On the optimal period of spanwise forcing for turbulent drag reduction

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ITI XI July 27-30 2025, Bertinoro

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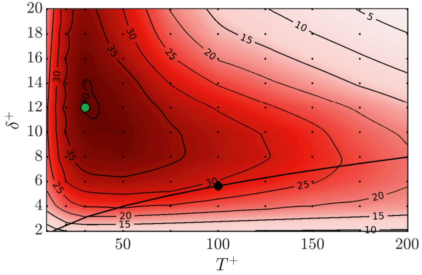
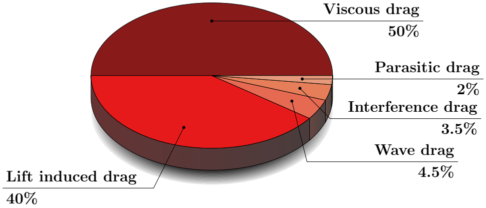


Control for turbulent skin-friction drag reduction

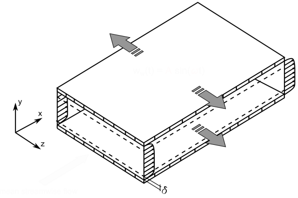
- 50% of an aircraft's drag comes from viscous effects
- An efficient drag reduction ($\mathcal{R}$) technology would have huge economic and environmental benefits

$$\mathcal{R} = \frac{C_{f,0} - C_f}{C_{f,0}}$$



Spanwise forcing

Oscillating wall (Jung et al., PoF 1992)



$$w_{wall}(t) = A \sin\left(\frac{2\pi}{T} t\right)$$

T: oscillation period

It works:

- at large Reynolds number
- at large Mach number
- with complex configurations

How does it work?

Answer this question means:

- designing more performing actuators
- getting insight of the turbulence working mechanism

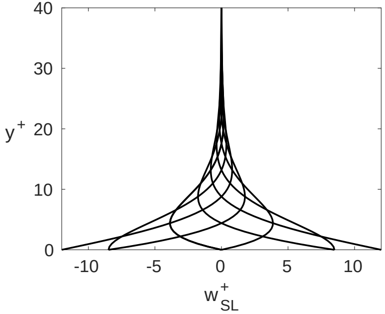
What we know

The wall oscillation generates a spanwise Stokes layer (SL)

$$w_{SL} = Ae^{y/\delta_{SL}} \sin\left(\frac{2\pi}{T}t - \frac{y}{\delta_{SL}}\right)$$

- T: oscillation period
- δ_{SL}: SL thickness

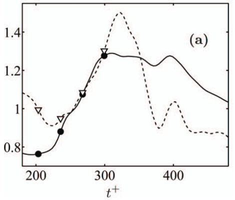
$$\delta_{SL} = \sqrt{\frac{\nu T}{\pi}}$$



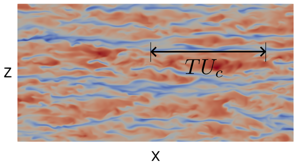
For maximum drag reduction: $T^+ \approx 100$, $\delta^+_{SL} \approx 5.6$

Possible interpretations of $T^+ \approx 100, \delta_{SL}^+ \approx 5.6$

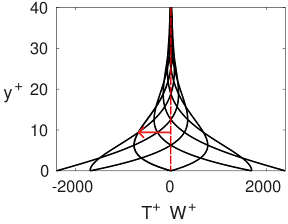
- Time scale



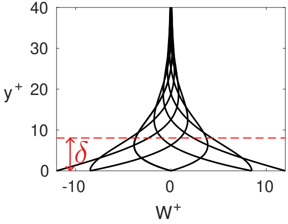
- Longitudinal length scale



- Lateral displacement scale



- Penetration depth scale



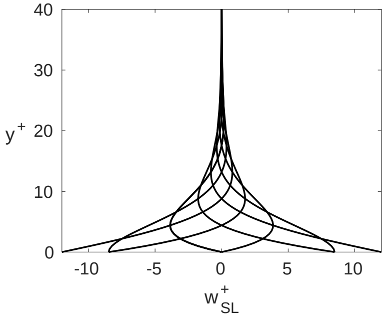
Decoupling temporal and spatial scales

Extended Stokes layer

$$w_{SL} = Ae^{y/\delta_{ESL}} \sin\left(\frac{2\pi}{T} t - \frac{y}{\delta_{ESL}}\right)$$

