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SYNERGIES BETWEEN PARTICLE-IN-A-BOX METHODS AND HIGH-FIDELITY METHODS FOR LONG-TERM LEO DEBRIS EVOLUTION

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Abstract

The growing accumulation of orbital debris is a significant concern for the sustainability of space activities. Inactive satellites and fragmentation debris from collisions and explosions pose risks to operational spacecraft. Statistical approaches, like Particle-in-a-Box (PIB) and the hybrid COMETA model developed by Politecnico di Milano, offer more efficient simulations by tracking debris in bins or as continuous density distributions. This work extends PIB to multi-dimensional variables, including semi-major axis, eccentricity, and inclination, and applies it to Low Earth Orbit under atmospheric drag. This extension improves collision and fragmentation calculations and incorporates disposal strategies. Its performance and scalability are compared with COMETA to develop a robust framework supporting debris mitigation and sustainable space operations.

List of Acronyms

ADR Active Debris Removal. 3, 9

LEO Low Earth Orbit. 1–3, 7

MOC Method of Characteristics. 6, 7

MRO Mission Related Object. 2, 3, 6

ODE Ordinary Differential Equation. 2

PIB Particle-in-a-Box. 1–9

PMD Post-Mission Disposal. 1, 3–9

SBM Standard Breakup Model. 5

1. Introduction

The rapid growth of satellite launches in recent years, driven by reduced launch costs and the accelerated deployment of large constellations, has significantly increased the population of objects in Low Earth Orbit (LEO). This surge, visible in the peak of launches shown in Figure 1 [1], has increased concerns regarding the accumulation of orbital debris.

Although launch activity has risen sharply over the past decade, only a small fraction of in-orbit objects remains operational. Today, debris constitutes over 90% of catalogued LEO objects. Even in the absence of future launches, explosions, and collisions among existing objects could further worsen the deterioration of the orbital environment. Collisions are particularly hazardous because of the large number of fragments they generate, initiating a chain reaction, known as the Kessler Syndrome, that further saturates orbital space.

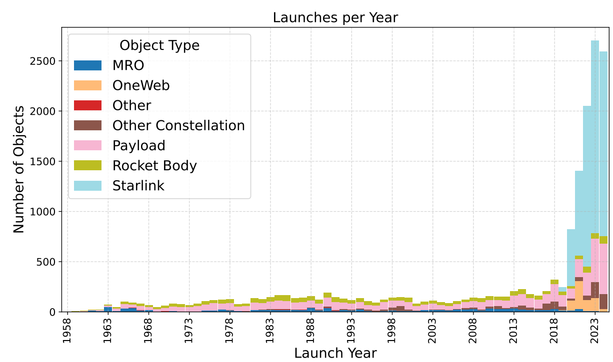

Fig. 1: Number of active objects in orbit per year divided by category (from DISCOS database [2]).

Mitigation measures such as collision avoidance manoeuvres and Post-Mission Disposal (PMD) are now essential to limiting the growth of debris. Numerous studies have quantified the associated risks and proposed strategies, policies, and metrics to address them. This challenge has led to the development of various models aimed at predicting debris evolution and assessing collision risks for active spacecraft. As the density of LEO objects continues to rise, accurately modelling and understanding this environment is critical for ensuring sustainable space operations.

2. Existing models

To predict the future state of the debris environment, two broad classes of evolutionary modelling methods have emerged: deterministic models, which track individual objects or fragments with high fidelity, and statistical/probabilistic approaches that simulate population-level dynamics through simplified equations. Each methodology has distinct advantages and trade-offs in terms of computational cost, accuracy, and applicability.

Deterministic methods model the evolution of each object's orbit using high-precision numerical propagators. These techniques include Monte Carlo simulations and similar approaches that simulate a wide range of future scenarios by propagating uncertainties in initial conditions and physical parameters. Generally, they include the full propagation in the six-dimensional space of orbital parameters under the effect of a detailed dynamic model, including all the main perturbations. Models like NASA's LEGEND [3], CNES's STELA [4], and Southampton's DAMAGE [5] fall into this category. These tools are highly valuable for short-term collision avoidance, conjunction analysis, and detailed space traffic management. However, they are computationally intensive, especially when simulating long-term evolution or sampling large uncertainty spaces, because of the need for small time steps and fine-grained accuracy. The high resource cost limits their utility for exploratory analyses or large-scale sensitivity studies.

