Definition of Pruned Frequency-Domain Volterra Models Based on Knowledge About the Input Spectrum

Marco Faifer, *Senior Member, IEEE*, Christian Laurano, *Member, IEEE,* Roberto Ottoboni, *Fellow, IEEE,* Sergio Toscani, *Senior Member, IEEE* and Michele Zanoni

***Abstract*—The Volterra representation is one of the most widely employed approaches to the behavioral modeling of nonlinear time invariant systems in the frequency domain. Its main drawback is that the input-output relationship is defined by a set of coefficients whose cardinality rapidly grows with the considered nonlinearity degree and with the number of input harmonics. The purpose of this work is proposing a method that, assuming to know which are the strongest spectral components in the typical input signals, allows writing a subclass of Volterra models whose behaviors are defined by a dramatically lower number of coefficients, with minor impact on accuracy.**

According to this information, input spectral components are classified into "large", "small" and "linear". The output spectrum is computed by considering all the possible interactions between "large" components, according to the Volterra theory. On the contrary, "small" components interact only with "large" components, but not with each other. "Linear" components are linearly transferred to the output. The effectiveness of the pruning technique is evaluated with both numerical simulations and experiments. Results highlight the advantages and the flexibility enabled by the proposed approach, which become even more evident in the presence of significant noise during identification.

***Index Terms*—Nonlinear system identification; Nonlinear systems; Volterra series; Wiener-Hammerstein systems; Behavioral modeling; Harmonic distortion**

I. INTRODUCTION

MANAGING nonlinear models has always been a challenging task for scientists and engineers. This is the main reason why, when nonlinear effects are weak enough with respect to the target accuracy, an approximated linear time invariant (LTI) representation is preferred. For example, when a frequency domain approach is adopted, an attractive possibility is estimating its Best Linear Approximation (BLA) [1]-[4]; it provides the best prediction of the output for a given class of input signals, while nonlinearities can be qualified and quantified with respect to measurement noise.

However, if nonlinearity becomes relevant with respect to the required accuracy, or when nonlinear effects must be studied, using a nonlinear model becomes mandatory.

Let us focus on nonlinear behavioral modeling, which is employed if there is not enough information about the operating principle, or when physics-based modeling is too complex to be tackled. In this respect, several approaches have been proposed in the literature, together with proper identification techniques [5], [6]: they include polynomial models [7], lookup tables [8], fuzzy logic and artificial neural networks [9], [10], [11], support vector regression-based models [12], etc...

Among them, Volterra-Wiener models [13], [14] are extremely attractive, since they can be derived as the most natural extension of the well-known theory of LTI systems. In fact, the output corresponds to a sum of contributions coming from homogenous nonlinear subsystems; each of them is obtained as a multifold convolution between the input and the generalized impulse response characterizing its behavior. Thanks to its flexibility, this approach is employed in a wide range of applications. Considering the most recent literature, they include the characterization of electronics devices [15], [16], the modeling electrical machines [17], of electrochemical energy storages [18], of physiological phenomena [19], [20] (in this respect, it is worth citing a dedicated book [21]), the equalization of radio-frequency amplifiers [22], sensors [23] and actuators [24].

In some cases, the target is predicting the steady-state response to periodic inputs: it is generally favorable to employ the (truncated) Volterra representation in the frequency-domain [13]. Each output spectral component is obtained as a linear combination of proper products between up to I input spectral components [25]. The set of the weight coefficients of these products among input components characterizes the behavior of the peculiar system, while I is the degree (or order) of the Volterra representation. Increasing the degree allows reaching a more detailed modeling of the nonlinear behavior, just as a higher-order Taylor polynomial enables a better approximation of a function.

This work has been partially financed by the Research Fund for the Italian Electrical System under the Three-Year Research Plan 2022-2024 (DM MITE n. 337, 15.09.2022), in compliance with the Decree of April 16th, 2018.

Marco Faifer, Christian Laurano, Roberto Ottoboni and Sergio Toscani are with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, 20133 Milan, Italy (e-mail: marco.faifer@polimi.it; christian.laurano@polimi.it; roberto.ottoboni@polimi.it; sergio.toscani@polimi.it)

Michele Zanoni is with Ricerca sul Sistema Energetico RSE SpA, 20134, Milan, Italy (e-mail: michele.zanoni@rse-web.it)

Color versions of one or more of the figures in this article are available online at http://ieeexplore.ieee.org

In this respect [26], [27] analyze the frequency components present in the steady-state output of truncated Volterra systems when the input is a linear combination of sinewaves. Moreover, [28] proposes a method that allows deriving the structure of the coefficients defining the steady-state response of truncated Volterra systems (characterized by an arbitrary degree) to band-limited periodic inputs. However, it is worth highlighting that the number of coefficients defining such models grows more than exponentially with the degree and the number of spectral components in the input signal. Therefore, 2nd or 3rd degree models are employed in most applications [29], [30], [31]: higher degrees may result in troublesome and time-consuming identification procedure. It is worth highlighting that the different nonlinear output contributions often have a very unequal impact on the overall behavior of the system. On the one hand, this means that such behavior can be well approximated with a considerably simpler structure. Furthermore, considering the practical implementation, modeling the weakest output contributions is prone to be highly affected by measurement disturbances occurring during identification, thus severely reducing robustness.

For this reason, several techniques that allow simplifying (or pruning) Volterra models with the least impact on overall accuracy have been proposed. Most of them refer to the time-domain formulation; simplifications are introduced by exploiting some kind of *a priori* knowledge about the system to be modeled, or by using matching pursuit, compressed sensing and optimization techniques [32]-[37], assuming a sparse structure for the underlying model [38]. They are often employed in the modeling of radiofrequency and microwave devices [39], but in principle they are general and thus they abstract from the peculiar application. When nonlinearity is significant, but memory effects can be considered as short, it could be advantageous adopting the modified Volterra series [40], formulated in terms of dynamic deviation of the input signal.

In some important cases, such as in the modeling of ac power system devices, the input contains a frequency component which is much larger than the others. Considering the Volterra representation, products involving more than one "small" input component have significantly lower magnitudes. In turn, their output contributions are also expected to be weaker. In this context, simplified Volterra models under quasi-sinusoidal conditions have been defined by neglecting these terms: the number of coefficients can be reduced dramatically, thus enabling the employment of frequency-domain Volterra models of unprecedented degrees [41], [42]. This simplification has been applied to the modeling of voltage transformers [43] and diode rectifiers [44], resulting in an exemplary tradeoff between accuracy and complexity.

In [45] the previously discussed approach has been somewhat extended: by exploiting *a priori* knowledge about the typical amplitude of the input spectral components, they can be categorized into the "large" and "small" subsets, similarly to what happens as long as the spectral linearization approximation (employed to introduce the X-parameters, namely the extension of the S-parameters to the nonlinear case) is applied [46], [47]. Output contributions involving the product of more than an input spectral component belonging to the "small" subset have been neglected. The resulting simplified Volterra representations have been defined, and achieved performance has been evaluated.

When considering once again applications where input contains a unique largely prevailing input component, a further simplification can be introduced [48]. In this case, all the output contributions due to the products involving the main component and the smaller ones have been neglected. The resulting representation does not consider any interaction (intermodulation) between different input components, hence it is called harmonic distortion (HD) model.

The present paper is the technical extension of [45]. The target is defining a broader and more flexible class of pruned frequency-domain Volterra models, so that it is also able to include and generalize the previously mentioned quasi sinusoidal and HD models. The input spectral components are now split into three subsets:

- "Large" input components, whose interactions between each other are considered as in conventional Volterra models.
- "Small" input components, which interact only with "large" input components.
- "Linear" input components, which do not interact neither with "large", nor with "small" input components.

Thanks to the proposed approach, the user gains more freedom in tailoring the complexity of the nonlinear model, according to the features of the input spectrum and the accuracy requirements in predicting the output frequency components.

After a brief recall about the frequency-domain Volterra representation, the proposed class of *a priori* pruned models is presented, together with a procedure that enables defining their structure. The achievable performance has been firstly evaluated through numerical simulations by adopting different configurations of the proposed approach to model the behavior of two Wiener-Hammerstein systems. The impact of noise during identification is assessed, showing that using a properly pruned model could be beneficial under these conditions. Finally, the pruning technique has been experimentally validated. Results are in full agreement with the expectations, thus confirming the capability of the proposed method to reach a favorable tradeoff between model accuracy and complexity that suits the specific requirements.

