Accurate Measurement of Optical Resonator Finesse

Gianluca Galzerano, Edoardo Suerra, Dario Giannotti, Francesco Canella, Edoardo Vicentini, and Simone Cialdi

***Abstract*— We report on a novel and general measurement technique to characterize the finesse of optical resonators. The method retrieves the cavity transmission profile coefficient by using an amplitude modulation of the probe beam and a synchronous detection scheme. Both the real and imaginary parts of the transmission profile can be accurately measured. Validation of the method is performed using two different cavity configurations, a linear and a ring resonator, probed by both continuous-wave and mode-locked laser sources. A comparison with respect to conventional cavity-ring-down measurement scheme shows the accuracy of the proposed method.**

***Index Terms*— Optical measurements, optical resonators, resonator finesse and quality factor, synchronous detection schemes.**

I. Introduction

OPTICAL resonators, due to their fundamental properties, are widely exploited in a large variety of scientific fields, ranging from quantum electrodynamics [1], [2] to gravitational-wave detection [3], [4]. For example, in optical clock and frequency metrology applications, stable and ultranarrow reference cavities are currently employed to reduce the emission linewidth of the probing lasers interacting with narrow atomic/molecular transitions [5], [6]. In high-sensitivity laser spectroscopy, optical resonators are used to enhance the interaction path length between the intracavity radiation and the absorbing sample [7]. Nonlinear optics takes advantage of the enhancement of circulating power inside an optical resonator to increase the nonlinear effects, such as the generation of high-order harmonics [8]. Moreover, the extreme sensitivity to phase fluctuations and intracavity losses of optical resonators is used in the optical sensor technology based on microresonators and Fabry–Pérot fiber resonators [9], [10]. The whole characterization of optical resonators is, therefore, a fundamental step to fully understand the potentials of these very versatile optical tools.

In this article, we report on a new and straightforward method to measure the finesse of an optical resonator, both in the linear and ring configurations, by detecting the response of the cavity transmission to an amplitude-modulated input electric field. In particular, the method retrieves both the amplitude and the phase of the cavity transmission by means of a synchronous detection of the cavity transmission at the modulation frequency when the optical carrier is kept in resonance with the cavity. Due to the properties of the synchronous detection scheme, the technical noise scales with the demodulation bandwidth of the adopted lock-in amplifier [11], increasing significantly the signal-to-noise ratio compared with direct measurement schemes such as [12] and [13]. When compared with the most used method of the cavity-ring-down (CRD) [14], which is insensitive to both laser amplitude and frequency noises [15], the proposed technique, although is not completely immune to the noise, allows evaluating not only the cavity finesse but also other parameters such as cavity resonance phase dispersion (imaginary part of the transmission coefficient) and transmission line profile (real part of the transmission coefficient). The direct measurement of the cavity resonance dispersion and transmission profiles finds interesting applications also in cavity-enhanced spectroscopy of gas samples with narrow resonance lines [16]. The proposed method is experimentally validated using two different cavity configurations with the common characteristic of transmission linewidth narrower than 50 kHz (i.e., comparable to the linewidths of the laser sources used to probe the resonators): a linear cavity with a nominal finesse of $\sim$75 000 and a free spectral range (FSR) of 1.5 GHz (corresponding to a linewidth of $\sim$15 kHz) and a ring cavity with a lower finesse of $\sim$2000 and an FSR of 100 MHz (corresponding to a linewidth of $\sim$50 kHz). The measured values of the resonator finesse are 66 137 $\pm$ 700 and 1903 $\pm$ 54 for the linear and ring cavity resonators, respectively. Compared to the values measured with the conventional CRD characterization method [14], 64 834 $\pm$ 900 and 1987 $\pm$ 89, respectively, the measurements, assumed uncorrelated, are in good agreement and result compatible within $2-\sigma$ intervals.

