Wiener filtering in wall turbulence

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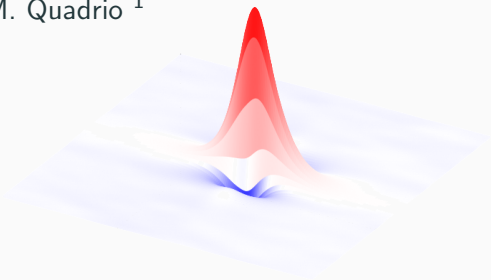

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Linear Estimation and Control of Turbulent Channel Flow

Optimal linear control applied to turbulent flows dates back to early 2000s.
Estimators/controllers:

- designed based on a linear model of the system
- applied to the true nonlinear system

Successful examples:

- Chevalier et al., JFM, (2006): Kalman filters
- Högberg et al., JFM, (2003): full state LQR controllers

Linear Estimation and Control of Turbulent Channel Flow

Optimal compensator were designed from linearized Navier–Stokes equation:

- Resort to time domain and state space formulation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{n}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{d}$$

- Turbulence was state (white!) noise

Our aim:

Develop the best possible linear, time-invariant filters for turbulent channel flow

From time to frequency domain

Standard time domain

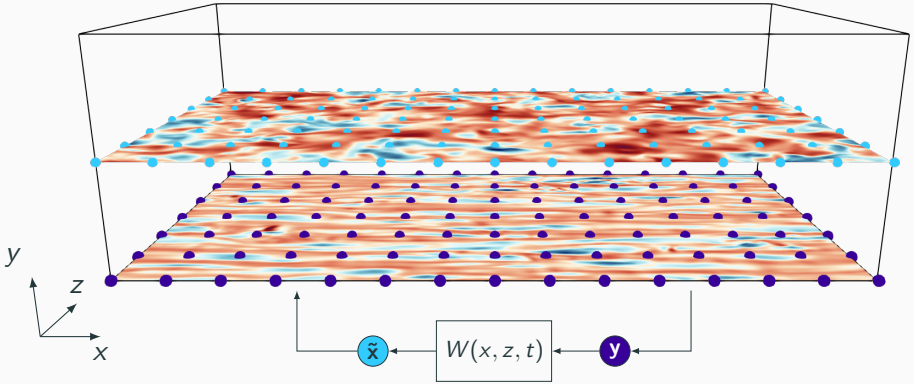
- Resort to separation principle
- Causality comes for free
- Naturally accounts white noise
- Solve a Riccati equation $O({N_s}^3)$

Frequency domain

- Input-output formulation
- Causality has to be enforced
- Naturally accounts colored noise
- Solve a Wiener-Hopf equation $O({N_f}^2)$

➤ Frequency is well-suited for *under*-sensed/actuated systems (as wall turbulence).

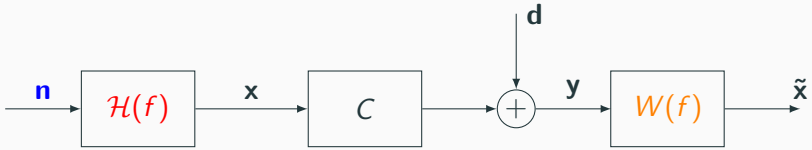
Optimal Linear Estimation of Turbulent Channel Flow



$$\tilde{\mathbf{x}}(x,z,t) = \int \mathbf{y}(x',z',t')W(x-x',z-z',t-t')dx'dz'dt'$$

Chevalier et al., *JFM*, 2006

Estimation: block scheme



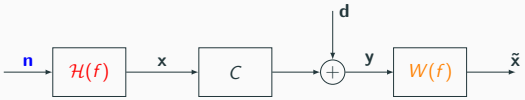
- $\mathcal{H}(f)$: linearized model of the turbulent flow
- n: non linear terms become forcing noise
- C: measurements matrix

Goal: find causal estimator $W(f)$ such that $e = \mathbf{x} - \tilde{\mathbf{x}}$ is minimized