II. The Frequency-Domain Volterra Representation

As anticipated during introduction, the Volterra approach arises as the most natural generalization of LTI systems theory to the nonlinear case [13], [14]. Supposing that $u(t)$ and $y(t)$ are, respectively, the system input and output, their relationship in the Volterra framework results

$$y(t) = \sum_{i=1}^{\infty} y^i(t)$$

$$y^i(t) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} h^i(\tau_1, \dots, \tau_i) u(t-\tau_1) \dots u(t-\tau_i) \, d\tau_1 \dots d\tau_i .$$

(1)

As from (1), the output is the sum of infinite terms $y^i(t)$, which are the contributions coming from an infinite number of homogeneous subsystems, each characterized by its nonlinear degree i. In turn, $y^i(t)$ is given by the i-fold convolution between the ith degree kernel $h^i(\tau_1,\dots,\tau_i)$ and the input u. In order to stress the analogy with the LTI case, $h^i(\tau_1,\dots,\tau_i)$ is also known as ith degree generalized impulse response. In practical applications, the summation in (1) must be bounded to the Ith degree, such as a truncated Taylor expansion is used to approximate a nonlinear function.

When periodic inputs characterized by the fundamental angular frequency ω_0 are considered and the target is obtaining the corresponding steady-state responses, a frequency-domain approach is favorable. In this case, the two-sided input signal spectrum is discrete, namely defined by the set Γ of harmonic indexes (let R be its cardinality) and by the corresponding set of harmonic phasors $\{U(n)\}_{n\in\Gamma}$, each characterized by the angular frequency $n\omega_0$. Adopting an Ith degree Volterra model, (1) can be manipulated to obtain the phasor of the generic mth order output harmonic, which is

$$Y(m)=\sum_{i=1}^{I}Y^i(m)$$
$$Y^i(m)=\sum_{\substack{n_1,\dots,n_i\in\Gamma\\ m=\sum_{j=1}^{i}n_j}}H^i(n_1,\dots,n_i)\prod_{j=1}^{i}U(n_j) \tag{2}$$

where $H^i(n_1,\dots,n_i)$ is the i-dimensional Fourier transform of $h^i(\tau_1,\dots,\tau_i)$ evaluated in $n_1\omega_0,\dots,n_i\omega_0$. Similarly to (1), according to (2) the generic output phasor $Y(m)$ is given by the sum of I contributions $Y^i(m)$, $i\in\{1,\dots,I\}$, each being the output of a nonlinear homogeneous subsystem. $Y^i(m)$ is a weighted sum of all the possible products between groups of i input phasors (with repetitions) whose sum of the harmonic indexes $n_1,\dots,n_i$ equals m; the complex-valued weight coefficient of the generic product is $H^i(n_1,\dots,n_i)$. The first-degree subsystem is LTI and thus $H^1(n_1)$ represents its frequency response function evaluated in $n_1\omega_0$. In order to highlight the formal similarity with respect to the LTI case, $H^i(n_1,\dots,n_i)$ is the ith degree generalized frequency response function (GFRF) evaluated in the set of frequencies $n_1\omega_0,\dots,n_i\omega_0$.

The set of GFRF values (we often refer to them as coefficients) determines the system behavior, but the same input-output behavior corresponds to different sets of GFRFs [14]. In order to establish a one-to-one relationship, a peculiar structure for the GFRFs must be selected. The triangular representation is considered here: $H^i(n_1,\dots,n_i)$ could be nonzero only when arguments are sorted in non-ascending order.

Equation (2) can be conveniently rewritten using vector notation [42] as

$$Y(m)=\mathbf{W}^T(m)\mathbf{H}(m)=\begin{bmatrix}\mathbf{W}_m^1\\ \vdots\\ \mathbf{W}_m^I\end{bmatrix}^T\begin{bmatrix}\mathbf{H}_m^1\\ \vdots\\ \mathbf{H}_m^I\end{bmatrix} \tag{3}$$

where $\mathbf{W}^i{}_m$ is a column vector whose entries are all the possible products between i input components (with repetition) whose sum of the harmonic indexes equals m (thus, according to (2), it affects the mth order output harmonic). The column vector $\mathbf{H}^i{}_m$ includes the corresponding ith degree GFRF coefficients, which define the model behavior. This compact form highlights that the Volterra model is linear in the coefficients. Therefore, identification can be tackled by exciting the system with a proper set of periodic input signals and observing the corresponding steady-state periodic outputs. For a generic mth order output harmonic, using (2) for each signal results in a linear system of equations whose unknowns are the entries of $\mathbf{H}(m)$. Assuming that it is overdetermined (thanks to the number and spectral content of the input signals) $\mathbf{H}(m)$ can be estimated as the least squares (LS) solution.

The total number of coefficients that allows obtaining the output spectrum from the input, thus defining the behavior of the Ith degree Volterra model is

$$C=\sum_{i=1}^{I}\frac{(R)_i}{i!} \tag{4}$$

where $(\cdot)_i$ denotes the Pochhammer polynomial. It should be noticed that if real-valued input and output signals are assumed, their spectra are inherently Hermitian. Hence, the number of independent coefficients is about half of C.

III. Pruning Volterra Models by Exploiting knowledge About the Input Spectrum

Let us consider a generic Ith degree Volterra system subject to real-valued, periodic input signals; the target is computing the corresponding steady-state response. As aforementioned, the study can be conveniently carried out in the frequency domain. Let us assume that some *a priori* knowledge about the typical spectral content of the input signals is available. In particular, let us suppose knowing that there is a set of spectral components whose magnitudes are likely to be significantly larger with respect to the others. Let us consider the contribution to the output spectrum from the ith degree ($i\geq 2$) homogeneous nonlinear subsystem According to (2), it is a linear combination of all the possible products between sets (with repetitions) of i input spectral components. Of course, products involving only "large" spectral components have higher magnitudes, hence they are likely to highly affect the nonlinear input/output behavior of the system. Products involving also "small" components have weaker magnitudes, with those not including any "large" component having the weakest expected impact on the output. Starting from these considerations, information about the typical spectral characteristics of the input signal can be exploited by neglecting all the output contributions involving more than one "small" component. This allows considerably reducing the complexity of the Volterra representation (namely the number of coefficients defining its behavior) with limited impact on the output, thus optimizing the tradeoff between accuracy and computational cost.

The idea summarized in the previous paragraph has been firstly presented in [45] and it leads to polynomial models that can be considered as the extension of the class of simplified

Volterra models under quasi-sinusoidal conditions [39], [42]. In fact, this refers to the specific case where input contains a fundamental component having a much higher magnitude with respect to its harmonics, as typically happens in ac power systems applications. Under this condition, a further simplification can be introduced if needed: all the output contributions involving the fundamental and at least a harmonic are neglected. The resulting model takes into account the HD produced by the fundamental, while intermodulation is neglected: the other input harmonics are linearly transferred to the output, without interacting with each other, nor with the fundamental. For this reason, we refer to them as HD models: in some cases, they result in satisfactory performance (with respect to a predetermined accuracy target) albeit they are defined by an extremely small set of coefficients [48]. It should be noticed that this simplification cannot be included in the class of polynomial models defined in [45]. Therefore, one of the targets of this paper is further generalizing the approach introduced in [45], so that it is capable of considering also this kind of simplification, thus increasing flexibility.

A. Classification of the input spectral components: large, small and linear subsets

Let us introduce the set of the spectral components that may appear in the periodic input. Since the input signal is assumed to be real-valued, the two-sided input spectrum is made of a constant term (zeroth order harmonic) and pairs of complex conjugate components, thus the set Γ contains zero and couples of opposite harmonic indexes. For achieving the generalization introduced in the previous paragraph, Γ must be split into 3 non-overlapping subsets. The first one includes the spectral components which, according to *a priori* knowledge, are expected to be significantly stronger than the others. Their harmonic indexes are thus the elements of Γ_{L}, which is the subset of the large input (LI) components.

Some of the other input harmonics are likely to interact only with the LI components, but not with each other. They are called small input (SI) components, and their harmonic indexes form the set Γ_{S}. Contributions to the output spectrum due to products involving more than one component belonging to Γ_{S} are neglected.

Conversely, the rest of the input harmonics is supposed not to interact with any other spectral component. Therefore, all the possible output contributions due to products between multiple harmonics that involve at least one of these components are discarded. From another point of view, such components never appear at the input of nonlinear homogeneous subsystems ($i\geq 2$). Since they are linearly transferred to the output, they represent the linear input (LinI) subset; the set of their harmonic indexes are the elements of Γ_{Lin}.

