

Efficient reliability assessment of multi-performance multi-state systems using dynamic matching of source and load feasible regions

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ABSTRACT

Multi-performance multi-state systems (MPMS) that consider performance conversion processes represent a current modeling approach for engineering systems, where different types of performance can be exchanged and converted. However, in large-scale MPMS, the complexity of performance variables and performance conversion characteristics significantly increases the computational burden of accurate reliability assessment. To address this challenge, this paper proposes an efficient reliability assessment method for large-scale MPMS considering performance conversion, based on the concept of feasible regions. The proposed method introduces the MPMS model and classifies the system into two categories: source-side MPMS and load-side MPMS, based on the characteristics of performance variables. The states and performance conversion characteristics of MPMS are described using feasible regions, while system adequacy is determined through the spatial relationship between feasible regions. The performance loss of the system is quantified by calculating the shortest Euclidean distance between the feasible regions of the two types of MPMS. Finally, system reliability is evaluated through dynamic matching of feasible regions. The proposed model and method are validated through two case studies, demonstrating their efficiency and effectiveness in assessing the reliability of MPMS.

1. Introduction

Multi-state systems (MSS) are commonly utilized as reliability models to characterize engineering systems in real-world scenarios [1]. In MSS, components are represented by a finite set of states, each reflecting a specific performance level, ranging from full functionality to complete failure [2]. As engineering systems become increasingly intricate, those operating across multiple states often exhibit multi-performance characteristics by nature [3]. This means that the performance of both components and systems must be evaluated not only through multiple states but also across various performance dimensions. As a result, the conventional MSS framework must be expanded into a multi-dimensional structure, referred to as a multi-performance multi-state system (MPMS). For example, multi-energy carrier system (MECS) is a representative case of MPMS, requiring four performance variables, electricity, natural gas, heat and cooling to comprehensively describe their operations [4].

Furthermore, as systems evolve with greater complexity and diversity, performance conversion processes have become critical in

ensuring the execution of system tasks in multi-performance system (MPS) [5]. A performance conversion process refers to the ability of a component to transform one type of performance into another. By integrating performance conversion processes into the MPMS model, the representation of complex engineering systems becomes more precise and versatile, offering a practical and robust framework to analyze real-life MPS.

Significant progress has been made in the reliability assessment of MPMSs. The universal generating function (UGF) technique, known for its versatility and convenience, is one of the most widely used methods, as it simplifies system state enumeration by avoiding complex combinatorial processes [6]. Xia et al. [7] and Ma et al. [8] have developed reliability models for systems integrating electricity with natural gas or heat supply, incorporating performance conversion in combined heat and power (CHP) plants. Bao et al. [9] proposed a probabilistic energy flow method to analyze the effects of temperature and gas pressure on system reliability, while Shariatkhah et al. [10] integrated dynamic thermal loads using Markov chains and Monte Carlo simulation (MCS) for deeper insights. Gao et al. [11] evaluated the reliability of multi-state k-out-of-n systems and modeled their performance using the UGF

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Nomenclature		R	Spatial dimensions of performance flow (superscript)
Abbreviations		Symbols and parameters	
MPMS	Multi-performance multi-state system	$\mathbf{A}_g$	EH port-branch incidence matrix
MSS	Multi-state system	$\mathbf{H}_g$	Converter characteristic matrix
S-FR	Source-side feasible region	$\mathbf{Z}_g$	Performance conversion matrix
L-FR	Load-side feasible region	$\mathbf{C}_{in}/\mathbf{C}_{out}$	The input/output incidence matrix
UGF	Universal generating function	$\mathbf{V}_{in}/\mathbf{V}_{out}$	Performance input/output flows
MECS	Multi-energy carrier system	$\mathbf{V}$	Performance flow vector
OPF	Optimal power flow	Γ_V	Feasible set of performance flows
LP	Linear programming	$\Delta P_{*,Load}^t(t)$	Transferable performance demand value
MCS	Monte Carlo simulation	$\Delta P_{*,Load}^r(t)$	Replaceable performance demand value
IDR	Integrated demand response	$P_{*,Load}^s(t)$	Stable performance demand value
CHP	combined heat and power	$\zeta_*^{cha/dis}(t)$	Operating state of storage devices
P2G	power-to-gas	$P_*^{cha/dis}(t)$	Charge/discharge performance
G2P	gas-to-power	$\eta_{x,y}$	Performance conversion efficiency
DR	Demand response	$E_*(t)$	Storage device capacity at time t
FR	Feasible region	$\hat{\mathbf{Z}}_g$	Performance conversion matrix after failure
EH	Energy hub	κ_g	Component status
SR	Security region	η_*^{dis}	Discharge efficiency of storage
ETC	Electricity to cooling	Θ_l	Feasible region on load side
ETH	Electricity to heating	$\Theta^n(t)$	Feasible region on source side
CTE	Cooling to Electricity	$\mathbf{P}_{load}^{pre}(t)$	Load after pre-scheduling
CTH	Cooling to heating	$\mathbf{e}_i$	The unit basis vector
HTC	Heating to Cooling	$\mathbf{G}_j$	The facet of feasible region
HTE	Heating to Electricity	$\mathbf{F}^*$	The surface set
Subscripts		V_{ip}	Interior point of feasible region
*	Index of energy types (subscript)	H_j	Axis intercept of the feasible region surface
g	Index of EH nodes (subscript)	$\mathbb{N}^K$	Hyperspace of performance output
s	Stable load (superscript)	$\mathbb{N}^R$	Hyperspace of performance flows
t	Transferable load (superscript)	$P_{\Theta_r}^i$	Source side feasible region point set
r	Replaceable load (superscript)	$q_{\Theta_l}^j$	Load side feasible region point set
r	Resource side (subscript)	$\mathbf{P}_{load,cut}(t)$	System performance cutting value
l	Load side (subscript)	N	Number of system states
dis	storage charging (superscript)	T	
cha	System state (superscript)	$S_{*,min/max}$	Scheduling cycle
n	storage discharging (superscript)	ζ_t	Unit compensation factor of transferable load
i	Source side feasible region points set (superscript)	ζ_r	Unit compensation factor of replaceable load
j	load side feasible region points set (superscript)		
K	Spatial dimensions of performance output (superscript)		

approach. For integrated power and gas systems, a UGF-based method addressing performance substitution and sharing mechanisms was proposed [12]. Additionally, Shabani-Poodeh et al. [13] developed a reliability-constrained stochastic model for optimizing power-to-gas (P2G) and gas-to-power (G2P) equipment placement, and Hasanzad et al. [14] introduced a robust optimization model to enhance the resilience of integrated systems to seismic events.

However, existing MPMS reliability models and methods face challenges in evaluating the reliability of large-scale MPMS, particularly when performance conversion processes are considered. In large-scale MPMS, the number of multi-performance components (MPCs), performance variables, and possible states can become significantly large, greatly increasing the computational burden of reliability evaluation [15]. Additionally, different components may exhibit varying performance conversion characteristics. Reliability assessment of large-scale MPMS that incorporate performance conversion processes requires substantial computational resources. Therefore, developing efficient reliability evaluation methods for MPMS is of critical importance.

To address the challenges in large-scale MPMS reliability assessment considering performance conversion, several efficient methods have been proposed. Huang et al. [16] uses multi-parametric linear

programming to linearize multi-energy carrier system models. Lei et al. [17] improves efficiency with a decoupled power flow method and incremental impact enumeration. Zeng et al. [18] accelerates load shedding calculations using piecewise linearization and extends reliability metrics for gas systems. Chen et al. [19] proposes a topological simplification to decouple power and gas system evaluations. Ding et al. [20] extends UGF for integrated heat and power systems, employing hydro-thermal decoupling to decompose scheduling models into linear programming models. Hu et al. [21] applies Gaussian approximation and clustering to classify component states and performance conversion processes, reducing evaluation complexity. Wang et al. [22] introduces a rapid method for supply reliability evaluation in integrated power-gas systems, simplifying consequence analysis and enhancing MCS efficiency with cross-entropy-based resampling.

The reliability assessment of MPMS involves two key steps: First, generating different system states under potential failure scenarios based on the failure probabilities of components; second, analyzing each system state to evaluate the difference between the system's supply capacity and demand, and calculating the performance loss or shortfall when the system's supply capacity is insufficient to meet the required performance metrics. However, the above-mentioned accelerated MPMS

reliability assessment methods primarily focus on simplifying the optimal performance loss model to enhance the efficiency of system state analysis. Yet, these performance loss-based methods cannot evaluate the adequacy of the system before executing calculations and require the computation of the optimal performance loss for each system state, resulting in low time efficiency.

System adequacy assessment is a critical step in improving the efficiency of MPMS reliability evaluation. By explicitly defining the relationship between system supply capacity and performance demand, system adequacy can be evaluated efficiently without the need for extensive performance loss calculations. To achieve this, the feasible region (FR) model offers an effective approach to characterize the system's performance levels, taking into account the multi-performance conversion processes within the system. The feasible region provides a clear representation of the system's performance supply capabilities under different states, enabling a determination of whether the demand is met. Chen et al. [23] proposes a nonlinear programming framework to determine the boundary points of the MPMS feasible region. Su et al. [24] develops a vertex search algorithm for rapidly establishing the secure region (SR) considering N-1 contingencies. Zhang et al. [25] formulates the SR of an MPMS with CHP units and proposes a time-varying model to obtain equivalent SR hyperplanes. Despite these advancements in describing the performance supply capabilities of MPMS, their application in MPMS reliability assessment remains limited.

This paper proposes a reliability evaluation method for MPMS based on dynamic feasible region matching, enabling efficient assessment of large-scale MPMS. The system is categorized into source-side and load-side MPMS based on performance variables. The source-side feasible region (S-FR) represents the achievable performance range under different states while ensuring system safety and stability, while the load-side feasible region (L-FR) reflects achievable performance levels through integrated demand response (IDR). Both feasible regions are constructed offline, significantly reducing computational demands during online evaluation.

The method defines system state analysis and performance loss calculation based on the feasible region model. By examining the interaction between source-side and load-side feasible regions, system adequacy is pre-determined, enabling effective state screening. For inadequate states, the optimal performance loss is determined as the shortest Euclidean distance between the two feasible regions. This approach effectively characterizes MPMS performance, providing guidance for system planning, operational optimization, and ensuring safe operation of real-world systems.