In contrast, statistical models approximate debris dynamics using population-based Ordinary Differential Equations (ODEs). Instead of tracking individual objects, these models represent the debris environment through aggregated populations, such as payloads, derelict satellites, and debris fragments, and simulate their interactions over time. A classic formulation is the "particle-in-a-box" (PIB) model, first introduced by Talent [6]. Tools like FADE [7], STAT [8], MOCAT [9], and MISSD [10] use variations of these models to project long-term trends in debris accumulation. Statistical models are computationally efficient and enable rapid assessment of many scenarios, including parametric studies, policy evaluation, and orbital capacity estimation. However, they often rely on simplifying assumptions, such as average orbital parameters, altitude bands, and coarse object size groupings, which may reduce fidelity compared with deterministic models. As a consequence, they are generally used only to capture overall population trends under predefined conditions, rather than to provide comprehensive analyses. These methods provide a powerful and highly scalable way to assess the long-term stability of the LEO environment.

3. COMETA and Particle-in-a-Box

Research at Politecnico di Milano has focused on developing COMETA [11], a long-term probabilistic debris propagation tool that blends characteristics of deterministic and probabilistic approaches. COMETA classifies intact objects into four categories: payloads, rocket bodies, mission-related objects (Mission Related Objects (MROs)), and constellations, each with distinct mission profiles in terms of lifetime, post-mission disposal rate, duration, and disposal strategy. This enables independent assessment of each category's environmental impact.

For fragments, COMETA adopts a continuum approach: instead of modelling each piece individually, the debris cloud is treated as a continuous distribution in orbital phase space. Its density evolves according to the continuity equation [12, 13], with perturbations driving its evolution. This method makes computational time independent of the smallest fragment size considered, enabling scalability to any size range. Intact objects are still tracked individually, preserving one of the strengths of deterministic models, although without full high-fidelity propagation for performance reasons—a feature that could be expanded in future work.

In parallel, a simplified one-dimensional PIB model has been developed to study space debris mitigation strategies [14]. While effective for its specific purpose, this simplified formulation produces results that are not directly comparable with those from COMETA.

The present work aims to extend the PIB model to bridge the gap with COMETA's semi-analytical framework. COMETA will serve as a reference to calibrate certain parameters and to assess the validity of the extended PIB model.

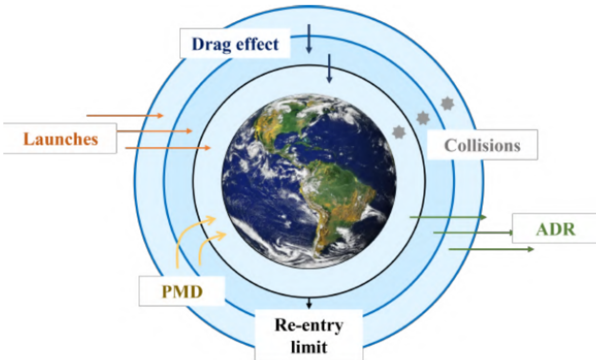

Fig. 2: Graphical representation of the model, with division in shells only in semi-major axis, including all the sources and sink mechanisms [14].

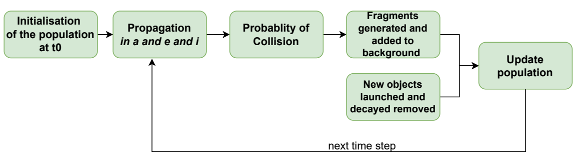

Fig. 3: Block diagram of the model.

3.1 *Description of the model*

The basic schematic of the model is shown in Figure 2, while Figure 3 shows the block diagram of the model. In it, the orbital space is divided into altitude shells. In the extended formulation, the LEO regime is discretised into bins defined by semi-major axis (a), eccentricity (e), and inclination (i), all equally spaced. The remaining orbital parameters are treated as randomised and not explicitly resolved, since their inclusion would require an extremely large number of bins and result in prohibitive computational costs.

In the previous version of the model, only two species, intact objects and fragments, were included, primarily for simplicity. This approach did not distinguish between active objects (which are not subject to decay during their operational lifetime and may be capable of collision-avoidance manoeuvres) and passive ones. In the extended version, additional species are included, with separate categories for payloads, rocket bodies, mission-related objects (MROs), and inactive objects. The latter three could, in principle, be grouped to reduce computational costs (fewer bins and lower memory requirements) and to avoid categories with very few and widely scattered objects. The drawback of grouping is that, without a division into A/m bins, the model may overestimate or underestimate average values, which directly affects decay-time accuracy.