II. Measurement Method

The cavity finesse, defined as the ratio between the frequency spacing of the fundamental (longitudinal) resonator modes (called the cavity FSR) and their full-width at half-maximum (FWHM) bandwidth, is the main performance parameter of an optical resonator; it is related only to the cavity mirror reflectivity (and losses) and intracavity losses using the following relation [17]:

$$F = \frac{\mathrm{FSR}}{\Delta\nu} = \frac{\pi\sqrt[4]{R}}{1-\sqrt{R}} \tag{1}$$

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where in the case of linear and ring resonators we have, respectively, $\mathrm{FSR}_{\mathrm{FP}} = c/(2L)$ and $\mathrm{FSR}_{\mathrm{ring}} = c/L$ with L the cavity length and c the speed of light in vacuum. $\Delta\nu$ is the cavity FWHM linewidth and R stands for the product of the reflectivity of all cavity mirrors ($R = \Pi_i R_i$). Intracavity losses can be considered by using an equivalent value of R, such as $R_{\mathrm{eq}} = R(1-\alpha)$, where $\alpha \ll 1$ are the round-trip linear losses. One simple method to measure the cavity finesse is the recording of the cavity transmission intensity profile by scanning the frequency of the probe laser around the resonance condition and measuring the FWHM linewidth of the resulting spectral profile, which represents the spectral convolution between the laser emission and cavity resonance profiles. When the resonance linewidths are narrower than those of the probe lasers, the cavity finesse is usually measured by the noise-immune CRD technique, which consists in the measurement of the decay time of the cavity transmission intensity, which turns out to be insensitive to both laser amplitude and frequency noises [15]. The cavity decay time is, indeed, related to the resonator finesse and linewidth by the following relation: [17]

$$\tau_{\mathrm{decay}} = \frac{\tau_{\mathrm{rt}}}{2(1-\sqrt{R})} = \frac{\tau_{\mathrm{rt}} F}{2\pi \sqrt[4]{R}} = \frac{1}{2\pi\, \Delta\nu \sqrt[4]{R}} \tag{2}$$

where τ_{rt} is the cavity round-trip time and it is $\tau_{\mathrm{rt}} = (1/\mathrm{FSR})$. However, the measurement of the cavity transmission and reflection profiles are more complete with respect to CRD, allowing to evaluate not only the finesse but also the cavity resonance phase dispersion (imaginary part of the transmission coefficient) and the cavity transmission profile (real part of the transmission coefficient). Fig. 1 shows the measurement principle of the proposed cavity characterization method based on the accurate recording of the cavity transmission to an intensity-modulated input laser beam when the laser frequency is in resonance with a fundamental cavity mode. A frequency locking servo keeps the resonance condition between the laser angular frequency ω_0 and the resonator mode ω_{cavity}. Considering the resonant condition from now, we have $\omega_{\mathrm{cavity}} = \omega_0$.

An intensity modulator is used to modulate the amplitude on the laser electric field as follows:

$$\widetilde{E}_{\mathrm{in}}(t) = E_0[1 + a\cos(\omega_m t)]e^{-i\omega_0 t} \tag{3}$$

where E_0 is the laser field, a is the amplitude modulation coefficient, and ω_m is the modulation (angular) frequency. The input beam is, therefore, represented by the following three spectral lines:

$$\begin{aligned}\widetilde{E}_{\mathrm{in}}(t) &= E_0 e^{-i\omega_0 t} + \frac{a}{2}E_0 e^{-i(\omega_0-\omega_m)t} \\ &\quad + \frac{a}{2}E_0 e^{-i(\omega_0+\omega_m)t} \\ &= [E_0 + E_{\mathrm{in}}(-\omega_m, t) + E_{\mathrm{in}}(\omega_m, t)]e^{-i\omega_0 t}.\end{aligned} \tag{4}$$

Introducing the transmission coefficients of the cavities in terms of electric field as

$$T_{\mathrm{FP}}(\omega_m) = \frac{\sqrt{1-R_1}\sqrt{1-R_2}\; e^{i\frac{\omega_m}{c}L}}{1-\sqrt{R}\; e^{i\frac{\omega_m}{c}2L}} \tag{5}$$

$$T_{\mathrm{ring}}(\omega_m) = \frac{\sqrt{1-R_1}\sqrt{1-R_2}\; e^{i\frac{\omega_m}{c}L_{12}}}{1-\sqrt{R}\; e^{i\frac{\omega_m}{c}L}} \tag{6}$$