B. Model structure and complexity

Assuming that the LI components represent a small subset of Γ, the previously described simplification is capable of dramatically reducing the number of coefficients that rule the input-output behavior. A new subclass of Volterra systems is thus defined, whose input-output relationship can be written in the form (3), but with much shorter vectors $\mathbf{W}_{\mathrm{P}}(m)$ and $\mathbf{H}_{\mathrm{P}}(m)$ replacing $\mathbf{W}(m)$ and $\mathbf{H}(m)$, respectively. However, a procedure that allows defining the structure of such vectors is required. For this purpose, it is convenient considering (2) while applying the proposed classification of the input components and pruning of the corresponding output contributions. In turn, the resulting output can be split into the sum of 3 terms from different subsystems, as shown in Fig. 1.

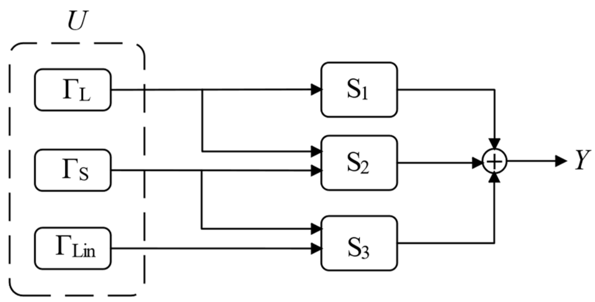

Fig. 1. Block diagram of the proposed class of pruned Volterra systems.

In particular, we have

- A conventional Ith degree Volterra system S_1, whose input is represented by the set of components having indexes belonging to Γ_{L}. It allows considering the pure output contributions of the LI terms.
- A pruned Ith degree Volterra system S_2, whose generic output contribution linearly depends on a component belonging to the SI subset.
- A LTI system S_3, whose input is represented by the union of the SI and LinI subsets.

C. Volterra subsystem S_1

Let us consider the previously introduced decomposition of the proposed class of simplified Volterra models. The first output term comes from a conventional Ith degree Volterra system S_1. Its input consists of the LI components, whose harmonic indexes are the L elements of Γ_{L}. The algorithm firstly proposed in [28] can be employed to define the structure of the ith degree contributions affecting the input-output behavior of S_1; the block diagram reported in Fig. 2 summarizes the procedure, which is also summarized in the following. Let $H^i{}_1(n_1,\ldots,n_i)$ be the generic coefficient of the ith degree GFRF of subsystem S_1. Having considered a triangular GFRF representation, $H^i{}_1(n_1,\ldots,n_i)$ is meaningful only if $n_1\geq n_2\geq\ldots\geq n_i$. This guarantees that, for a given set $\{n_j\}_{j\in\{1,\ldots,i\}}$ of input harmonic indexes, there is a one-to-one relationship with a corresponding GFRF value. Therefore, for a given mth order output harmonic, the structure of the ith degree contribution is fully defined by all the products between sets of i input components (with repetitions) whose sum of their harmonic indexes equals m, which are the elements of vector $\mathbf{W}^i{}_{1,m}$.

Let us represent Γ_{L} as a vector containing the L LI harmonic indexes, sorted in ascending order, with the highest being N_L. A set of i counters $\{k_j\}_{j\in\{1,\ldots,i\}}$ is introduced and initialized to zero (Fig. 2, step a). The state of the counters identifies a set of i

input components, whose generic harmonic index n_j (with $j \in \{1,\ldots,I\}$) is the $(L-k_j)$th element of Γ_L, indicated as $\Gamma_L(L-k_j)$ (Fig. 2, step b). Then, the output harmonic order m affected by this set of components is obtained as the sum of the elements in $\{n_j\}_{j\in\{1,\ldots,i\}}$ (Fig. 2, step c). If the output signal is real-valued, its spectral content is completely defined by nonnegative frequencies. In that case, only if $m \geq 0$ a new element is appended to vector $\mathbf{W}^i_{1,m}$, containing the product between the considered set of i LI components (Fig. 2, step d). A new entry is appended to $\mathbf{H}^i_{1,m}$, which contains the coefficient $H^i_1(n_1,\ldots,n_i)$.

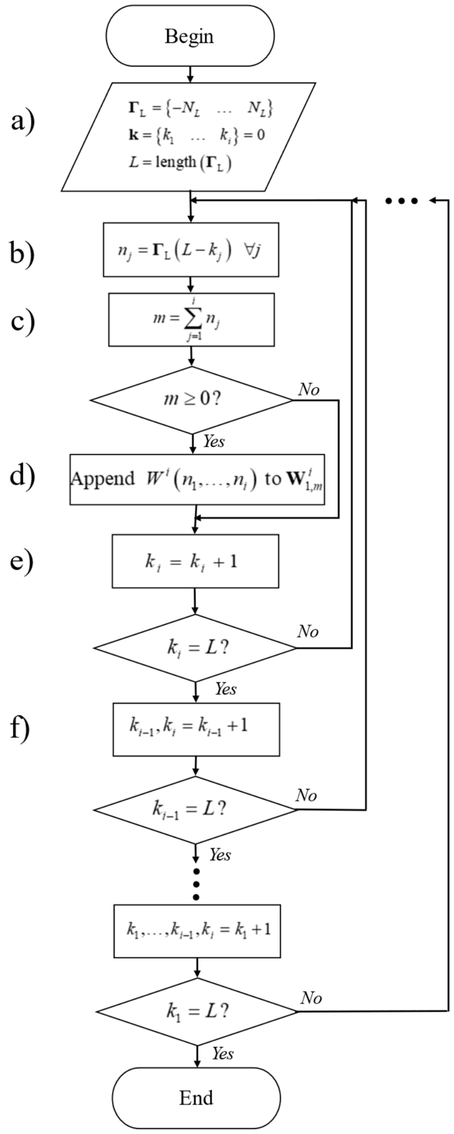

Fig. 2. Flowchart to define the structure of the ith degree output contributions in a conventional Volterra system.

The last counter k_i is then increased by one (Fig. 2, step e); if its value does not exceed $L-1$, the procedure is reiterated from step b. Otherwise, counter k_{i-1} is incremented (Fig. 2, step f) and, if its value is less or equal than $L-1$, the algorithm returns to step b. This task consisting in counter update and comparison with $L-1$ is repeated for all the other counters, down to k_1. The procedure is completed when all the counters equal L.

Having run the algorithm, the structure of the ith degree output contributions from LI components are defined. In order to derive the structures of the contributions from all the nonlinear homogeneous subsystems, the most efficient approach is using the previously presented algorithm to derive those coming from the highest degree, thus setting $i=I$. According to [28], it is straightforward to obtain the structures of the lower-degree contributions (namely those of the vectors $\mathbf{W}^i_{1,m}$ and $\mathbf{H}^i_{1,m}$ with $1 \leq i \leq I-1$) without running again the iterative algorithm. Finally, the total number of coefficients defining subsystem S_1 is obtained using (4), thus

$$C_{S_1} = \sum_{i=1}^{I} \frac{(L)_i}{i!} \tag{5}$$

that has to be roughly halved having assumed real-valued signals.

D. Pruned Volterra subsystem S_2

According to Section III-B, the second output term is provided by a pruned Ith degree Volterra subsystem that takes into account also the S SI components, whose harmonic indexes are the elements of Γ_S. In this respect, the generic ith degree contribution to the mth order output harmonic comes from the product between $i-1$ components belonging to Γ_L and one belonging to Γ_S, whose sum of the harmonic indexes equals m. Therefore, the structure of the ith degree output contributions from S_2 can be obtained by firstly considering all the possible sets of $i-1$ LI components that have been already generated according to the algorithm described in Section III-C. In turn, each set of $i-1$ LI components (with harmonic indexes $\{n_j\}_{j\in\{1,\ldots,i-1\}} \subset \Gamma_L$) is paired with another component having harmonic index $n_s \in \Gamma_S$. A new element is appended both to vector $\mathbf{W}^i_{2,m}$ (with $m=n_1+\ldots+n_{i-1}+n_s$) containing the product between the i involved components, and to vector $\mathbf{H}^i_{2,m}$, which contains the corresponding GFRF value. As before, if signals are real-valued, a contribution is considered only if $m \geq 0$. According to this procedure, that has to be carried out for i ranging from 2 to I, and reminding (4), the number of coefficients defining the behavior of subsystem S_2 (almost halved in case of real-valued signals) is

$$C_{S_2} = \sum_{i=1}^{I-1} \frac{(L)_i}{i!} S. \tag{6}$$

E. Linear subsystem S_3

The third output term is produced by the linear subsystem S_3, whose frequency-domain input is represented by all the components belonging to the SI or LinI subsets. Therefore, for each $m \in (\Gamma_{Lin} \cup \Gamma_S)$, its behavior is defined by a frequency response value $H^1_3(m)$, for a total of $S+Q$ coefficients (where Q is the cardinality of Γ_{Lin}).