Compared with the earlier PIB model, the extended version offers several accuracy advantages:

- More realistic area-to-mass (A/m) ratios without requiring bin-level distinctions, reducing lifetime over- or underestimation.

- Differentiation between active and inactive objects, enabling category-specific explosion and collision rates, post-mission disposal (PMD) compliance, and active debris removal (Active Debris Removal (ADR)) parameters.

The entire object population is distributed across these bins. The exact orbital shape of each object is not preserved; instead, objects are assumed to follow representative paths consistent with the parameter ranges of their bins.

For orbital propagation, the contraction method formulas developed by King-Hele [15, 16] are applied to propagate a and e under atmospheric drag, using the same formulation as in COMETA. Only bin centres are propagated in time. After propagation, two redistribution strategies are available:

- Rigid translation in a–e space [8]: the N objects in a bin are proportionally redistributed among overlapping bins. This approach neglects bin distortion over time and is therefore an approximation. Differential algebra mapping computes a local expansion around the propagation point to capture bin-domain deformation. This method has higher fidelity but is limited by the convergence radius of the polynomial expansion, requiring either small bin sizes or high polynomial orders. In the present study, this method was not used because it significantly increases computation time. With appropriate time steps, the translation approach keeps errors within acceptable bounds [17].

- An alternative is to apply the continuity equation, modelling the objects as a phase space density function:
$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{F}) = 0 \qquad (1)$$
With $n(\mathbf{x}, t)$ the phase space density and $\mathbf{F}(\mathbf{x}, t)$ representing the dynamics of the system, solvable with

the method of characteristics:

$$\frac{dn}{dt} = -n \operatorname{tr} \mathbf{J}, \tag{2a}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}, \tag{2b}$$

$$J_{i,j} = \frac{\partial F_i}{\partial x_j} \tag{2c}$$

with tr the trace. Here, the propagated density field is sampled and re-binned, similar to COMETA's treatment of debris cloud propagation.

In addition to semi-analytical propagation, the model includes source and sink mechanisms that add or remove objects at discrete time steps:

$$\frac{dN(a,e,i)}{dt} = \frac{dN_d}{dt} + \frac{dN_c}{dt} + \frac{dN_e}{dt} + \frac{dN_l}{dt} + \frac{dN_{pmd}}{dt} \tag{3}$$

where dN_d, dN_d, dN_c, dN_e, dN_l and dN_{pmd} represent the variation at each time step of the number of objects in a specific bin due to drag, collisions, explosions, launches, and PMD respectively. Instead of the total number of elements, the densities n can be used.

The future state is computed using an explicit Euler method:

$$N_{k+1} = N_k + \frac{dN(t_k, N_k)}{dt}\Delta t \tag{4}$$

For future launch traffic, the most common strategy, used, for example, in ESA's Environmental Report [1], is to cyclically repeat data from the previous years, typically the last eight, in a conservative approach. Given the sharp recent increase in launches, a shorter cycle of five years could also be considered to reduce the oscillations. Alternative models [18,19] extrapolate future launch trends and may incorporate additional factors such as GDP. These approaches allow the analysis of the effects of different traffic assumptions. An example of an exponential fit compared with a cyclic model is shown in Figure 4.

PMD is implemented by marking as inactive all objects launched more than a fixed operational lifetime (typically 8 years) before the current simulation time. For these, a new circular orbit is computed, and they are removed from the active category. Inactive objects are reassigned to the bin from which they will re-enter within the disposal timeframe, with a disposal success rate given as an input parameter.

Fragment generation is split into collision and explosion sources, both based on the NASA breakup model. The collision probability (P_c) is calculated with a method based

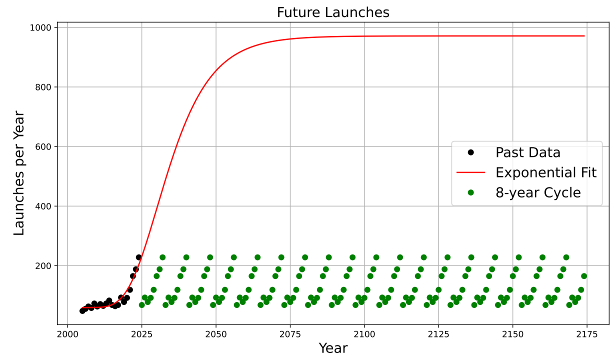

Fig. 4: Gompertz fitting of launch traffic as described in [20].