The contributions of this paper are summarized as follows:

- (1) A feasible region modeling method for MPMS reliability assessment is proposed, decoupling the MPMS feasible region into S-FR and L-FR, enabling offline modeling and online application of the MPMS feasible region.
- (2) To improve computational efficiency, a system state analysis model based on dynamic matching of feasible regions is introduced. By analyzing the interaction between S-FR and L-FR, system adequacy is assessed, enabling effective filtering of system states.
- (3) A fast method for optimal performance loss is proposed by calculating the minimum Euclidean distance between S-FR and L-FR, simplifying the reliability assessment process.

The remainder of this paper is organized as follows: Section 2 introduces the structure and the topology of MPMS; Section 3 presents the modeling of source and load side feasible regions for MPMS reliability assessment; Section 4 describes an efficient operational reliability evaluation model for MPMS based on dynamic matching of source and load side feasible regions; Section 5 presents the case studies and Section 6 concludes the paper.

2. Multi-performance multi-state systems

2.1. Introduction to MPMS

MPMSs are advanced modeling frameworks that represent the complex interactions of systems and devices in engineering applications, characterized by multiple performance variables and discrete states. MPMSs account for the dynamic performance of components and the ability to exchange or convert different types of performance. A multi-energy carrier system (MECS) serves as a typical example of an MPMS, integrating various energy forms such as electricity, natural gas, heat and cooling across multiple stages, including energy production, conversion, transmission, storage and consumption.

MECSs exploit physical interconnections and multi-performance conversion processes between systems and components, enabling synergy across energy stages. These systems facilitate significant energy and information exchange, embodying the "smart energy" paradigm. Based on the characteristics of performance variables, MECSs can be categorized into source-side MPMS and load-side MPMS. Source-side MPMS refers to the part of the system that provides energy or services, typically including power sources, generators, or other energy generation devices. The state of the source side is mainly influenced by external resource supply, environmental conditions, and internal system allocation. In contrast, load-side MPMS refers to the part of the system that receives energy or services from the source side, usually including various load devices and user demands. The state of the load side is influenced by equipment usage, changes in load demand, and system operating strategies. In an MPMS, the source side and load side MPMS are tightly coupled. The source side MPMS provides energy, while the load side MPMS consumes it based on demand.

As shown in Fig. 1, an MECS includes four main components: energy suppliers, energy microgrids, load aggregators, and MECS operators. Energy suppliers consist of systems such as the power grid and natural gas network. Energy microgrids, positioned near consumers, enable energy conversion using technologies such as P2G, CHP, gas boilers (GB), electric boilers (EB), absorption chillers (AC), electric refrigerators (ER) and storage components for electricity and heat. On the load side, electrical, thermal and cooling loads are aggregated and submitted to MECS operators, who are responsible for system scheduling and reliability management.

The categorization of MECSs into source-side and load-side MPMS enables a comprehensive analysis of the system's multi-performance conversion processes and performance variables. This approach enhances energy system efficiency, reliability, and flexibility, offering a solid framework for managing the complexities of multi-energy systems. Through real-time data exchange and coordination, the source-side and load-side MPMS ensure overall system optimization. For example, the source-side MPMS provides the load-side MPMS with power supply capacity, while the load-side MPMS adjusts its demand to match the available supply, maintaining system balance. In response to fluctuations in power demand, both MPMS systems collaborate via demand response and load scheduling mechanisms to ensure stability and efficiency. The source-side adapts its generation or supply strategies based on load-side demand changes, and the load-side adjusts its load according to scheduling instructions from the source-side.

2.2. Energy Hub Framework

The energy hub (EH) model provides a unified framework for describing the coupling relationships between various energy forms within a single model [26], clearly representing energy flows between different carriers [27]. In many cases, the EH model can substitute for MECS. According to EH theory, in an MECS, performance conversion is characterized by the distribution of input energy on the source side to meet demand on the load side, guided by the operational characteristics of conversion equipment and system scheduling strategies. Fig. 2

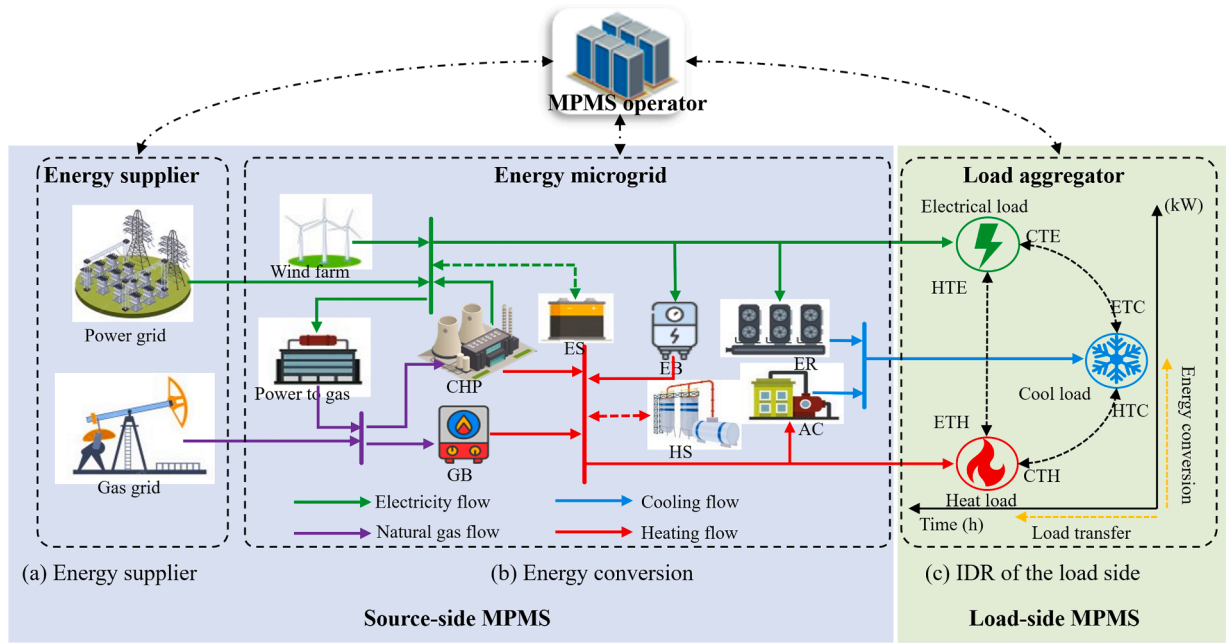


Fig. 1. MECS topology structure.

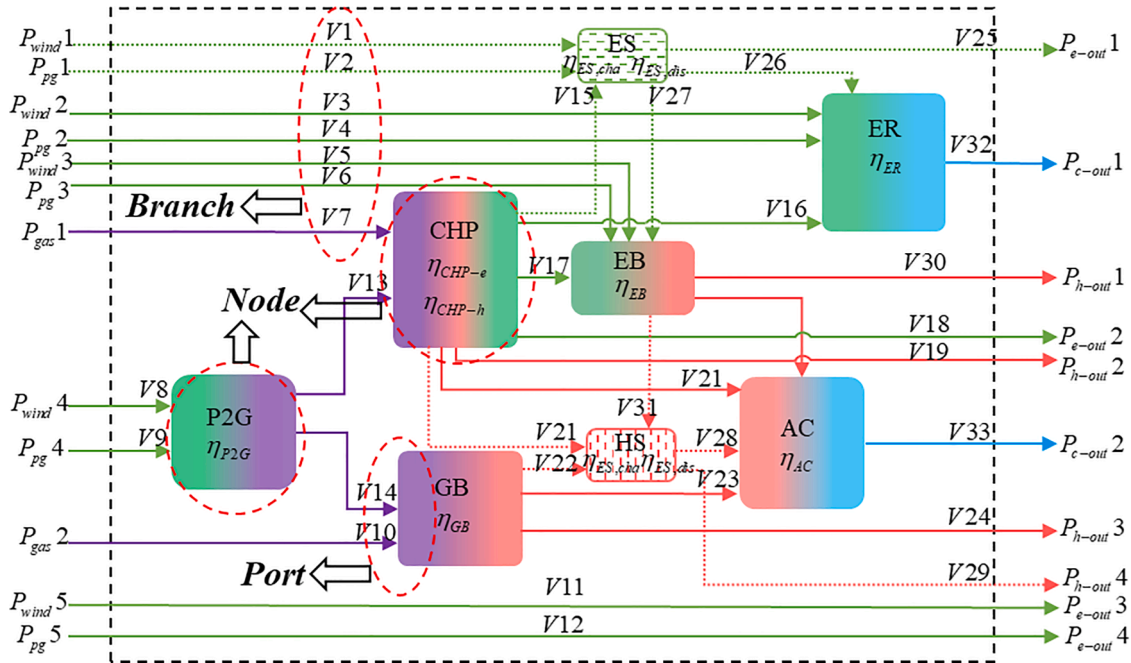


Fig. 2. Structure of EH.

illustrates the topology of an MECS modeled as an MPMS, showcasing its source-side and load-side configurations.

The topology diagram of the EH consists of a directed graph with branches, nodes, and ports. Inputs and outputs are represented as distinct nodes, each with multiple performance exchange ports. Performance flows between different variables, represented by branches, are treated as independent variables of the system. A performance flow vector, denoted by V , is formed by summing all performance exchanges within the MECS. Incidence vectors and matrices are employed to describe the connections between the elements, analogous to methods used in power system analysis.

3. Source-side and load-side MPMS feasible region modeling for reliability assessment

In MPMS, the concept of feasible regions is a powerful tool for characterizing the performance levels of systems under various operating states. For reliability assessment, feasible region modeling allows the system's performance supply and demand to be evaluated comprehensively. Specifically, the MPMS is categorized into two main components based on performance variables: source-side MPMS and load-side MPMS, each with distinct roles and characteristics.

3.1. Analysis and Modeling of Source-Side MPMS

3.1.1. Flexibility analysis of source-side MPMS considering performance conversion processes

The flexibility of the source side MPMS is primarily reflected in the stages of performance conversion and energy storage. The coupling and conversion between multiple performance variables eliminate technical barriers between different performance networks at the physical level, enabling interconnectivity and interoperability among various performance types, thereby enhancing the overall system flexibility. Additionally, the coordinated operation of storage components with performance supply and conversion equipment maximizes the potential of system flexibility. Based on the EH framework, the flexibility of the source-side MPMS is modeled using a standard matrix form [6].

