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Interplanetary Transfers by the Automatic Search of Earth and Earth/Moon Resonant Arcs

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Abstract

Interplanetary trajectories to inner or outer planets - except Venus and Mars - often require multiple gravity assists. Most of the time, the first arc after launch sends the spacecraft either towards Venus or Mars. The required escape velocity declinations can, however, significantly deviate from the Earth's equator, posing a challenge for launch vehicles such as Ariane 5 ECA, whose performance drops sharply outside of these conditions.
Considering an earlier departure, followed by one or more Earth-resonant arcs, often represents a useful trick, as it can allow to de-couple the "true" interplanetary transfer from the launch constraints, at the cost of an increased time of flight: starting from an optimal launch in the equator, the last gravity assist targets the required declination. Independently from Ariane 5 ECA, an advantage of multiple initial Earth gravity assists is to enable V_∞ leveraging, which increases the final payload mass, and decreases the launch window ΔV cost.
Moreover, meeting the Earth multiple times also opens the door for another opportunity: a Lunar-Earth Gravity Assist (LEGA). Developed for the first time for ESA's JUICE mission, this type of encounter exploits the Moon's relative motion around the Earth to increase the spacecraft's Earth-infinite velocity at virtually no deterministic ΔV expense. The increase in the stochastic ΔV leads to a global trade-off.

The focus of this work is on the development of an approach to automatically identify promising LEGAs and Earth resonant sequences that connect the launch to the first "true" interplanetary leg. In the proposed approach full and pseudo resonances, as well as pi-transfers and V_∞ leveraging transfers, are considered. The combinatorial nature of resonance problems is exploited, and the search space is reduced from combinatorial-continuous to discrete, allowing to investigate all the feasible alternatives. A preliminary design technique to formulate a first guess for an efficient LEGA is also outlined and incorporated into the search algorithm. Initial guesses are then refined exploiting ESA's flight dynamics software Godot, to generate fully optimised transfers.
The computed possibilities for resonant sequences will provide the mission analyst with a bird's eye view on the feasible alternatives, substituting lengthy and possibly suboptimal manual searches. To validate the results, the tool is tested using the JUICE mission as a case study, demonstrating good accuracy in finding baseline and backup trajectories developed by ESOC, both with and without LEGA.

Keywords: Interplanetary transfer, Resonance, LEGA, V_∞ leveraging

Nomenclature

Throughout the text the following naming convention is used for position and velocity vectors:

$$^{RF}\underline{r}_{from}^{to},\ ^{RF}\underline{v}_{from}^{to}$$

Where RF specifies the reference frame.
For hyperbolic infinite velocities,

$$^{body}\underline{v}_{\infty}^{-},\ ^{body}\underline{v}_{\infty}^{+}$$

Where the superscript "+" or "-" indicates the incoming or outgoing leg of the hyperbola

Acronyms/Abbreviations

CReMA – Consolidated Report on Mission Analysis
EGA – Earth Gravity Assist
ESA – European Space Agency
ICRF – International Celestial Reference Frame
JUICE – JUpiter ICy moons Explorer
LEGA – Lunar-Earth Gravity Assist
LVLH – Local Vertical Local Horizontal
MIDAS – MIssion Design and Analysis Suite
VILT – V-Infinity Leveraging Transfer
WORHP – We Optimise Really Huge Problems

1. Introduction

Earth resonant transfers can broadly be understood as same-body transfers in which a simple mathematical relationship, such as an integer ratio, exists between the orbital periods of the Earth and of the spacecraft around the Sun. Sequences of such resonant orbits often represent an effective intermediate step to bridge the gap between the escape velocities achievable by a launcher vehicle and the Earth outgoing conditions required for the interplanetary transfer.

For **full resonances**, each individual transfer can be identified through a "resonant ratio", which corresponds to the ratio between the two periods of the spacecraft and the gravity assist body:

$$\frac{T_{sc}}{T_{ga}} = \frac{N}{M}, \ \ with\ N, M \ \in \ \mathbb{N} \tag{1}$$

which means that the spacecraft will leave and return to the gravity assist body (Earth) at the same relative position with respect to the central body (the Sun). This leaves the inclination of the orbit free to change, making this particular type of resonance very useful to circumvent launcher declination constraints [1].
Staring from a full resonance, by increasing the period of the spacecraft, we can make it miss the encounter after one revolution, but subsequently meeting the Earth at the other intersection point. A similar result can be obtained by a period decrease. This results in what is typically defined as a **pseudo resonant** orbit, which is by construction constrained to the gravity assist's body plane. A pseudo resonance is identified by the direction of the period change ("plus" or "minus"). Figure 1 shows a 1:1 Earth full resonance and its relative pseudo resonances

