On the safety of damaged masonry structures: a first probabilistic approach

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Abstract. The topic of structural safety is an extremely important subject and is studied mainly with reference to structures and infrastructures made with materials other than masonry.

Masonry is a composite material made of a variety of materials - including brick, stone, and mortar - with dry joints or mortar joints. This results in complex mechanical behaviour, featured primarily by low tensile strength and good compressive strength, and the presence of finite friction and cohesion at the interface joints. Given the above, masonry has inherent uncertainties in its mechanical characterization.

To these uncertainties must be added those related to the evolution of degradation and natural aging of the component materials. These aspects complicate the evaluation of the structural response in terms of safety, reliability or robustness in historic masonry buildings.

This paper presents the first step for the safety assessment of a masonry building subjected to seismic action, considering a certain level of damage through a significant performance parameter. This parameter is able to describe the reduction of the characteristic strength values due to the presence of a crack pattern induced by live-loads changing and its evolution over time.

The evolution of the damage affects the global safety factor, SF_G, significant parameter in the evaluation of the safety of a building in a seismic zone. In this paper, the assessment of time required to reach a SF_G level of possible collapse is approached for the first time in a probabilistic way.

Keywords: Masonry, Seismic Behaviour, Safety Factor, Probabilistic Approach, Fragility Curves.

1 Introduction

In an extended campaign to assess the vulnerability of the existing building heritage, it is considered useful to have a quick and minimally invasive investigation method, which enables the acquisition of a non-exhaustive, yet trustworthy, picture of the overall level of vulnerability of the residential built heritage of a city. The results obtained with this method must be the starting point for targeted and more in-depth investigations on the heritage that already seems to have critical issues.

Exploiting a model present in the literature [1-3] and developing a probabilistic approach for the evaluation of the global safety factor and the static safety factor when the seismic, topographic and subsoil conditions of the site vary, some scholars [4] highlighted the importance of some performance parameters present in the calculation procedure proposed in [1-3], whose randomness can influence the estimate of these factors.

A fast approach to the characterization of masonry can only rely on the values provided by the regulations. In [5], reference values are proposed for the main types of masonry present in the Italian national territory to be used as a stochastic characterization of their mechanical properties. These values are obtained from those proposed in Table C8A.2.1 in [6]. Although they have not been updated with the new table C8.5.1 of Circular n. 7 of 21.1.2019, they remain a valid reference for statistical, reliable data.

From these records it emerges that for each masonry typology the parameters of characteristic strength present a certain degree of uncertainty. In [7], the influence of the randomness of these parameters on the evaluation of the global and static safety factors is investigated. Assuming the masonry typology is known, the randomness of the shear and compression strength characteristics of this typology was introduced by referring to the indications reported in [5] and their influence on the variation of the safety factors is studied through the construction of fragility curves.

An important aspect in the evaluation of the safety and reliability of a structure is also the possibility of evaluating its future life based on the level of damage already reached.

The construction of the safety factor requires that a certain level of damage already reached by the building can be taken into account through the performance parameter, f, of reduction of the characteristic strength values based on the presence or absence of a crack pattern, see [4]. The evolution of the crack pattern requires the definition of an instant of damage triggering and a subsequent temporal evolution that is intrinsically not deterministic.

In order to capture the randomness of the damage evolution and its influence on the estimation of the safety factors, in this work it has been assumed that the above mentioned parameter, f, varies over time, according to a variation law derived from long-term behaviour tests carried out in the laboratory on existing masonry samples [8,9].

2 A time-varying law for the performance parameter *f*

The methodology proposed by Borri and De Maria in [1-3] develops the assessment of the global and static safety factors by introducing in the formulas a performance parameter, f, capable of taking into account the integrity or damage of the masonry which contributes to the seismic behaviour of the building.