1) Performance conversion matrix

The performance conversion matrix is used to model the interactions and coupling between different performance types, forming the basis for analyzing the flexibility and operational characteristics of the source-side MPMS. The performance conversion matrix is derived by multiplying the port-branch incidence matrix by the converter characteristic matrix of node g :

$$\mathbf{Z}_g = \mathbf{H}_g \mathbf{A}_g \quad (1)$$

where $\mathbf{Z}_g$ represents the performance conversion matrix of node g , $\mathbf{A}_g$ is the port-branch connection matrix used to indicate the connectivity between node g and all its branches, and $\mathbf{H}_g$ is the conversion characteristic matrix of node g , which represents the performance conversion efficiency within node g .

$$\mathbf{A}_g(k, b) = \begin{cases} 1 & b \text{ is attached to } g\text{'s input port } k \\ -1 & b \text{ is attached to } g\text{'s output port } k \\ 0 & b \text{ is not attached to any port of } g \end{cases} \quad (2)$$

$$\mathbf{H}_g(s, k) = \begin{cases} \eta_s, & \text{input port } k \text{ is related to performance conversion process } s \\ 1, & \text{output port } k \text{ is related to performance conversion process } s \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

2) Performance flow equation

The performance flow equation links the inputs, outputs, and conversion processes within the source-side MPMS, ensuring that the balance of performance variables is maintained under operational constraints. Matrix $\mathbf{Z}_g$ describes the performance conversion characteristics of node g from the perspective of branches. By integrating all performance conversion matrices, we obtain the performance conversion matrix $\mathbf{Z}$ for source-side MPMS.

$$\mathbf{Z} = [\mathbf{Z}_1^T, \mathbf{Z}_2^T, \dots, \mathbf{Z}_G^T]^T \quad (4)$$

Based on the definitions of the matrices, the performance conversion equation for source-side MPMS can be expressed as:

$$\mathbf{Z}\mathbf{V} = \mathbf{0} \quad (5)$$

where the branch energy flows make up the performance flow vector $\mathbf{V}$, which is composed of the performance conversion matrix $\mathbf{Z}$. Let $\mathbf{V}_{in}$ and $\mathbf{V}_{out}$ stand for the input and output flows of performance, respectively. The following definitions apply to the input incidence matrix $\mathbf{C}_{in}$ and the output incidence matrix $\mathbf{C}_{out}$:

$$\mathbf{C}_{in}(i, b) = \begin{cases} 1, & \text{if input node } i \text{ is the source of branch } b \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

$$\mathbf{C}_{out}(i, b) = \begin{cases} 1, & \text{if output node } i \text{ is the sink of branch } b \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Then, the total performance flow equations of the source-side MPMS can be expressed in matrix form:

$$\mathbf{Z}\mathbf{V} = \mathbf{0} \quad (8)$$

$$\mathbf{C}_{in}\mathbf{V} = \mathbf{V}_{in} \quad (9)$$

$$\mathbf{C}_{out}\mathbf{V} = \mathbf{V}_{out} \quad (10)$$

$$\mathbf{V} \in \Gamma_V \quad (11)$$

where Γ_V represents the feasible set of performance flows, which is constrained by inequalities related to branch performance transfer limits. Additionally, the capacity constraints of performance conversion components within source-side MPMS are embedded in Γ_V through constraints on the input and output branch performance flows, denoted as $\mathbf{V}_{min} \leq \mathbf{V} \leq \mathbf{V}_{max}$. Eq. (8) represents the performance balance within the source-side MPMS, equations (9) - (10) represent the performance input and output of the source-side MPMS, and Eq. (11) represents the capacity constraint of the performance within the source-side MPMS.

3.1.2. Feasible region modeling for Source-side MPMS

The performance supply capability of the source-side MPMS is defined as vector $\mathbf{Y}_r$, where each component of $\mathbf{Y}_r$ represents the output performance flows. Vector $\mathbf{X}_r$ represents the state variables of the MPMS. The source-side feasible region (S-FR) of the MPMS can be described as follows:

$$\Theta_r = \{\mathbf{Y}_r | f(\mathbf{X}_r, \mathbf{Y}_r) \leq 0\} \quad (12)$$

where $f(\mathbf{X}_r, \mathbf{Y}_r) \leq 0$ represents the operational safety constraints of the source-side MPMS. For any $\mathbf{Y}_r \in \Theta_r$, there exists at least one $\mathbf{X}_r$ that satisfies both equality and inequality constraints.

According to the flexibility model of source-side MPMS, Eq. (12) can be described as:

$$\Theta_r = \{\mathbf{V}_{out} | \text{Eqs. (8) - (11)}\} \quad (13)$$

Further, when a component failure occurs within the IES, the source-side MPMS should utilize its flexibility potential to address the current performance demand with maximum flexibility, minimizing performance shedding and improving MPMS reliability. Considering the operational status of the components within the MPMS, the performance conversion matrix at this node is described as:

$$\hat{\mathbf{Z}}_g = \kappa_g \mathbf{H}_g \mathbf{A}_g \quad (14)$$

$$\kappa_g = \begin{cases} 1 & \text{if the note } g \text{ status is normal} \\ 0 & \text{if the note } g \text{ status is faulty} \end{cases} \quad (15)$$

where $\hat{\mathbf{Z}}_g$ represents the performance conversion matrix of the source-side MPMS after a device failure, and κ_g denotes the component status at node g .

The system's performance conversion matrix can be characterized by the following equation:

$$\hat{\mathbf{Z}}^n \mathbf{V} = \mathbf{0}, \quad n \in N \quad (16)$$

$$\hat{\mathbf{Z}} = [\hat{\mathbf{Z}}_1^T, \hat{\mathbf{Z}}_2^T, \dots, \hat{\mathbf{Z}}_G^T]^T \quad (17)$$

where n denotes the current operational state of the system, N represents the set of all operational states of the system, and $\hat{\mathbf{Z}}^n$ indicates the performance conversion matrix of the MPS when operating in system state n .

When a component failure occurs at time t within the MPMS, the

storage components should take on the role of performance sources, fully leveraging their functionality to maximize the flexibility of the MPMS. Based on the pre-scheduling, the capacity state of the storage components within the scheduling period can be characterized by:

$$P_*^{ES}(t) = \eta_*^{dis} E_*^{pre}(t-1) \quad (18)$$

$$0 \leq P_*^{ES}(t) \leq P_{*,max}^{dis} \quad (19)$$

where $P_*^{ES}(t)$ represents the output performance of the storage component at time t , and $E_*^{pre}(t-1)$ denotes the pre-scheduled capacity state of the storage component at time $t-1$.

After a component failure in the MPMS, the system operator should reschedule the components based on the current component connectivity topology to maximize the system's flexibility potential and mitigate the risk of performance loss due to component failure. By constructing the S-FR of the MPMS, the system's performance supply capability can be depicted and characterized. As described above, at time t in system state n , the S-FR of the MPMS can be characterized as:

$$\Theta_r^n(t) = \left\{ \begin{array}{l} \hat{\mathbf{Z}}^n \mathbf{V}^n(t) = \mathbf{0}, \\ \mathbf{C}_{in} \mathbf{V}^n(t) = \mathbf{V}_{in}^n(t), \\ \mathbf{C}_{out} \mathbf{V}^n(t) = \mathbf{V}_{out}^n(t), \\ \mathbf{V}^n(t) \in \Gamma_V, \\ \text{Eqs. (18) - (19)} \end{array} \right\}, n \in N \quad (20)$$

3.2. Analysis and Modeling of load-Side MPMS

3.2.1. Flexibility analysis of load-side MPMS considering performance conversion processes

The integration of multiple performance forms introduces significant flexibility at the load side of MPMS. In conventional single-energy systems, specific demands, such as electrical loads, can only be met by one type of energy. However, in MPMS, performance demands include various performance forms, such as electricity, cooling and heating, which are interrelated and interconvertible. For instance, heating demands can be satisfied through electric heating, heat pumps or district heating systems. This complementarity and substitutability of performance variables provide considerable flexibility for optimizing load-side strategies during normal operation, demand response, or emergency.

This paper considers integrated demand response (IDR) in the load-side MPMS, encompassing both temporal and spatial dimensions, as shown in Fig. 1(c). Temporally, users adjust performance consumption based on price signals, enabling performance shifting over time. Spatially, different types of performance demands can substitute for one another while maintaining the same service quality. Load-side MPMS performance demands are categorized into three types: stable performance demands, transferable performance demands and replaceable performance demands. Transferable performance demands can be adjusted in response to changes in demand and real-time instructions, thus supporting both load balance and the system's flexibility requirements. Replaceable performance demands provide backup support in situations where primary load demands cannot be met, ensuring system stability and continuous operation. Stable performance demands contribute the system's base load, ensuring a stable foundation for consistent operation.

$$P_{*,Load}(t) = P_{*,Load}^s(t) + \Delta P_{*,Load}^t(t) + \Delta P_{*,Load}^r(t) \quad (21)$$

$* \in \{\text{Electricityload, Heatload, Coolload}\}$

where $P_{*,Load}(t)$ represents the value of the $*$ -th type of performance load at time t , $P_{*,Load}^s(t)$ represents the fixed load of the $*$ -th type of performance load at time t , $\Delta P_{*,Load}^t(t)$ represents the transferable load of the $*$ -th type of performance load at time t , and $\Delta P_{*,Load}^r(t)$ represents the substitutable load of the $*$ -th type of performance load at time t .

1) Stable performance load $P_{*,Load}^s(t)$

Stable performance loads, such as electrical, heating, or cooling demands, do not participate in demand response (DR) under either temporal or spatial dimensions. These loads must be satisfied immediately without any flexibility.

2) Transferable performance load $\Delta P_{*,Load}^t(t)$

Transferable performance loads can be shifted in the time dimension during the scheduling period. The model for this type of load is as follows:

$$\Delta P_{*,Load}^t(t) = \zeta_*^{t,in}(t) P_{*,Load}^{t,in}(t) - \zeta_*^{t,out}(t) P_{*,Load}^{t,out}(t) \quad (22)$$

$$\zeta_*^{t,in}(t) + \zeta_*^{t,out}(t) = 1 \quad (23)$$

$$|\Delta P_{*,Load}^t(t)| \leq \Delta P_{*,Load,max}^t \quad (24)$$

$$\sum_{t=1}^{24} \Delta P_{*,Load}^t(t) = 0 \quad (25)$$

where $P_{*,Load}^{t,in}(t)$ and $P_{*,Load}^{t,out}(t)$ represent the input and output values of the transferable load of the $*$ -th type at time t , respectively. $P_{*,Load}^{t,in}(t)$ represents the amount of load transferred to time t from other times for the $*$ -th type of load at time t , whereas $P_{*,Load}^{t,out}(t)$ indicates the load transferred from time t to other times. $\zeta_*^{t,in}(t)$ and $\zeta_*^{t,out}(t)$ are binary variables representing the transferable load input and output parameters for the $*$ -th type of load at time t . Eq. (24) represents the power transfer constraints for transferable performance loads. Eq. (25) ensures that the total amount of transferable performance load within a scheduling period remains constant.