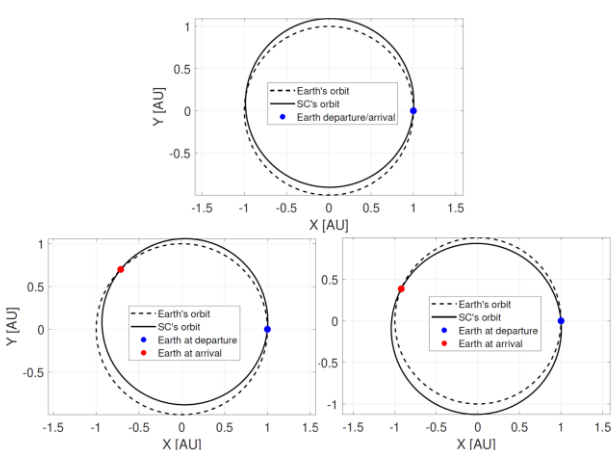

Figure 1 - 1:1 full resonance (above), 1:1- pseudo resonance(left), 1:1+ pseudo resonance(right)

A special case of pseudo resonance is represented by **backflip transfers**, also referred to as "Pi transfers", in which the departure and arrival are separated in true anomaly by π radians. By definition, the departure and arrival points lie on the same line as the main attractor, meaning that much like fully resonant transfers, they can be inclined with respect to the gravity assist's body orbit. In this case, however, the inclination is not free, but is a function of the v_∞ velocity magnitude of the spacecraft with respect to the Earth. For a more comprehensive examination of the analytical theory of resonant transfers, the interested reader is referred to works of Strange [2].
While transfers strictly belonging to the resonant class are purely ballistic, thrusting burns are often exploited in Earth-to-Earth arcs. Hollenbeck [3], first introduced the concept of a "Δv-EGA" in the seventies, which would later be generalised into the definition of a **V-infinity leveraging transfer**, or "VILT" [4,5]. These transfers, which are in literature largely used for moon tours and endgame problems, *leverage* a relatively small Δv burn at one of the apse points, to obtain a larger change in v_∞ velocity at the following planetary encounter. The higher the resonance ratio, the higher the efficiency of the manoeuvre. Additionally, due to the opposite effects of perihelion and aphelion leveraging manoeuvres on the duration of the transfer arc, VILTs prove useful in Earth resonant arcs to flatten launch window costs [6].

Finally, the latest addition to the framework of Earth resonant sequences is represented by the possibility of performing a **Lunar-Earth Gravity Assist**, or "LEGA". Meeting the Earth multiple times can in fact, in some cases, allow to tweak the trajectory in such a way that a close Moon encounter occurs either on arrival or at departure from an Earth swing-by. This concept was introduced for the first time for the European Space Agency's JUICE mission [6] and successfully performed its world premiere this past August. Flying by the Moon allows the spacecraft to exploit the relative motion of the satellite with respect to the planet, and to obtain, virtually "for free", an increase in the infinite velocity with respect to Earth, depending on the geometry of the encounter, as shown in Figure 2.

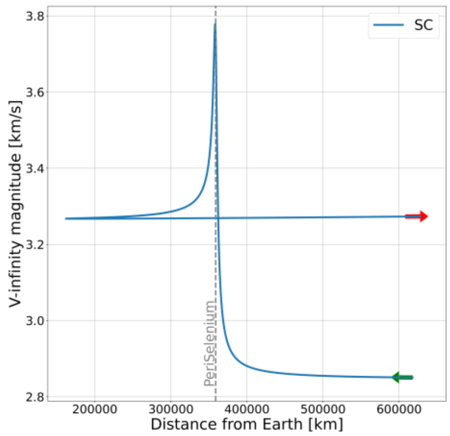

Figure 2 - Instantaneous infinite velocity evolution through a LEGA

The efficiency of a LEGA is closely tied to the amount of rotation of the spacecraft's infinite velocity vector in the Moon's orbital plane [6]. This implies that LEGAs are particularly advantageous when the spacecraft's v_∞ is lower, as this allows for greater deflection. This typically occurs during the first resonant arcs if lower launch escape velocities are used and offers diminishing returns at higher speeds. Additionally, because the out-of-plane rotation required to target Earth does not contribute to Δv gains, LEGAs are most effective when the Moon-side interplanetary leg lies close to the Moon's orbital plane.

A trade-off must be made on the stochastic costs, as the quick succession of the Moon and Earth close encounters forbids to perform an intermediate manoeuvre. This results in the dispersion caused by the first swing-by to be amplified by the second, and therefore to a larger clean-up burn.

As opposed to the optimisation of a full interplanetary transfer, the combinatorial nature of transfers pertaining to the resonant family and the reduced search space of post-launch sequences, make the problem suitable for an automatic investigation.

The aim of this work is therefore the creation of a tool capable of exploring the solution space and identifying the most promising alternatives, allowing the mission analyst to have a "bird's eye" view on all the alternatives. An approach is proposed for the preliminary design of double Earth-Moon flybys, and this possibility is included in the search space.

