration-based damage detection" (VBDD) to a prestressed concrete box girder to detect and locate flexural cracks. VBDD was also used by Zhou et al. [11] to detect and localize artificially flexural damage (spalling damage) in the girder by removal of cubic portions of concrete at 12 different locations at the extrados of the girder. With respect to all aforementioned studies, other important outcomes of this study arise from the dynamic identification of prestressed beams affected by shear cracks pattern.

The fundamental frequency (f_1) and the relevant mode shape (ϕ_1) of

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https://doi.org/10.1016/j.engstruct.2024.117572
Received 12 August 2023; Received in revised form 12 December 2023; Accepted 23 January 2024
Available online 9 February 2024

simply supported beam can be calculated through these simple equations [13]:

$$f_1 = \left(\frac{\pi^2}{L^2}\cdot\sqrt{\frac{E_c J}{m}}\right)/2\pi \tag{1}$$

$$\phi_1(x) = sen(\pi x/L) \tag{2}$$

being: L the length of the beam; m is the mass per unit length; E_c is the elastic modulus of the concrete; J is the equivalent homogeneous elastic inertia moment of the full cross-section; x the longitudinal coordinate within the beam length L.

Concerning the calculation of the parameter J of prestressed concrete beams (PCB), it is worth to clarify that: (1) the beam cross-section is assumed to be fully integer (i.e., uncracked); (2) the coefficient n ("Method-n"), defined as the ratio between the steel E_s and concrete E_c elastic modulus (experimental values), is employed to calculated the transformed section properties; (3) in case of unbonded tendons, the actual area of the concrete should be taken into account excluding the holes for housing ducts.

Three short dissertations, as critical reviews of the study [14] about the influence of the axial load on the dynamics of concrete beams, are reported in [15]. All the authors agree on the fact that the compression level has a negligible influence on the fundamental frequency of beams

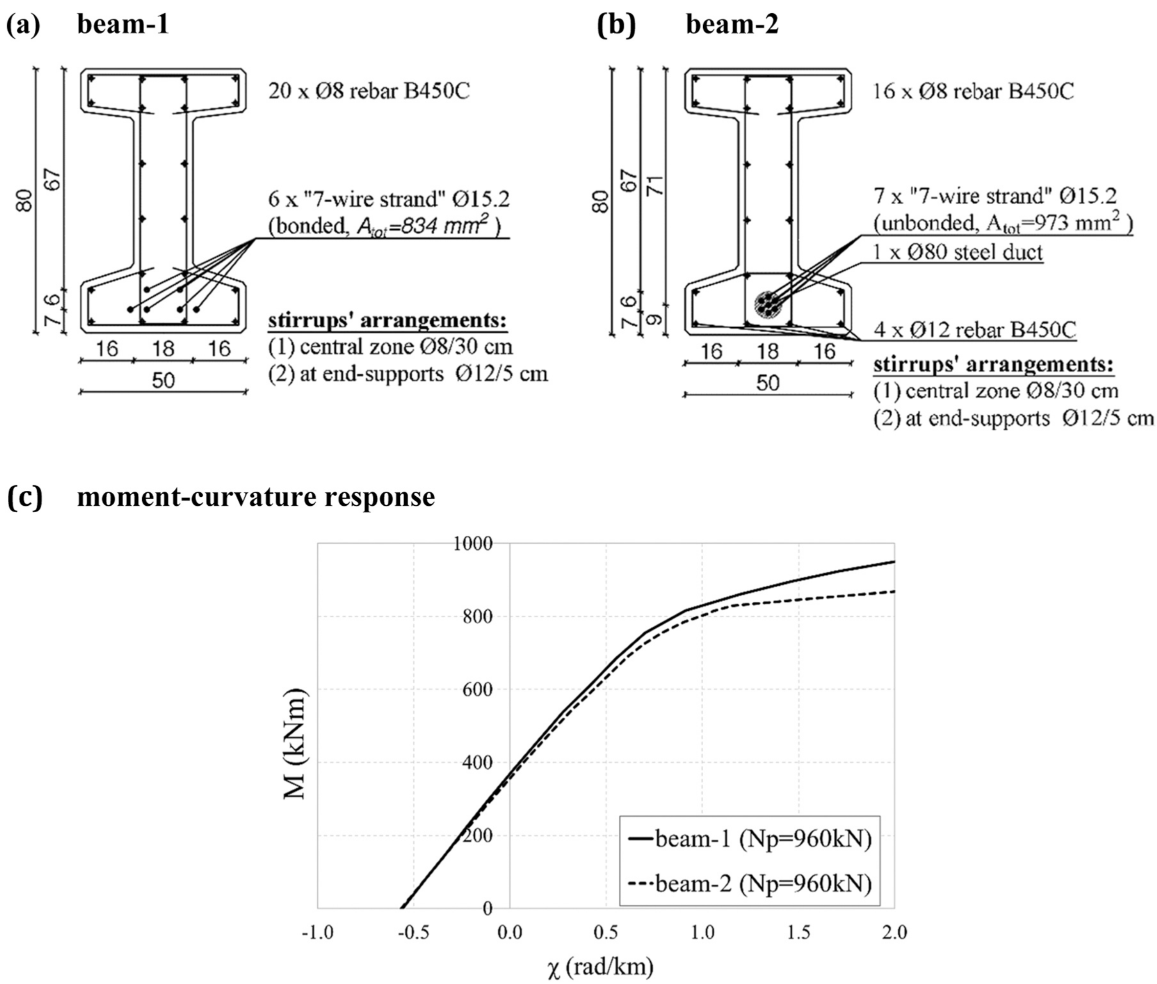

Fig. 1. Case-study beams: (a) beam-1 cross-section; (b) beam-2 cross-section; (c) comparison of the two moment-curvature (M-χ) responses. Notes: (i) cross-section geometric parameters expressed in cm; (ii) rebars' diameter in mm.

Table 1
Case-study beams' properties.

parameter	unit	beam-1	beam-2
mass per unit length	m [kg/m]	586	563
inertia moment	J [m^4]	0.0190	0.0176
concrete C50/60 elastic modulus	$E_{c,exp}$ [GPa]	36.2	35.8
concrete C50/60 compression strength	$f_{c,exp}$ [MPa]	61.2	75.5
rebars B450C elastic modulus (Φ8 / Φ12)	$E_{s,exp}$ [GPa]	196.9	196.9 / 196.2
rebars B450C yielding strength (Φ8 / Φ12)	$f_{s,y,exp}$ [MPa]	532.8	532.8 / 525.8
rebars B450C ultimate strength (Φ8 / Φ12)	$f_{s,u,exp}$ [MPa]	635.3	635.3 / 641.3
strands S15 elastic modulus	$E_{s,exp}$ [GPa]	202.4	202.4
strands S15 yielding strength	$f_{s,y,exp}$ [MPa]	1783	1783
strands S15 ultimate strength	$f_{s,u,exp}$ [MPa]	1947	1947
prestressing internal action	N_P [kN]	960	0÷1130

