Linear Impulse Response Function of a Turbulent Channel Flow: measurement and possible uses

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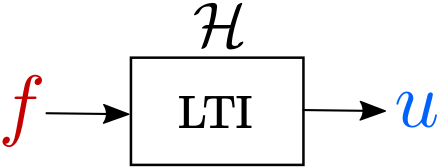
Motivation

Aim: linear statistical description of a turbulent channel flow

- linear mechanisms are central to the near-wall turbulence cycle
- linear models of turbulence may suffice for flow control purposes

LIRF: Linear Impulse Response Function

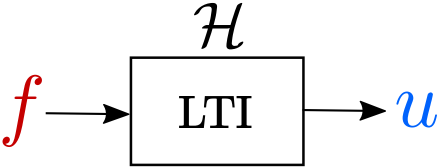
Relationship between the input and the output of the linear system



$$u(t) = \int_{-\infty}^{+\infty} \mathcal{H}(t-\tau)\; f \; d\tau \qquad\qquad f = \epsilon\delta(\tau)$$

LIRF: Linear Impulse Response Function

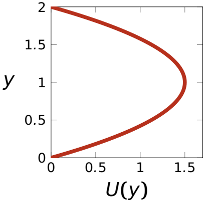
Relationship between the input and the output of the linear system



$$u(t) = \int_{-\infty}^{+\infty} \mathcal{H}(t-\tau)\ f\ d\tau \qquad f = \epsilon\delta(\tau)$$

What about measuring the LIRF of a non-linear turbulent channel flow ?

LIRF for a **laminar** channel flow



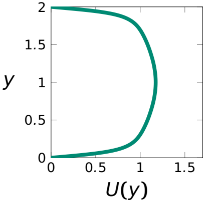
Stability theory: Jovanovic and Bamieh 2005, JFM
Control theory: Höpffner et al. 2005, JFM

- Non-linear Navier–Stokes Equations
- no fluctuations: $u = U + H \underbrace{\epsilon\delta(t)}_{f}$

$$\downarrow$$

- ϵ needs to be small enough for the response to be linear

LIRF for a pseudo-turbulent channel flow



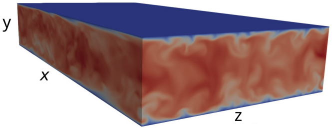
Resolvent analysis: McKeon & Sharma 2010, JFM

- Non-linear Navier–Stokes Equations
- no fluctuations: $u = U + H \underbrace{\epsilon\delta(t)}_{f}$

$$\downarrow$$

- ϵ needs to be small enough for the response to be linear

LIRF for a turbulent channel flow



Control: Luchini, Quadrio & Zuccher 2006, PoF

- Non-linear Navier–Stokes Equations
- fluctuations: $u = U + u' + H \underbrace{\epsilon\delta(t)}_{f}$

$$\downarrow$$

- ϵ needs to be small enough for the response to be linear
- ϵ needs to be large enough for $f > u'$

How to measure the LIRF

- Direct measurement

$$\mathcal{H}(t - t_f) = \frac{u(t)}{f} \qquad f = \epsilon\delta(t - t_f)$$

small S/N ratio

✓ laminar/pseudo-turbulent

✗ turbulent

How to measure the LIRF

- Direct measurement

$$\mathcal{H}(t - t_f) = \frac{u(t)}{f} \qquad f = \epsilon\delta(t - t_f)$$

small S/N ratio

✓ laminar/pseudo-turbulent

✗ turbulent

- Through input-output correlation

$$\mathcal{H}(t - t_f) = \frac{\langle u(t) f(t_f) \rangle}{\epsilon^2} \qquad f = \text{white noise}$$