Finally, the proposed tool is validated against known alternatives for the JUICE trajectory.

2. Framework

2.1 Transfer parametrisation

A ballistic resonant arc can be fully defined by the outgoing conditions of the spacecraft at the gravity assists leading to it. A convenient parametrisation is to describe the v_∞ vector through a set of spherical coordinates defined by the crank and pump angles κ and α, as shown in Figure 3:

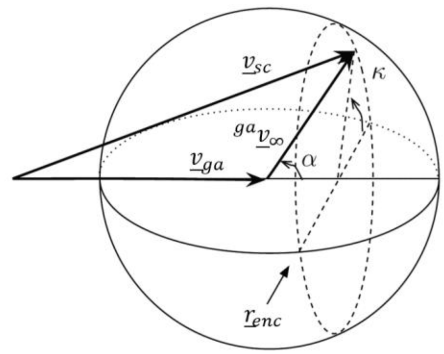

Figure 3 - crank and pump parametrisation of the v-infinity vector

Where $\underline{v}_{ga}$ is the velocity of the gravity assist body (Earth) around the main attractor (Sun), $\underline{v}_{sc}$ that of the spacecraft, and $\underline{r}_{enc}$ is the position vector of the gravity assist body with respect to the main attractor. Considering a pump and crank parametrisation is particularly fitting for resonant orbits, as the pump angle can be related to the semi-major axis a_{sc} of the orbit through the following:

$$v_\infty^2 + v_{ga}^2 + 2v_\infty v_{ga}\cos(\alpha) = \mu_{cb}\left(\frac{2}{r_{enc}} - \frac{1}{a_{sc}}\right) \tag{2}$$

Where μ_{cb} represents the gravitational parameter of the central body. Eq. 2 shows how, for a given v_∞ magnitude, the pump angle is directly tied to the semi-major axis of the resonant orbit, and therefore the orbital period. This means that, given a v_∞ magnitude for the encounter, choosing the resonance type means fixing the pump angle. Conversely, it can be shown that the crank angle is tied to the spacecraft's orbital inclination i_{sc}:

$$sin(\kappa) = tan\,(i_{sc})\left(\frac{\frac{v_{ga}}{v_\infty} + cos\,(\alpha)}{sin(\alpha)}\right) \tag{3}$$

2.2 Definition of the search space

Before discussing methods for an automatic exploration of resonant possibilities, we must define our search space, and discuss its bounds. In particular, the following solutions are considered as distinct.

For full resonant transfers, to each resonant ratio is associated a pump angle α, representing the discrete design parameter, and an interval of crank angles, the continuous component. In the frame of this work, two cases are considered for each ratio, which share the same inclination but differ in Earth outgoing direction: inbound, if the spacecraft departs from the gravity assist body towards the central body (i.e., $\cos(\kappa) < 0$) and outbound, otherwise. The two alternatives shall be kept separate as different resonances are reachable trough an Earth swing-by depending on the inbound/outbound characteristics of the transfer.

Moreover, the distinction is necessary when considering the potential encounter with the Moon near the Earth flyby hyperbola, as it significantly impacts the LEGA.

Pseudo resonant and backflip transfers retain a similar discrete nature with respect to the pump angle but lose the degree of freedom on the inclination (and therefore on the crank), which is zero for the former and function of the v_∞ level for the latter. A pseudo resonance can be unambiguously identified through its reference resonance ratio and the +/- characteristic, as the latter determines if the transfer is inbound or outbound. Conversely, two possibilities exist for a backflip transfer, with opposite inclination, one leaving the gravity assist body directed "above" its orbital plane, and the other directed "below". The two alternatives are kept separate.

Lastly v_∞ leveraging manoeuvrers can be added to resonant arcs to increase the infinite velocity with respect to the Earth. The transfers that are obtained in this way can therefore be identified by the reference resonant transfer, by whether the manoeuvre is performed at perihelion or aphelion, and by the targeted increase in the v_∞ level. Figure 4 summarizes the elements of the resonant solution space (i.e., the considered transfer types) as they were described.

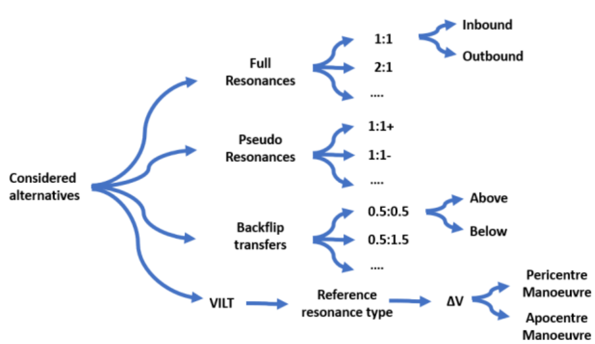

Figure 4 - Structure of the resonant solution space: considered transfer types

It shall be pointed out how, while virtually infinite resonant transfers can be described by eq. 2, only a small subset of these are of interest for the problem at hand. A first constraint is on the trade-off with the time of flight. The latter can quickly become unacceptable for mission requirements, especially considering that this time is added on top of that of the actual interplanetary transfer. As a result, Earth resonances with N > 3 are typically beyond the relevant range, although different limits could be set for different scenarios.