In [1-3] the parameter, f, is assumed equal to 1 for the undamaged masonry and 0.8 for the damaged masonry. The assumption of two distinct values for the parameter f leads to a deterministic evaluation of the safety of the building, even though the randomness of the characteristic strength values has clearly highlighted the importance of moving to a probabilistic approach, streamlined and not completely exhaustive, such as the construction of fragility curves.

The application of the variation of the value of $f < 1$ implies the knowledge of the trigger time of one or more aggressive actions on the building that in the long term can cause an evolutionary crack pattern (replacement of floors with increase in weight, change of intended use, insertion of large openings on the ground floor with reduction of masonry piers, etc.). Starting from these actions, the progress of the crack pattern could follow an evolutionary law that is not easy to define. There are indeed numerous uncertainties at play:

• the instant of cracking start (a historical record can help define one or more instants of damage initiation);

• damage evolution law (laboratory and on-site tests, even if accelerated, can provide useful indications on the evolution of a given damage process);

• collapse time.

The study of the history of a building, from its construction to the present day, following its transformations over the centuries and especially investigating the most recent interventions, can provide indications on the moments in which the stresses on the masonry walls may have undergone a significant variation that, in the long term, can lead to a weakening and the formation of cracks.

From the instant the cracking pattern is triggered, the performance parameter f must detect this variation by decreasing according to the damage evolution law.

The variation can be defined as:

$$f(t) = f_0[1 - \delta(t)] \tag{1}$$

where f_0 is the value of the parameter f at the time t=0, $1 - \delta(t)$ measures the decrease of f over time, following a defined damage law.

The latter must be sufficiently flexible to be able to adapt to the randomness of the evolutionary behaviour, if it can be defined in a sufficiently reliable way.

Among others, a law that responds to these needs could be that proposed by [10]:

$$\delta(\tau) = \begin{cases} \omega^{1-\rho}\,\tau^{\rho} & , \tau \leq \omega \\ 1 - (1-\omega)^{1-\rho}\,(1-\tau)^{\rho} & , \omega < \tau < 1 \\ 1 & , \tau \geq 1 \end{cases} \tag{2}$$

where ρ and ω represent the parameters describing the shape of the damage evolution law, while $\tau = \frac{t}{\mathrm{T}_f}$ is the normalized evolution time with T_f= failure time. The shape parameters are assumed to have a probability distribution law of the normal type. An important uncertainty is the definition of the failure time, T_f, which certainly varies from case to case. Sometimes in literature the failure time is assumed to have a probability distribution law of the Gamma or Weibull type [10]. However, it is better if the choice of the type of probability distribution could be made based on physical knowledge of the phenomenon.

For the law to be reliable, it is necessary that the shape parameters can be defined on the basis of plausible evolutionary behaviour.

In the case of the historic masonry in [8,9], the values of the variation of the strain rate due to the permanence of long-term static loads, recorded on samples of existing masonry, are reported. The variation of the strain rate over time is a symptom of a worsening of the crack pattern (Fig. 1).

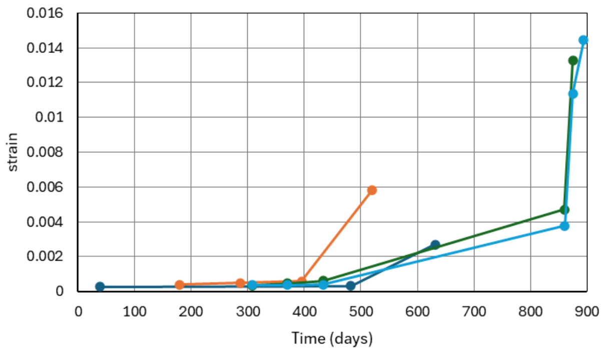


Fig. 1. Variation of the strain value with time for long-term constant static load. Each colour refers to a different specimen (data from laboratory tests in [8,9]).