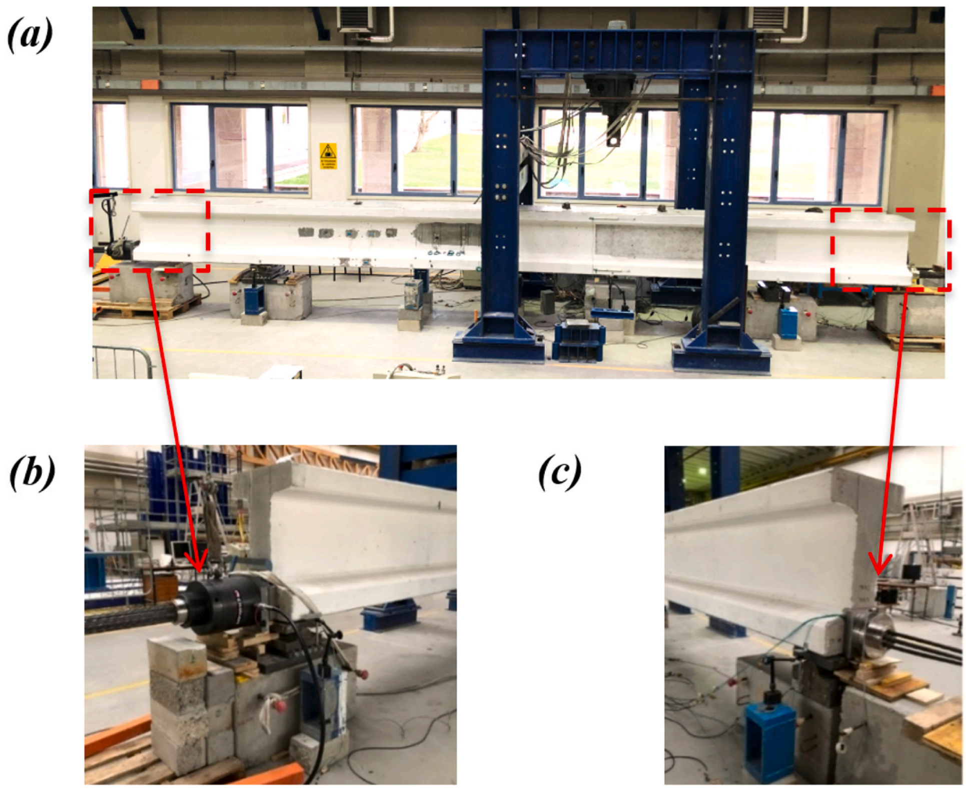

Fig. 2. Case-study beam-2: (a) overall longitudinal view; (b) sliding support bearing and horizontal hydraulic-jack used to vary the precompression load ("active head"); (c) hinge support bearing and strands' anchoring plate with load-cell at the "passive beam head".

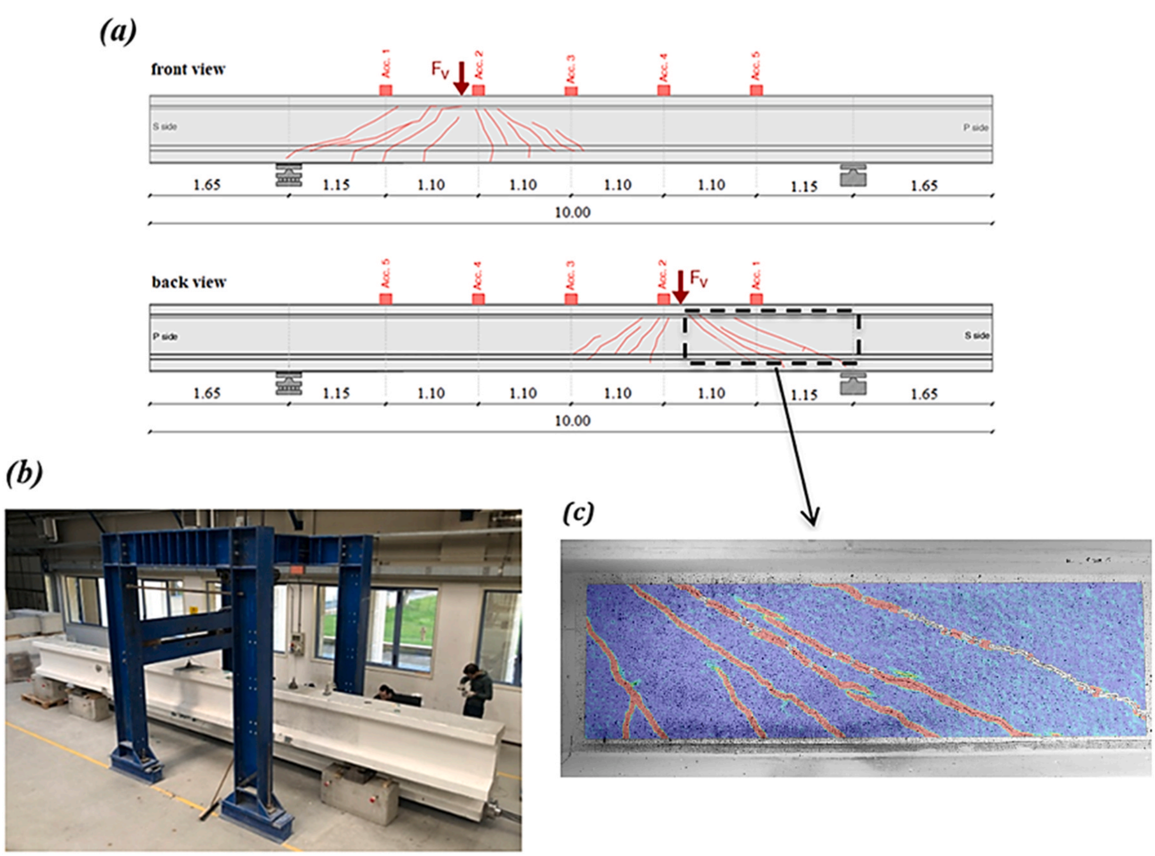

Fig. 3. Dynamic characterizations of beam-1: (a) accelerometers' longitudinal arrangement and detected cracks' pattern; (b) picture of the beam in virgin condition; (c) shear cracks at $F_v = 800\ kN$ detected through the DIC system.

with bonded strands. In these cases, a loss of prestress is likely to alter the natural frequencies of the beam through the opening of microcracks in the concrete. On the contrary, small reductions of the natural frequencies with the increase the compression level (N_P) can be appreciated for beams with unbonded strands (external load). In this regard, according to Jain and Goel (within the aforementioned technical note edited by Deak [15]), this phenomenon can be taken though this formulation:

$$f_1 = \sqrt{\frac{1}{m}\left(\frac{\pi^4 E_c J}{L^4} - \frac{\pi^2 N_P}{L^2}\right)} / 2\pi \tag{3}$$

while a slight variation has been proposed by Dell'Asta and Dezi [15]:

$$f_1 = \sqrt{\frac{1}{m}\left[\frac{\pi^4}{L^4}\left(E_c - \frac{N_P}{A_c}\right)J - \frac{\pi^2 N_P}{L^2}\right]} / 2\pi \tag{4}$$

being: N_P the compression load; A_c the equivalent overall (concrete plus steel rebars) area of the beam cross-section.

It is worth noting that, despite differing for the presence of the term related to the precompression stress $\sigma_{PC} = (N_P/A_c)$, since the elastic modulus of ordinary concrete (E_c) is in the range $30 \div 40GPa$ while σ_{PC} is usually smaller than $15MPa$ (i.e., three orders of magnitude lower), Eqs. (3) and (4) should lead to essentially identical results.