Finally, vectors $\mathbf{W}_P$ and $\mathbf{H}_P$ of the pruned Volterra model are obtained by concatenating the contributions of all degrees from the three subsystems S_1, S_2 and S_3. The overall number of coefficients thus results

$$C_P = Q + \sum_{i=1}^{I}\left[\frac{(L)_i}{i!} + \frac{(L)_{i-1}}{i-1!}S\right]. \tag{7}$$

IV. Case Studies and Simulation Results

The target of this section is evaluating through numerical simulations the effectiveness of the proposed pruning technique for frequency-domain Volterra models. In this respect, two Wiener-Hammerstein systems have been selected as case studies, in which a static nonlinearity $h(u_1)$ is sandwiched between two LTI systems characterized by the transfer functions (TFs) $F_1(s)$ and $F_2(s)$, respectively. The generic block diagram is depicted in Fig. 3.

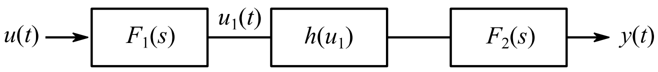

Fig. 3. Block diagram of a Wiener-Hammerstein system.

A. Class of input signals and pruned Volterra models

The reference system is supposed to be fed with real-valued periodic waveforms having 50 Hz fundamental frequency, an average component and harmonics up to the 5th order, namely $\Gamma=\{-5,\ldots,0,\ldots,5\}$. The class of these input signals is statistically defined by independent probability density functions (pdfs) of harmonic amplitudes and phases. The amplitude of the fundamental component is uniformly distributed between 0.8 and 1.2, while that of the 3rd order harmonic has uniform distribution between 0.5 and 0.8. The constant term is uniformly distributed between −0.05 and 0.05 and the 2nd order harmonic has uniform amplitude distribution between 0.02 and 0.05. The 4th and 5th order harmonic magnitudes are characterized by rectangular pdfs between 0.01 and 0.03. Phases have independent uniform probability distributions in $[-\pi, \pi)$.

Let us suppose to roughly know in advance what are the expected amplitudes of the input spectral components. According to Section III, the first step to build the proposed pruned Volterra models is exploiting this knowledge for classifying such components into non-overlapping subsets Γ_L, Γ_S and $\Gamma_{Lin}=\Gamma\backslash(\Gamma_L\cup\Gamma_S)$. However, it is worth highlighting that such *a priori* information about expected amplitudes may not directly translate into a unique composition for the sets Γ_L, Γ_S and Γ_{Lin}, but instead it typically suggests few model configurations that the user may consider, as it will be shown in the following. The best structure among them can be selected *ex post*, comparing the achieved results with the peculiar accuracy target. Moreover, these results may also suggest refining the classification of the input spectral components, leading to further model structures.

Since real-valued inputs are considered, according to the adopted formalism Γ_L, Γ_S and Γ_{Lin} shall include both positive harmonic indexes and their opposites. However, from here on, only nonnegative harmonic indexes will be indicated for the sake of a more compact notation. Considering the previously defined class of input signals, the fundamental component has the highest expected amplitude. Therefore, a first possibility is choosing $\Gamma_L=\{1\}$; $\Gamma_S=\{0,2,\ldots,5\}$, thus corresponding to a quasi-sinusoidal Volterra model [39], [42]. However, also the 3rd order harmonic has a very significant magnitude: it seems reasonable to include it into the large input subset, leading to $\Gamma_L=\{1,3\}$. In turn, different possibilities can be taken into account as far as the small input subset. Since the other components are much smaller with respect to the fundamental and the 3rd order harmonic, a first possibility is to include none of them into Γ_S, which leads to the lowest complexity. The average term and the 2nd order harmonic can be almost twice as large with respect to the 4th and 5th; therefore, another choice is $\Gamma_S=\{0,2\}$. Alternatively, all of the previous components can be included into the small input subset, thus $\Gamma_S=\{0,2,4,5\}$, but resulting in considerably higher complexity. For all these pruned Volterra representations, nonlinearity degrees I=3 and 5 have been considered. Complete 3rd and 5th degree Volterra models and the best linear approximation [1] have been also taken into account as performance benchmarks. The complexities of the different models have been compared in terms of the number of coefficients defining their behaviors, and the result is summarized in Fig. 4.

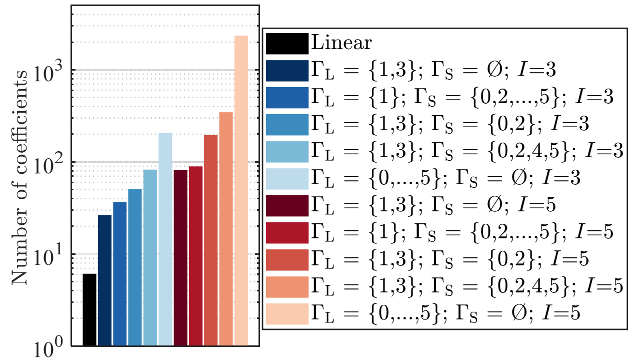

Fig. 4. Number of coefficients defining the considered models.

A conventional 5th degree Volterra model ($\Gamma_L=\{0,\ldots,5\}$; $\Gamma_S=\varnothing$; I=5) would surely enable the best representation of the reference system, but as from Fig. 4, it requires more than 2300 coefficients. The most detailed among the considered pruned representations ($\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2,4,5\}$; I=5) is defined by 340 coefficients, slightly higher than those of a 3rd degree complete Volterra model ($\Gamma_L=\{0,\ldots,5\}$; $\Gamma_S=\varnothing$; I=3). Reducing the small-signal subset to $\Gamma_S=\{0,2\}$, the number of coefficients is almost halved. Considering $\Gamma_L=\{1,3\}$ but no small input components leads to just 88 coefficients, very close to that of a 5th degree quasi sinusoidal Volterra model. As far as the 3rd degree models are considered, a similar complexity reduction can be observed. It should be noticed that, when comparing (4) to (7), it becomes evident that the reduction in the number of coefficients achievable with the proposed pruning approach is as effective as the LI components are a small subset of the overall input harmonics. Even stronger complexity reduction occurs if the maximum input harmonic index is larger than 5 (as often happens in practical applications) and for a nonlinearity degree I above 5.

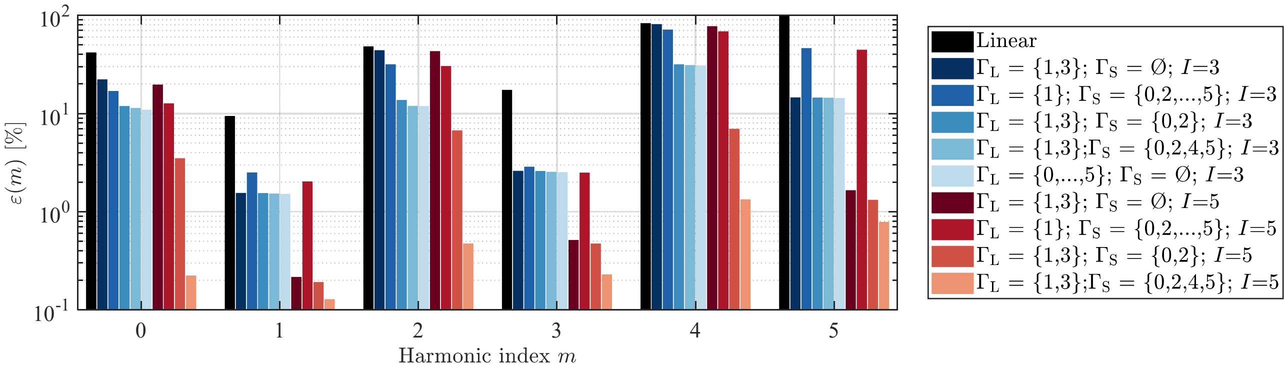

Fig. 5. Error ε for the considered pruned Volterra models at the different harmonics (reference system with polynomial nonlinearity).