A damage curve that satisfies a trend like that of Fig. 1 is reported in Fig. 2, where the black curve δ is obtained by considering the mean values of the shape parameters. The dashed curves represent the limit curves characterized by the mean values of the shape parameters plus and minus their standard deviations. In the interval thus described, the uncertainty that characterizes the estimate of the parameters ρ and ω is taken into account. This interval can be considered exhaustive of all possible curves describing the phenomenon.

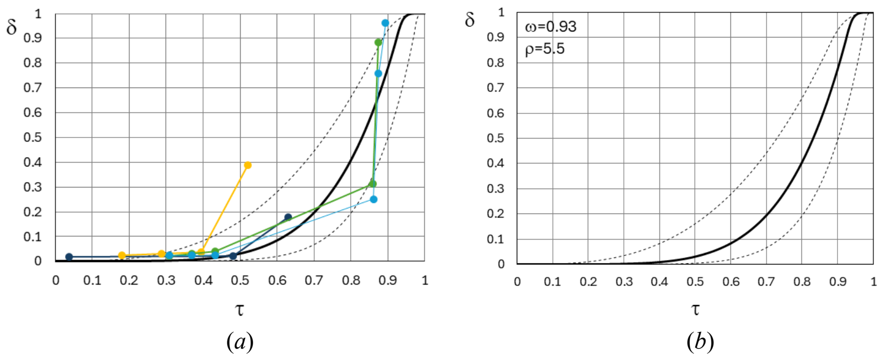


Fig. 2. Study of the damage law as a function of laboratory results: *a*) comparison of the theoretical law with normalized laboratory values; *b*) final damage law for δ associated with the mean ω and ρ values (in black) and the limits of the interval (dashed lines), $\delta_{\min}$ and $\delta_{\max}$, including all possible degradation curves.

3 Estimate of the global safety factors, SF_G, and the static safety factors, SF_S, as a function of both the parameter *f* and time

The uncertainties in the estimation of the parameter *f* and of the safety factors SF_G and SF_S, as already mentioned, are mainly related to the instant of damage triggering, mainly due to variations in the original static arrangement. For a first assessment of the trend of the safety factors with the random variation of the parameter *f* after the instant of damage triggering, the case study of a typical historic masonry building of the Lombard lowland area of the last centuries was assumed, which in previous analyses had shown a certain fragility even adopting the parameter $f = 1$ [4].

The building considered is designed in seismic zone 2 with peak ground acceleration between $0.15g < a_g \leq 0.25g$. The seismic characteristics assumed for these analyses are those of the municipality of Salò (BS), the most earthquake-prone area in Lombardy: topography T1 - with spectral coefficient $S_T = 1$ - flat and subsoil type C - medium-densified coarse-grained and very consistent fine-grained with spectral coefficient S_S=1.39, seismic return period equal to 475 years, peak acceleration equal to 0.158g, spectral amplification factor assumed equal to 2.483 and spectral value T_C* (characteristic of the constant velocity section) equal to 0.271.

Assuming the damage initiation at an instant T_1 of the structure's life time from a plausible Gamma distribution curve of the structure life time, eleven instants, *t*, were extracted following the instant T_1 assumed to correspond to $\tau = 0.4$ of the damage curve (Fig. 2); these instants are, then, normalized to τ according to the previous definition $\tau = \frac{t}{T_f}$. For each instant τ, from (2) the value of the decrement, $1 - \delta(t)$, for the factor, $f(\tau)$, due to the cracking initiation can be derived and the time variation of *f* will be given by Eq. (1).

The choice to make the onset time of the first damage, T_1, correspond to $\tau = 0.4$ is due to considerations deriving from the laboratory tests on which the construction of the damage curve was based, see Eq. (2). The onset time of the damage can also be associated with the value τ=0.0 or with other values of τ, if more precise information is available on the evolution of the damage.

