Turbulence without spectra

Looking at turbulent structures in inhomogeneous anisotropic flows

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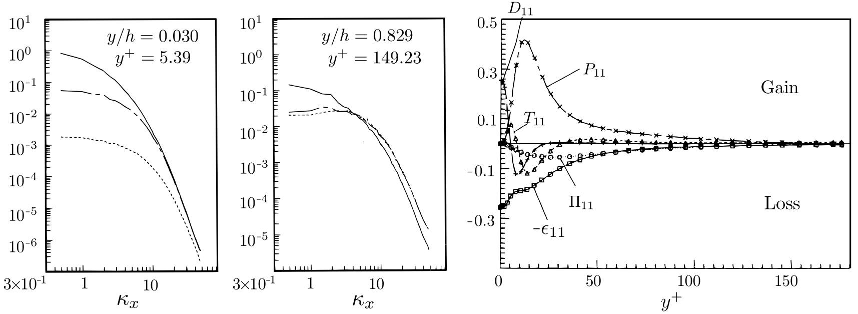
Scale space or physical space?

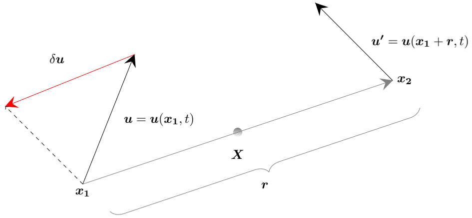


Spectra: from Kim et al JFM 1987. Single-point budget: from Mansour et al JFM 1988

Both: the Generalized Kolmogorov Equation!

Structure function of the turbulent kinetic energy:

$$\langle \delta q^2 \rangle = \left\langle \left(u_i'\left(\boldsymbol{X} + \frac{\boldsymbol{r}}{2}, t\right) - u_i'\left(\boldsymbol{X} - \frac{\boldsymbol{r}}{2}, t\right) \right)^2 \right\rangle$$



GKE is obtained via manipulation of the NS equations:

$$\boxed{\frac{\partial \langle \delta q^2 \rangle}{\partial t} + \frac{\partial \phi_k}{\partial r_k} + \frac{\partial \psi_k}{\partial X_k} = P + \epsilon}$$

The (generalized) Kolmogorov equation

- Several forms (and names: KE, GKE, KHMH)
- Increasing popularity after Hill, JFM 2001
- Advantage: no FT is required
- Exact budget equations, with kinematics and dynamics
- Not fully equivalent to spectral Reynolds stress budget equations (fluxes/redistribution)

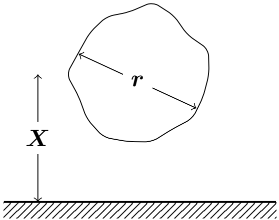
Since energy is the trace of the Reynolds stress tensor ...

AGKE: Anisotropic Generalised Kolmogorov Equations

$$\left\langle \delta u_i' \delta u_j' \right\rangle = \left\langle \left[u_i'\left(\boldsymbol{X} + \frac{\boldsymbol{r}}{2}, t\right) - u_i'\left(\boldsymbol{X} - \frac{\boldsymbol{r}}{2}, t\right)\right]\left[u_j'\left(\boldsymbol{X} + \frac{\boldsymbol{r}}{2}, t\right) - u_j'\left(\boldsymbol{X} - \frac{\boldsymbol{r}}{2}, t\right)\right]\right\rangle = \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$

$$\boxed{\frac{\partial \left\langle \delta u_i' \delta u_j' \right\rangle}{\partial t} + \frac{\partial \phi_{ij,k}}{\partial r_k} + \frac{\partial \psi_{ij,k}}{\partial X_k} = P_{ij} + \Pi_{ij} + \epsilon_{ij}}$$

- Budget equation for each component of $\left\langle \delta u_i' \delta u_j' \right\rangle$.
- Production, transport, redistribution and dissipation of the turbulent stresses in the space of scales/positions.