B. Identification and validation procedure

For a given reference system, the first step is identifying the coefficients characterizing the different behavioral models. The considered Wiener-Hammerstein system is thus fed with P=1000 input signals belonging to the previously defined class, evaluating the corresponding steady-state output harmonics. The input-output relationship of all the considered models can be expressed in the form (3), namely the collected data enables writing a linear system of P equations for each output component. Since P is always considerably greater than the maximum length of the vector of the unknowns, given the characteristics of the class of the injected signals, these systems of equations are overdetermined. Therefore, the LS estimate of the model coefficients can be obtained thanks to the Moore-Penrose inverse.

Once having estimated the coefficients of all the considered models, (3) enables reconstructing each output harmonic for the whole set of P input signals. In order to evaluate the accuracy of each model, predicted output harmonics have been compared with the actual output of the reference system. For each model and harmonic, the following error $\varepsilon(m)$ has been evaluated:

$$\varepsilon(m) = \frac{\left\| \mathbf{Y}_e(m) - \mathbf{Y}(m) \right\|}{\left\| \mathbf{Y}(m) \right\|}. \tag{8}$$

$\mathbf{Y}_e(m)$ and $\mathbf{Y}(m)$ represent the vectors concatenating the predicted and the actual mth order output harmonic for all the input signals, while $\|\cdot\|$ denotes the Euclidean norm.

C. Wiener-Hammerstein System with polynomial nonlinearity

In the first test case, the Wiener-Hammerstein system is defined as follows. $F_1(s)$ corresponds to a second-order Bessel low-pass filter having 150 Hz cutoff frequency; $F_2(s)$ is the TF of a second-order, Chebyshev low-pass filter characterized by 3 dB bandpass ripple and 350 Hz cutoff frequency. The static nonlinearity is represented by a 5th degree polynomial function $h(u_1)$, written in the form

$$h(u_1) = \sum_{i=0}^{I} \gamma_i u_1^i. \tag{9}$$

The values of the coefficients γ_i are reported in TABLE I.

TABLE I. COEFFICIENTS OF THE 5TH DEGREE POLYNOMIAL FUNCTION

γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
0	1	-0.04	0.05	0.01	0.03

Having defined the reference system, the next step is representing its frequency-domain behavior by using the pruned Volterra models that can be defined on the basis of the proposed approach, considering the expected amplitude of the input components. It is worth highlighting that Wiener-Hammerstein systems having Ith degree polynomial nonlinearity are a subclass of the Ith degree Volterra systems. Therefore, the different behaviors between the reference system and the identified 5th degree pruned Volterra models are purely due to undermodeling (or definitional uncertainty) produced by the neglected interactions between input harmonics. In the following, each Ith degree simplified Volterra model will be denoted with the subsets of large and small components (Γ_L and Γ_S). Harmonics present in the input which are not elements of Γ_L or Γ_S belong to Γ_{Lin}.

The considered behavioral models have been identified and validated according to the procedure described in Section IV-B. For each harmonic, the errors $\varepsilon(m)$ defined as in (8) are reported in Fig. 5. According to the previous considerations, the employment of a full 5th order Volterra model (not shown) would have resulted in theoretically zero error.

Results highlight that the reference model is characterized by a highly nonlinear behavior: in fact, the accuracy achieved by a linear model is poor, with ε above 80 % at the 4th and 5th order harmonics. Errors are significantly lower at the largest injected spectral components, namely the fundamental and the 3rd order harmonic: nonlinear output contributions have a smaller relative impact in this case. As expected, 5th degree models have significantly better performances than the 3rd degree ones. Even more interesting, the accuracy of the full 3rd degree Volterra model is very close to the 3rd degree pruned models Γ_L={1,3}; Γ_S={0,2,4,5} and Γ_L={1,3}; Γ_S={0,2}, requiring just 81 and 50 coefficients, respectively. Therefore, under these conditions, Γ_L={1,3}; Γ_S={0,2} is the most effective 3rd degree pruned Volterra model. The 5th degree quasi-sinusoidal Volterra representation ($\Gamma_L=\{1\}$; $\Gamma_S=\{0,2\ldots5\}$) does not achieve very good performance, especially when compared to models including the 3rd order harmonic in the large input subset. As expected, the best accuracy is achieved by model $\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2,4,5\}$; $I=5$, which is also the most complex pruned Volterra model, albeit it has 1/7 of the coefficients of a full Volterra system with the same degree. Very good results can be achieved also by model $\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2\}$; $I=5$, which is noticeably worse only at even-order components.

In order to assess the overall accuracy achieved by the different models in predicting the output waveforms, the normalized root mean square error (NRMSE) has been computed for each pth excitation signal, $p\in\{1,\ldots,P\}$. It is defined as

$$\mathrm{NRMSE}^{[p]}=\sqrt{\frac{\int_T\left[y_e^{[p]}(t)-y^{[p]}(t)\right]^2dt}{\int_T\left[y^{[p]}(t)\right]^2dt}} \tag{10}$$

where T is the period, $y^{[p]}(t)$ is the time-domain output waveform of the reference system corresponding to the pth input signal, while $y_e^{[p]}(t)$ is that predicted by one of the considered models. Since we are under the assumptions of the sampling theorem, the corresponding frequency-domain expression has been employed for a more effective calculation. The average values computed over the P signals (indicated as NRMSE_{avg}) are reported in Fig. 6.

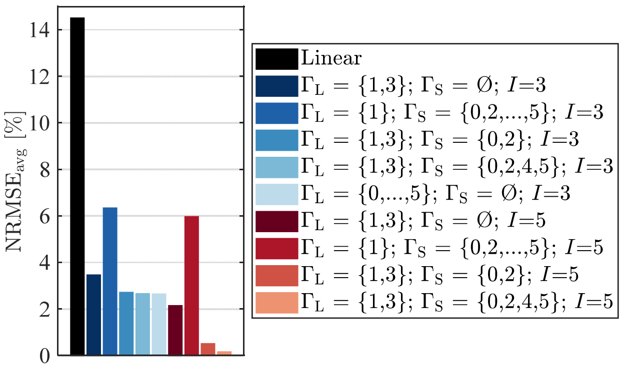

Fig. 6. NRMSE_{avg} for the different pruned Volterra models (reference system with polynomial nonlinearity).

As expected, the linear model provides a rough approximation of the output waveform, with NRMSE_{avg} reaching 14 %. Poor performance is also achieved by quasi-sinusoidal models ($\Gamma_L=\{1\}$; $\Gamma_S=\{0,2\ldots5\}$) with similar values of about 6 % for both the 3rd and 5th degree. This clearly suggests that at least the interaction between the fundamental and the third order harmonic plays a major role. All the other representations are characterized by much better performance, but the best overall accuracies are reached by models $\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2\}$ and $\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2,4,5\}$ with $I=5$. NRMSE_{avg} values are about 0.5 % and 0.16 %, respectively, namely far better with respect to all the others. In particular, the structure $\Gamma_L=\{1,3\}$; $\Gamma_S=\{0,2\}$; $I=5$ achieves a very favorable tradeoff between complexity and accuracy.

D. Wiener-Hammerstein Benchmark

The second test case is based on the benchmark for comparing black-box nonlinear system identification methods presented in [49]. The reference system is a Wiener-Hammerstein model, with $F_1(s)$ being the TF of a third order lowpass Chebyshev filter with 0.5 dB passband ripple and 4.4 kHz cutoff frequency. $F_2(s)$ corresponds to a third order inverse Chebyshev filter with 40 dB stopband attenuation at 5 kHz. The static nonlinearity is due to a voltage divider made of a 1 kΩ resistor and a 10 kΩ resistor that is parallel connected to a diode. Its current-voltage characteristic has been represented with the Shockley equation, whose parameters have been extracted from the 1N4148 datasheet. The key difference with respect to the previous test case is that now the reference system no longer belongs to the class of truncated Volterra systems. Therefore, an Ith degree Volterra representation inherently result in undermodeling; for this very reason, the full 5th degree Volterra system is included in the comparison.

LS identification of the behavioral models and performance assessment have been carried out according to the procedure defined in Section IV-A. After that, the spectral components of the P input-output identification waveforms have been corrupted by independent additive noise with circularly symmetric complex normal distribution, characterized by the standard deviation σ of the real and imaginary parts. Two noise levels have been considered, with σ corresponding to 0.5 % and 1 % of the average rms value of the signals. For quantifying performance degradation, the identification procedure and the corresponding validation (the latter performed under noiseless condition) have been repeated 100 times for each noise level, evaluating the rms value of the error $\varepsilon(m)$. Obtained values are reported in Fig. 7: solid bars refer to noiseless identification, whiskers show the increase of $\varepsilon(m)$ for the two noise levels.