In more recent experimental studies, some doubts arise about the possibility of estimating prestressing force using dynamic acquisitions on beams with unbonded tendons. Some authors [16,17] argued that the natural frequency remains relatively constant when changes in prestress force are induced. On the contrary, according to others [18–20], the fundamental frequency change with the prestressing force, but it is also affected by other parameters and/or phenomena, such as the tendon eccentricity and the formation of micro-cracks, making this parameter of dubious usefulness for the assessment of the actual prestress level of the beam.

Within this framework, the main goals of this study are: (1) the first application of OMA procedures for identification of the dynamic properties of PCBs affected by a shear cracks pattern (Section-3) or almost negligible, i.e., reversible, flexural damages (Section-4); (2) dispel the doubts, through an ad-hoc set of experimental tests, regarding suitability of the fundamental frequency as a parameter for the assessment of residual tendons' prestressing load (Section-5).

2. Case-study beams

Two different case-study beams, namely "beam-1″, and "beam-2″, are considered in this study. Both beams feature a "double T" cross-section *50* × *80 cm* made of concrete *C50/60* (see the Italian Building Code "NTC 2018″ [CSLP, 2018]). The beams cross-sections and the corresponding moment-curvature diagrams are represented in Fig. 1-a-c. Beam-1 is equipped with *6 x S15* bonded strands with an initial design pre-tension load $N_{pd} = 1130kN$ (i.e., $\sigma_{pd} = 1360MPa$ and $\varepsilon_{pd} = 6.8mm/m$). Otherwise, beam-2 is characterized by *7 x S15* unbonded strands housed in a steel duct.

At the time of tests (28 days from casting), the residual internal action of beam-1 is $N_P = 960kN$ (i.e., $\sigma_p = 1150MPa$) that has been estimated taking into account these prestress losses: (i) 1.5% due to the initial slip between barrels and wedges at strands' anchor; (ii) 3.6% caused by the initial elastic deformations of the beam; (iii) 1.2% ascribed to strands relaxation; (iv) 0.6% and 8.2% due to concrete shrinkage and creep, respectively.

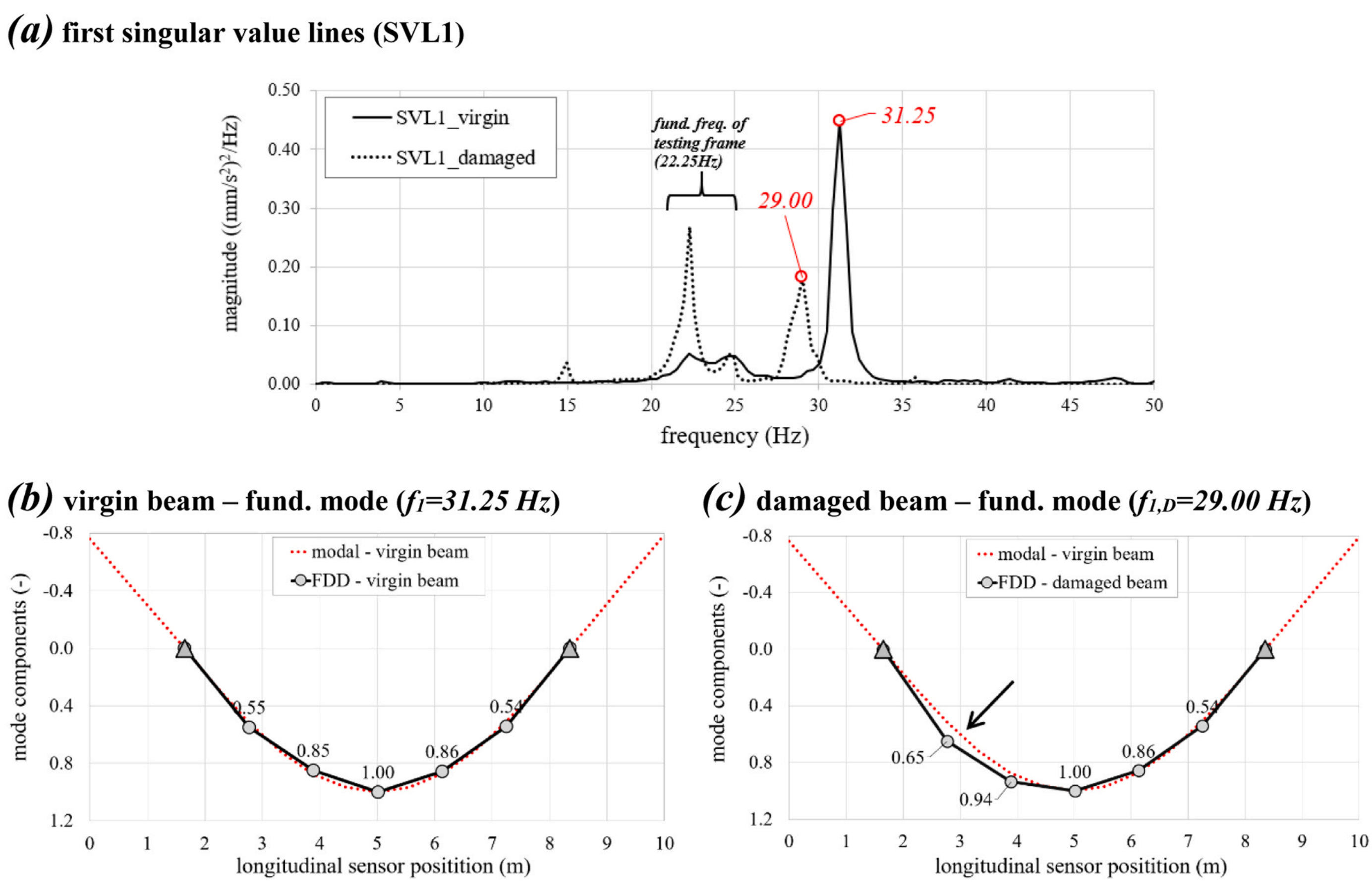

Fig. 4. Results of ambient vibration records on beam-1 processed by FDD technique: (a) first singular value line (SVL1) of the power spectral density matrix in both virgin and damaged conditions; (b) detected mode shape in virgin condition at $f_1 = 31.25$ *Hz*; (c) detected mode shape in the damaged condition at $f_{1,D} = 29.00$ *Hz*.