$$\delta_{ESL} \neq \sqrt{\frac{\nu T}{\pi}}$$

$T^+ = 100, \delta^+ = 5.6$



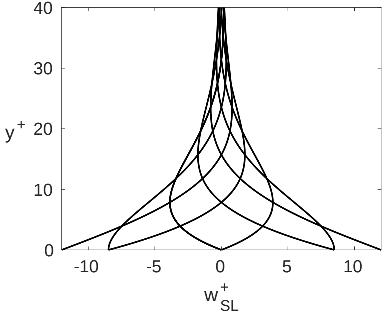
Decoupling temporal and spatial scales

Extended Stokes layer

$$w_{SL} = Ae^{y/\delta_{ESL}} \sin\left(\frac{2\pi}{T}t - \frac{y}{\delta_{ESL}}\right)$$

$$\delta_{ESL} \neq \sqrt{\frac{\nu T}{\pi}}$$

$T^+ = 100, \delta^+ = 10$



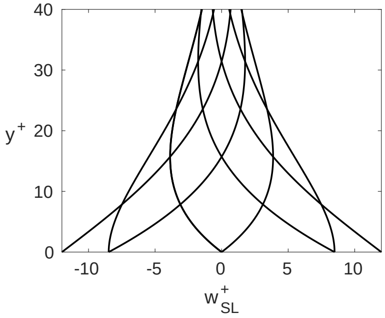
Decoupling temporal and spatial scales

Extended Stokes layer

$$w_{SL} = Ae^{y/\delta_{ESL}} \sin\left(\frac{2\pi}{T}t - \frac{y}{\delta_{ESL}}\right)$$

$$\delta_{ESL} \neq \sqrt{\frac{\nu T}{\pi}}$$

$T^+ = 100, \delta^+ = 20$



Numerical implementation

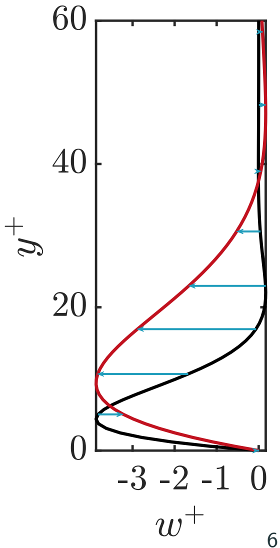
B.C. $\quad y = 0 \quad \langle w \rangle_{x,z}(t) = A \sin\left(\frac{2\pi}{T} t\right)$

$+$

Volume force $\quad y \neq 0 \quad \frac{\partial \langle w \rangle_{x,z}}{\partial t} + \frac{\partial \langle vw \rangle_{x,z}}{\partial y} = -\nu \frac{\partial^2 \langle w \rangle_{x,z}}{\partial y^2} + \langle f_z \rangle_{x,z}(t, y)$

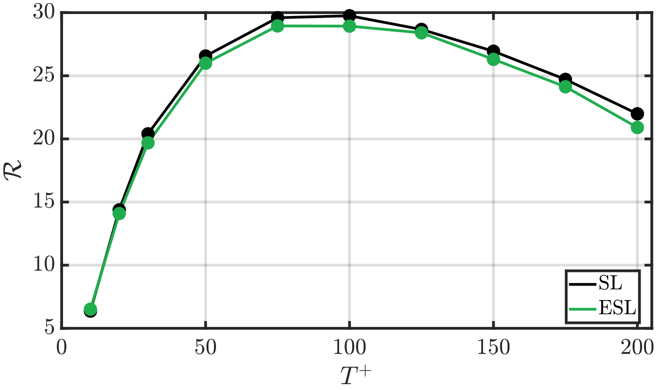
$=$

ESL $\quad \forall y \quad \langle w_{ESL} \rangle_{x,z} = A e^{y/\delta_{ESL}} \sin\left(\frac{2\pi}{T} t - \frac{y}{\delta_{ESL}}\right)$ with $\delta_{ESL} \neq \sqrt{\frac{\nu T}{\pi}}$