3) Replaceable load $\Delta P_{*,Load}^r(t)$

Replaceable performance loads allow users to choose among different performance supply methods to meet their performance demands within the same time period. This flexibility is commonly observed in practical scenarios. For instance, domestic hot water demand can be met using thermal power, electric water heaters, or gas water heaters. Similarly, cooling demands can be satisfied via cooling power supplied by the MPMS or electric air conditioners. When replaceable loads participate in DR, the user's total performance demand remains constant, ensuring no degradation in the quality of service.

The model for replaceable performance loads is as follows:

$$\begin{cases} \Delta P_{*,Load}^r(t) = \Delta P_{*,Load}^{r,EH}(t) + \Delta P_{*,Load}^{r,EC}(t) \\ \Delta P_{*,Load}^{r,EH}(t) = \Delta P_{*,Load}^{r,HE}(t) + \Delta P_{*,Load}^{r,HC}(t) \\ \Delta P_{*,Load}^{r,EC}(t) = \Delta P_{*,Load}^{r,CE}(t) + \Delta P_{*,Load}^{r,CH}(t) \end{cases} \quad (26)$$

$$\Delta P_{*,Load}^r(t) = -\eta_{xy} \Delta P_{*,Load}^r(t) \quad x, y \in * \quad (27)$$

$$\eta_{xy} = \frac{W_x \cdot V_x}{W_y \cdot V_y} \quad x, y \in * \quad (28)$$

$$0 \leq |\Delta P_{*,Load}^r(t)| \leq \Delta P_{*,Load,max}^r \quad (29)$$

where $\Delta P_{*,Load}^{r,EH}(t)$ and $\Delta P_{*,Load}^{r,EC}(t)$ represent the replaceable performance conversion between electrical load and heating or cooling loads, η_{xy} denotes the performance conversion efficiency, W represents the thermal value per unit of performance, V indicates the utilization rate per unit of performance and $\Delta P_{*,Load,max}^r$ denotes the maximum value of the replaceable load.

By incorporating transferable and substitutable performance loads,

the load-side MPMS achieves significant flexibility, optimizing system performance while ensuring service reliability and user satisfaction.

3.2.2. Feasible region modeling for load-side MPMS

Load-side MPMS represents the system's performance demand and response processes. The feasible region of the load-side MPMS (L-FR) describes the achievable performance levels of multi-performance variables, such as electrical, thermal and cooling loads, under various demand scenarios through demand response and optimization strategies. By considering the optimized scheduling of load-side IDR in MPMS, performance shifting and substitution are implemented to achieve peak shaving and valley filling. When a component failure occurs in the system, the load side should leverage its flexibility potential to mitigate the risk of performance loss caused by system component failures, reducing the impact of these failures on its performance demand. Performance substitution, achieved by utilizing different performance types to meet the same performance requirements, ensures reliable operation without compromising user experience. Based on the performance substitution demand response of the load side, the L-FR is established.

$$\Theta_l = \{\mathbf{Y}_l | g(\mathbf{X}_l, \mathbf{Y}_l) \leq 0\} \quad (30)$$

where $\mathbf{Y}_l$ denotes the performance demand vector on the load side, $\mathbf{Y}_l$ represents the demand for electrical, thermal and cooling performance variables, $\mathbf{X}_l$ indicates the state variables, and $g(\mathbf{X}_l, \mathbf{Y}_l) \leq 0$ represents the substitutable performance demand response constraints. For any $\mathbf{Y}_l \in \Theta_l$, there exists at least one $\mathbf{X}_l$ that satisfies both equality and inequality constraints.

Based on the above introduction, the L-FR at time t can be described as:

$$\Theta_l(t) = \left\{ \mathbf{P}_{load}(t) \left| \begin{array}{l} \mathbf{P}_{load}(t) = \mathbf{P}_{load}^{pre}(t) + \Delta \mathbf{P}_{r,load}(t), \\ 0 \leq |\Delta \mathbf{P}_{r,load}(t)| \leq \Delta \mathbf{P}_{r,load,max} - |\Delta \mathbf{P}_{r,load}^{pre}(t)|, \end{array} \right. \right\} \quad (31)$$

Eqs.(26) – (28)

where $\Theta_l(t)$ represents the L-FR at time t , $\mathbf{P}_{load}(t)$ represents the performance demand vector of the load side at time t , $\mathbf{P}_{load}^{pre}(t)$ denotes the performance demand on the load side after system pre-scheduling at time t , $\Delta \mathbf{P}_{r,load}(t)$ represents the performance value of the load side participating in substitutable DR at time t and $\Delta \mathbf{P}_{r,load}^{pre}(t)$ is the performance value of the load side participating in substitutable DR during system pre-scheduling at time t .

The construction of the L-FR for the load-side MPMS is based on system pre-scheduling. Therefore, it is necessary to consider the performance values of substitutable DR during the pre-scheduling phase to avoid exceeding the constraints imposed by substitutable DR on the load-side. By incorporating these constraints, the L-FR ensures that load-side flexibility can be utilized while maintaining system reliability and respecting pre-scheduled performance levels.

3.3. Adjusting the feasible region model to account for fluctuations

The large-scale integration of renewable energy into MPMS introduces variability in performance supply and uncertainty in performance demand, posing challenges to the system's safe operation and reliability assessment. Using the S-FR and L-FR models proposed in this paper, deviations between the actual performance output of renewable energy units, performance demand values and their forecasts at time t can be addressed. The feasible region model adjusts by modifying constraints based on day-ahead scheduling and intraday rolling optimization results, accounting for these fluctuations:

$$\tilde{\Theta}_r^n(t) = \left\{ \mathbf{V}_{out}^n(t) \left| \begin{array}{l} \hat{\mathbf{Z}}^n \mathbf{V}^n(t) = \mathbf{0}, \\ \mathbf{C}_{in} \mathbf{V}^n(t) = \mathbf{V}_{in}^n(t), \\ \mathbf{C}_{out} \mathbf{V}^n(t) = \mathbf{V}_{out}^n(t), \\ \mathbf{V}^n(t) \in \Gamma_V, \\ \tilde{\mathbf{P}}_*^{ES}(t) = \eta_*^{dis} \mathbf{E}_*^{roll}(t-1) \\ 0 \leq \tilde{\mathbf{P}}_*^{ES}(t) \leq \mathbf{P}_{*,max}^{dis} \\ 0 \leq \mathbf{P}_{wind}(t) \leq \mathbf{P}_{wind}^{pre}(t) + \Delta \mathbf{P}_{wind}(t) \end{array} \right. \right\}, n \in N \quad (32)$$

$$\tilde{\Theta}_l(t) = \left\{ \mathbf{P}_{load}(t) \left| \begin{array}{l} \mathbf{P}_{load}(t) = \mathbf{P}_{load}^{pre}(t) + \Delta \mathbf{P}_{r,load}(t) + \Delta \mathbf{P}_{load}(t), \\ 0 \leq |\Delta \mathbf{P}_{r,load}(t)| \leq \Delta \mathbf{P}_{r,load,max} - |\Delta \mathbf{P}_{r,load}^{roll}(t)| \end{array} \right. \right\} \quad (33)$$

Eqs.(26) – (28)

where $\tilde{\Theta}_r^n(t)$ and $\tilde{\Theta}_l(t)$ represent the adjustments to the S-FR and L-FR of the MPMS to account for renewable energy performance supply and demand performance fluctuations, respectively. $\mathbf{P}_{wind}^{pre}(t)$ denotes the amount of wind power absorbed as determined by pre-scheduling, $\mathbf{E}_*^{roll}(t-1)$ indicates the variation in wind power output, $\Delta \mathbf{P}_{wind}(t)$ represents the capacity status of storage components at $t-1$ as determined by intraday rolling optimization, $\Delta \mathbf{P}_{load}(t)$ is the amount of demand performance variation, and $\Delta \mathbf{P}_{r,load}^{roll}(t)$ signifies the quantity of substitutable DR determined by intraday rolling optimization.

4. Efficient Reliability Assessment of MPMS Based on S-FR and L-FR Dynamic Matching

4.1. Reliability assessment based on S-FR and L-FR dynamic matching

Reliability assessment of a MPMS evaluates its ability to perform specified functions under given conditions within a set timeframe [28]. MPMS reliability measures the system operator's capacity to meet diverse performance demands, including electricity, heat and cooling, through performance conversion components. Existing reliability assessment methods primarily fall into three categories: MCS, analytical methods and hybrid approaches. These methods aim to minimize performance shedding while leveraging the flexibility of MPMS to meet performance demands even under $N-1$ faults. However, traditional approaches often rely on complex performance shedding calculations, resulting in high computational costs and inefficiencies.

The core of MPMS reliability assessment lies in evaluating how well performance demand aligns with supply capacity under varying conditions. By effectively representing performance supply capacity and performance demand, surplus states can be assessed without resorting to complex performance shedding calculations, thereby improving assessment efficiency.

Building on this concept, this paper proposes using feasible regions to represent the system's performance supply capability and performance demand. Through the dynamic matching method of the S-FR and the L-FR, as illustrated in Fig. 3, system states are analyzed, and reliability indicators are calculated accordingly. This approach leverages the flexibility of both the source and load sides of the MPMS, the need for performance shedding calculations in surplus states and significantly improving the efficiency of reliability assessments.