Moreover, for any given magnitude of the v_∞ with respect to Earth, not all resonances are reachable. Figure 5 shows how as the resonant ratio moves away from 1, higher v_∞ magnitudes become necessary.

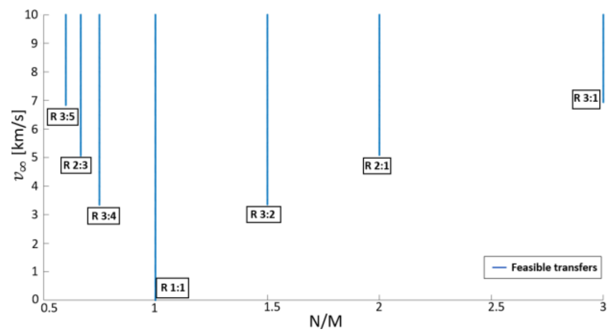

Figure 5 - Reachable Earth full resonances for a given v-infinity level

As the scope of this work is limited to post-launch Earth resonant sequences, the relevant range is considered as that within 6 km/s. This choice is driven by the fact that reaching higher energy levels without exploiting any other-body Gravity Assist and by means of chemical propulsion alone exceeds the practical capabilities of real missions.

3. Automatic Search

The proposed approach is composed of three main steps:

1) **Resonant initial guess generation:** a set of initial guesses for all possible resonant trajectories is generated under simplifying assumptions
2) **Numerical optimisation:** The initial guesses are set up as a multiple shooting problem and numerically optimised using ESA's non-linear programming solver WORHP
3) **Incorporation of LEGAs:** lastly, the possibility to add LEGAs to the trajectories is investigated, and the new possibilities are re-optimised

Each step is analysed in further detail in the following paragraphs.

3.1 Resonant initial guess generation

In a first approximation we can consider the Earth resonant sequence as a separate entity from the properly said interplanetary transfer, with the only point of contact between the two being represented by the set

$$\left\{t_{PP}, \alpha^{o}_{PP}, \kappa^{o}_{p}, \left\|v_{\infty,PP}\right\|\right\} \tag{4}$$

Where the subscript "PP" refers to the "Patching Point" between the two, i.e. the last Earth encounter before leaving towards a different body, the superscript "o" refers to the outgoing leg of the flyby hyperbola and t_{PP} is the epoch of the Patching Point.

The first step in computing a resonant sequence is therefore to try and prepend a single resonant arc before the Patching Point. To compute the set of resonant transfers that can lead to these outgoing conditions through a swing-by, we must recall that the maximum deflection δ_m attainable through a gravity assist cannot be arbitrarily large. In the considered v_∞ parametrisation this constraint can be translated as:

$$\begin{cases} |\alpha_i - \alpha_o| \leq \delta_m \\ \cos(\Delta\kappa) = \dfrac{\cos(\delta_m) - \cos(\alpha_i)\cos(\alpha_o)}{\sin(\alpha_i)\sin(\alpha_o)} \end{cases} \tag{5}$$

where the subscripts "i" and "o" respectively refer to the incoming and outgoing conditions at the encounter, and $\Delta\kappa$ is the maximum attainable change in crank.

For each of the considered resonance types, an interval of reachable crank angles exists, therefore for each of them a manifold of trajectories that satisfies eq. 5 is retrieved. At this point, to continue in the process, we can keep backpropagating each manifold through eq. 5, bearing in mind that at each node (i.e. Earth encounter) each alternative branches out, each branch being a new manifold representing a different resonance type.

This process of backward propagation can then be repeated until a user-imposed bound is met, such as the maximum duration or maximum number of Earth swing-bys. However, simply backpropagating a solution might lead to an unfeasible sequence, as constraints also exist on the departure point of each sequence, i.e. at launch. Two constraints must be considered:

$$\left\{\kappa_{Launch}, \left\|v_{\infty,Launch}\right\|\right\} \tag{6}$$

Where κ_{Launch} , i.e. the crank angle of the launch outgoing infinite velocity, is function of the launch date (sequence dependent) and the attainable equatorial declinations at launch (launcher dependent). Therefore, a second forward propagation could be initiated for each sequence, and the "solution" sequences provided as output would be the intersection of the two. In Figure 6, this reasoning is shown graphically, considering only full resonant transfers for simplicity:

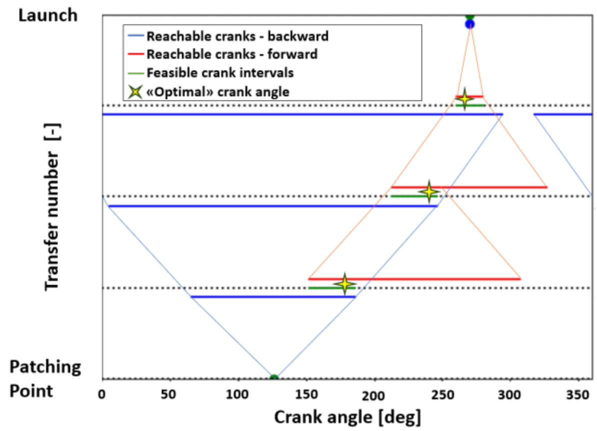

Figure 6 - Propagation of the crank angle manifold

The result of the search would therefore be a set of all feasible combinations of resonant transfers, each one of them containing not single trajectories but manifolds of varying crank angle. However, to avoid wasting computational effort on suboptimal alternatives, a search space reduction is performed such that, during the forward propagation, a single crank value is chosen from the feasible intervals.

The adopted choice criterion was that of choosing the alternative that maximises the altitude at the resonant swing-bys, as this allows to move the initial guess away from the unfeasible bounds, while reducing the stiffness of the problem. Moreover, low altitude flybys represent a sub-optimal alternative in terms of navigation costs and shall therefore be avoided when possible. For more complex scenarios, dynamic programming has shown to be a suitable approach for reducing a manifold of resonant trajectories to a single "best" trajectory [7]. However, due to the simpler nature of the problem at hand, the optimal solution can be found either by imposing the inclination of the arc to be the average of the previous and following one, when these are prescribed, or by a coarse grid search if they are not.

A parenthesis shall be opened for the case of $v_{\infty,}$ leveraging, as while analytical methods for finding initial guesses exist [5], zero eccentricity of the gravity assist body is assumed. When considering Earth's orbit, the impact of this assumption on the quality of the initial guess makes this approach not suitable for the following unsupervised optimisation. ESA's Mission Design and Analysis suite "MIDAS" is therefore exploited to numerically generate initial guesses starting from a reference ballistic resonant transfer.

Finally, a last remark shall be made on $v_{\infty,}$ leveraging arcs. Given the two v-infinity magnitudes at Patching

Point and at launch, a constraint was imposed that the sum of the amount of leveraging in the different arcs of the trajectory adds up to the difference of the two. Once again, how to distribute the amount of leveraging in the different arcs can be arbitrarily chosen from a continuous set of alternatives. In more complex scenarios such as giant planets moon tours this represents a design choice that modifies the reachable alternatives for *resonance hopping* [8]. Due to the smaller set of relevant resonances as discussed in paragraph 2.2 however, this is issue remains less significant for the problem at hand. Additionally, by starting the backpropagation from the highest v-infinity level (i.e. at patching point) we are sure that all the reachable resonances are investigated. A simpler approach that can be employed is then that of considering that the Δv is equally distributed among all the VILT arcs. This generates an initial guess in the neighbourhood of the optimum, such that the optimal Δv distribution can then be retrieved numerically.

3.2 Numerical optimisation

Once the initial guesses have been generated, an optimisation problem is setup as a multiple shooting problem, where a control point is set at each Earth (and Moon, for LEGA sequences) encounter. The problem is setup using ESA's flight dynamics software Godot and solved through the non-linear programming solver WORHP, with the objective of minimising the spacecraft's Δv. The considered dynamics are a full ephemeris model considering the gravity of Earth, the Moon and the Sun. The following constraints are imposed:

Conditions at launch:

- Launch infinite velocity is fixed
- Launch declination constrained to the launcher optimum for the considered infinite velocity

Conditions at patching point:

- Outgoing $\boldsymbol{v}_\infty$ vector with respect to Earth is fixed. For sequences that introduce an Earth-Moon swing-by at the patching point, the $\boldsymbol{v}_\infty$ is computed after the Moon encounter
- Epoch at patching point is fixed

Minimum altitude requirements:

- Perigee altitude along the flyby hyperbola higher than 350 km
- Periselenium altitude along the flyby hyperbola higher than 300 km

And the free parameters chosen as:

- Incoming and outgoing $\boldsymbol{v}_\infty$ vector at flybys
- Δv magnitude, declination and right ascension of the manoeuvres
- Epoch at flybys and launch

Lastly, to help convergence, zero delta-v manoeuvres were added at all apse points. It shall be pointed out how the launch v-infinity is constrained. A first approach that was considered to solve the problem was to let the v-infinity at launch vary freely. The launch mass could then be computed as a function of this value, and the best solution would maximise mass instead of minimising Δv. However, as the computed EGA sequences will be used as first guesses for their respective LEGA sequences, a trade-off was made on the computational time by fixing the v-infinity and performing a coarse grid search iteration over the feasible range. This approach in fact produces more alternatives of the same sequence, in which the energy levels at each flyby are different. This is more suitable for the individuation of Moon-Earth flybys, as the optimal LEGA sequence can be retrieved from a sub-optimal EGA alternative.