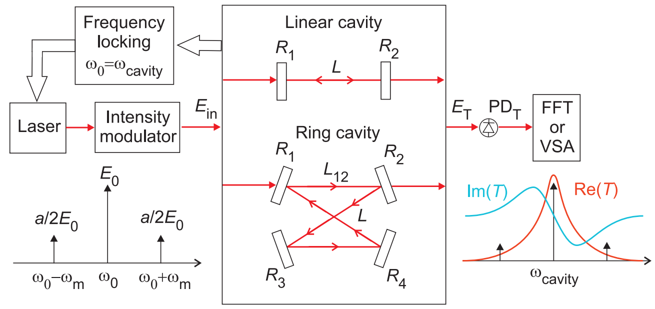

Fig. 1. Measurement principle for the accurate characterization of the transmission coefficient of an optical resonator. PD: photodetector. FFT: fast Fourier transform spectrum analyzer. VSA: vectorial spectrum analyzer.

where L_{12} is the distance between the input and output mirrors in the ring cavity configuration, and it is therefore possible to calculate the component at the fundamental modulation frequency of the intensity transmitted by the cavity as

$$\begin{aligned}I_T(\omega_m, t) = [T(0)E_0] \cdot [&T(\omega_m)E_{\mathrm{in}}(\omega_m, t) \\ &+ T(-\omega_m)E_{\mathrm{in}}(-\omega_m, t)] + c.c.\end{aligned} \tag{7}$$

Considering the symmetric relation $T(-\omega_m) = T^*(\omega_m)$, after some algebra, the component at the fundamental frequency can be written as

$$\begin{aligned}I_T(\omega_m, t) &= 2aE_0^2\, T(0)\mathrm{Re}[T(\omega_m)e^{-i\omega_m t}] \\ &= 2aE_0^2\, T(0)\, \left|T(\omega_m)\right| \cos[\omega_m t - \phi_T(\omega_m)] \\ &= 2aE_0^2\, T(0)\Big\{\mathrm{Re}[T(\omega_m)]\cos(\omega_m t) \\ &\qquad + \mathrm{Im}[T(\omega_m)]\sin(\omega_m t)\Big\}\end{aligned} \tag{8}$$

where $T(\omega_m) = |T(\omega_m)|\, e^{i\phi_T(\omega_m)}$. Equation (8) represents the main result of the proposed technique since it allows to record the transmission (in-field) coefficient of the cavity under test, as a function of the modulation angular frequency, by using either a synchronous detection scheme, which retrieves the real (in phase with the intensity modulation) and imaginary (in-quadrature) components of the cavity transmission, or a direct measurement of the amplitude of the oscillating component of (8) directly linked to the modulus of the cavity transmission, $\left|T(\omega_m)\right|$.

III. Experimental Results

To validate the proposed method, we performed two different experimental measurements using either a linear or a ring resonator. In the linear resonator case, the cavity is constituted by two broad-band high-reflectivity (HR) mirrors at 1.5 μm (a plane and a spherical, 1 m radius of curvature, fused-silica mirror, ATFilms & Precision Photonics) with a nominal finesse of ~75 000 and an FSR of 1.5 GHz (corresponding to a linewidth of ~20 kHz). The probe laser, a single-frequency Er-fiber laser, is frequency stabilized against one of the fundamental cavity modes using the Pound–Drever–Hall (PDH) method [18]. The PDH, operating at a modulation frequency of 12 MHz, servo locking system acts on a piezoelectric laser actuator for slow frequency control (~2-kHz bandwidth) and an external acousto-optic frequency shifter for fast control (∼200-kHz bandwidth). In frequency locking configuration, the 10-kHz free-running laser emission linewidth is narrowed down to 200 Hz (up to 0.1-s integration time). By means of an additional acousto-optic modulator, the intensity of the laser beam can be sinusoidally modulated to implement the proposed characterization method or, alternatively, turned off to measure the CRD time. The cavity transmission is measured using a low-noise photodetector connected to a vector analyzer to retrieve both the in-phase and in-quadrature components reported in (8). For CRD measurement, the same photodetector is connected to a 12-bit digital oscilloscope to measure the temporal decay of the intensity of the cavity transmitted beam. Fig. 2(a) and (b) shows the measured real and imaginary components, respectively, as a function of the modulation frequency, f_m ($\omega_m = 2\pi f_m$). Assuming balanced mirror reflectivity, $R_{\mathrm{FP}} = R_1 = R_2$, the absence of intracavity losses, from (8), we obtain the following relations for the cavity transmission:

$$T(0)_{\mathrm{FP}} = \frac{\sqrt{1-R_1}\sqrt{1-R_2}}{1-\sqrt{R_1R_2}} = 1 \tag{9}$$

$$\begin{aligned}\mathrm{Re}[T(f_m)]_{\mathrm{FP}} &= \frac{\sqrt{1-R_1}\sqrt{1-R_2}(1-\sqrt{R_1R_2})\cos\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}{1+R_1R_2-2\sqrt{R_1R_2}\cos\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}\\ &= \frac{(1-R_{\mathrm{FP}})^2\cos\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}{1+R_{\mathrm{FP}}^2-2R_{\mathrm{FP}}\cos\left(\frac{2\pi f_m}{\mathrm{FSR_{FP}}}\right)}\end{aligned} \tag{10}$$

$$\begin{aligned}\mathrm{Im}[T(f_m)]_{\mathrm{FP}} &= \frac{\sqrt{1-R_1}\sqrt{1-R_2}(1+\sqrt{R_1R_2})\sin\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}{1+R_1R_2-2\sqrt{R_1R_2}\cos\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}\\ &= \frac{(1-R_{\mathrm{FP}})(1+R_{\mathrm{FP}})\sin\left(\frac{\pi f_m}{\mathrm{FSR_{FP}}}\right)}{1+R_{\mathrm{FP}}^2-2R_{\mathrm{FP}}\cos\left(\frac{2\pi f_m}{\mathrm{FSR_{FP}}}\right)}.\end{aligned} \tag{11}$$

From the fits reported in Fig. 2(a) and (b), cavity reflectivity values of $R_a = 0.999953(1)$ and $R_b = 0.999952(1)$ are obtained from the real and imaginary components, respectively. Using in (1), the average value $R = 0.9999525(5)$, a cavity finesse of $F = 66,137(700)$, and an FWHM linewidth of 22.7(3) kHz are deduced, where the $1-\sigma$ uncertainty is dominated by Type A statistical contribution due to the fit residuals [as shown in the bottom of Fig. 2(a) and (b)]. For comparison, Fig. 2(c) shows the temporal behavior of the cavity transmission when the laser intensity is turned off applying to the acousto-optic modulator a square wave (with typical rise times of ∼100 ns). The intensity decay time retrieved by using the single exponential decay, $I(t) = I_0\exp[-(t-t_0)/\tau_{\mathrm{FP}}]$ (red dashed curve), is $\tau_{\mathrm{FP}} = 6.915(1)\ \mu$s. By ten independent repeated measurements, the mean decay time of $\tau_{\mathrm{FP}} = 6.826(90)\ \mu$s is obtained corresponding to a mirror reflectivity $R = 0.999951(1)$ and a cavity finesse of 64,834(900), consistent within the $2-\sigma$ coverage factor with the value measured by the analysis of the cavity transmission coefficient.