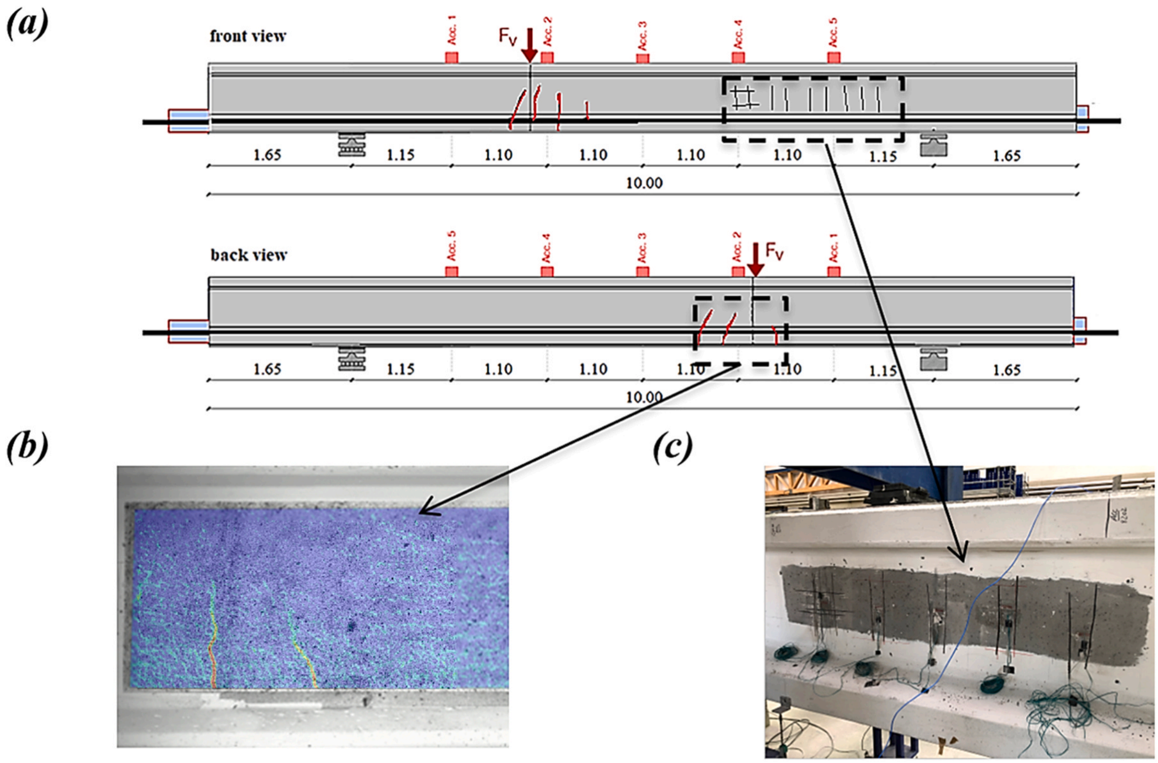

Fig. 5. Dynamic characterizations of beam-2: (a) accelerometers' longitudinal arrangement and detected cracks' pattern; (b) flexural cracks at $F_v = 420\ kN$ detected through the DIC system; (c) cuts for the execution of the "stress-release" tests.

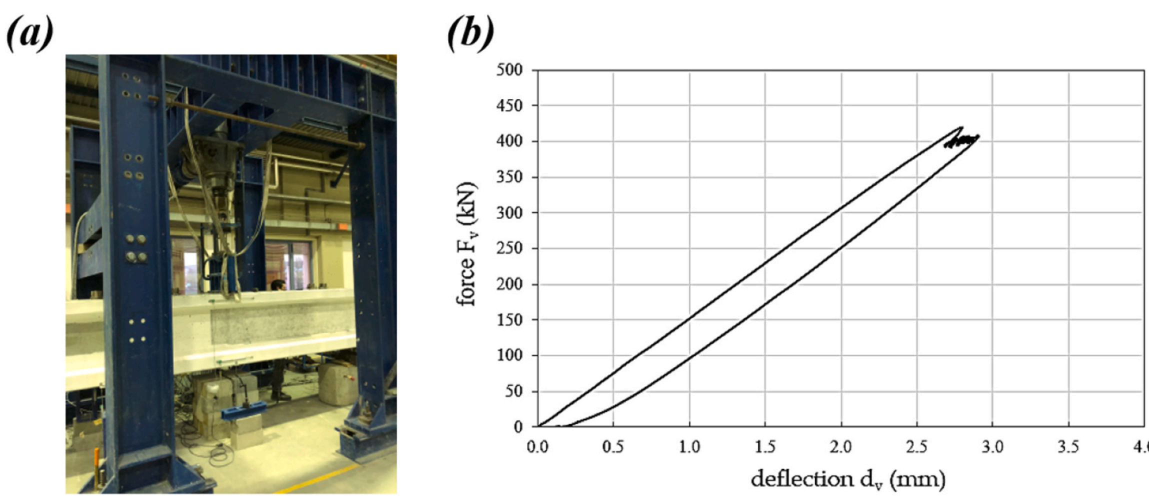

Fig. 6. Static test on beam-2: (a) experimental set-up; (b) recorded force-displacement loop.

In addition, beam-1 provides *20 x Φ8* while beam-2 is equipped with *16 x Φ8* plus *4 x Φ12* longitudinal rebars of steel grade B450C. Two legged stirrups (*Φ8/30 cm*) are arranged along with the two beams while additional shear reinforcements (Φ12/5 cm) are disposed for about 1.5 m from the beam heads with the aim to diffuse the precompression load and withstand the higher shear reaction force of the support bearings.

Being a fundamental parameter for a fine assessment of the dynamic response of structural system, the elastic modulus of the concrete ($E_{c,\exp}$) of each beam has been experimentally identified, according to the EN12390–13 [22], as the mean value over three specimens taken from the three concrete castings and cured until few hours before the execution of OMA tests. Tensile tests on steel rebars B450C were carried out according to the ISO 6892–1 [23] on a minimum of six specimens for both diameters (i.e., Φ8 and Φ12). Otherwise, the mechanical properties of strands S15 were taken from the certificate provided by the manufacturer according to the ISO 15630–3 [24].

Materials' mechanical properties are reported in Table 1 together with other case-sensitive parameters like the mass per unit length (m), the inertia moment (J, calculated according to the assumptions reported in the Introduction), and strands' tensile load (N_P).

Fig. 1-c shows the slightly different moment-curvature responses of beam-1 and beam-2 calculated by means of the FEM code VecTor2 [21] assuming, for this purpose, the same level of compression load (i.e., $N_P = 960kN$). In agreement with values reported in Table 1, one can note that elastic flexural stiffness of beam-1, i.e., $M/\chi = J \bullet E_{c,\exp} = 688(kNm/(\frac{rad}{km})) = 0.688GPa \bullet m^4$, is only marginally higher (+8.4%)