4.2. System state analysis based on S-FR and L-FR dynamic matching

Based on the S-FR and L-FR modeling method proposed in Section 3, system state analysis is performed through dynamic matching of the S-FR and L-FR. By analyzing the interaction between the S-FR and L-FR, system state analysis includes surplus assessment and performance shedding calculations. The dynamic matching process is designed to

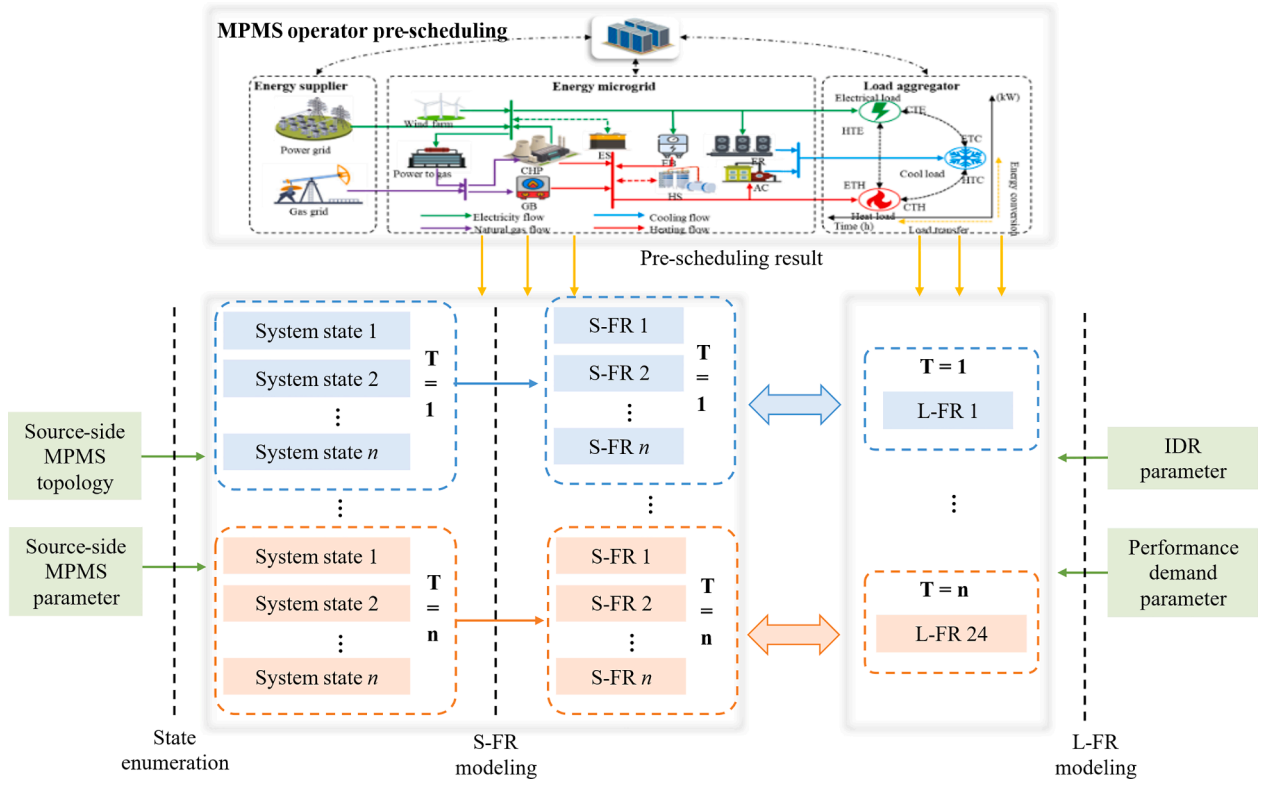


Fig. 3. MPMS operational reliability framework based on source-load feasible region dynamic matching.

identify the current state within the feasible regions of both the S-FR and L-FR by analyzing the conditions of devices within the MPMS. This process involves determining the S-FR and L-FR from their respective libraries. The relationship between these feasible regions, specifically their spatial configuration, is then evaluated to assess the system's sufficiency. In cases of inadequacy, the system's load reduction is represented by the shortest Euclidean distance between the feasible regions. The Manhattan distance between the closest pair of points is subsequently calculated to determine the required load reduction value.

4.2.1. System state surplus assessment

The main advantage of the MPMS state analysis model based on S-FR and L-FR dynamic matching, compared to traditional optimization-based models like optimal power flow (OPF), lies in its ability to efficiently assess system surplus conditions. It simplifies the problem of optimal performance shedding during system inadequacy into calculating the shortest Euclidean distance between two polyhedral, thereby

avoiding complex optimization procedures and reducing the computational complexity of state analysis.

In reliability assessment, higher-probability system states often involve lower-order faults, where surplus insufficiencies are uncommon. By omitting performance shedding calculations in surplus states, computational costs are significantly reduced, improving overall assessment efficiency.

For example, in an MPMS involving three types of performance, its feasible region can be represented by a three-dimensional polyhedron. The feasible region represents the system's ability to meet the performance demands for electrical, thermal and cooling, as shown in Fig. 4. The spatial relationships between S-FR and L-FR can be classified into three categories: inclusion (Fig. 4(a)), intersection (Fig. 4(b)) and disjoint (Fig. 4(c)).

- Inclusion: the L-FR is fully contained within the S-FR, indicating sufficient supply capability with no need for performance shedding.

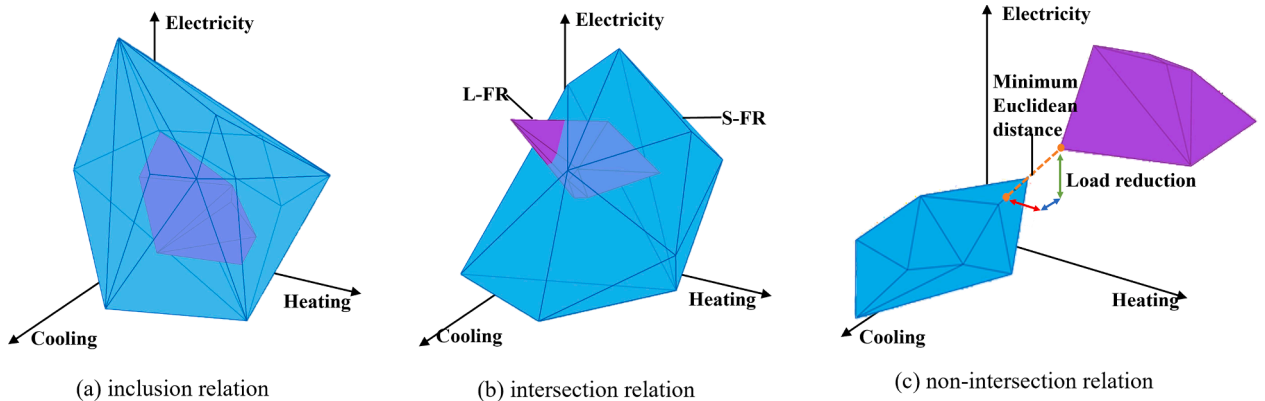


Fig. 4. Spatial position relation of source-load side feasible region of MPMS.

- Intersection: the S-FR and L-FR partially overlap, also representing a surplus state with no need for performance shedding.
- Disjoint: the S-FR and L-FR do not overlap, indicating insufficient supply capability and requiring performance shedding.

By identifying these spatial relationships, the surplus states of MPMS can be efficiently analyzed, eliminating unnecessary performance shedding calculations in inclusion and intersection cases, which constitute the majority of system states under normal conditions.

4.2.2. Performance shedding calculation

Euclidean distance accurately expresses the shortest path between two points in continuous space, as shown in Fig. 4(c). Therefore, in this study, use Euclidean distance to represent the geometric relationship between two non-intersecting polyhedra in three dimensions. The calculation formula is as follows:

$$\begin{aligned} \min_{i,j} d(p_{\Theta_r}^i, q_{\Theta_l}^j) \\ \text{s.t. } p_{\Theta_r}^i \in \Theta_r^n(t) \\ q_{\Theta_l}^j \in \Theta_l(t) \end{aligned} \quad (34)$$

$$d(p_{\Theta_r}^i, q_{\Theta_l}^j) = \sqrt{(x \cdot p_{\Theta_r}^i - x \cdot q_{\Theta_l}^j)^2 + (y \cdot p_{\Theta_r}^i - y \cdot q_{\Theta_l}^j)^2 + (z \cdot p_{\Theta_r}^i - z \cdot q_{\Theta_l}^j)^2} \quad (35)$$

where $p_{\Theta_r}^i$ and $q_{\Theta_l}^j$ represent the sets of points on the surfaces of the S-FR and L-FR, respectively.

In the reliability analysis and multi-criteria decision-making of the MPMS, the performance demands in different dimensions (e.g., electricity, heat, and cold) may have different importance. Weighted Euclidean distance can effectively evaluate the overall performance of multi-attribute objectives, and its general form is as follows:

$$d(p_{\Theta_r}^i, q_{\Theta_l}^j) = \sqrt{\omega_x (x \cdot p_{\Theta_r}^i - x \cdot q_{\Theta_l}^j)^2 + \omega_y (y \cdot p_{\Theta_r}^i - y \cdot q_{\Theta_l}^j)^2 + \omega_z (z \cdot p_{\Theta_r}^i - z \cdot q_{\Theta_l}^j)^2} \quad (36)$$

where, ω_x , ω_y and ω_z are the weights corresponding to the x, y, and z axes, respectively.

In the reliability evaluation of the MPMS system, the Manhattan

distance based on this shortest point pair $(p_{\Theta_r}^i, q_{\Theta_l}^j)$ to obtain the performance shedding calculation in the x, y, and z axis directions (i.e., electricity, heat, and cold energy).

$$P_{load, cut} = D_{Manhattan} = |x_{p_{\Theta_r}^i} - x_{q_{\Theta_l}^j}| + |y_{p_{\Theta_r}^i} - y_{q_{\Theta_l}^j}| + |z_{p_{\Theta_r}^i} - z_{q_{\Theta_l}^j}| \quad (37)$$

where, $P_{load, cut}$ is the performance shedding amount, $x_{p_{\Theta_r}^i}$, $y_{p_{\Theta_r}^i}$, $z_{p_{\Theta_r}^i}$ represent the coordinates of point $p_{\Theta_r}^i$, $x_{q_{\Theta_l}^j}$, $y_{q_{\Theta_l}^j}$, $z_{q_{\Theta_l}^j}$ represent the coordinates of point $q_{\Theta_l}^j$.

4.3. Efficient reliability assessment process for MPMS based on S-FR and L-FR dynamic matching

An efficient reliability assessment process for MPMS based on S-FR and L-FR dynamic matching is proposed, as illustrated in Fig. 5. The specific steps are as follows:

Off-line Modeling Calculation Process:

Step 1: Input the source-side MPMS topology, component parameters, pre-scheduling results, and IDR results.

Step 2: At time t, generate the source-side MPMS state and establish the S-FR for the current time step based on the system state, establish the L-FR based on IDR parameters and pre-scheduling results.

Step 3: When time step t = T, output the combined S-FR and L-FR libraries for reliability analysis.

Online Evaluation Process:

Step 4: Initialize $SENS_{IES}$ and SIF_{IES} reliability indicators.

Step 5: Perform Monte Carlo simulations to generate random system states for the scheduling period.

Step 6: Search and load the feasible regions of the current time and state from the S-FR and L-FR libraries, achieving dynamic matching between S-FR and L-FR.

Step 7: Evaluate system adequacy and calculate performance degradation based on the spatial relationship between S-FR and L-FR.

Step 8: Update $SENS_{IES}$ and SIF_{IES} reliability indicators.