The second experimental validation is performed using a ring resonator constituted by four HR mirrors (two plane mirrors, the input one with 99.8% reflectivity and the second

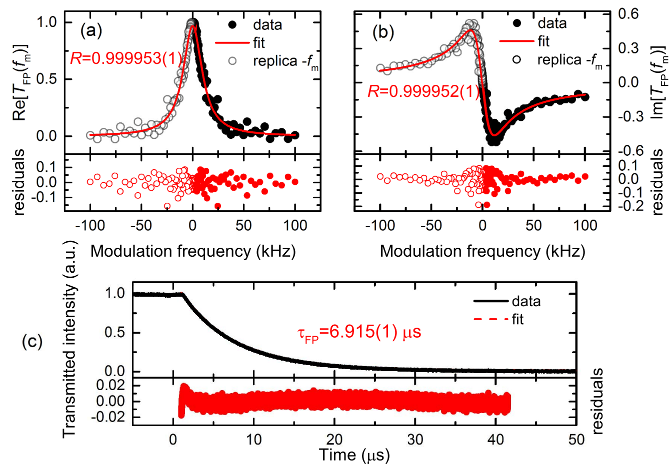

Fig. 2. (a) Real and (b) imaginary components of the linear cavity transmission coefficient as a function of the modulation frequency f_m. The error bar associated at each data point is at the level of 5%. Red curves represent the data fits using (10) and (11), and the bottom reports fit residuals. (c) Temporal profile of the ring down intensity transmitted by the linear resonator (red dashed curve is the fit exponential decay profile and the bottom reports the fit residuals).

one with the same reflectivity of the curved mirrors, and two curved ones, 750 mm radius of curvature, Layertec) in a bow-tie configuration with an FSR of 100 MHz and a nominal finesse of ∼2000. As probe laser, a mode-locked Yb-fiber laser at 1030 nm with a pulse repetition rate of 100 MHz and a pulse bandwidth of 4 nm is used. In this validation experiment, the ring resonator resonance is stabilized against the probe source using again a PDH method (operating at a modulation frequency of 3.5 MHz) acting on a piezoelectric actuator attached to one cavity mirror (typical control bandwidth ∼5 kHz). Fig. 3(a) shows the amplitude of the photocurrent oscillating at the modulation frequency, which based on (8) is directly linked to the modulus of the cavity transmission, $|T(\omega_m)|$, as a function of the modulation frequency. Red curves represent the best fit of the data using the normalized modulus of the cavity transmission

$$|T(f_m)|_{\mathrm{ring}} = \frac{1-\sqrt{R_{\mathrm{ring}}}}{\left[1+R_{\mathrm{ring}}-2\sqrt{R_{\mathrm{ring}}}\cos\left(\frac{2\pi f_m}{\mathrm{FSR_{ring}}}\right)\right]^{1/2}}. \tag{12}$$

From the fits reported in Fig. 3(a), the retrieved R_{ring} value is 0.9968(1), corresponding to a cavity finesse $F_{\mathrm{ring}} = 1,982(61)$ and an FWHM resonance linewidth of 50.5(1.5) kHz [as obtained using (2)]. Each point in Fig. 3(a) is the modulus of the FFT obtained by the oscilloscope making the average of ten acquisitions. We also measured the phase of the transmitted signal at the modulation frequency with the respect of the phase of the transmitted signal at the resonance frequency ($f_m = 0$). From (6), we have

$$\Phi_T(f_m)_{\mathrm{ring}} = \arctan\left[\frac{\sqrt{R_{\mathrm{ring}}}\sin\left(\frac{2\pi f_m}{\mathrm{FSR_{ring}}}\right)}{1-\sqrt{R_{\mathrm{ring}}}\cos\left(\frac{2\pi f_m}{\mathrm{FSR_{ring}}}\right)}\right]. \tag{13}$$

Fig. 3(b) shows the measured phase of the ring cavity transmission, average of ten acquisitions, as a function of the