3.3 Incorporation of LEGAs

Starting from the optimised resonant sequences, the next step in the algorithm is that of evaluating the possibility of incorporating one or more Moon flybys in the already present Earth gravity assists (EGAs).
WHORP, as any local optimisation algorithm, finds the optimum in the neighbourhood of a first-guess solution that is provided as input. Therefore, the aim of this analysis is that of providing the tool with a preliminary design for a Lunar Earth Gravity Assist, that is as close as possible to the global optimum. To do this, the following assumptions are considered:

- Due to the high relative velocities involved, a patched conics approach can be applied, where each arc is propagated under the dynamics of the two-body problem ("2BP") around the instantaneous principal body.
- The LEGA is "similar" to the original Earth flyby, i.e. the incoming and outgoing infinite velocities do not change in direction, only in magnitude.
- The Earth-Moon leg lies on the Moon's orbital plane. This is not only done as it simplifies the analysis but because, as previously discussed, the Moon's orbital plane is the locus of the most efficient solutions. This is however not always possible, as a small out of plane component is often needed to target specific points in the B-plane, therefore a small position discontinuity will exist in the guess.
- The Moon flyby altitude is fixed. As it is known that higher Moon deflections lead to higher gain in v_∞ [2], the Moon altitude is taken as the minimum acceptable for navigation purposes, i.e. 300 km [3]

It must be remarked how two possibilities exist, depending on whether the spacecraft meets first the Earth or the Moon. The main drawback of an Earth-first LEGA is that by meeting the most massive body first, navigation costs can quickly grow. For the sake of simplicity, the following subsections describe the preliminary design for a Moon-first flyby. The solution for the Earth-first case can be found following the same steps in a symmetric fashion.

3.3.1 Phasing

Firstly, the encounter epoch is modified in order to allow meeting the Moon. The assumption is that the duration of the previous transfer can be easily adjusted through an apse point manoeuvre, as can the position of the incoming infinite velocity on the Earth's B-plane. Let us define for convenience a "Moon Local" inertial reference frame as the Moon's Local Vertical Local Horizontal (LVLH) reference frame at a fixed time "t":

$$\hat{x}_{ML} = \frac{\underline{r}_E^M}{\|\underline{r}_E^M\|}$$
$$\hat{y}_{ML} = \hat{z}_{ML} \times \hat{x}_{ML}$$
$$\hat{z}_{ML} = \frac{\underline{r}_E^M \times \underline{v}_E^M}{\|\underline{r}_E^M \times \underline{v}_E^M\|} \tag{8}$$

A first guess on the epoch is generated such that the Moon and the spacecraft share the same right ascension in the Moon Local reference frame at the time the spacecraft is at Moon distance from Earth. An important remark is that two encounters spaced by a 28-day interval are always possible, where one can be obtained by shifting the encounter date forward and one by shifting it backward. A possible approach is that of only considering the closest in time, but limitations will be highlighted. A first guess for the encounter epoch is then retrieved such that the Moon is as close as possible to be met along the original Earth flyby hyperbola, as shown in Figure 7.

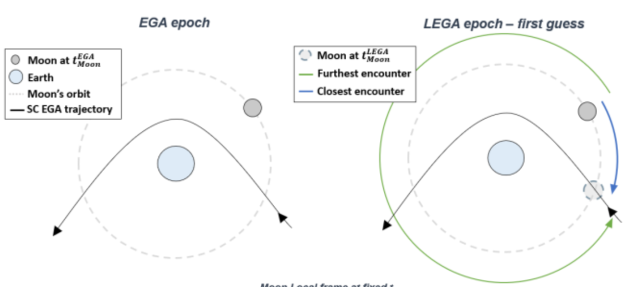

Figure 7 - Possible encounters for a LEGA, starting from an EGA initial guess

A boundary case is that in which the original flyby has a perigee altitude higher than that of the Moon. To modify the hyperbolic path of such a high altitude EGA to meet the Moon, the perigee must clearly be lowered. However, doing so means to increase the deflection that is provided by the Earth flyby, not preserving the direction of the incoming and outgoing infinite velocity vectors and not satisfying the assumptions. Therefore, the only possible LEGA geometry is that in which the deflection performed by the Moon and that of the Earth counteract each other, as is shown in Figure 8.

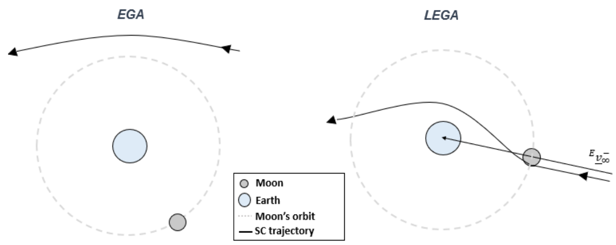

Figure 8 - Phasing for a LEGA stemming from a high altitude EGA

Due to the low deflection available from the Moon, a good initial guess for the LEGA's epoch is the one for which the Moon-Earth vector is aligned with the incoming EGA Earth v-infinity velocity $^E\underline{v}_\infty^-$.