The nonlinear behavior of the reference system is rather strong, since linear modeling leads to huge errors. This is especially true for the output components corresponding to a weaker expected input magnitude, where nonlinear contributions have stronger impact in relative terms. In fact, while error at fundamental and 3rd order harmonics is below 8 %, it exceeds 90 % as far as the other components. Noise has a negligible impact in this case.

When moving to a frequency-domain Volterra model, the accuracy improvement is remarkable, with errors mostly below 10 % for all the output harmonics. However, they highly differ both in terms of performance, complexity and robustness to noise. It is not surprising that, under noiseless conditions, the best accuracy is reached by the full 5th degree Volterra model, defined by the largest set of coefficients. Error almost reaches 10 % at the 5th order harmonic, while being much smaller for the other components (around 2 % or significantly better). On the other hand, the accuracy of such model exhibits the highest noise sensitivity. The reason lies in the large number of weak output contributions, which are prone to be corrupted by noise during identification.

Considering the 3rd degree models, accuracy is noticeably improved when moving from a quasi-sinusoidal model

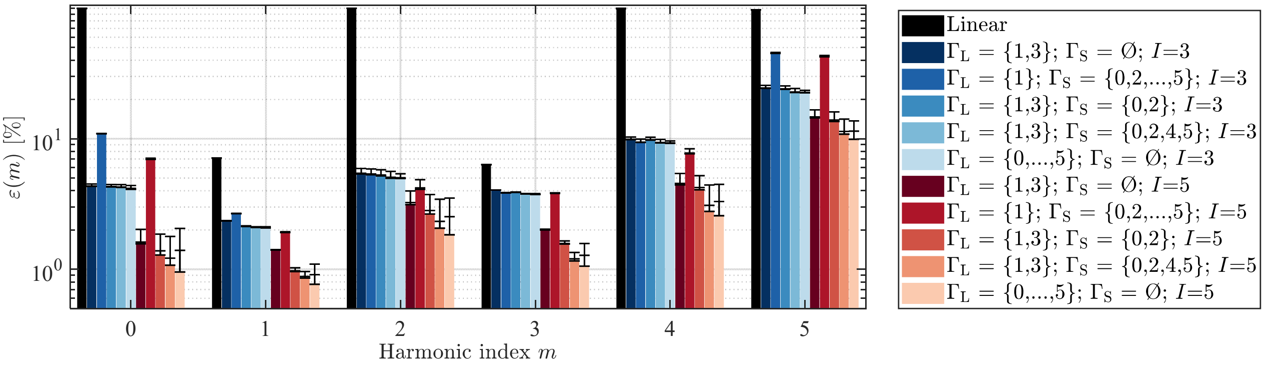

Fig. 7. Error ε for the considered pruned Volterra models at the different harmonics; whiskers show performance degradation corresponding to the two noise levels (Wiener-Hammerstein benchmark).

(Γ_L={1}; Γ_S={0,2,…,5}), to those including at least the 3rd order harmonic in the large signal subset (the simplest being Γ_L={1,3}; Γ_S= $\varnothing$). In this respect, performance is very similar, albeit their number of coefficients varies by almost a factor 8. On the one hand, these numbers show how the proposed pruning approach enables reducing model complexity with weak impact on performance. On the other hand, this means that in this case the bottleneck is having neglected higher-degree nonlinear contributions: better performance is viable only by increasing I. Impact of noise is rather limited for all the 3rd degree models.

Considering I=5 under noiseless conditions, Fig. 7 suggests that the best balance between accuracy and complexity is obtained with the model defined by Γ_L={1,3}; Γ_S={0,2}, achieving almost the same performance as the full Volterra model, but defined by less than 200 coefficient instead of more than 2300. Further expanding either the large signal or the small signal subset results in limited improvement. Moreover, whiskers highlight that the additional output contributions greatly boost noise sensitivity. In fact, it may happen that a pruned 5th degree model achieves better performance than a full Volterra model when identification is carried out in the presence of significant disturbances.

Therefore, the proposed pruning technique allows considering in a simple way the most significant output contributions, which are also those characterized by the highest signal to noise ratio during identification. This enables adopting high nonlinear degrees without stepping into huge complexity, which may result not just in unnecessarily high computational burden, but also in degraded performance due to overparameterization and poor robustness to noisy identification data.

Finally, Fig. 8 compares a typical output waveform of the reference system and three corresponding reconstructions obtained with different behavioral models: linear, a pruned Volterra model (Γ_L={1,3}; Γ_S={0,2}; I=5) and a full 5th degree Volterra model. As expected, the linear model achieves a very poor output prediction, while the waveforms obtained with both the full and the pruned Volterra models are hardly distinguishable from the actual output.

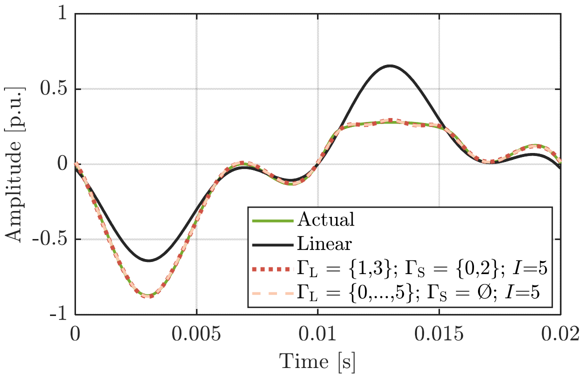

Fig. 8. Time-domain output signal and their reconstructions obtained with a linear model, a pruned Volterra (Γ_L={1,3}; Γ_S={0,2}) and full Volterra model with I=5 (Wiener-Hammerstein benchmark).

V. Experimental Validation

A. Experimental setup

The proposed pruning technique has been experimentally applied to represent the behavior of a Wiener-Hammerstein system subject to periodic input signals belonging to the class previously defined in Section IV-A.

In this case, the input LTI system is a 2nd order Butterworth lowpass filter having 250 Hz cutoff frequency. Its output voltage feeds a static nonlinearity, whose structure resemble that adopted in the Wiener-Hammerstein benchmark [49]. It consists of a voltage divider, made of a 1 kΩ resistor and a 10 kΩ resistor that is parallel connected to an IR 11DQ05 Schottky diode. In turn, the voltage across the diode represents the input of another 2nd order Butterworth lowpass filter with 200 Hz cutoff frequency. Generation of the input signal and acquisition of the resulting output have been provided by a NI USB–6356 data acquisition board. For each signal injection, M=100 steady-state periods of the corresponding output have been acquired with 100 kHz rate, namely 2000 samples per period. Spectral components have been extracted by computing the DFT on each observed period of the output, then averaged over the M periods to minimize the impact of measurement noise.

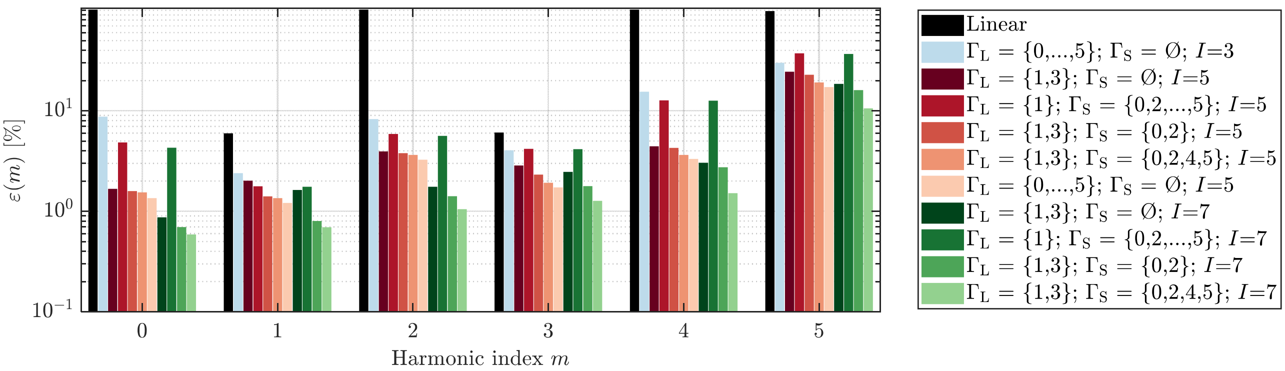

Fig. 9. Error ε for the considered pruned Volterra models at the different harmonics (experimental results).