on the kinetic theory of gases. In the earlier PIB version, the collision probability was calculated via the and debris generation was modelled in the same altitude shell, considering only intact–intact collisions. In the extended version, collisions between all species are considered (with options to ignore or limit the risk of the species capable of collision avoidance). The collision probability is then passed to a Poisson extractor to determine whether an event occurs, requiring multiple simulation runs for averages. The Poisson distribution is used to describe probabilistically how often an event occurs in an interval of time over which is accumulated [21]. The P_c computation uses the Cube algorithm (as in deterministic models [22]), though PIB cannot use the standard 10 km cubes due to the full-LEO coverage. The probability of collision is computed for each species with respect to each species in the same bins (a,e,i), interactions between bins with different eccentricities but the same perigee are also considered.

$$N(i,j,k,s_1,s_2) = \sigma N_{s_1} N_{s_2} V v_{rel} \Delta t \tag{5}$$

with $N_{s_1} N_{s_2}$ the number of objects in the bin belonging to species s_1 and s_2, v_{rel} the relative velocity (in the majority of works approximated to the orbital velocity at the specific altitude) and the volume V is an approximation of the volume of the shell delimited by the minimum and maximum semimajor axis and the average eccentricity. Sigma contains the geometrical information of the hard bodies and is calculated as:

$$\sigma = (r_1 + r_2)^2 \tag{6}$$

with $r_1 + r_2$ the sum of the radii of the two species in the selected bin. The number of fragments per collision is derived from the NASA standard breakup model [23, 24], which classifies events as catastrophic or non-catastrophic

Table 1: Simulation settings.

Parameters	Values
End of propagation (t_f)	2125
Sat. lifetime	8 years
PMD time	25 years
PMD rate	0.9

based on impact energy. The Standard Breakup Model (SBM) is reformulated as a probability distribution function as described in [25]. Each fragment's properties, area-to-mass ratio (A/M), mass (M), and velocity change (Δv), are computed as functions of its characteristic length (L_c). Rather than generating distributions per fragment after assigning L_c, the model first computes global distributions for L_c, A/M, and Δv within their ranges, then assigns fragment properties from these. This approach is more efficient, particularly for large numbers of fragments [26,27].

Explosions are treated as continuous events with species-specific rates that may be taken from literature or extracted from COMETA simulations. In COMETA, explosion probability depends on object type, inherent risk, and time in orbit. In the current PIB implementation, explosions are excluded for simplicity, since PIB does not track individual object histories.

3.2 Analyses and limitations of the model

For the initial analysis, only objects with altitudes up to 2000 km and diameters larger than 10 cm were considered. The initial population was derived from the DISCOS database, excluding satellite constellations. Constellations exhibit distinct lifetimes and launch patterns compared to other payloads, and therefore require a separate treatment. Unless otherwise specified, the parameters used in the simulations are summarised in Table 1. For all simulations, multiple runs were performed. In the figures, the darker lines represent the average results across runs, while the lighter, more transparent lines (when present) indicate individual cases.

Comparison with the 1D model. The multidimensional model was first compared against the one-dimensional implementation to assess consistency and potential improvements. As shown in Fig. 6, when collisions are not included, both models produce nearly identical population trends, with only minor discrepancies arising from the chosen bin division. For example, as can be seen in the figure, the faster decrease of fragments can be explained by taking into account the division in bins in eccentricity as well, whereas the 1D model erroneously assumes all items are in circular orbit, which is not the case, especially for the fragments (Fig. 5).

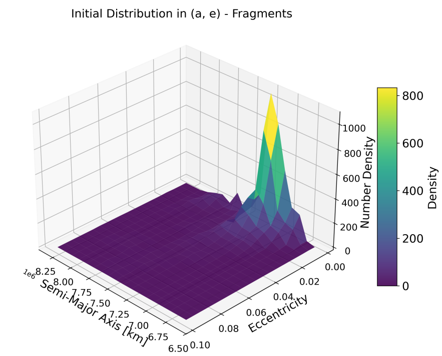

Fig. 5: Initial fragments distribution in semi-major axis and eccentricity.

However, once collisions are introduced, the one-dimensional model becomes significantly less stable (Fig. 7), whereas the multidimensional approach exhibits more stable behaviour, as seen in Figure 8.