Step 9: Calculate and output system reliability indicators.

4.3.1. Offline modeling

During the offline modeling phase, S-FR and L-FR of the MPMS are established. The S-FR is constructed by enumerating the fault states of

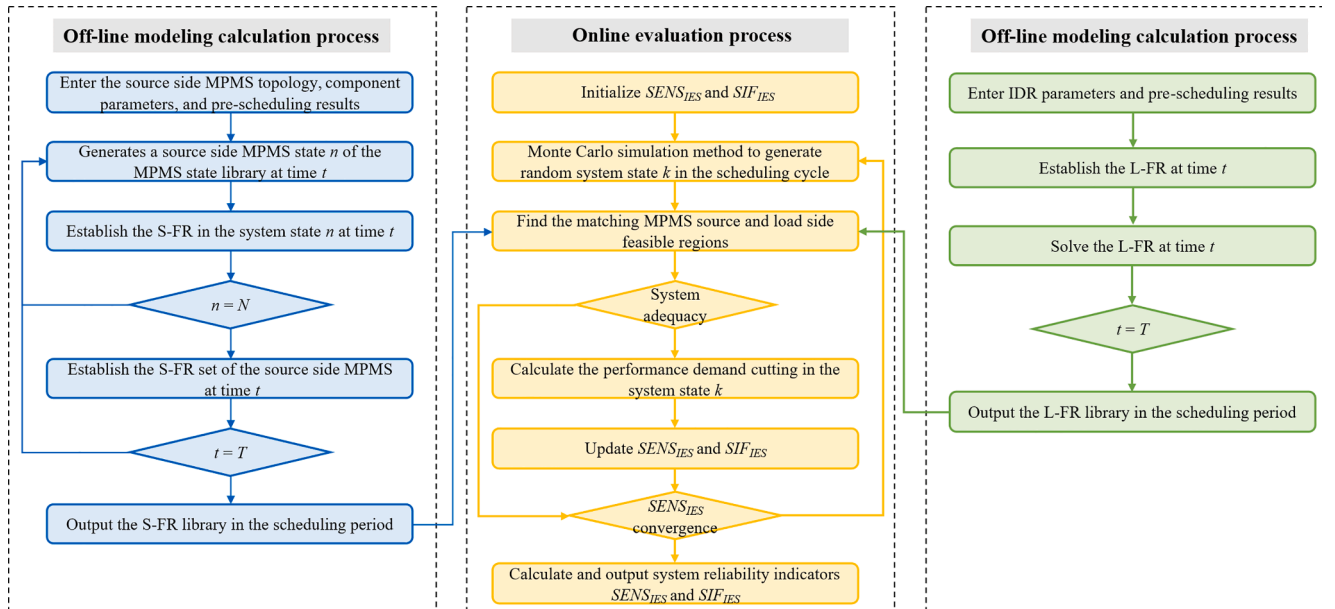


Fig. 5. Efficient reliability assessment process for MPMS based on S-FR and L-FR dynamic matching.

components within the MPMS, generating a library of system states. For each enumerated system state, the S-FR captures the system's performance supply capabilities under all possible fault conditions. Meanwhile, the L-FR is modeled for each time point within the scheduling period, representing the performance demands of the load side for electricity, heat, and cooling. The L-FR incorporates temporal flexibility and substitutable performance demands to account for the adaptability of the load side.

4.3.2. Online reliability assessment

In the online reliability assessment phase, MCS is used to generate system states. For each simulated system state, the corresponding S-FR is retrieved from the S-FR library, while the L-FR for the current time point is located from the L-FR library. The system state analysis method, based on dynamic matching of S-FR and L-FR, is then applied. If the system is in surplus, such as when the S-FR includes or intersects with the L-FR, no performance shedding is calculated. However, for inadequate states, the system requires load shedding. By calculating the shortest Euclidean distance between the feasible regions of the source side and the load side, the pair of points with the shortest distance can be identified. The load shedding amount is then determined based on the Manhattan distance between this closest pair of points. Finally, all system state analysis results are summarized to calculate the system reliability metrics.

The formula for calculating the system reliability metrics is as follows.

$$SENS_{IES} = \sum_{n=1}^N \left(\sum_{t=1}^T \mathbf{P}_{load,cut}(t) \right) / N \quad (38)$$

$$SIF_{IES} = \sum_{n=1}^N \left(\sum_{t=1}^T \text{flag}(\mathbf{P}_{load,cut}(t)) \right) / N \quad (39)$$

where $SENS_{IES}$ represents the system's performance insufficiency, SIF_{IES} denotes the system's performance supply interruption frequency, N is the number of Monte Carlo sampling iterations, $\mathbf{P}_{load,cut}(t)$ is the performance shedding amount at time t under system state n , and T is the reliability assessment period. The definition of $\text{flag}(x)$ is as follows:

$$\begin{cases} x > 0, \text{flag}(x) = 1 \\ x \leq 0, \text{flag}(x) = 0 \end{cases} \quad (40)$$

5. Case Studies

In this section, case studies are presented to demonstrate the application of the proposed reliability assessment method for MPMS based on S-FR and L-FR dynamic matching. The case studies in this paper are based on a MECS, where energy conversion, transmission, and storage are modeled by the EH model, which transforms the load curtailment minimization problem of MECS into a linear programming (LP) problem. Therefore, this paper conducted a comparative analysis between the reliability evaluation method based on LP model [31] and the proposed reliability evaluation method based on dynamic matching of feasible regions.

5.1. Representative example analysis

To improve the model's readability and practical engineering value, a representative example is provided to demonstrate how to model the S-FR and conduct system state analysis for a given system state. The test system is a simple multi-energy system that includes devices such as transformers, CHP, GB, ES, and EB. The system topology is shown in Fig. 6, and the system parameters are listed in Table 2. Assuming a failure of the GB, we establish the source-side feasible region and perform the system state analysis accordingly.

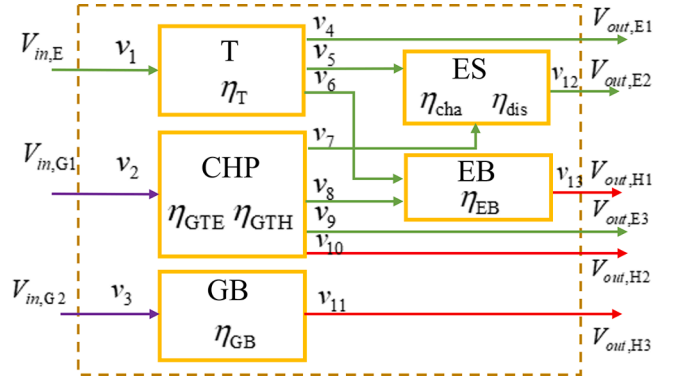


Fig. 6. Schematic diagram of different distances in three-dimensional space.

5.1.1. Source-Side Feasible Region Modeling

First, based on the model presented in Section 3.1, we have established the source-side MPMS feasible region boundary when GB fails. When GB fails, the system parameters can be calculated according to the system topology and device parameters.

$$\mathbf{C}_{in} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (41)$$

$$\mathbf{C}_{out} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad (42)$$

$$\hat{\mathbf{Z}}^n = \begin{bmatrix} \eta_T & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \eta_{GTE} & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & \eta_{GTH} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \eta_{EB} & 0 & \eta_{EB} & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (43)$$

$$\mathbf{V}^n = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6 \ v_7 \ v_8 \ v_9 \ v_{10} \ v_{11} \ v_{12} \ v_{13}]^T \quad (44)$$

$$\mathbf{V}_{in} = [V_{in,E} \ V_{in,G}]^T \quad (45)$$

$$\mathbf{V}_{out} = [V_{out,E} \ V_{out,H}]^T \quad (46)$$

$$v_{12} = \eta_{dis} E(t-1) \quad (46)$$

By organizing the matrices involved in formula (20) from the original manuscript, the source-side MPMS feasible region when GB fails can be obtained, as shown in the Fig. 7.

As shown in the Fig. 7, although the source-side MPMS feasible region has many boundaries, the number of truly effective boundaries is quite small. From the source-side MPMS feasible region shown, the three

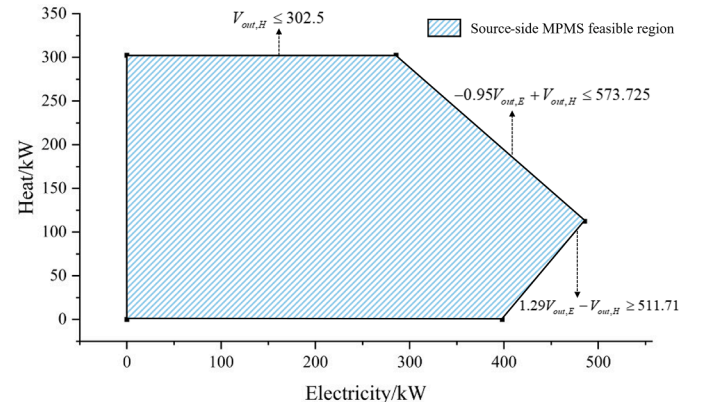


Fig. 7. Source-side MPMS feasible region when GB fails.

boundaries from top to bottom represent system thermal energy output capacity limitation, EB generation capacity limitation, and ES discharge capacity limitation.

5.1.2. System state analysis

After establishing the source-side MPMS system state feasible region, we used the dynamic matching-based system state analysis method to quickly analyze the system state under different load conditions, judge the system state's sufficiency, and compute the optimal load shedding amount.

In the example, we did not consider the integrated demand response of the load-side MPMS, so the load-side feasible region is a single load point. When the original given load is a certain value, the optimal load shedding result calculated by the proposed model is shown in Fig. 8. By calculating the shortest Euclidean distance between the original load and the source-side MPMS feasible region, the current reductions in electrical and thermal loads are 28.6 kW and 30.11 kW, respectively. These values are consistent with the optimal load reduction results obtained from the LP model.

5.2. Analysis of small-scale test system A

5.2.1. Case introduction of test system A

The MECS topology of the small-scale test system A, representing a MPMS, is illustrated in Fig. 1. The operational reliability of the MPMS is evaluated over a 24-hour dispatch period. Table 1 and Table 2 present the parameters of the MPMS [29,30], whereas Figs. 9 and 10 show the pre-dispatch results for a typical day in this test system.

5.2.2. Analysis of the S-FR and L-FR dynamic matching process

Based on the pre-dispatch results of the MPMS, an offline calculation is performed to establish a S-FR and L-FR database for the system. The analysis is conducted during a performance demand valley period ($T=5h$) and a performance demand peak period ($T=12h$), considering $N-1$, $N-2$, and $N-3$ contingencies within the system. The S-FR and L-FR databases are constructed for these two time periods, and dynamic matching between the S-FR and L-FR is carried out to filter the system states, as illustrated in Fig. 11.