In order to take into account the randomness of the value of $\delta(\tau)$ at each instant τ, a random extraction of 100 values of δ between $\delta_{\min}$ and $\delta_{\max}$ is made (Fig. 2). For each instant τ and for each value of δ, $f(\tau)$ is calculated from Eq. (2); subsequently - with the application of the methodology proposed by [1-3] and implemented with a progressive calculation macro - the safety factors SF_G and SF_S associated with each τ and each $f(\tau)$ are evaluated. The SF_G and SF_S parameters resulting from the application of the method proposed by [1-3] are expressed as a percentage respectively of the initial spectral acceleration $a_g \cdot S$ (Eq. 3) and the compressive stress limit σ_{RD} (Eq. 4):

$$SF_{G,i} = \frac{a_{SLU,\,i}}{a_g \cdot S} \qquad (3)$$

where $a_{SLU,\,i}$ is the acceleration recorded at each floor and referred to the ultimate limit state; a_g is the seismic acceleration referred to the seismic zone and the return period considered, and S is the spectral coefficient that characterizes the area from the topographic and subsoil point of view (for the case study a_g=0.158g and S=$S_T \cdot S_S$=1.39).

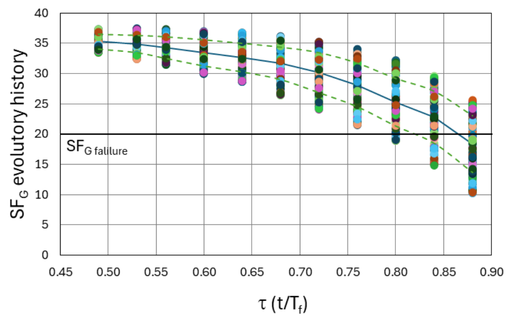
To be on the safe side, the minimum value among the SF_G values obtained at each floor is assumed, $SF_{G,min} = \min[SF_{G,i}]$;

$$SF_{S,min} = \frac{\sigma_{RD}}{\sigma_{0,i}} \tag{4}$$

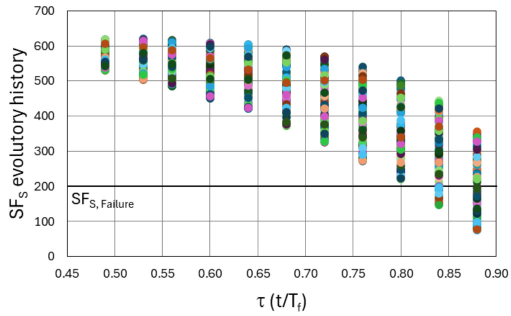
where σ_{RD} the compressive stress limit (National regulation NTC2018 [11]) and $\sigma_{0,i}$ is the compressive stress recorded at every floor.

To be on the safe side, the minimum value among the SF_S values obtained at each floor is assumed, $SF_{S,min} = \min[SF_{S,i}]$.

The evolution of the performance loss described by the two safety factors SF_G and SF_S is shown in Fig. 3.



(*a*)



(*b*)

Fig. 3. Time evolution of the safety factors SF_G (*a*) and SF_S (*b*), as the performance parameter $f(\tau)$ varies. In (*a*) the mean value (solid line) as well as the range described by the mean + and − the standard deviation (dashed lines) are marked. The solid black horizontal lines indicate the limit thresholds of possible failures.

Since the SF_G factor always records lower values than the SF_S factor, SF_G should be investigated as a possible significant performance factor indicating a probable critical condition.

4 System reliability and probability of failure

The next step is to estimate the probability of failure and the resulting loss of reliability and safety.

The reliability of a system is described as the probability that the system will not collapse at time *t* [12]. This definition is extended here, indicating with $R(\tau)$ the probability that a system registers an SF_G value higher than the chosen collapse threshold, by considering a certain significant damage threshold at time τ.

The random variable assumed is time $\bar{\tau}$:

$$R(\tau) = \Pr\{\bar{\tau} > \tau\} = 1 - F_{SF_G}(\tau) \quad (5)$$

The reliability study requires the probabilistic modelling of the SF_G values recorded at each instant τ. In the presented case study, the distribution chosen for modelling $SF_G(\tau)$ is the LogNormal distribution.