In the 1D model, fragment generation is handled deterministically. At each time step, the increase in fragment number is computed as

$$\Delta N_{\text{frag}} = P_c \cdot N_{\text{frag/coll}} \cdot \Delta t, \quad (7)$$

where P_c is the collision probability within the bin, $N_{\text{frag/coll}}$ the expected fragment yield per collision, and Δt the integration step. This formulation injects a fractional number of fragments at every step, effectively distributing collisions continuously over time and removing the randomness associated with rare events.

By contrast, the multidimensional model employs a stochastic treatment. For each bin and time step, the expected number of collisions is

$$\lambda = P_c \cdot \Delta t, \quad (8)$$

which serves as the mean of a Poisson distribution. A random integer $k \sim \text{Poisson}(\lambda)$ is drawn, representing the actual number of collisions in that step. If $k > 0$, the corresponding number of fragments is injected in a discrete burst. This preserves the intermittent character of collisions: many steps contain no events, while others yield one or more depending on the random draw.

Two main differences emerge between the deterministic and stochastic approaches:

1. Suppression of low rates. In the deterministic scheme, the N^2 dependence always produces a non-zero contribution, so even sparsely populated bins generate

fragments fractionally. In contrast, Poisson sampling often yields zero events for bins with very low expected rates, preventing artificial inflation.

2. Treatment of rare events. When the expected number of collisions per bin per step is $\ll 1$, the 1d approach continuously accumulates fragments, while Poisson sampling instead produces rare discrete bursts. This results in a more realistic representation of collision intermittency.

In summary, the overestimation observed with coarse binning is primarily a consequence of the deterministic formulation. The stochastic Poisson sampling mitigates this effect by suppressing many low-probability contributions, thereby reducing the bias introduced by bin size. As shown in Fig. 8, the total number of generated fragments remains highly sensitive to the chosen bin resolution. Although stochastic treatment removes part of the problem, this dependency is still present, highlighting the importance of selecting an appropriate bin size for reliable results.

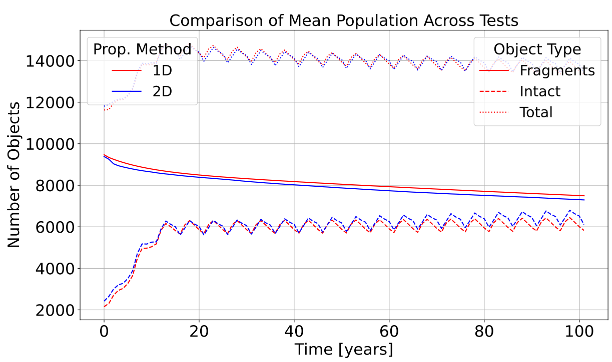

Fig. 6: Comparison between the multidimensional and 1D PIB models with no collisions or explosions.

Comparison with COMETA. In contrast, the comparison with the COMETA model reveals the largest discrepancies. When collisions are excluded, both models predict a similar decay trend in the fragment population. However, the total number of intact objects (payloads, rocket bodies, and MROs) is consistently lower in the PIB results, even though the overall trend remains comparable for different PMD ratios, as seen in the following figures.

This behaviour is stable across different initial populations and launch traffic models, and it does not depend on the propagation method, as both the Method of Characteristics (MOC) and the simple propagation and translation of bin method show similar results (Figure 9).

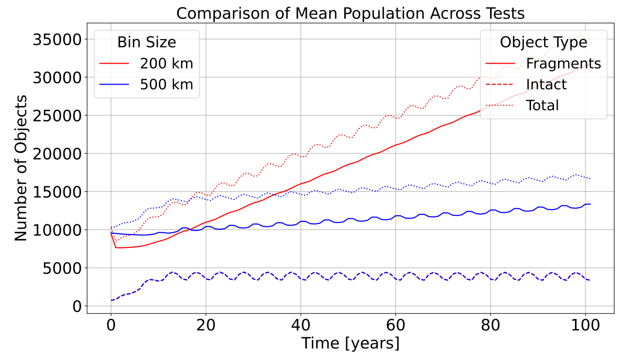

Fig. 7: Fragments generated in the 1D model with different bin sizes.