Estimation: cost function

Cost function to minimize:

$$J = Tr[\sigma_{ee}^2] = \int_{-\infty}^{+\infty} Tr[\phi_{\mathbf{ee}}(f)]df,$$



$$\mathbf{x} = \mathcal{H}\mathbf{n}$$

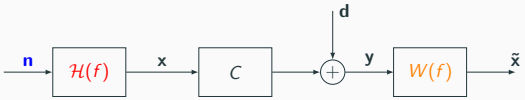
$$\tilde{\mathbf{x}} = \mathbf{W}(\mathbf{C}\mathcal{H}\mathbf{n} + \mathbf{d})$$

$$J = \int_{-\infty}^{+\infty} \mathrm{Tr}\big[\mathbf{W}\ \ \phi_{dd}\mathbf{W}^H + \mathbf{W}\ \ \mathbf{C}\phi_{nn}\mathcal{H}^H\mathbf{C}^H\mathbf{W}^H + \mathcal{H}\phi_{nn}\mathcal{H}^H + \dots$$
$$\cdots - \mathcal{H}\phi_{nn}\mathcal{H}^H\mathbf{C}^H\mathbf{W}^H - \mathbf{W}\ \ \mathbf{C}\phi_{nn}\mathcal{H}^H\big] \qquad df$$

Estimation: cost function with causality

Cost function to minimize:

$$J = Tr[\sigma_{ee}^2] = \int_{-\infty}^{+\infty} Tr[\phi_{\mathbf{ee}}(f)]df,$$



$$\mathbf{x} = \mathcal{H}\mathbf{n}$$

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$$J = \int_{-\infty}^{+\infty} \mathrm{Tr}\big[\mathbf{W}_+\phi_{dd}\mathbf{W}_+^H + \mathbf{W}_+\mathbf{C}\phi_{nn}\mathcal{H}^H\mathbf{C}^H\mathbf{W}_+^H + \mathcal{H}\phi_{nn}\mathcal{H}^H + \dots$$
$$\cdots - \mathcal{H}\phi_{nn}\mathcal{H}^H\mathbf{C}^H\mathbf{W}_+^H - \mathbf{W}_+\mathbf{C}\phi_{nn}\mathcal{H}^H\big] + \mathrm{Tr}[\Lambda_-\mathbf{W}_+]\ df$$

Estimation: final equation

Minimizing J wrt $\mathbf{W}_+^H$ leads to the Wiener-Hopf equationx

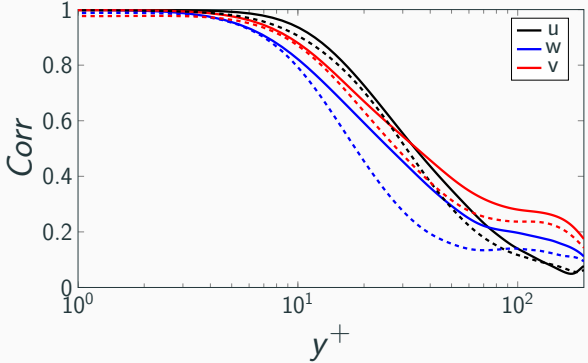
$$\mathbf{W}_+(\mathbf{C}\mathcal{H}\phi_{nn}\mathcal{H}^H\mathbf{C}^H + \phi_{dd}) + \Lambda_- = \mathcal{H}\phi_{nn}\mathcal{H}^H\mathbf{C}^H$$

to be solved for every wavenumber couple.

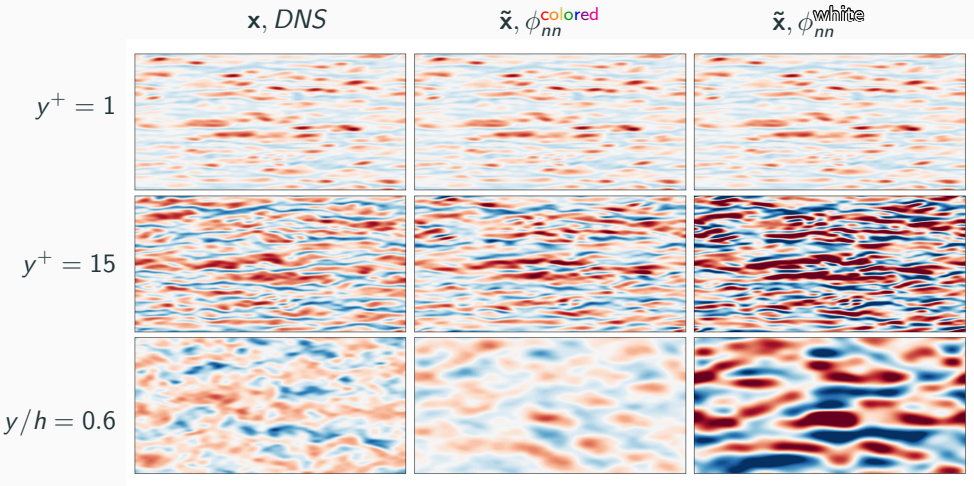
- Easily handle colored forcing noise ϕ_{nn}
- Lower computational cost than time domain couterpart (Riccati)