3.3.2 Solution in velocity

Once a preliminary epoch for the encounter has been chosen, the two flyby hyperbolas need to be characterised. Figure 9 summarises the velocity triangles for the LEGA, considering a planar case for readability.

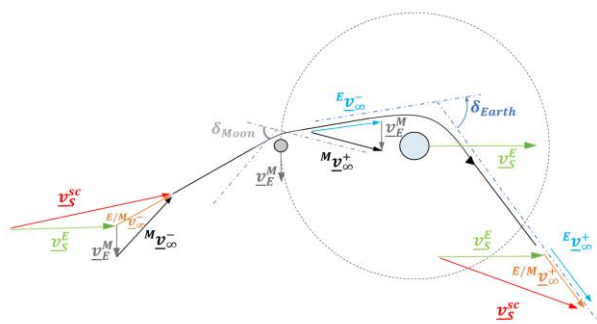

Figure 9 - Velocity triangles for a Moon-Earth LEGA

Due to the assumptions on the Moon's flyby altitude (i.e. on the deflection angle) and of the Moon-Earth leg lying on the Moon's orbital plane, knowing the incoming v_∞ velocity vector means that only two possibilities can be considered for $^M\underline{v}_\infty^+$, i.e. the intersections of the cone of constant deflection with the Moon's orbital plane, as shown in Figure 10:

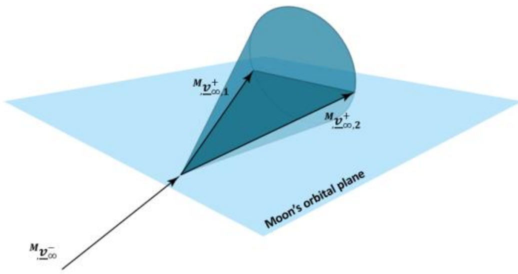

Figure 10 - Cone of outgoing flyby v_∞

As the scope of a LEGA is (in the considered case) that of increasing the infinite velocity with respect to Earth, we can compute the latter for both cases and choose the solution that maximises it. The opposite could be done e.g. to slow down a spacecraft for an Earth sample return mission. With this strategy and leveraging the relationships shown in Figure 9, we can compute the infinite velocities for the Moon and Earth encounters:

$$^{M}\underline{v}_{\infty}^{-},\ ^{M}\underline{v}_{\infty}^{+},\ ^{E}\underline{v}_{\infty}^{-},\ ^{E}\underline{v}_{\infty}^{+} \tag{9}$$

and fully characterise the two flyby hyperbolas.

3.3.3 Solution in position

While the retrieved solution will be continuous in velocity, it will present discontinuities in position as no constraint has been imposed. In Figure 11, an example partial result of this kind is shown, and by propagating the two flyby hyperbolas it is evident how they do not meet.

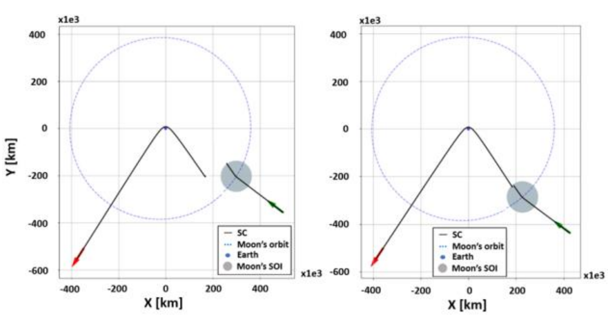

Figure 11 - LEGA position discontinuity(left) and correction (right)

Once again, it is possible to simply change the epochs of encounter to fix the discontinuity. A simple approach, similar to the one used for the preliminary phasing can be employed. Both hyperbolae can be propagated until the same distance from Earth "d". An intuitive choice for d is such that the Moon side hyperbola propagation reaches the bound of the Moon's sphere of influence. By considering the two position vectors representing the position of the spacecraft at "d" along the Earth ("E") and Moon ("M") hyperbola:

$$^{E,ML}\underline{r}_{E}^{sc@d},\ ^{M,ML}\underline{r}_{E}^{sc@d} \tag{10}$$

We can then calculate the angular distance along the Moon's orbital plane $\Delta\theta$ between the two and then shift both encounters by the quantity

$$\Delta t = -sign\left[\left(^{E,ML}\underline{r}_{E}^{sc@d} \times {}^{M,ML}\underline{r}_{E}^{sc@d}\right)\cdot \hat{z}_{ML}\right]\Delta\theta \frac{T_{Moon}}{2\pi} \tag{11}$$

A remark shall be made, as from a change in encounter epoch stems a change in the velocity vector of the Moon, therefore modifying the velocity triangles shown in Figure 9. Consequently, by closing the gap in position, a small discontinuity in velocity is introduced. It is easy to understand how this can become an iterative procedure, as convergence can be reached by repeating the steps shown in this and the previous paragraph. In the results presented in this work, a single iteration approach has been considered, as this already leads to an acceptably high-quality initial guess.