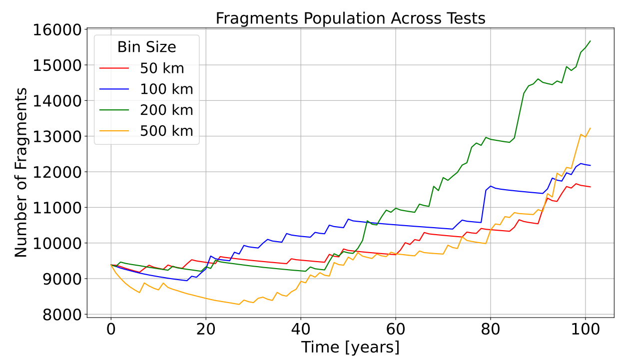

Fig. 8: Evolution of the fragments in the 3D PIB generation using different bin sizes in a and keeping constant e and i.

The discrepancy arises from a fundamental modelling difference: COMETA propagates intact objects individually, while the PIB approach evolves population bins using averaged values of mass and area. Since active satellites typically have higher area-to-mass ratios, the bin-averaged treatment in PIB accelerates their orbital decay, especially in the lower-altitude bins, leading to a reduced number of surviving objects. An additional contributing factor that can influence this discrepancy is related to the PMD strategy. Although both models apply the same disposal strategy (calculating the orbit required for natural disposal within a defined timeframe and assuming the manoeuvre occurs instantaneously), discretisation of orbits into shells and the use of averaged mass and area values introduce additional errors.

Together, these two mechanisms act as sinks that reduce the population in the PIB model relative to COMETA.

This highlights the need to introduce a finer differentiation in A/m values within the PIB framework, which appears to be more critical than further refining the bin size alone. The limitation is present under both deterministic and stochastic propagation schemes.

As a result, direct comparison of collision rates and fragment generation between the two models is challenging, given the systematically higher total population in COMETA. Nevertheless, some general trends emerge: with the chosen assumptions, excluding large constellations such as Starlink, restricting collisions to intact–fragment and intact–intact events, and accounting for collision avoidance capabilities of active satellites, the total number of fragments predicted by COMETA tends to decrease over time as can be seen in Figures 10 and 11. However, PIB consistently predicts a higher frequency of collisions, even at smaller bin sizes (Figures 12 and 13). This behaviour reflects a known tendency of PIB-type models, and of the Cube Algorithm more generally, to overestimate collision probabilities, an issue also noted in other frameworks such as in [27] and [26].

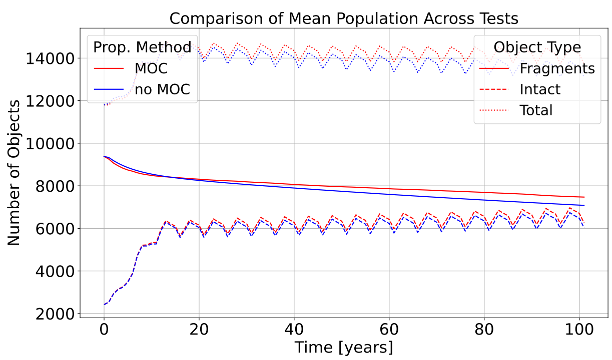

Fig. 9: Propagation with the PIB model using both MOC and without.

Impact of Cube Size on Collision Modelling. A fundamental limitation of the Cube algorithm is its simplification of orbital dynamics: collisions are computed assuming objects can be located anywhere within a cube, and the expected number of collisions is derived from kinetic gas theory. While computationally efficient, this approach may underestimate the true collision probability by neglecting actual orbital motion. The performance of the Cube algorithm is highly sensitive to the choice of cube size. Smaller cubes require more time for two objects to occupy the same cube, reducing the likelihood of detecting collisions. However, when two objects do fall within the same small cube, the estimated collision count can be artificially high due to the restricted volume. Conversely, very large cubes in-

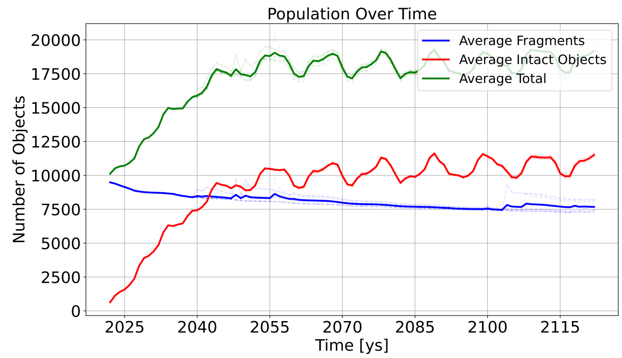

Fig. 10: Propagation for 100 years using the COMETA tool.