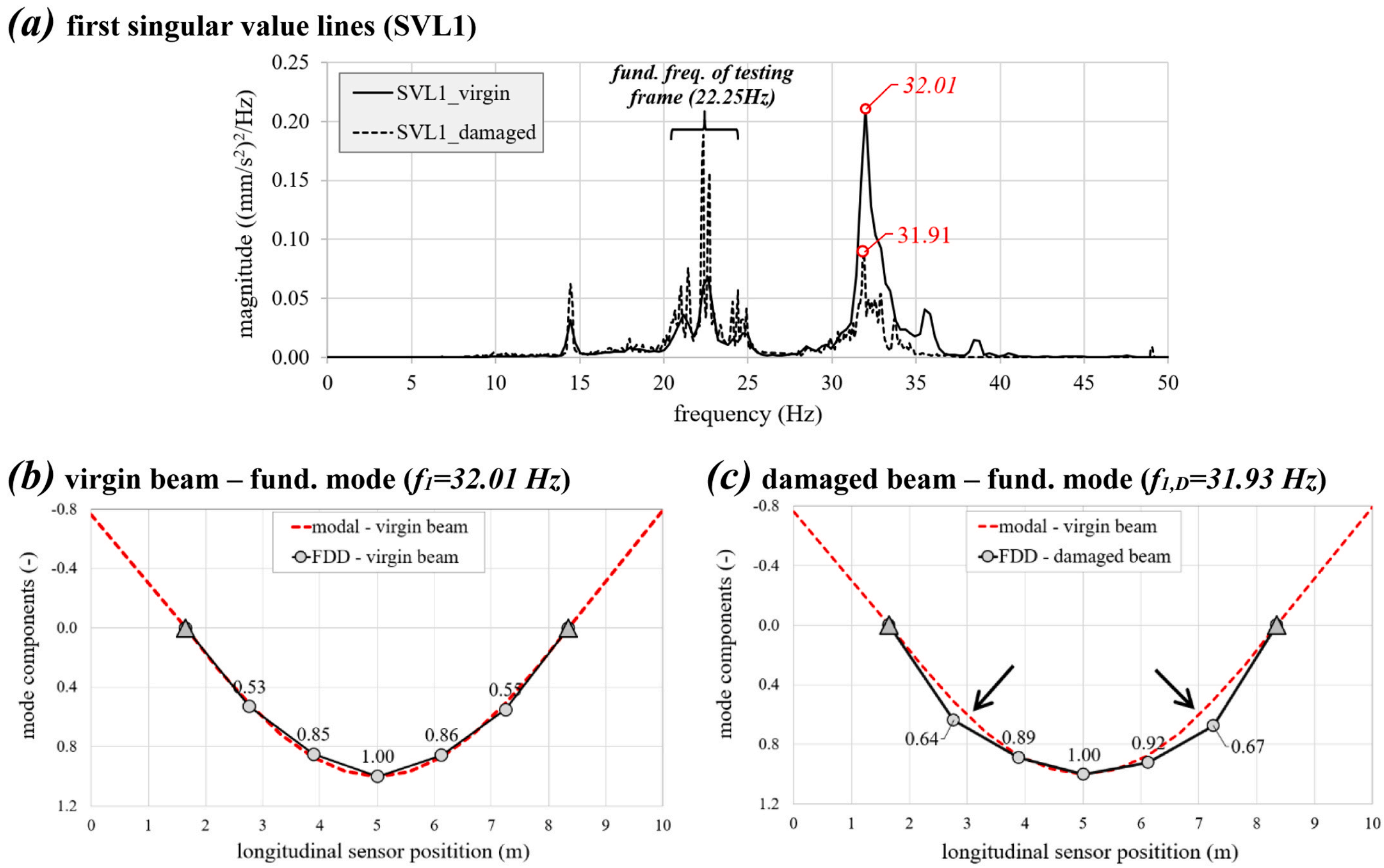

Fig. 7. Results of ambient vibration records on beam-2 processed by FDD technique: (a) first singular value line (SVL1) of the power spectral density matrix in both virgin and damaged conditions; (b) detected mode shape in virgin condition at $f_1 = 32.01$ *Hz*; (c) detected mode shape in the damaged condition at $f_{1,D} = 31.93$ *Hz*.

than that of beam-2 equal to $0.630 GPa \bullet m^4$. However, since according to Eq. (1) the fundamental frequency of simply supported beams is directly proportional to the factor $\sqrt{(E_{c,\mathrm{exp}} \bullet J)/m}$, the dynamics of beam-1 and beam-2 is practically identical because the relevant factors differ of about 2.3% only. The cracking moment M_{cr}, here conventionally identified as the bending moment causing the achievement of the concrete cracking strain (i.e., $\varepsilon_{cr} = f_t/E_{c,\mathrm{exp}} = \frac{(0.33 \bullet \sqrt{f_{c,\mathrm{exp}}})}{E_{c,\mathrm{exp}}}$) at bottom fibers [21], is equal to 685 kNm and 655 kNm for beam-1 and beam-2, respectively.

To investigate the effect of the compression level on the dynamic response of the beam the strands' tensile load is varied from 0 kN to 1130 kN. For this purpose, beam-2 is equipped with the "ad-hock" set-up shown in Fig. 2. At one head of the beam (active side), the strands are fixed to a hydraulic actuator to vary their actual strain level (ε_p) while at the opposite side (passive head) the strands are constrained by a steel plate equipped with a load cell (model: nbc TOR 1600 kN) to monitor the instantaneous tensile load.

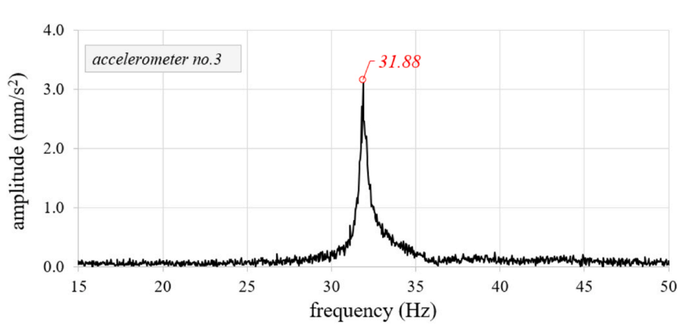

Fig. 8. Fourier spectrum obtained through hammer test on damaged beam-2.

3. Detection of shear cracks

In this Section the results of dynamic identification tests performed on beam-1 in both virgin and damaged conditions are discussed. The second configuration, as shown in Fig. 3-a,b, was obtained by applying a vertical load (F_v) through an hydraulic actuator located at 2.2 m from the left-side support bearing. A peak force of 800 kN was gradually reached to generate the multiple shear cracks shown in Fig. 3-a,c. These were detected through the "Digital Image Correlation (DIC)" technique processing the pictures taken by a Reflex Canon EOS D5 through the software GOM [25].

With respect to a possible flexural failure, this cracks' pattern was eased by two factors: (1) both support bearings were moved from the beam heads towards the midspan to reduce their effective distance to 6.7 m; (2) the residual (estimated) precompression load of the bonded strands was quite high ($N_P = 960$ *kN*).

The two dynamic identifications (i.e., in virgin and damaged conditions) were performed by recording the random ambient vibrations (i.e., "white noise") and processing the same through the "Frequency Domain Decomposiotion (FDD)" technique implemented by the Authors in MatLab environment [26] allowing to obtain both the natural frequencies and the modal shapes of the beam [2]. The goal of this investigation is to determine the possibility to detect (early warning) a potential failure or a significant damage of the beam due to the presence of shear cracks through a variation of the fundamental frequency. Moreover, if possible, another target is to locate the position of the damage by analysing the variations of the components of the relevant mode shape.

In both conditions, i.e., virgin and damaged, ambient vibrations were recorded on completely unloaded beam (except for its self-weight) through five piezoelectric accelerometers (Wilcoxon 731 A model)

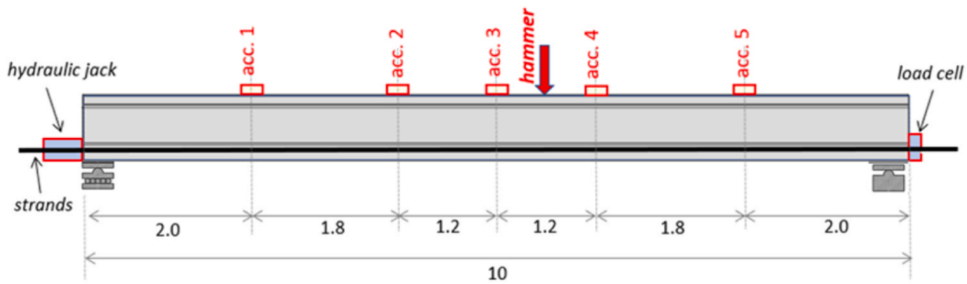

Fig. 9. Longitudinal arrangement of the accelerometers for hammer tests carried out on beam-2 at different levels of compression load.