Gatti et al. JFM 2020

AGKE: Space and physical fluxes

$$\boxed{\frac{\partial \phi_{ij,k}}{\partial r_k} + \frac{\partial \psi_{ij,k}}{\partial X_k} = P_{ij} + \Pi_{ij} + \epsilon_{ij}}$$

$$\phi_{ij,k} = \underbrace{\left\langle \delta U_k \delta u_i' \delta u_j' \right\rangle}_{\text{Mean transport}} + \underbrace{\left\langle \delta u_k \delta u_i' \delta u_j' \right\rangle}_{\text{Turbulent transport}} \underbrace{-2\nu \frac{\partial}{\partial r_k} \left\langle \delta u_i' \delta u_j' \right\rangle}_{\text{Viscous diffusion}} \qquad k = 1, 2, 3$$

$$\psi_{ij,k} = \underbrace{\left\langle U_k^* \delta u_i' \delta u_j' \right\rangle}_{\text{Mean transport}} + \underbrace{\left\langle u_k^* \delta u_i' \delta u_j' \right\rangle}_{\text{Turbulent transport}} \underbrace{+ \frac{1}{\rho} \left\langle \delta p' \delta u_i' \right\rangle \delta_{kj} + \frac{1}{\rho} \left\langle \delta p' \delta u_j' \right\rangle \delta_{ki}}_{\text{Pressure transport}} \underbrace{- \frac{\nu}{2} \frac{\partial}{\partial X_k} \left\langle \delta u_i' \delta u_j' \right\rangle}_{\text{Viscous diffusion}}$$

f^*: arithmetic average of f between $\boldsymbol{X} \pm \boldsymbol{r}/2$.

AGKE: Production, pressure strain and dissipation

$$\boxed{\frac{\partial \phi_{ij,k}}{\partial r_k} + \frac{\partial \psi_{ij,k}}{\partial X_k} = P_{ij} + \Pi_{ij} + \epsilon_{ij}}$$

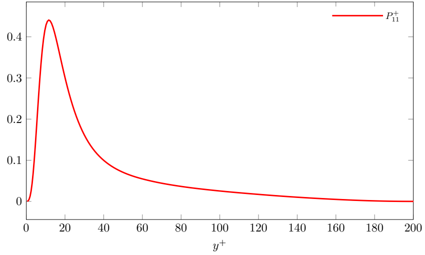
$$P_{ij} = -\left\langle u_k^* \delta u_j' \right\rangle \delta \left(\frac{\partial U_i}{\partial x_k} \right) - \left\langle u_k^* \delta u_i' \right\rangle \delta \left(\frac{\partial U_j}{\partial x_k} \right) - \left\langle \delta u_k' \delta u_j' \right\rangle \left(\frac{\partial U_i}{\partial x_k} \right)^* - \left\langle \delta u_k' \delta u_i' \right\rangle \left(\frac{\partial U_j}{\partial x_k} \right)^*$$

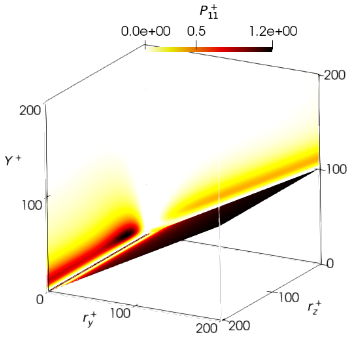
$$\Pi_{ij} = \frac{1}{\rho} \left\langle \delta p' \frac{\partial \delta u_i'}{\partial X_j} \right\rangle + \frac{1}{\rho} \left\langle \delta p' \frac{\partial \delta u_j'}{\partial X_i} \right\rangle$$