By dynamically matching the source-load feasible regions at the same time point, the system's state analysis can be performed. During the performance demand valley period ($T=5h$), even when an $N-3$ contingency occurs within the MPMS, the system can maintain adequacy without any performance shedding, as illustrated in Fig. 11(a)-Fig. 11(c). During the performance demand peak period ($T=12h$), the system remains adequate without performance loss under an $N-1$ contingency, as illustrated in Fig. 11(d). However, when $N-2$ or higher contingencies occur, the system's adequacy is compromised, leading to performance

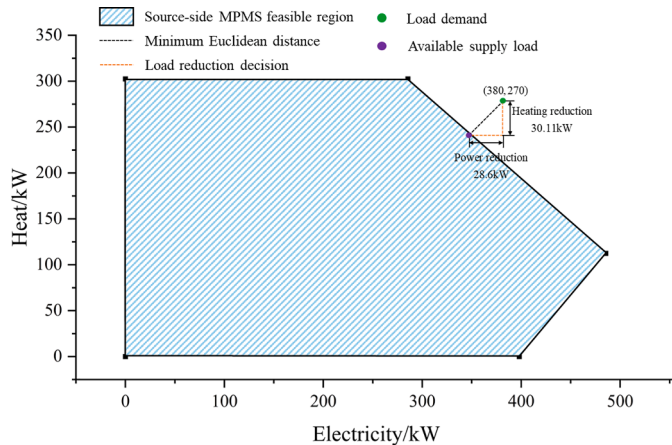


Fig. 8. Load shedding calculation when GB fails and the load is certain.

Table 1

IDR parameters on the load-side MPMS.

Alternative IDR			Transferable IDR	
Converter	Capacity (kW)	Efficiency	Performance type	Capacity (kW)
ETC	100	2	$\Delta P_{E,Load,max}^f$	60
ETH	100	1.2		
HTC	60	0.9	$\Delta P_{H,Load,max}^f$	50
HTE	60	0.8		
CTH	80	0.9	$\Delta P_{C,Load,max}^f$	50
CTE	60	0.8		

Table 2

Parameters of performance conversion components in MPMS.

Converter	Capacity(kW)	Efficiency	Failure rate (times)
T	500	$\eta_T = 0.9$	2
CHP	250	$\eta_{GTE} = 0.35, \eta_{GTH} = 0.45$	3
P2G	350	$\eta_{P2G} = 0.75$	2
GB	600	$\eta_{GB} = 0.9$	3
EB	200	$\eta_{EB} = 0.95$	2
ER	200	$\eta_{ER} = 2$	3
AC	200	$\eta_{AC} = 1.2$	2
ES	Max: 160* Min: 40*	$\eta_{ES,cha} = 0.95, \eta_{ES,dis} = 0.95$	2
HS	Max: 240* Min: 60*	$\eta_{HS,cha} = 0.9, \eta_{HS,dis} = 0.9$	2

* The capacity of storage components denotes the performance capacity (kWh)

shedding as shown in Fig. 11(e) and Fig. 11(f).

By analyzing the interaction between the S-FR and L-FR, the system states are filtered to retain only those states where adequacy is insufficient, enabling the calculation of performance shedding for these specific conditions. For the system states shown in Fig. 11(e) and Fig. 11(f), the performance shedding values are calculated using the minimum Euclidean distance between the S-FR and L-FR in three-dimensional space, as defined by Eqs. (36) and (37). The results of the system state analysis are presented in Table 3.

5.2.3. System state analysis based on S-FR and L-FR dynamic matching

The system state analysis method proposed in this paper leverages the dynamic matching of the S-FR and L-FR in the MPMS. This approach eliminates the need for performance-shedding calculations when the system is deemed adequate. Performance-shedding calculations are only required when the S-FR and L-FR do not intersect, indicating system inadequacy.

To validate the accuracy and efficiency of the proposed dynamic matching model, two case studies are conducted to analyze specific system states. The computation results are summarized in Table 4.

Case 1: System state analysis using the proposed dynamic matching model based on S-FR and L-FR.

Case 2: System state analysis using the LP model [31].

Case 1 is based on the reliability assessment method proposed in this paper and includes both offline modeling time and online analysis time. The offline modeling time refers to the time required to build the feasible region libraries for both the source side and load side, which primarily supports data provision for the subsequent online analysis phase. The online analysis time, on the other hand, refers to the time required for system state analysis after offline modeling is complete. At this stage, system state analysis is carried out by analyzing the spatial relationship between the feasible domains of the source side and load side, eliminating the need for further optimization calculations, thereby significantly reducing the time required for online analysis.

Case 2 adopts a conventional optimization-based reliability assessment method and includes only the system state analysis time, without involving offline modeling time.

The results demonstrate that while maintaining consistent load-

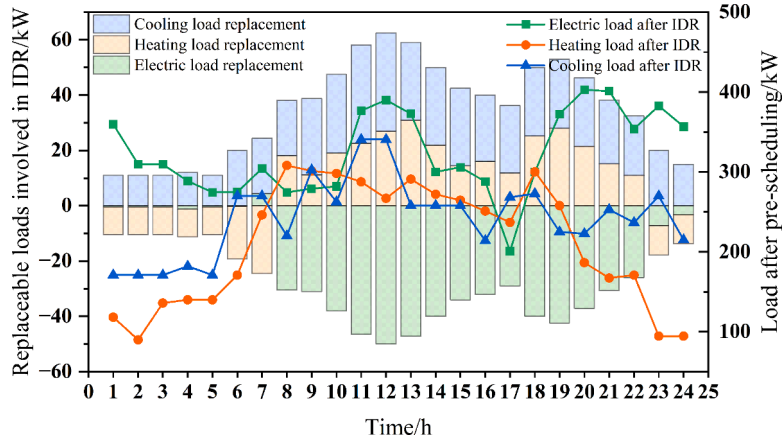


Fig. 9. The load curve after IDR and the value of replaceable load participating in IDR.

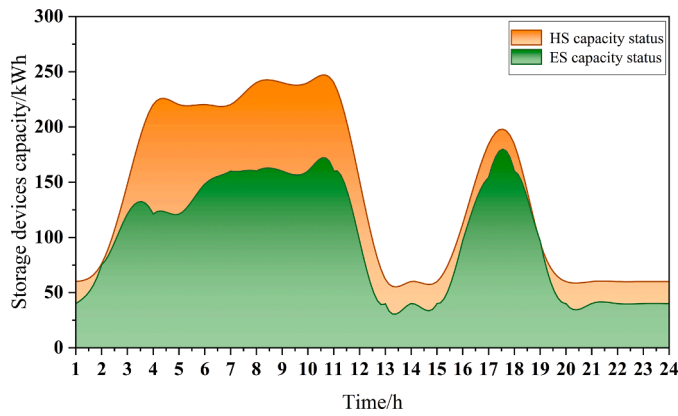


Fig. 10. Capacity status of the storage component during the scheduling period.

shedding amounts, the proposed dynamic matching model improves system state analysis time by approximately 2.7 times compared to the LP model. Although the construction of the S-FR and L-FR requires additional computational time, this process can be performed offline, significantly reducing the impact on real-time calculations.

To validate the accuracy and general applicability of the dynamic matching model based on feasible regions across all system states, an enumeration and comparative analysis of all possible states of the system in MPMS was conducted. The enumerated system states include performance conversion components, storage components and wind power units within the system, generating a total of 12,288 system states. The proposed method was applied for system state analysis, and the arithmetic mean of the performance-shedding amounts across all system states was calculated. The results are summarized in Table 5.

As shown in the Table 5, the dynamic matching model based on the S-FR and L-FR (Case 1) maintains an accuracy of system state analysis comparable to the LP model (Case 2). However, in terms of computational efficiency, Case 1 requires additional offline modeling time for constructing the S-FR and L-FR databases, whereas Case 2 does not. During the system state analysis, Case 1 pre-filters the system states by eliminating the need for performance-shedding calculations in cases where the system is adequate, reducing the original 12,288 system states to 5,760. This significantly lowers the computational burden for performance shedding calculations.

Additionally, by calculating the shortest Euclidean distance between the S-FR and L-FR in inadequate states, Case 1 avoids the complex optimization process inherent in the LP model, further improving computational efficiency. The computational efficiency of Case 1 is

enhanced by 82.54% compared to Case 2, demonstrating the effectiveness of the proposed dynamic matching model in reducing the computational burden and improving the efficiency of system state analysis.

5.2.4. Reliability assessment results based on S-FR and L-FR dynamic matching

Based on the system state analysis results, the reliability indices of the MPMS were calculated. The reliability indices were computed for the two case studies discussed earlier, and a comparative analysis was performed to evaluate both the reliability indices and computation times, as shown in Table 6.

The results demonstrate that the proposed reliability assessment method based on dynamic matching of S-FR and L-FR achieves the same level of accuracy as the LP model, validating the accuracy of the proposed method. Furthermore, the proposed method significantly outperforms the LP model in terms of computation time.

The LP model lacks the capability to pre-determine the adequacy of a system state or whether performance shedding is necessary. Consequently, it performs optimal performance shedding optimization calculations even for system states where adequacy is maintained, leading to substantial unnecessary computations. In contrast, the proposed method can pre-determine the adequacy of system states by analyzing the interaction between the S-FR and L-FR. This eliminates unnecessary performance-shedding calculations in states where adequacy is maintained, significantly reducing computational costs.

For inadequate system states, the proposed method simplifies the optimal performance shedding problem into a calculation of the shortest Euclidean distance between the S-FR and L-FR. By avoiding the complexity of solving an optimization problem, this transformation streamlines the reliability assessment process, reducing its computational complexity and improving efficiency. Overall, the results confirm that the proposed method offers a more efficient and effective approach for MPMS reliability assessment compared to the LP model.

5.3. Analysis of large-scale test system B

5.3.1. Case introduction of test system B

To validate the scalability of the proposed reliability assessment model for MPMS based on S-FR and L-FR dynamic matching, a larger test system B was used for verification. The analysis primarily focuses on the coupling relationships between three interconnected MPMSs and their users. The performance within these three MPMSs can be interconnected and shared, with a multi-performance storage system further enhancing operational flexibility. The component parameters for the large-scale test system are consistent with those used in the small-scale test system, as shown in Fig. 1.