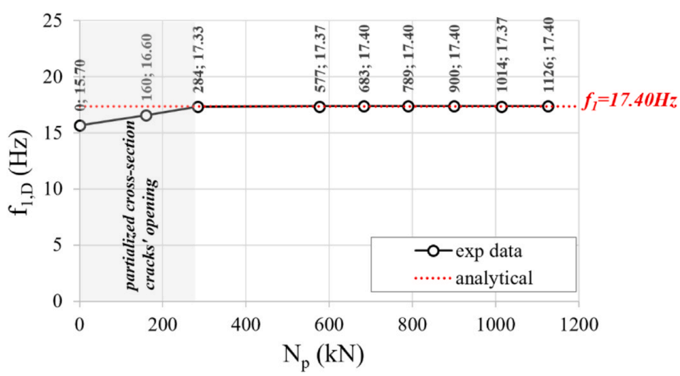

Fig. 10. Fundamental frequency of the damaged beam ($f_{1,D}$ detected through "hammer tests") as a function of the applied precompression load and comparison with the analytical prediction in virgin condition.

arranged along the upper flange and symmetrically positioned with respect to the midspan. The cantilever portions of the beam beyond the two support bearings were not instrumented since unuseful for the purposes of this investigation. The sampling frequency was set to 600 Hz.

The comparison between the first singular value line (SVL1) calculated through the FDD technique in virgin and damaged condition is shown in Fig. 4-a. First, it is important to point out some preliminary observations: (1) the frequency related to the major peak of the SVL1 in virgin condition (f_1*=31.25 Hz*) is almost coincident to the fundamental frequency (f_1*=31.45 Hz*) calculated through a modal analysis carried out with the FEM software Telaio2D [27]. This small discrepancy (*Δf=0.20 Hz,* i.e. *0.64%*) can be due to several factors (e.g., small errors in the measurement of the beam weight and/or the effective distance between the support bearings, etc.); (2) the slightly lower peak at *22.25 Hz*, more evident from the records in damaged conditions, is due to the ambient vibration of the blue testing frame used to restrain the vertical hydraulic actuator (see Fig. 3-b). This evidence, after several hypothesis on the source of this "anomaly", was proven thanks to execution a hammer test directly on the testing frame.

Eventually, most important, some important outcomes can be depicted by comparing the SVL1 of the beam in virgin and damaged conditions, and the calculated mode shapes that are associated to the relevant peaks: (a) the presence of a wide (shear) cracks pattern, extended for about 3.3 m from the support, causes a significant drop of the fundamental frequency of the beam (i.e., $\Delta f_1 = f_{1,D} - f_1 = -2.25Hz$, being $f_{1,D} = 29.00Hz$ the "damaged frequency"); (2) there is also a discrepancy between the two mode shapes calculated through the FDD. In virgin condition, the fundamental mode (see Fig. 4-b) is perfectly overlapped with that calculated by the FEM software Telaio2D [27]. On the contrary, in the damaged condition, there are two differences in the modal components associated to the position of accelerometer no. 1 and no. 2 that indeed are located exactly above the cracked region of the beam (see Fig. 4-c). Despite the quite high value (0.996) of the "Modal Assurance Criterion (MAC)", a synthetic index (0 <MAC<1) used to verify the similarity of the two mode shapes (i.e., in virgin and damaged conditions) [28], a visual comparison of the two plots allows to define the position of shear cracks that have caused the drop of the natural frequency. This becomes more outstanding when considering possible on-field applications like the permanent monitoring of multi-span bridges since the FDD could allow to limit the portion of the bridge to be visually inspected by technicians to assess the level of damage.

4. Detection of small flexural cracks

In this Section the results of dynamic identifications performed on beam-2 in the original virgin condition and after the formation of small flexural cracks (damaged condition) are presented. As in previous tests on beam-1, the span between the supports was 6.7 m and the damage was triggered by applying a vertical load (F_v) at 2.2 m from the left-side (see Fig. 5-a) up to a peak force of F_v*= 420 kN* generating a bending moment of 595 kNm. Moreover, the precompression load was reduced from the initial maximum design value (i.e., 1130 kN) to N_P*= 678 kN*. This combination of boundary conditions caused the formation of a small flexural damage, i.e., the sub-vertical cracks below the load point that were detected through DIC technique shown in Fig. 5-b. Afterwards, the vertical load was released and the precompression level was reset to the initial value (more precisely to 1110 kN). Looking at Fig. 6-b, showing the loop F_v-d_v (i.e., force vs. vertical deflection) recorded during the static test, one can state that the induced flexural damage is almost negligible as confirmed by the small residual displacement (about 0.15 mm) after the removal of the vertical load. Indeed, with respect to previous experimental study [8,9,11,12], the novelty of this Section is the evaluation of the effectiveness of "vibration-based damage detection" (VBDD) procedures when applied to beams affected by minor flexural cracks.

In both conditions, i.e., virgin and damaged, ambient vibrations were recorded (at a sampling frequency of 600 Hz) on completely unloaded beam (except for its self-weight) through five piezoelectric accelerometers (Wilcoxon 731 A model) arranged along the upper flange and symmetrically according to the layout shown in Fig. 5-a. The ambient vibrations' records (i.e., "white noise") were again processed through the FDD technique [2].

Some concerns arise about the identification of the exact natural frequency of the first mode shape in virgin conditions. The value obtained processing records of "ambient vibrations" is f_1*= 32.01 Hz* as hardly comparable to that obtained through a modal analysis. Indeed, due to the presence of the additional masses connected to the two "cantilever heads" of the beam (i.e., horizontal hydraulic actuator 175 kg, load cell plus plate 30 kg, strands 105 kg, etc.) whose effective participating mass is unclear, a modal analysis cannot help to solve this issue but only to establish a possible range within 30.90 Hz (considering 100% of the additional masses) and 32.14 Hz (mass of the beam only). On the contrary, there are no uncertainties about the mode shape (see Fig. 7-b) identified through the FDD that indeed is perfectly overlapped with that calculated numerically by means of the software Telaio2D