B. Experimental results

The experimental setup described in Section V-A has been employed to measure the steady-state output spectra of the Wiener-Hammerstein system subject to P=1000 input voltages belonging to the class introduced in Section IV-A. Using these data, the full and pruned frequency-domain Volterra models introduced in Section IV-A have been identified and validated, following the procedure of Section IV-B. The adopted Schottky diode has a very steep *v-i* characteristic, resulting in significant high-order nonlinear contributions and thus challenging conditions. For this reason, four pruned 7th degree Volterra models were added to the comparison, defined by the reasonable classifications of the input spectral components (on the basis of their expected magnitudes) previously discussed in Section IV-A. Therefore, we have the 7th degree Volterra models Γ_L={1,3}; Γ_S=∅ (97 coefficients), Γ_L={1}; Γ_S={0,2,…,5} (121 coefficients), Γ_L={1,3}; Γ_S={0,2} (299 coefficients) and Γ_L={1,3}; Γ_S={0,2,4,5} (525 coefficients). A 7th degree Volterra model would have resulted in huge complexity, hence it has not been considered.

Figure 9 shows the obtained results in terms of error $\varepsilon(m)$ defined by (8). As happened in the previous case studies, the quasi-sinusoidal pruned models (characterized by Γ_L={1}) cannot reach good accuracy, while errors decrease significantly as the third order harmonic is added to the large subset. This means that the interaction between fundamental and 3rd order harmonic produces significant output contributions; neglecting them results in a severe performance cap.

It is worth noting that because of the abrupt nonlinearity of the reference system, all the 3rd degree models perform similarly, but they achieve rather poor accuracy. For this reason, we report only the results obtained with a full 3rd Volterra model. Moving to I=5, performance improves noticeably, while it is possible to see that a full model performs just marginally better than that pruned with the proposed approach having Γ_L={1,3}; Γ_S={0,2,4,5}, in spite of the significant difference in complexity (7 times more coefficients, see Fig. 4). This, once again, confirms the effectiveness and flexibility of the pruning technique.

When considering I = 7, the pruned model defined by the sets Γ_L={1,3}; Γ_S= ∅ reaches a slightly worse results than the full 5th degree model, but with just 97 coefficients (i.e., about 24 times less). Moreover, the pruned Volterra model characterized by Γ_L={1,3} and Γ_S={0,2,4,5} is defined by 525 coefficients, but it performs significantly better than the more complex full 5th degree model. In particular, error values are more than halved, except for m = 3.

Finally, the overall accuracy achieved by the different models in predicting the output signals has been evaluated in terms of NRMSE, defined as in (10) but computed in the frequency domain. The average values over the P test signals have been obtained and results are reported in Fig. 10.

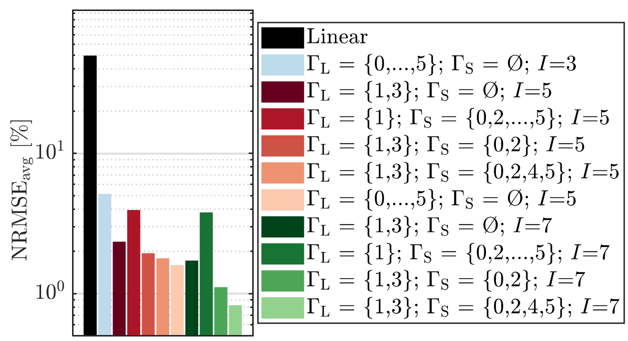

Fig. 10. $\mathrm{NRMSE}_{\mathrm{avg}}$ for the different pruned Volterra models (experimental results).

Also in this case, for the previously explained reasons, the performance achieved by the pruned 3rd Volterra models are not shown. These results further highlight the effectiveness of the proposed pruning approach. A full 5th degree Volterra model results in 1.58 % $\mathrm{NRMSE}_{\mathrm{avg}}$, just slightly lower than that obtained with a pruned 5th degree Volterra model with Γ_L={1,3}; Γ_S={0,2,4,5} or a pruned 7th degree Volterra model defined by Γ_L={1,3}; Γ_S=∅, having a much simpler structure. The best overall performance (0.81 % $\mathrm{NRMSE}_{\mathrm{avg}}$) is obtained adopting a pruned 7th degree model with Γ_L={1,3}; Γ_S={0,2,4,5}, which in this case represents an excellent tradeoff between accuracy and complexity.

VI. Conclusion

This work proposes a method that allows exploiting the knowledge about the presence of prevailing components in the input spectrum for substantially reducing the number of coefficients defining a frequency-domain Volterra model, with minor impact on accuracy. According to the presented approach, input spectral components are classified into three subsets: *large*, *small* and *linear*. Small components interact only with large components, while linear components are linearly transferred to the output. The classification is qualitatively driven by the expected magnitudes of the input components, which may suggest alternative configurations. Starting from them, the user can tune the balance between accuracy, complexity and noise robustness, according to the peculiar requirements and conditions. Once the input components have been classified, a simple algorithm enables obtaining the structure of the pruned model.

The performance that can be achieved by adopting the proposed pruning approach has been firstly evaluated through numerical simulation by means of two case studies, also in the presence of noise during identification. Then, the effectiveness of the pruning technique has been validated experimentally. Both simulation and experimental results highlight the flexibility of the approach, which enables obtaining pruned models suiting specific needs in terms of accuracy and complexity. This allows one to adopt high nonlinearity degrees that, with the conventional Volterra approach, would result in unmanageable complexity and troublesome identification. Moreover, neglecting weak output contributions could significantly enhance robustness to noise that may affect identification data, thus enabling better overall performance under these conditions.

References

[1] R. Pintelon and J. Schoukens, “System Identification. A frequency domain approach”, 2nd ed., Boston, MA, USA, Wiley, 2012.

[2] J. Schoukens, T. Dobrowiecki and R. Pintelon, “Parametric and nonparametric identification of linear system in the presence of nonlinear distortion – A frequency domain approach”, *IEEE Trans. Autom. Contr.*, vol. 43, no. 2, pp. 176-190, February 1998.

[3] J. Schoukens, R. Pintelon, T. Dobrowiecki and Y. Rolain, “Identification of linear systems with nonlinear distortion”, *Automatica,* vol. 41*,* pp.491-504, 2005.

[4] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "Characterization of voltage instrument transformers under nonsinusoidal conditions based on the best linear approximation," *IEEE Trans. Instrum. Meas.*, vol. 67, no. 10, pp. 2392-2400, Oct. 2018.

[5] F. Giri and E.W. Bai, “Block-Oriented Nonlinear System Identification,” Lecture Notes in Control and Information Sciences, London, U.K.: Springer-Verlag, 2010.

[6] O. Nelles, “Nonlinear system identification: from classical approaches to neural networks and fuzzy models”, Springer Science & Business Media, 2013.

[7] A. E. Abdelrahman, O. Hammi, A. K. Kwan, A. Zerguine and F. M. Ghannouchi, "A novel weighted memory polynomial for behavioral modeling and digital predistortion of nonlinear wireless transmitters," *IEEE Trans. Ind. Electron.*, vol. 63, no. 3, pp. 1745-1753, March 2016

[8] A. I. Dalbah, O. Hammi and A. Zerguine, "Hybrid look-up-tables based behavioral model for dynamic nonlinear power amplifiers," *IEEE Access*, vol. 8, pp. 53240-53249, 2020.

[9] Chun Che Fung, Kok Wai Wong and H. Eren, "Modular artificial neural network for prediction of petrophysical properties from well log data," *IEEE Trans. Instrum. Meas.*, vol. 46, no. 6, pp. 1295-1299, Dec. 1997.

[10] X. Hu *et al*., "Behavioral model with multiple states based on deep neural network for power amplifiers," *IEEE Microw. Wireless Compon. Letters*, vol. 32, no. 11, pp. 1363-1366, Nov. 2022

[11] H. Han, X. Wu and J. Qiao, "Nonlinear systems modeling based on self-organizing fuzzy-neural-network with adaptive computation algorithm," *IEEE Trans. Cybern.*, vol. 44, no. 4, pp. 554-564, April 2014.

[12] C. Peréz-Wences, J. R. Loo-Yau, P. Moreno and J. A. Reynoso-Hernández, "Digital predistortion of RF power amplifier using the NARX-SVR model," *IEEE Microw. Wireless Technol. Letters*, vol. 33, no. 4, pp. 475-478, April 2023.

[13] M. Schetzen, “The Volterra and Wiener Theories of Nonlinear Systems,” New York: Wiley, 2006.