$$\epsilon_{ij} = -4 \epsilon_{ij}'^*$$

Example 1: plane Poiseuille flow

Way more informative than single-point statistics



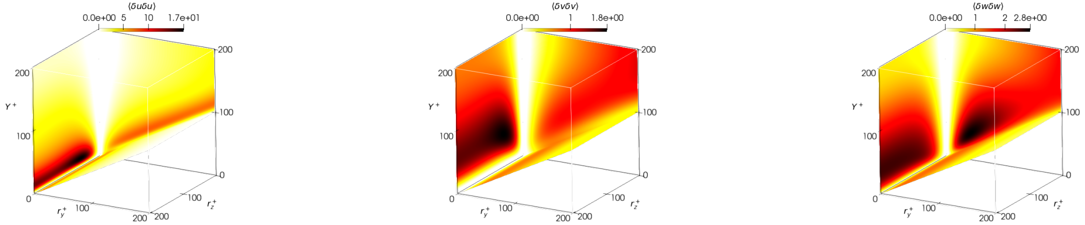


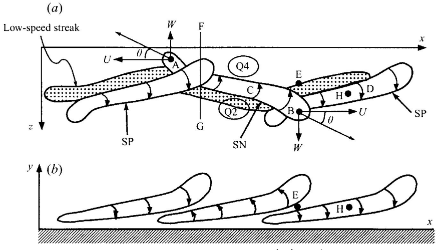
Single-point budget recovered at

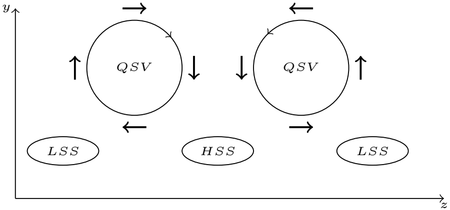
$r_x = L_x/2, r_y = 0, r_z = L_z/2$

Example 1: plane Poiseuille flow

Extract size and position of "structures"



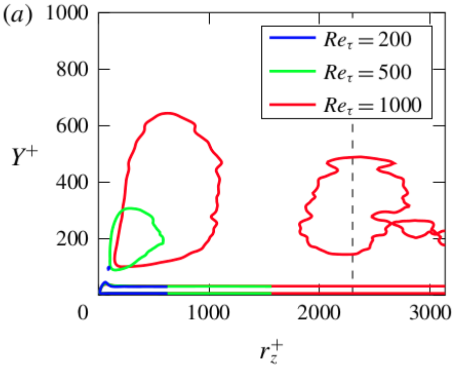




Example 1: plane Poiseuille flow

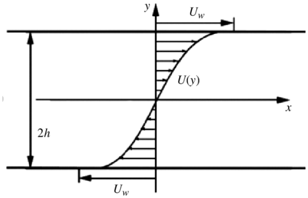
More sensitive than spectra

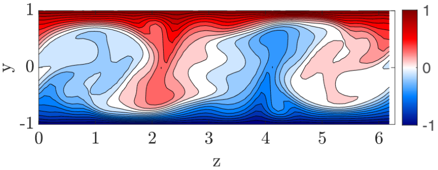
Contours of $\xi_{11} = 0$ at $Re_\tau = 200, 500, 1000$