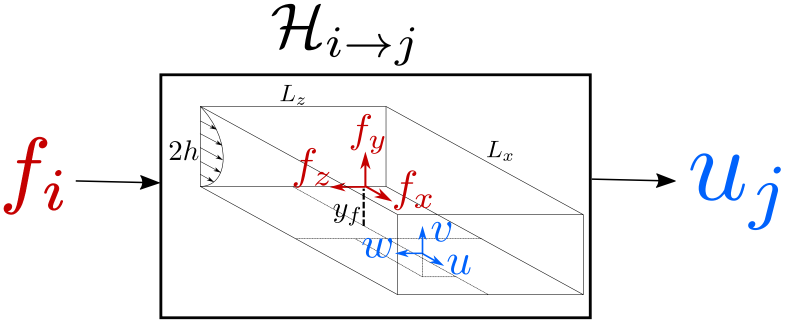
decent S/N ratio

✓ laminar/pseudo-turbulent

✓ turbulent

LIRF: Mean Linear Impulse Response Function

Relationship between the volume force and the velocity of the linear system

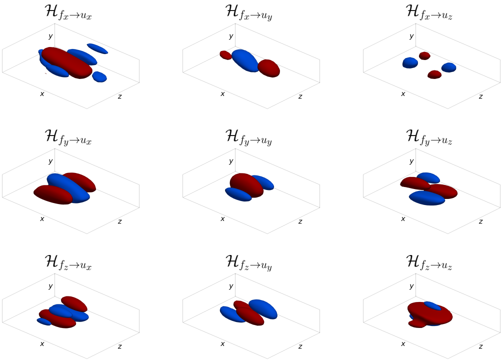


$$\mathcal{H}_{i\to j}(x, y, z, t-\tau; y_f)$$

Richness of information of the response

$$\mathcal{H}_{f_i \to u_j}(x, y, z, \overline{\tau}; \overline{y_c})$$

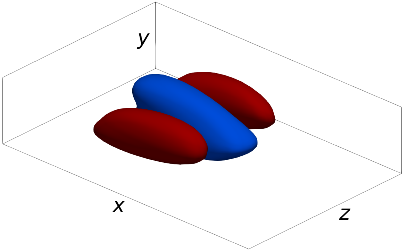
- anisotropy



Richness of information of the response

- anisotropy
- spatial structure

$$\mathcal{H}_{f_y \to u_x}(x, y, z, \overline{\tau}; \overline{y_c})$$

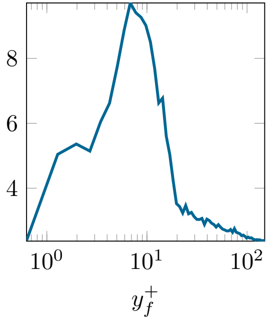


associated with
near-wall structures

Richness of information of the response

- anisotropy
- spatial structure
- dependence on forcing position

$\max_{x,y,z,\tau} \left| \mathcal{H}_{f_y \to u_x}(x, y, z, \tau; y_f) \right|$

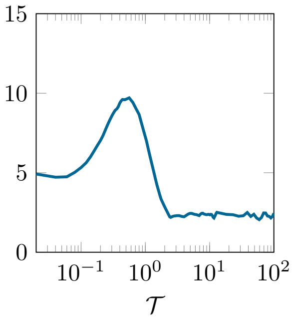


$y^+_{max} \approx$ buffer layer

Richness of information of the response

- anisotropy
- spatial structure
- dependence on forcing position
- evolution in time

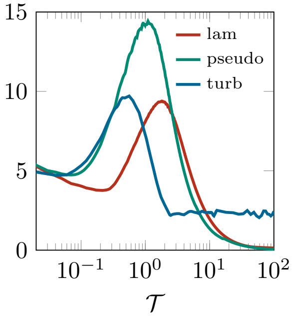
$$\max_{x,y,z,y_c} \left|\mathcal{H}_{f_y \to u_x}(x, y, z, \tau; y_f)\right|$$



transient growth

Richness of information of the response

$$\max_{x,y,z,y_c} \left|\mathcal{H}_{f_y \to u_x}(x, y, z, \tau; y_f)\right|$$



- anisotropy
- spatial structure
- dependence on forcing position
- evolution in time
- differences between approaches

turbulent diffusion

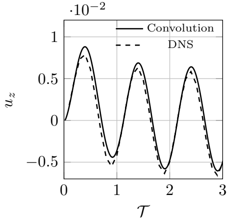
Possible uses of the measured mean turbulent LIRF

- cheap simulations

The linear response to a generic forcing can be computed without DNS

$$u_j = \int_{-\infty}^{+\infty} \mathcal{H}_{i \to j} \, \mathsf{f}_i \, d\tau$$

Example: $f_z = \sin(\omega t)$



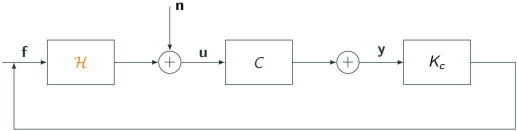
Possible uses of the measured mean turbulent LIRF

- cheap simulations
- enhanced linear model

We can control the flow field (u)
with a control input (f)

Example:

$$\begin{cases} \dot{u} = Au + Bf + n \\ y = Cu \end{cases}$$



M. Castelletti, Fri 29 h 10.10 (Session 10 Cat 18)

Conclusions

- First DNS-based measurement of the mean LIRF of turbulent channel flow with body forces
- The laminar/pseudo-turbulent LIRF are missing something
- Measurement of the best estimator of the linear behaviour of the turbulent channel flow
- The LIRF can be exploited for linear analysis and control