The areas enclosed by the densities that model $SF_G(\tau)$ and above the chosen failure threshold $SF^* = SF_{G,failure}$ (an example is reported in Fig. 4a) represent the experimental probabilities that contribute to the construction of Eq. (5) (Fig. 4b).

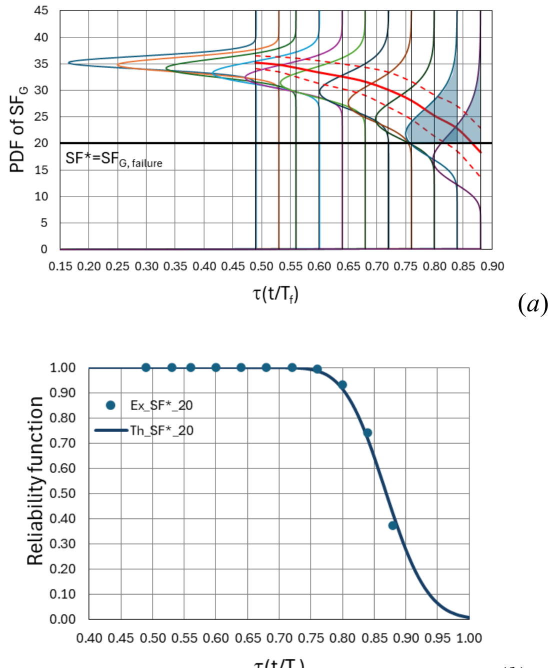


Fig. 4. (*a*) Probabilistic modelling of the distributions of the SF_G values at each instant τ and due to the random variation of the parameter *f*; (*b*) theoretical modelling (solid line) of the experimental reliability function (dotted).

In Fig. 4b, the experimental function is represented with points: each point at time τ quantifies the area enclosed by the densities that model $SF_G(\tau)$ and above the chosen failure threshold SF* of Figure 4a at the same time τ. The experimental reliability function was then modelled with a theoretical probability distribution of the Gamma type (solid line).

From the results obtained it is evident that the triggering of a damage can in the long run lead to a loss of performance even without further increase in loads. In this case, assuming that the damage was triggered at the instant $\tau^*=0.4$ and consequently $t^*=0.4T_f$ (or the life time of the structure) we can assume that the risk of reaching a global safety factor close to threshold SF*=20% of the initial spectral acceleration can occur in a time almost equal to the triggering time: $\tau_{\text{risk}}=0.8$ and consequently $t_{\text{risk}}=0.8T_f$. But surely in the remaining life the building will undergo other changes, and the performance of the reliability function could be modified as well as the failure prediction.

5 Concluding remarks and further developments

Extraordinary maintenance and retrofitting interventions on existing structures can have long-term consequences on the progression of damage in the masonry.

The situation worsens if the building is located in a seismic zone and already presents aspects of vulnerability never fixed.

In order to also take into account the aspect of long-term damage, this work addresses the problem of the variation of the static safety factor and the global safety factor when damage is triggered by a maintenance intervention.

Even considering that the intervention undergone by the building is the first since its construction and taking into account the uncertainties that the evolution of the damage may have (probabilistic approach), it can be noted how the two safety factors are subjected to a decreasing, while maintaining constant all the other parameters in game (geometric and seismic).

From this investigation it is possible to obtain a forecast of the decreasing level of reliability of the studied building system as a function of time, but there are still many uncertainties to be explored.

An aspect not yet investigated is how the reliability of the system changes if further modifications are made to the building system in the presence of an already active crack pattern, or with a performance factor $f < 1$, at the time of the new intervention. The influence of the temporal variation of safety factors must also be investigated as a function of the variability of the expected seismic acceleration. These aspects will certainly be a subject of future study.

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