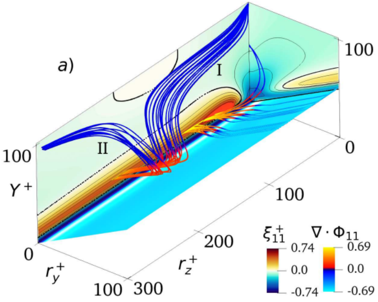


Example 2: plane Couette flow

Visualizing how energy flows





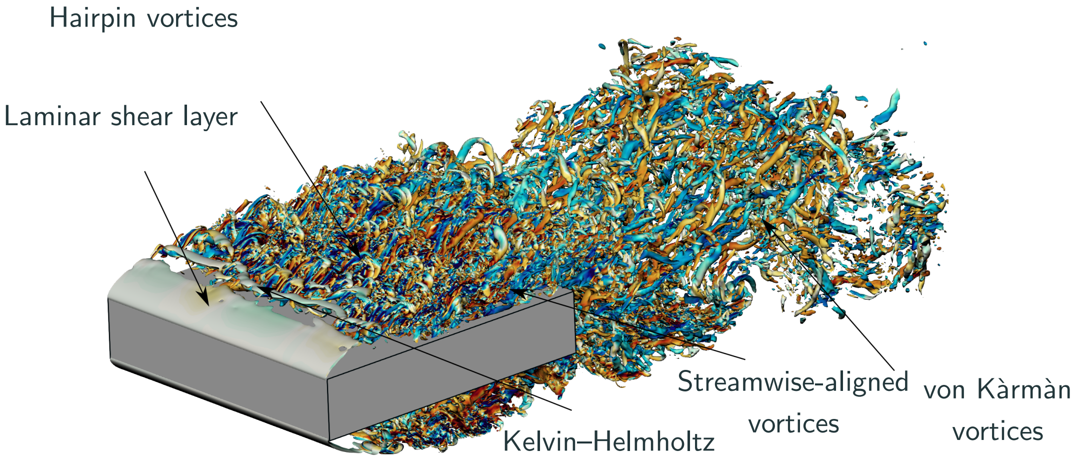


- Bottom-up interaction: transfer towards the large rolls
- Major contributors: viscous and turbulent transport

Chiarini et al JFM 2022a

Example 3: BARC

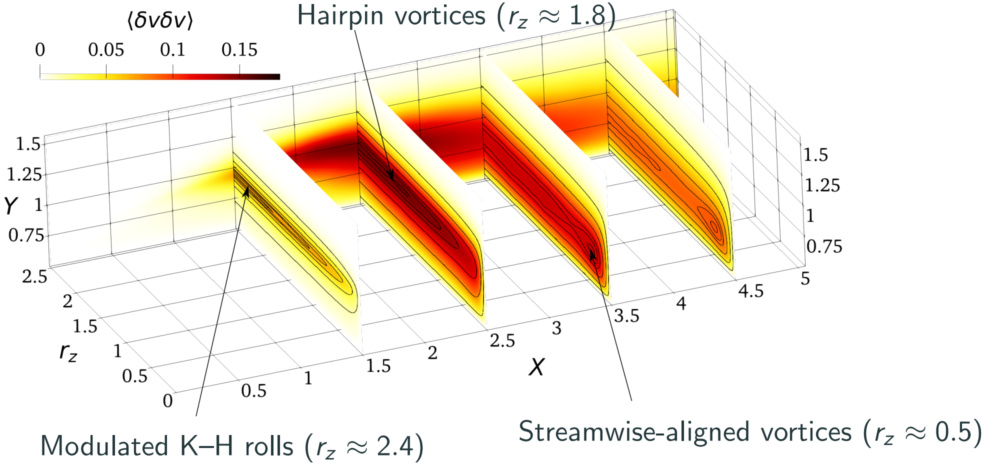
The structure of turbulence



Chiarini et al. JFM 2022b

Example 3: BARC

The structure of turbulence



The φAGKE

Triple decomposition: mean, coherent, stochastic

$$\boxed{u_i = U_i + \tilde{u}_i + u_i''}$$

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

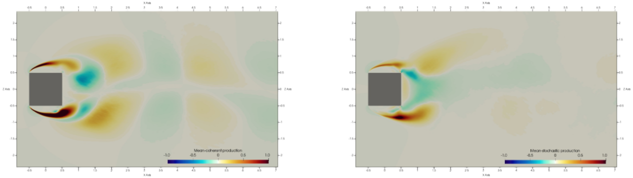
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u_i''\delta u_j''}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$

Gattere et al, JFM 2023

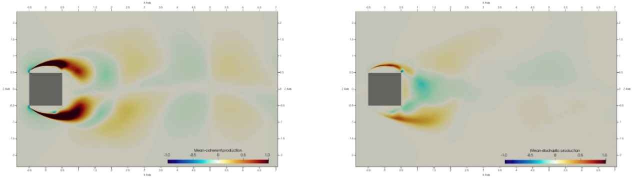
Example 4: turbulent wake after a cube

Interaction between small and large scales

Mean-coherent (p_{11}^{mc}) and mean-stochastic production (p_{11}^{cs}) of $\langle u''u'' \rangle$



(a) Phase 1



(b) Phase 2

Concluding remarks

- Heavy post-processing: memory and I/O intensive
- Importance of efficient code (pseudo-spectral when possible, subsampling, VoI)
- Data must be distilled into information