Estimation results: correlation between DNS and linear estimate

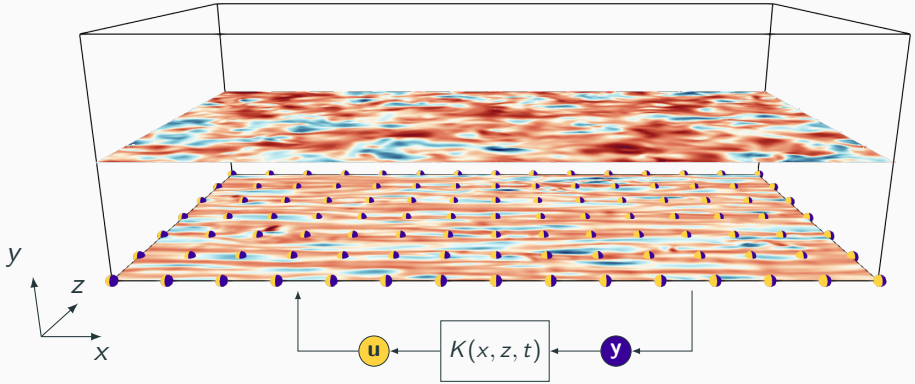
- Test case: channel flow $Re_\tau = 200$
- Measurements $\mathbf{y}$: τ_x, τ_z and p; $\phi_{dd} = 10^{-5}$
- $\phi_{nn}^{\text{colored}}$ ———; ϕ_{nn}^{white} - - - -



Estimation results: flow visualization



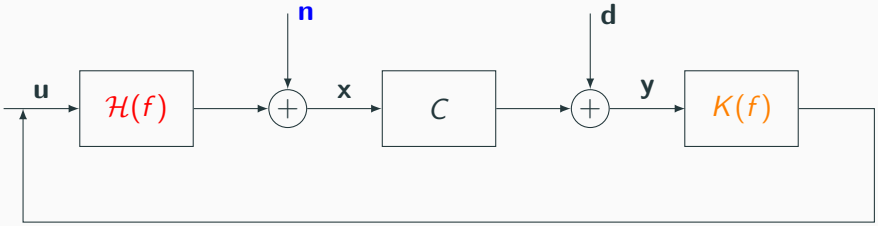
Optimal Linear Control of Turbulent Channel Flow



$$\mathbf{u}(x,z,t) = \int \mathbf{y}(x',z',t')K(x-x',z-z',t-t')dx'dz'dt'$$

Högberg et al., *JFM*, 2003

Control block scheme



- $\mathcal{H}(f)$: linearized model of the turbulent flow
- n: measured turbulent flow fluctuations

The goal is to design causal K such that

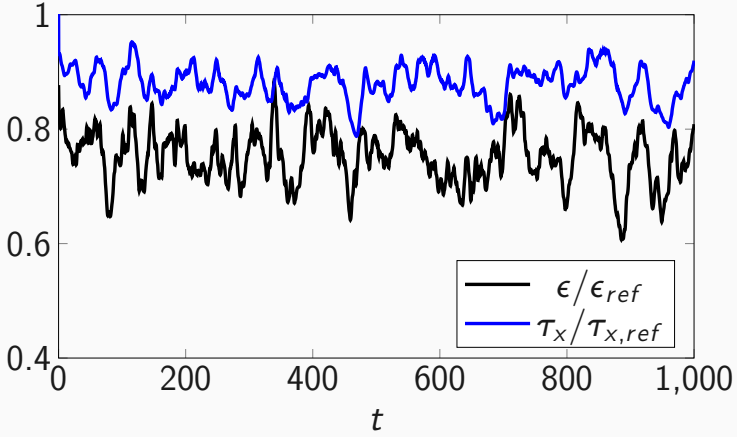
$$J = E\{x^H Q x + u^H R u\}$$

is minimized in the closed loop system

Preliminary control results

Dissipation reduction $\approx 25\%$

Drag reduction $\approx 12\%$



Conclusions & Outlook

- Linear estimators, based on wall-measurements, works close to the wall only.
- Wiener filtering for drag reduction looks promising.