[14] W. J. Rugh, “Nonlinear System Theory - The Volterra/Wiener Approach”, London: The Johns Hopkins Press Ltd., 1981.

[15] N. Bjorsell, P. Suchanek, P. Handel and D. Ronnow, "Measuring Volterra kernels of analog-to-digital converters using a stepped three-tone scan," *IEEE Trans. Instrum. Meas.*, vol. 57, no. 4, pp. 666-671, April 2008.

[16] K. Zhou, J. Qu, J. Zhao, X. Xu and Y. Shi, "Behavior modeling of the amplifier nonlinearity with the AC Josephson voltage standard," *IEEE Trans. Instrum. Meas.*, vol. 68, no. 6, pp. 1990-1995, June 2019.

[17] R. D. Fard, M. Karrari and O. P. Malik, "Synchronous generator model identification for control application using Volterra series," *IEEE Trans. Energy Convers.*, vol. 20, no. 4, pp. 852-858, Dec. 2005.

[18] V. Tartaglione, C. Farges and J. Sabatier, "Volterra equations for fractional behaviors modeling with application to lithium-ion cells," 2022 *American Control Conf.*, 2022, pp. 4107-4112.

[19] G. D. Mitsis, M. G. Markakis and V. Z. Marmarelis, "Nonlinear modeling of the dynamic effects of infused insulin on glucose: Comparison of compartmental with Volterra models," *IEEE Trans. Biomed. Eng.*, vol. 56, no. 10, pp. 2347-2358, Oct. 2009.

[20] G. D. Mitsis, "The Volterra-Wiener approach in neuronal modeling, " *Annu. Int. Conf. of the IEEE Engineering in Medicine and Biology Society*, 2011, pp. 5912-5915.

[21] Vasilis Z. Marmarelis, "Nonlinear Dynamic Modeling of Physiological Systems," IEEE Press Series on Biomedical Engineering, New York: Wiley-IEEE Press, 2004.

[22] D. R. Morgan, Z. Ma, J. Kim, M. G. Zierdt and J. Pastalan, "A generalized memory polynomial model for digital predistortion of RF power amplifiers," *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3852-3860, Oct. 2006.

[23] A. Carini, S. Cecchi, A. Terenzi and S. Orcioni, "A room impulse response measurement method robust towards nonlinearities based on orthogonal periodic sequences," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 29, pp. 3104-3117, 2021.

[24] A. Noll, C. D. Curiac, B. Gülecyüz and E. Steinbach, "Adaptive equalization of vibrotactile actuators," *IEEE Trans. Haptics*, vol. 14, no. 2, pp. 371-383, April-June 2021.

[25] L. O. Chua and C.-Y. Ng, “Frequency domain analysis of nonlinear systems: General theory,” *IEE J. Electron. Circuits Syst*., vol. 3, no. 4, pp. 165–185, Jul. 1979.

[26] H. L. Wei, Z. Q. Lang and S. A. Billings, "An algorithm for determining the output frequency range of Volterra models with multiple inputs," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 54, no. 6, pp. 532-536, June 2007.

[27] X. Wu, Z. Q. Lang and S. A. Billings, "Analysis of output frequencies of nonlinear systems," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3239-3246, July 2007.

[28] L. Cristaldi, M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "Iterative method for the definition of frequency-domain Volterra models," in *Proc. IEEE Int. Instrumentation and Measurement Technology Conf.*, 2017, pp. 1-6.

[29] S. W. Nam and E. J. Powers, “Application of higher order spectral analysis to cubically nonlinear system identification,” *IEEE Trans. Signal Processing*, vol. 42, no. 7, July 1994.

[30] C.-H. Tseng, “Identification of cubically nonlinear systems using undersampled data,” *IEE Proc.-Vision, Image and Signal Processing*, vol. 144, no. 5, pp. 267-277, Oct. 1997.

[31] M. Alizadeh, S. Amin and D. Rönnow, "Measurement and analysis of frequency-domain Volterra kernels of nonlinear dynamic 3×3 MIMO systems," *IEEE Trans. Instrum. Meas.*, vol. 66, no. 7, pp. 1893-1905, July 2017.

[32] S. A. Billings and S. Y. Fakhouri, "Identification of a class of nonlinear systems using correlation analysis," in *Proc. IEE*, vol. 125, no. 7, pp. 691-697, July 1978.

[33] C. Crespo-Cadenas, J. Reina-Tosina, M. J. Madero-Ayora and J. Munoz-Cruzado, "A new approach to pruning Volterra models for power amplifiers," *IEEE Trans. Signal Process*., vol. 58, no. 4, pp. 2113-2120, April 2010.

[34] A. Zhu and T. J. Brazil, "Behavioral modeling of RF power amplifiers based on pruned Volterra series," *IEEE Trans. Microw. Wireless Compon. Lett*., vol. 14, no. 12, pp. 563-565, Dec. 2004.

[35] A. Zhu, J. C. Pedro and T. R. Cunha, "Pruning the Volterra series for behavioral modeling of power amplifiers using physical knowledge," *IEEE Trans. Microw. Theory. Tech.*, vol. 55, no. 5, pp. 813-821, May 2007.

[36] J. A. Becerra-González, M. J. Madero-Ayora, J. Reina-Tosina and C. Crespo-Cadenas, "Digital predistortion of power amplifiers using structured compressed-sensing Volterra series," *Electronics Letter*, vol. 53, no. 2, pp. 89-91, 2017.

[37] J. A. Becerra, M. J. Madero-Ayora and C. Crespo-Cadenas, "Comparative analysis of greedy pursuits for the order reduction of wideband digital predistorters," *IEEE Trans. Microw. Theory Tech.*, vol. 67, no. 9, pp. 3575-3585, Sept. 2019,

[38] V. Kekatos and G. B. Giannakis, "Sparse Volterra and polynomial regression models: Recoverability and estimation," *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 5907-5920, Dec. 2011.

[39] O. A. Golovan and A. S. Korotkov, "Nonlinear analysis of the diode frequency mixers using Volterra series: Generalized matrix approach in the frequency domain," *Int. Conf. on Electrical Engineering and Photonics*, 2023, pp. 30-33.

[40] D. Mirri, G. Iuculano, F. Filicori, G. Pasini, G. Vannini, and G. Gabriella, “A modified Volterra series approach for nonlinear dynamic systems modeling,” *IEEE Trans. Circuits Syst. I*, vol. 49, no. 8, pp. 1118–1128, Aug. 2002.

[41] M. Faifer, R. Ottoboni, M. Prioli and S. Toscani, "Simplified modeling and identification of nonlinear systems under quasi-sinusoidal conditions," *IEEE Trans. Instrum. Meas*., vol. 65, no. 6, pp. 1508- 1515, June 2016.

[42] M. Faifer, C. Laurano, R. Ottoboni, M. Prioli, S. Toscani and M. Zanoni, "Definition of simplified frequency-domain Volterra models with quasi-sinusoidal input," *IEEE Trans. Circuits Syst. I*, *Reg. Papers*, vol. 65, no. 5, pp. 1652-1663, May 2018.

[43] M. Faifer *et al*., "Overcoming frequency response measurements of voltage transformers: An approach based on quasi-sinusoidal Volterra models," *IEEE Trans. Instrum. Meas.*, vol. 68, no. 8, pp. 2800-2807, Aug. 2019.

[44] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "Behavioral representation of a bridge rectifier using simplified Volterra models," *IEEE Trans. Instrum. Meas.*, vol. 68, no. 5, pp. 1611-1618, May 2019.

[45] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "A pruning technique for Volterra models: Exploiting knowledge about input spectrum," *IEEE Int. Instrumentation and Measurement Technology Conf.,* 2020, pp. 1-6.

[46] D. E Root, J. Verspecht, J. Horn and M Marcu, “X-Parameters: Characterization, Modeling, and Design of Nonlinear RF and Microwave Components”, Cambridge University Press, 2013.

[47] J. Verspecht and D. E. Root, "Polyharmonic distortion modeling," *IEEE Microwave*, vol. 7, no. 3, pp. 44-57, June 2006.

[48] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, “Harmonic distortion compensation in voltage transformers for improved power quality measurements,” *IEEE Trans. Instrum. Meas.*, vol. 68, no. 10, pp. 3823-3830, Oct.2019.

[49] J. Schoukens, J. Suykens, L. Ljung, "Wiener-Hammerstein Benchmark," *15th IFAC Symposium on System Identification*, 2009.