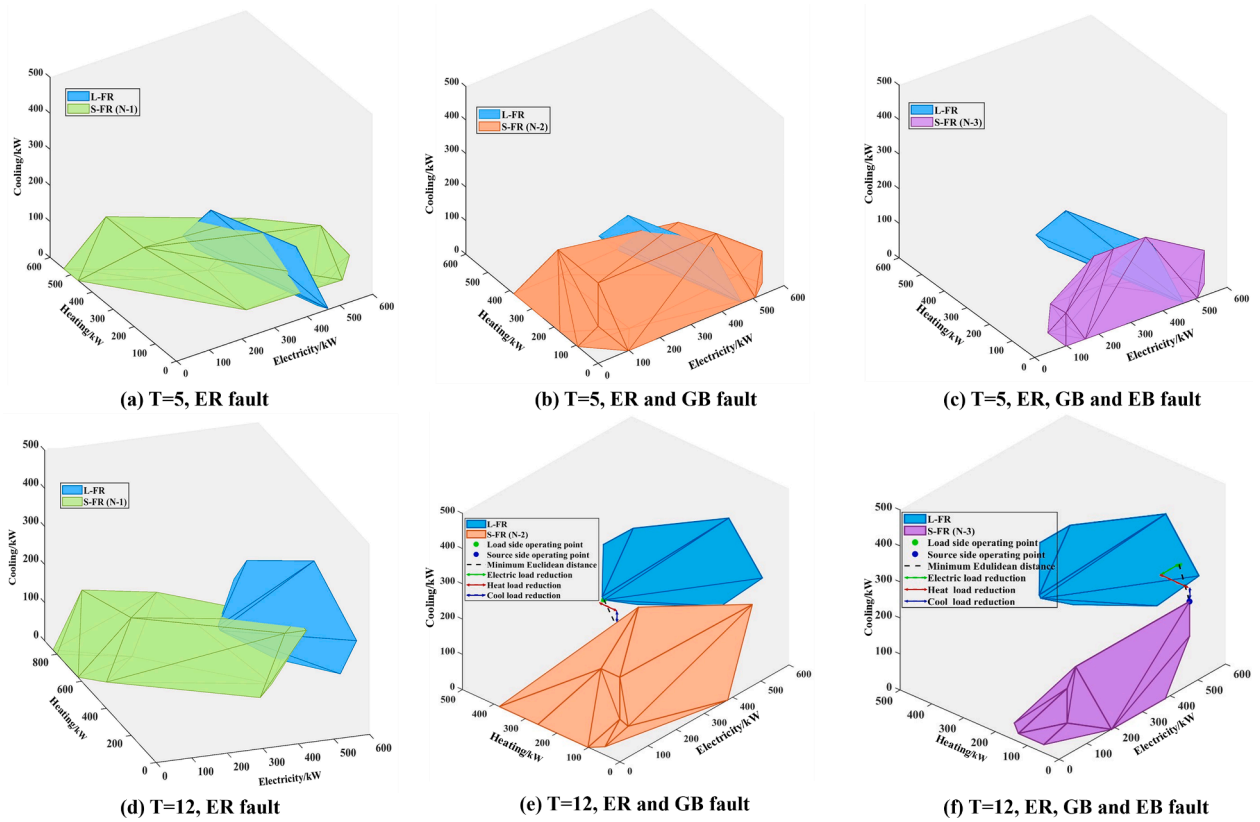


Fig. 11. MPMS source and load side feasible region database at time period 5 h and 12 h.

Table 3

System state analysis is based on S-FR and L-FR dynamic matching.

N-1 (ER fault)			
Time/h	Spatial position	System state	Load cutting/kW
5	intersect	adequacy	0
12	intersect	adequacy	0
N-2 (ER and GB fault)			
Time/h	Spatial position	System adequacy	Load cutting/kW
5	intersect	adequacy	0
12	non-intersect	inadequacy	Electric:16.50 Heat:55.02 Cool:36.99
N-3 (ER, GB and EB fault)			
Time/h	Spatial position	System adequacy	Load cutting/kW
5	intersect	adequacy	0
12	non-intersect	inadequacy	Electric:69.43 Heat:93.72 Cool:36.99

Table 4

Comparison of specific system state analysis results.

	Load cutting/kW	Off-line modeling time/s	System status analysis time/s
Case 1	108.51	0.0065	0.0019
Case 2	108.51	\	0.0051

5.3.2. System state analysis

In this large-scale test system, the contingency level for component failures is set to $N-5$, allowing the system to experience up to five simultaneous equipment failures. All possible system states are enumerated, including failures in performance conversion component, storage components and wind turbines, resulting in approximately $2.4 \times$

Table 5

Comparison of small-scale test system state analysis results.

	Average load cutting/kW	Number of MPMS states	Off-line modeling time/s	MPMS status analysis time/s
Case 1	136.08	5760	75.87	12.04
Case 2	136.08	12288	\	68.94

Table 6

Reliability evaluation results of small-scale test system A.

	Case 1	Case 2
$SENS_{IES}/kWh$	38.61	38.61
SIF_{IES}	3.36×10^{-2}	3.36×10^{-2}
Online reliability evaluation time/s	14.45	75.83

10^6 system states. The system state analysis was performed using the two aforementioned cases, with the results summarized in Table 7.

The proposed reliability assessment model based on S-FR and L-FR dynamic matching maintains high accuracy in the large-scale test system

Table 7

Comparison of large-scale test system B states analysis results.

	Average load cutting/kW	Number of MPMS states	Off-line modeling time/h	MPMS status analysis time/h
Case 1	115.67	36570	0.83	0.17
Case 2	115.67	2.4×10^6	\	3.12

while offering greater computational efficiency compared to the small-scale test system. During system state analysis, the computational efficiency of Case 1, based on S-FR and L-FR dynamic matching, improves by 94.56% compared to the LP-based model in Case 2. Even when accounting for the offline modeling time of S-FR and L-FR, the total computation time for Case 1 is still less than that of Case 2. For large-scale MPMSs, the computational complexity gap between the proposed FR-based dynamic matching model and the LP model becomes more pronounced.

Furthermore, the average performance shedding values obtained from the system state analysis indicate that the average performance shedding in the large-scale system is lower than in the small-scale system. This is attributed to the interconnection and mutual support between the interconnected MPMSs in the large-scale test system, which provides greater flexibility and reliability.

5.3.3. Reliability assessment results analysis

To further validate the accuracy and efficiency of the proposed reliability assessment model, the two case studies were applied to the large-scale test system, and the MCS method was employed to assess the system's reliability. The convergence precision of the MCS was set to 0.01. A comparative analysis of the reliability results and computation times is presented in Table 8.

The results show that the proposed dynamic matching model (Case 1) achieves the same level of accuracy as the LP model (Case 2) but with significantly faster computation times. Although the offline construction of the S-FR and L-FR requires additional time in Case 1, the online reliability assessment time using the dynamic matching method is only about 6.42% of the time required by the LP model. This demonstrates the efficiency of the proposed model in the operational reliability assessment of large-scale MPMSs.

5.3.4. Comparison of different test systems

The reliability calculation time for the S-FR and L-FR dynamic matching model can be divided into two parts: a relatively long offline preparation time and a significantly shorter online evaluation time. In contrast, the reliability calculation based on the LP model is entirely online, with no offline preparation. The calculation time for the LP model is significantly greater than the online evaluation time for the S-FR and L-FR dynamic matching model. Table 9 compares the computation times of the two case studies in both the small-scale and large-scale test systems.

As shown in Table 9, when considering both offline modeling and online reliability assessment time, the LP-based reliability assessment method performs better in the small-scale test system because the minimal difference in system state analysis time eliminates the need for offline modeling. However, in the large-scale test system, the reliability calculation model based on S-FR and L-FR dynamic matching demonstrates a significant advantage over the LP model, particularly in terms of computational efficiency and scalability.

6. Conclusions

This paper proposes a novel reliability assessment method for MPMS, leveraging the dynamic matching of the S-FR and the L-FR. The S-FR represents the multi-state performance supply capacity of MPMS under various operational conditions, while the L-FR captures the performance demand flexibility and interdependent dynamics of the load side, incorporating IDR. By analyzing the spatial relationships and interactions between the S-FR and L-FR, system adequacy is directly assessed, enabling the efficient filtering of system states and reducing the computational complexity of reliability evaluation. Additionally, performance shedding is quantified as the shortest Euclidean distance between the S-FR and L-FR, eliminating the need for complex optimization-based performance-shedding calculations and significantly improving computational efficiency.

Table 8

Reliability evaluation results of large-scale test system B.

	Case 1	Case 2
$SENS_{IES}/\text{kWh}$	27.82	27.82
SIF_{IES}	9.27×10^{-3}	9.27×10^{-3}
Online reliability evaluation time/h	0.24	3.74

Table 9

Reliability calculation time for different scale test systems.

	Time composition	Case 1	Case 2	Improved efficiency
A	Offline	75.87s	\	\
	Online	14.45s	75.83s	80.94%
B	Offline	0.83h	\	\
	Online	0.24h	3.74h	93.58%

Case studies conducted on small-scale and large-scale test systems validate the proposed method's accuracy and scalability. The results demonstrate that the method maintains high precision in reliability assessment while drastically reducing computational time, particularly for large-scale systems. This feasible region-based dynamic matching evaluation method offers a scalable, efficient, and practical solution for the operational reliability assessment of MPMS. It provides valuable guidance for system planning, scheduling, and operation, especially in scenarios involving complex performance dynamics and interdependencies.

Future research will extend the MPMS model presented in this paper to assess the reliability of a broader range of engineering systems, incorporating multiple performances and states. Currently, the load model in this study does not account for the uncertainty inherent in load forecasting. In future work, we aim to model the load as a probability distribution, represented in the form of feasible regions. Furthermore, integrating UGF technology with the reliability assessment method based on the feasible region model proposed here will facilitate the system state selection process, thereby enhancing the efficiency of reliability assessments.

CRedit authorship contribution statement

Yingqi Tie: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Data curation, Conceptualization. **Changzheng Shao:** Supervision, Investigation, Funding acquisition, Data curation. **Bo Hu:** Supervision, Resources, Investigation, Formal analysis. **Enrico Zio:** Writing – review & editing, Visualization, Supervision. **Wei Huang:** Writing – review & editing, Visualization.

Declaration of competing interest

I would like to declare on behalf of my co-authors that no conflict of interest exists in this submission. And we have no financial and personal relationships with other people or organizations that could inappropriately influence (bias) our work. This declaration has been approved by all authors for this publication.

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Data availability

The authors are unable or have chosen not to specify which data has been used.

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