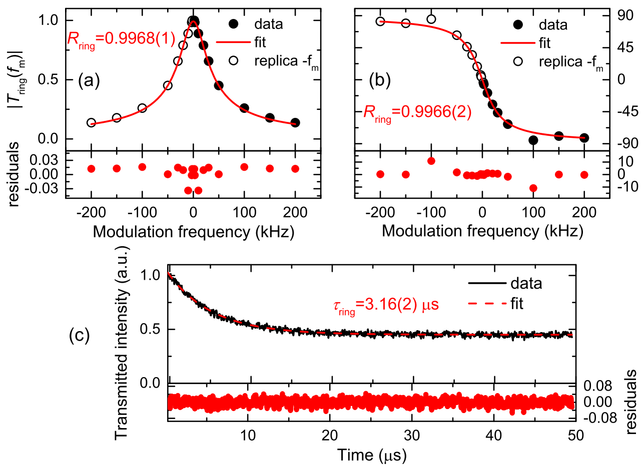

Fig. 3. (a) Modulus and (b) phase of the ring cavity transmission coefficient as a function of the modulation frequency f_m. The error bar associated at each data point is at the level of 5%. Red curves represent the data fits using (12) and (13). (c) Temporal profile of the ring down intensity transmitted by the ring resonator (red dashed curve is the fit exponential decay profile).

modulation frequency together with the fit curve corresponding (13). In this case, the retrieved value of R_{ring} is 0.9966(2), which corresponds to a cavity finesse $F_{\text{ring}} = 1,845(120)$, in good agreement with the values obtained by using the normalized modulus of the cavity transmission. To check the accuracy and reproducibility of these measurements, also in this case, the CRD times are measured, as shown in Fig. 3(c). With respect to the continuous-wave linear cavity case, here, the ring resonator is not completely depleted because the AOM cannot be entirely turned off; this is clear from Fig. 3(c), where the tail of the transmitted intensity does not go to zero. When the cavity is not completely depleted, straightforward calculations lead to the following exponential trend for the transmitted intensity:

$$I(t) = \left[A\exp\left(-\frac{t-t_0}{2\tau_{\text{ring}}}\right)+B\right]^2 = A^2\exp\left(-\frac{t-t_0}{\tau_{\text{ring}}}\right)+2AB\exp\left(-\frac{t-t_0}{2\tau_{\text{ring}}}\right)+B^2 \quad (14)$$

where A^2 is the transmitted intensity before AOM switching, B^2 is the residual intensity transmitted by the cavity, and τ_{ring} is the intensity decay time. The retrieved decay time, average of ten repeated measurements, is $\tau_{\text{ring}} = 3.16(2)$ μs, corresponding to a cavity finesse value of $F = 1,987(89)$, in excellent agreement with the value measured by the analysis of the cavity transmission coefficient ($1-\sigma$ compatibility).

To characterize the accuracy of the proposed method, the evaluation of the systematic contributions to the measurement uncertainty (Type B contribution) has to be performed. For the described methodology, the systematic contributions are related to the acquisition scheme of the cavity transmission (nonlinearity and amplitude quantization), the calibration of the modulation frequency, and the cavity FSR measurements. In general, the acquisition system nonlinearity (better than 10^{-5} in relative terms) and the calibration of the modulation frequency (at the level of 10^{-7}) give negligible contributions to the uncertainty of the finesse measurements for both the investigated cavity configurations. In the case of the linear cavity, the main systematic contribution comes from the calibration of the 1.5-GHz cavity FSR, which is measured with an uncertainty of ~200 kHz, leading to a type B relative uncertainty of 10^{-4}, two orders of magnitude lower than the statistical Type A contribution. By contrast, for the ring cavity characterization, the highest systematic contribution is due to the quantization level of the 8-bit acquisition system corresponding to a Type B relative uncertainty level of 10^{-3} in the measurement of the resonator finesse, but in any case more than one order of magnitude lower than the Type A contribution for the measurement of the linear cavity finesse.

IV. Conclusion

We presented and experimentally validated a new measurement method to accurately characterize the finesse and linewidth of optical resonators. Comparisons with the commonly used CRD method are also performed by validating the reproducibility and accuracy of the proposed methodology, which turns out to be more complete compared with the CRD technique. Indeed, the proposed method allows retrieving not only the mirror reflectivity and the cavity finesse but also the full cavity complex transmission profile, the real component corresponding to the intensity transmission profile and the imaginary component corresponding to the cavity resonance phase dispersion.

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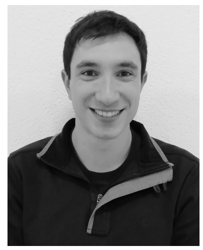

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