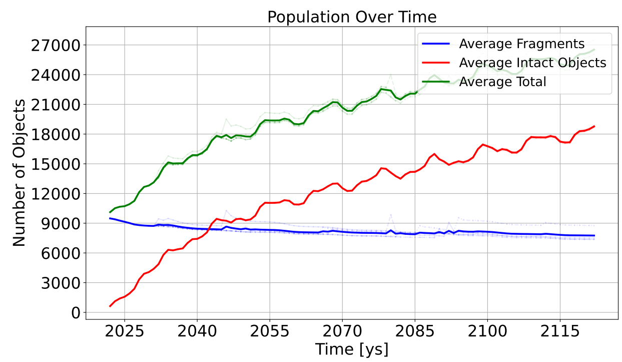

Fig. 11: Propagation with COMETA in the case of no PMD.

crease the risk of false positives, as objects whose orbits do not physically intersect may be considered colliding. This is particularly relevant for near-circular LEO orbits, where excessively large cubes may produce spurious collisions.

The effect of cube size is illustrated in Fig. 14. Large cubes tend to overestimate collisions and produce high variability across simulations, as many non-intersecting orbits are erroneously grouped. Interestingly, this trend is not strictly monotonic: for example, with a 500 km bin size, the average number of collisions decreases because the increased volume spreads objects too sparsely. Smaller cubes generally yield more stable results, though excessively fine binning can underestimate collisions due to too few objects per cube, and increases computational cost significantly.

Coarse binning also introduces uncertainties in orbital propagation. Bins covering a wide altitude range force objects with different decay rates to evolve identically, particularly affecting low-altitude orbits with rapid decay. In the two-dimensional model, broad eccentricity bins can assign objects to (a, e) combinations with perigees below 200 km, leading to premature removal. This issue does not arise in the one-dimensional model, which assumes purely circular orbits, but the latter has its own limitations.

These effects are evident when comparing PIB with the COMETA model. COMETA propagates each intact object individually, as deterministic models do, while PIB evolves bins using averaged mass and area values. This simplification leads to discrepancies in the total number of active objects, especially at low altitudes, highlighting the need for more detailed differentiation in object properties within the PIB framework. The results are presented in Figures 15 and 16. In the first figure, the evolution of the population exhibits a comparable trend across the different PMD rates, although notable differences arise in the absolute values. The second figure, by contrast, illustrates the distribution of generated fragments. The only case yielding a positive outcome corresponds to the PIB scenario without PMD. This finding suggests that, although the number of active objects is lower than in COMETA, the corresponding simulation substantially overestimates the number of collisions. Consequently, the introduction of a limiting factor is necessary to prevent the emergence of an unrealistic chain reaction.

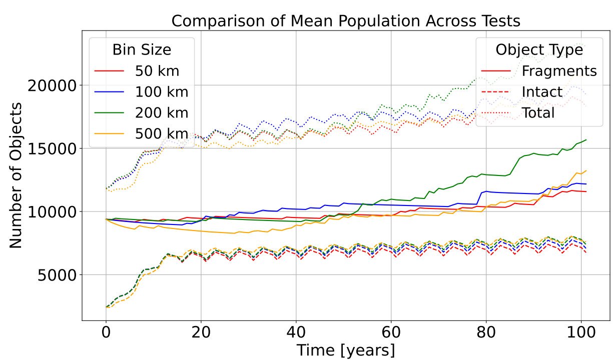

Fig. 12: Total population, fragments and payloads propagated with different bin sizes with PMD rate of 0.9 in the 3D PIB model.

4. Conclusion and future works

The purpose of this paper was the development and extension of a statistical model capable of incorporating the main features of high-fidelity models, keeping it as simple as possible to maintain low computational cost. As demonstrated in the previous sections, the current model is not yet complete and cannot be fully used for comprehensive analyses until all relevant elements have been incorporated.

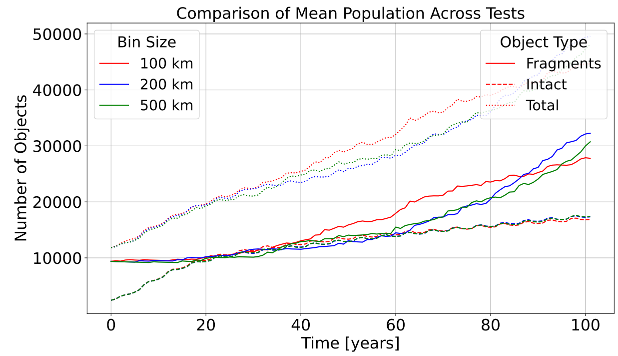

Fig. 13: Total population, fragments and payloads propagated with different bin sizes without PMD.