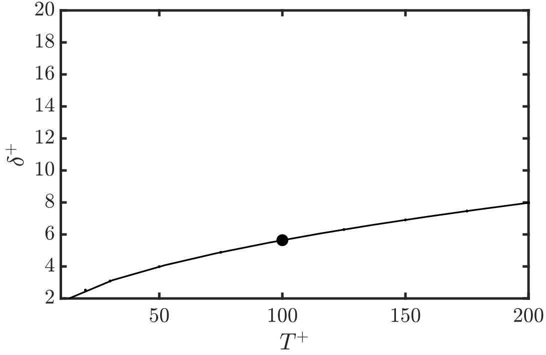


Validation

- Channel flow
- $A^+ = 12$
- $Re_\tau = 400$
- Constant flow rate

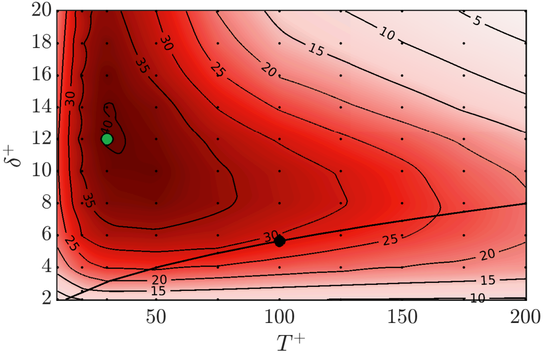


Drag reduction map



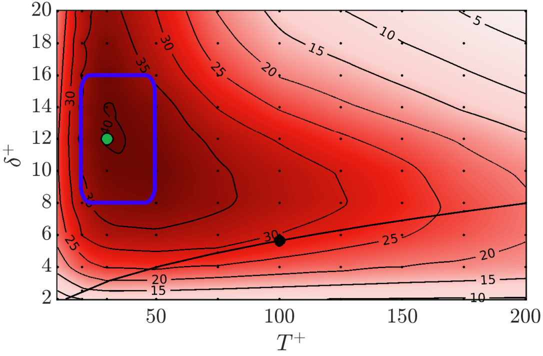
$T^+_{opt} = 100, \delta^+_{opt} \approx 5.6 \rightarrow \mathcal{R} \approx 30\%$

Drag reduction map



- $T^+_{opt} = 100, \delta^+_{opt} \approx 5.6 \rightarrow \mathcal{R} \approx 30\%$
- $T^+_{opt} = 30, \delta^+_{opt} = 12 \rightarrow \mathcal{R} \approx 40\%$

Drag reduction map



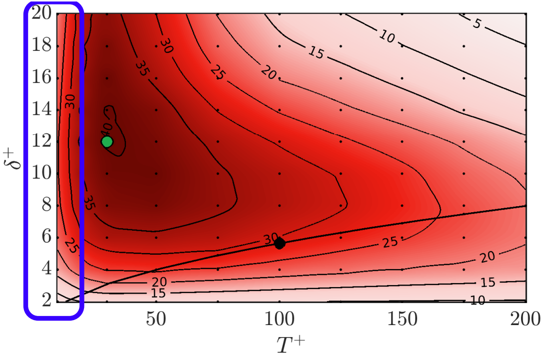
The area of the maximum $\mathcal{R}$

- $20 \leq T^+ \leq 50$
- $8 \leq \delta^+ \leq 14$

corresponds to

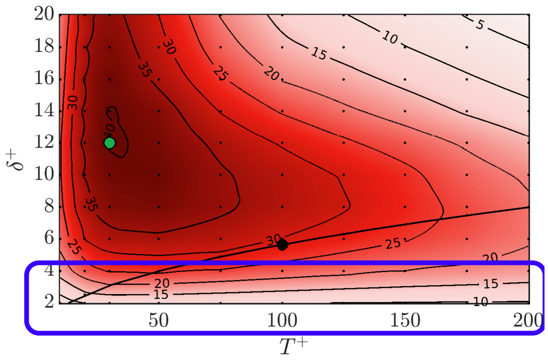
- the regeneration time-scale of the streaks
- the position of buffer layer