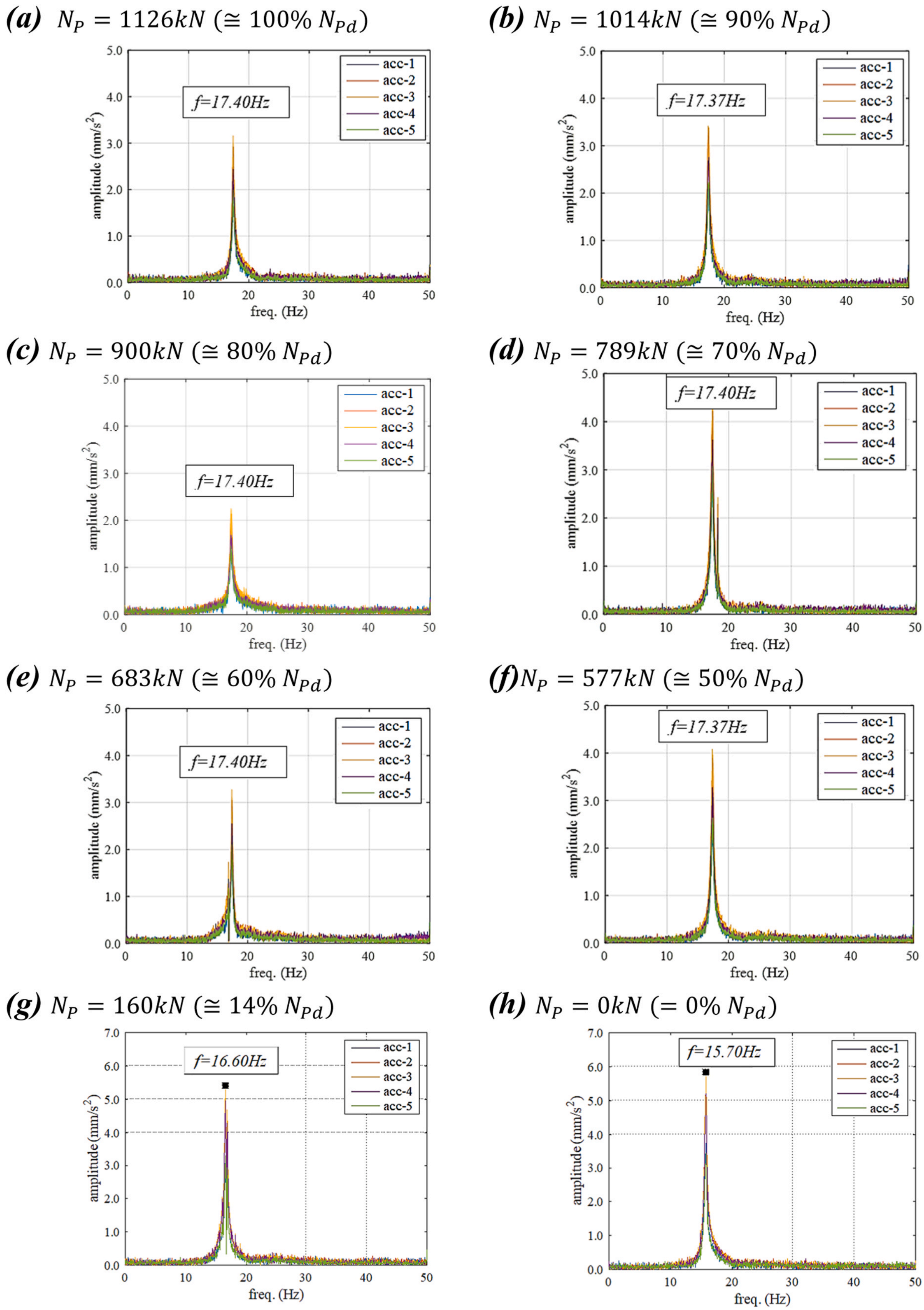

Fig. 11. Fourier spectra of beam-2 obtained at different level of precompression.

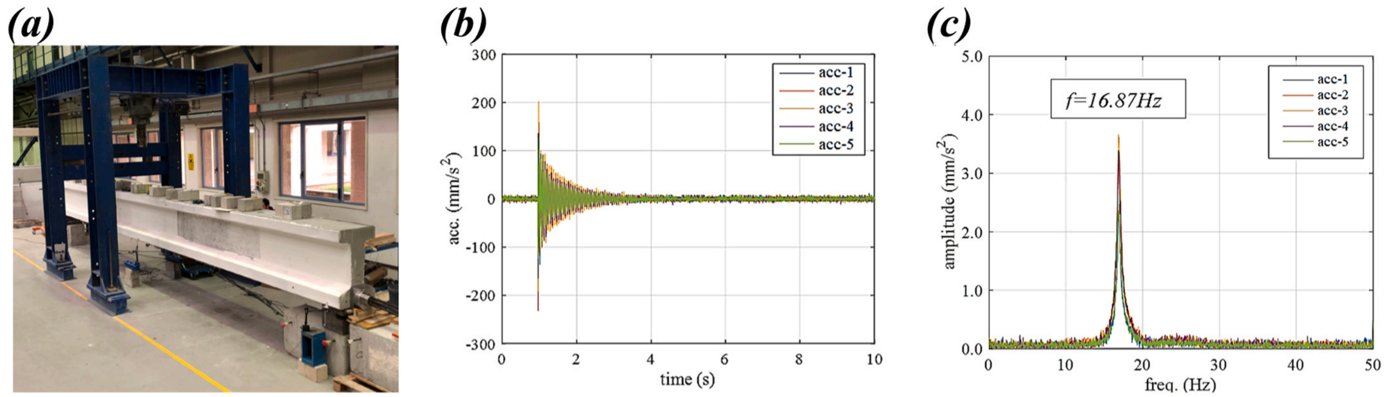

Fig. 12. Beam-2 with an additional distributed mass of 32.6 kg/m ($N_P = 1130kN$): (a) experimental set-up; (b) free-acceleration records induced by the "hammer test"; (c) relevant single-side Fourier spectrum.

[27].

In the damaged configuration, besides the small flexural cracks, another source of "anomaly" was introduced by some vertical cuts (about 30 cm long and 2.5 cm deep), close do the right-side support bearing (see Fig. 5-c), carried out through a flex to investigate (within another research project) the effectiveness of a new diagnostic technique to assess the actual prestress force acting on an existing beam [29].

As shown in Fig. 7(a), the FDD does not allow a persuasive detection of the "resonant frequency" of the damaged beam since the peak (at about $f_{1,D}$=*31.91 Hz*) is surprisingly lower than that of the testing frame (at *22.25 Hz*) and has jagged surrounding frequencies. However, the FDD allows again to identify the position of the two minor damages that indeed are both located where the mode-shape (see Fig. 7-c) of the damaged beam deviate (slightly higher components) from the plot obtained through the modal analysis for the virgin beam (see Fig. 7-b).

To dispel the uncertainty about the fundamental frequency in damaged conditions, a hammer test was also carried out to trigger higher excitation levels along with the beam. By processing the relevant acceleration-records through the "Fast Fourier Transform" (FFT) implemented in MatLab [26], it was possible to obtain the Fourier spectrum shown in Fig. 8 and to detect a clear peak at $f_{1,D}$= *31.88 Hz*.

One can note that both identified "damaged frequencies" (i.e., $f_{1,D}$=*31.91 Hz* through FDD of ambient vibrations and $f_{1,D}$=*31.88 Hz* through FFT of hammer test accelerations) are substantially identical and very close to that of the virgin beam (i.e., f_1=*32.01 Hz*). This could be explained by two reasons: (1) the flexural cracks were located in a small portion of the beam and, thanks to the high level of precompression load (1110 kN), did not affected the elastic flexural stiffness of the beam; (2) the cuts of "stress-release" tests are all located in the web and therefore do not significantly alter the inertia moment of the beam.