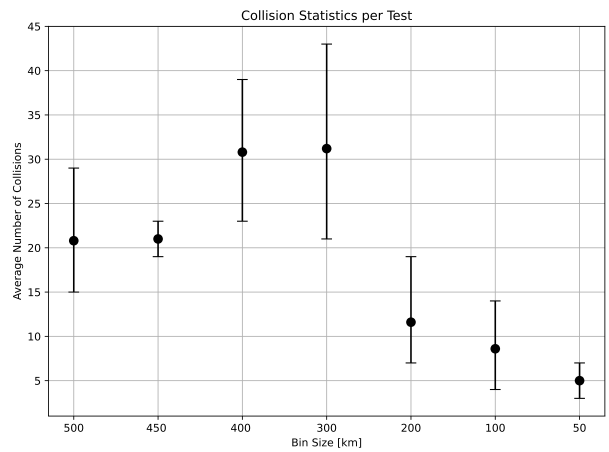

Fig. 14: Number of collisions estimated changing the bin size (in semi-major axis) 3D PIB model.

Comparison with a previously developed semi-analytical model highlights several limitations. The model strongly depends on the choice of parameters for both propagation and collision processes, which require calibration against more accurate models. Excessively fine divisions of bins or object species can compromise the model's efficiency, as the number of values that need to be calculated and stored grows with both the number of dimensions (a, e, i, area-to-mass ratio, species included) and the refinement of each dimension, as these factors multiply together to increase computational load. Therefore, a trade-off is necessary between achievable accuracy and computational simplicity.

Moreover, the next step is to implement a division of debris into area-to-mass ratio bins, which becomes critical when including sub-centimetre fragments in the simulation. This represents a key difference between this model

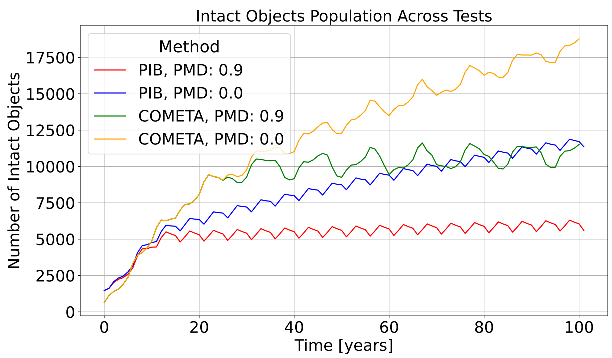

Fig. 15: Intact population propagated with PIB and COMETA and different PMD.

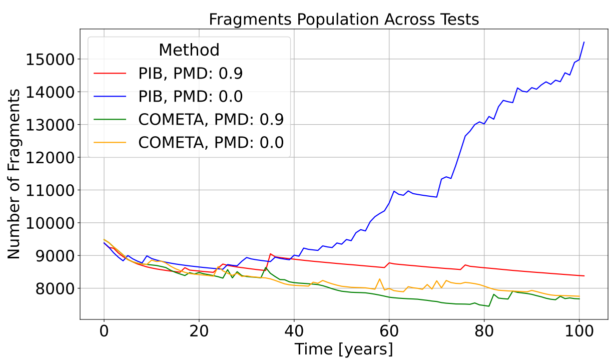

Fig. 16: Fragments propagated with PIB and COMETA and different PMD.

and existing approaches, as coarse propagation tends to either overestimate or underestimate the total number of objects.

Currently, all bins have the same size. A potential improvement would be to adapt the bin size based on the initial density of the population, using broader bins in regions with low debris density and finer bins where the population is concentrated.

Another area for improvement is the propagation time step, which is currently fixed at one year. Although a one-year step is a reasonable compromise based on previous studies and models, adaptive time-stepping could improve accuracy, especially for high-density or rapidly evolving regions.

The simulations conducted so far exclude explosions, which are one of the primary sources of fragmentation and must be incorporated in future work, using extrapolations of past data. Additionally, active debris removal (ADR) operations, not yet considered in these simulations, could be included to study their impact on the overall space debris environment. Incorporating these elements will allow for a more realistic and stable assessment of debris evolution, collision risk, and mitigation strategies.

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