Drag reduction map

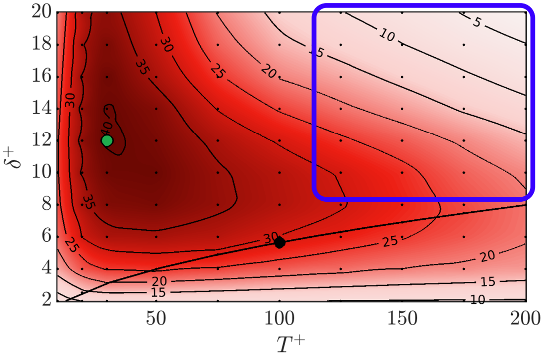


- $T^+ \leq 20$
- $\mathcal{R}$ small and almost constant with δ
- T is too small compared to the flow time scales

Drag reduction map

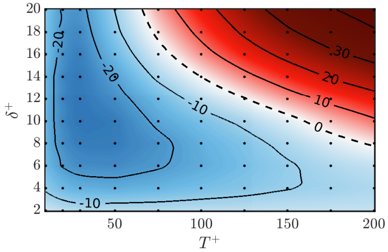


- $\delta^+ \leq 4$
- $\mathcal{R}$ small and almost constant with T
- The spanwise motion is confined in the viscous sublayer



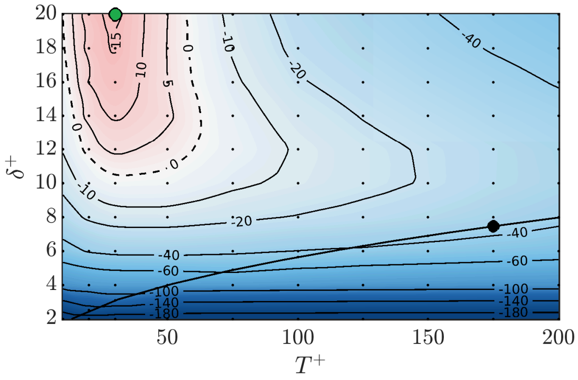
- For large T and δ, the control looses effectiveness

$$100\,\frac{\langle w^{rms}_{ESL}\rangle_{x,z,t} - \langle w^{rms}_{0}\rangle_{x,z,t}}{\langle w^{rms}_{0}\rangle_{x,z,t}}$$



Net power saving

$$
\begin{aligned}
P_{net} &= \mathcal{R} - P_c \\
P_c &= \mathcal{R} - (P_w + P_f) \\
&= \mathcal{R} - \frac{100}{P_0}\left(\langle 2\, w_w \tau_z \rangle_{x,z,t} + \left\langle \int_0^{2h} f_z\, w_{ESL}\, dy \right\rangle_{x,z,t} \right) \quad \text{with} \quad P_0 = 2\, U_b \, \langle \tau_x \rangle_{x,z,t}
\end{aligned}
$$



- $T^+_{opt} = 175, \delta^+_{opt} \approx 7.5 \rightarrow P_{net} \approx -30\%$
- $T^+_{opt} = 30, \delta^+_{opt} = 20 \rightarrow P_{net} \approx +17\%$

Conclusions

- Successfully decoupled the effect of T and δ
- $T^{+}_{opt} \approx 100$ and $\delta^{+}_{opt} \approx 5.6$ do not possess a special physical meaning
- Way paved for the design of alternative control strategies (actuation $\neq$ control)

Conclusions

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- $T^+_{opt} \approx 100$ and $\delta^+_{opt} \approx 5.6$ do not possess a special physical meaning
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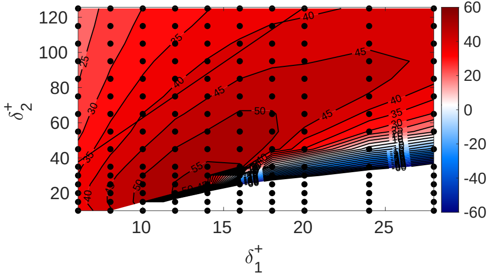
Thank you for the attention

Further extending the Stokes layer profile

$$w_{ESL} = Ae^{y/\delta_1} \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\delta_2}y\right)$$

- δ_1: how far from the wall the SL perturbs the flow
- δ_2: wavelength of the SL

$T^+ = 25$



$T^+ = 100$