This outcome (i.e., the virtual independence of f_1 from the presence of these small sources of damage) was also verified through further "hammer tests" executed by removing also the uncertainty related to the participating mass of the hydraulic actuators, load cell, and other plates, obtained by re-location the two support bearings at the end-sides of the beam (see Fig. 9). With this simple scheme, the aforementioned additional masses insist directly on the support bearings without affecting the dynamics of the beam and the effective span between supports' centreline is equal to 9.70 m. A total of nine hammer tests were executed at different precompression loads ranging from 0 kN up to 1130 kN. In Fig. 10 the identified natural frequencies (peaks of the Fourier spectra) are plotted as a function of the associated precompression load. One can note that for precompression loads in the range 284÷1126 kN (i.e., 1.20÷4,74 MPa ≈ 25÷100% of N_P) all "damaged fundamental frequencies" ($f_{1,D}$) are practically unvaried and equal to that of beam in virgin condition (f_1=*17.40 Hz*) that can be calculated analytically through Eq. (1). For lower precompression loads (i.e., <284 kN corresponding to 1.20 MPa) the flexural cracks tend to open leading to partialized cross-sections that hence to reduced natural frequencies ($f_{1,D}$=*15.70 Hz* for N_P=*0 kN*). Further details about the results of the "hammer tests" carried out at different level of precompression load are provided in the next Section.

5. Influence of the prestress level on the beam fundamental frequency

Due to the lack of a full agreement in the reference literature [16,17] about the influence of the prestress load on the fundamental frequency of beams with unbonded strands, further details about the results of the "hammer tests" carried out on beam-2 at different levels of precompression are provided hereafter. As shown in Fig. 10, the axial load provided by the jack positioned in the active head was decreased in nine steps from $N_P \cong 1130kN$ to $N_P \cong 0kN$ (i.e., σ_p from 1166 MPa to 0 MPa). It is worth noting that shorthand long-term prestress losses, in extreme unfavourable conditions, can induce a decrement of about 40–50% of the reference design value N_{pd}= *1130 kN* [30]. Free accelerations were triggered in the beam using a hammer Kiag Swiss 906B with a mass of 9 kg. The acceleration time-histories were acquired (sampling frequency 400 Hz) through five mems accelerometers (model PCB 3741B122G/−0001) arranged symmetrically with respect to the midspan of the beam (see Fig. 9). The identification of the first modal frequency was carried out by processing the records through "Fast Fourier Transform" (FFT) implemented in MatLab [26]. The whole set of the single-side Fourier spectra obtained at different levels of N_p are reported in Fig. 11. It can be observed that in the range 50÷100% of N_P there are no significant variations of the beam's fundamental frequencies (f_1) that are all comprised in the narrow range *17.37÷17.40 Hz*. Moreover, this matches exactly the natural frequency f_1= *17.39 Hz* of axially unloaded beam-2 calculated analytically through Eq. (1) [13] confirming, as already highlighted in other experimental studies [18–20], the independence (or negligible influence) of this parameter on the level of precompression load. However, it is worth noting that these results cannot be used to completely disavow the phenomenon behind Eqs. (3) and (4) [14]. Fore sake of clearness according to these theories, it should anyway be noted that the possible influence of N_P should lead to a very small discrepancy (Δf_1=*0.07 Hz*) between the fundamental frequencies at 100% and 50% of N_P.

For lower levels of precompression, i.e., N_p< *284 KN* (i.e., $\sigma_p < 293MPa$), the flexural cracks tend to open leading to cracked beam cross-sections properties and reduced inertia moments. This causes a sharp drop of the fundamental frequency down to the minimum value of f_1= *15.70 Hz* for N_p= *0 kN*. In this peculiar condition, the dynamics reflects that of an ordinary r.c. beam (i.e., without strands) experiencing the "breathing cracks" effect [31,32]. In particular, as usual in experimental tests, when the frequencies are identified with the structure under its dead load only, the dynamic motion implies opening and closing of cracks with a non-linear effect due to alternating contact conditions within the same. For a higher level of damage, a bigger drop of the fundamental frequency is expected since the cracks never close during the vibrations. For instance, this may happen when a heavy truck run over a bridge because the dynamic motion does not develop any upward displacement. A recent formulation [33], obtained through a best fit analysis of a wide set of experimental data, allows to estimate the "damaged frequency" ($f_{1,D}$) as a function of the ratio between the maximum moment experienced by the beam ($M_{\max}$) and its yielding strength (M_y). However, this model cannot be applied to this study because it was conceived for ordinary r.c. beams while beam-2 has been damaged under a precompression load $N_P = 678\ kN$.

To evaluate the sensitivity of the accelerometers enrolled for the dynamic identifications, a distributed mass of 32.6 kg/m (corresponding to an increase of 5.6% of the beam self-weight) was added by placing uniformly distributed concrete blocks (40 cubes, each of 8.15 kg) on the upper flange of the beam (see Fig. 12-a). To close the cracks the prestressing load was set again to $N_P = 1130kN$. The identified natural frequency was $f_1 = 16.87\ Hz$ (see Fig. 12-c) that is almost identical to that ($f_1 = 16.91\ Hz$) analytically predicted from Eq. (1) confirming the fine accuracy of the adopted dynamic identification system.

6. Conclusions

In this paper the results of dynamic identification tests carried out on two prestressed concrete beams (PCB), either with bonded or unbonded strands, have been presented. In particular, the Frequency Domain Decomposition (FDD) technique has been employed to detect, through the analysis of ambient vibration records, early warning signs (i.e., variation of natural frequencies and/or relevant mode shapes) associated to the onset of structural damages. With respect to similar past experimental studies, the main novelties presented hereafter concern the application of the FDD to the following situations: (1) a PCB equipped with bonded strands and affected by a "almost pure" shear-cracks pattern; (2) a PCB equipped with unbonded strands and affected by an almost negligible (i.e., reversible) flexural damage. Among the outcomes of this study, it was found that:

a) in the first case, characterized by a quite wide shear-cracks pattern, the damage is associated both to a significant variation of the PCB fundamental frequency (about −7%) and to a small alteration of the relevant mode shape;
b) in the second case, the assessment of variations in the PCB natural fundamental frequencies, is not an effective index of damage. Indeed, once the source of damage is removed (i.e., the concentrated vertical load), the cracks tend to completely restore the condition of fully reacting uncracked cross-section. On the contrary, a qualitative comparison between the undamaged and damaged identified mode shapes can be helpful even for the spatial location of the damage;
c) eventually, despite the diverging opinions in the reference literature, it is found that OMA technique is not a suitable tool to assess the potential influence of variations of the prestressing action on the PCB fundamental frequencies. This has been proven through dynamic tests carried out on a PCB with unbonded strands that, from time to time, were tensioned at different load levels. In particular, from 1126 kN to 577 kN (i.e., - 49%) it was not possible to detect significant variations of the fundamental frequency that, on the contrary, remained practically unaltered (from 17.40 Hz to 17.37 Hz, i.e., -0.17%).

CRediT authorship contribution statement

Minelli Fausto: Writing – review & editing, Supervision, Funding acquisition. **Mantelli Stefano:** Investigation, Data curation, Conceptualization. **Rossini Giulia:** Investigation, Data curation, Conceptualization. **Gandelli Emanuele:** Writing – original draft, Methodology, Investigation, Data curation, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

The authors thank M.Sc. Eng. Matteo Lamberti and Francesco Guarino for the work performed in the laboratory during the tests as well as for their contribution to data processing. A special thank goes also to the technician Mr. Andrea Del Barba for his assistance and suggestions provided during the tests. The studies presented here were carried out as part of the activities envisaged by the Agreement between the High Council of Public Works (CSLLPP) and the ReLUIS Consortium implementing Ministerial Decree 578/2020 and Ministerial Decree 240/2022. The contents of this paper represent the authors' ideas and do not necessarily correspond to the official opinion and policies of CSLLPP.

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