4. Results and validation

In this section, the proposed method for the individuation of resonant sequences is applied to the test cases from JUICE. During the trajectory analysis for the mission, possible resonant sequences that would lead to the needed Earth departure conditions were manually investigated, and the results reported in the Consolidated Report on Mission Analysis (CReMA) [3].

The considered test cases are taken from the family of solutions "15", which is the one containing the baseline and backup trajectories. The term "family" refers to the fact that trajectories belonging to the same group share the same proper interplanetary arc but differ in the Earth-Resonant sequence. Hereafter, the main resonant sequences investigated for the alternative 15 are reported:

- **150l0a:** consisting of two 1:1 resonances, with a Moon-Earth LEGA after the first. This case was considered the baseline until 150lb was found. It shall be pointed out how, since this case contains two resonances of the same type, the pump angle is not modified by the first gravity assist. Since also the change in inclination (i.e. in crank) is small, if no Moon swing-by was considered the trajectory would present a high altitude EGA, which makes it a good test case for the boundary case described in paragraph 3.3.1.

- **150lb:** consists of a 1:1- pseudo resonance followed by a Moon Earth LEGA. Interestingly, this scenario was not initially investigated as, being bound to the ecliptic, it seemed unfeasible with a near equatorial launch. Only later in the design phase it was noticed how, coincidentally, the launch date fell near the spring equinox, enabling such a launch, and took the place of 150l0a as baseline. This underscores the importance of a complete investigation of the resonant space, and represents part of the motivation for the present work

- **150la:** the backup trajectory, consisting of a launch in August 2023 into a full 1:1 resonance, terminating in a Moon-Earth LEGA

For trajectories belonging to the same family, the outgoing conditions at Patching Point (i.e. the last Earth encounter in the resonant sequence) are almost, although not exactly, the same. Therefore, all alternatives can in practice be found by running the algorithm once. Table 1 summarises the considered outgoing conditions at the patching point in ICRF:

Table 1 - Outgoing conditions at patching point

Epoch	2024-08-20T21:38 TDB
Earth v_∞ norm [km/s]	3.30
Right ascension [deg]	106.1
Declination [deg]	24.12

For the sake of simplicity, the search is limited to two resonant arcs, and only 1:1 resonances and their relative pseudo-resonances are investigated, as these boundaries comprehend all the considered JUICE solutions. v_∞ manoeuvres are employed to bring the spacecraft to the needed infinite velocity level with respect to Earth. Only one LEGA is added in each sequence, and only the closest encounter is investigated. The results of this first reduced search are summarised in Figure 12.

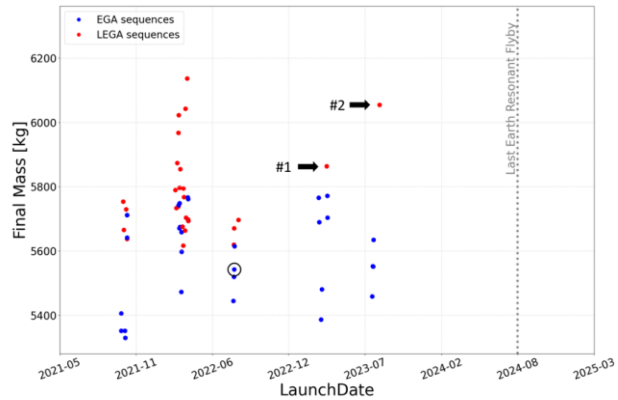

Figure 12 - Retrieved solutions for the "15" family of alternatives

In the figure, each dot represents a possible sequence, sorted by launch date and final mass at patching point. Alternatives higher on the y-axis clearly represent more promising solutions as they allow a higher mass budget, that can be exploited for additional payload or fuel. Alternatives with earlier launch dates pay the price of an increased time of flight, even though they can perform better in terms of Δv, as they can exploit two smaller v_∞ leveraging manoeuvres. Lastly, dots coloured in red mark sequences that exploit a Moon-Earth flyby.

LEGA alternatives that are outperformed by the original EGA sequence, and sequences that provide less than 5200 kg of final mass are not shown for clarity. While it must be kept in mind that considering stochastic costs and the full Launch window might lead to different trade-offs, the advantage of considering Moon encounters is made very evident in Figure 12, as they often allow for significant increase in the mass budget.

A remark shall be made on the solution space, as the retrieved alternatives seem to be aligned in "columns". The columns share the same number and type of resonances, as is highlighted in Figure 13.

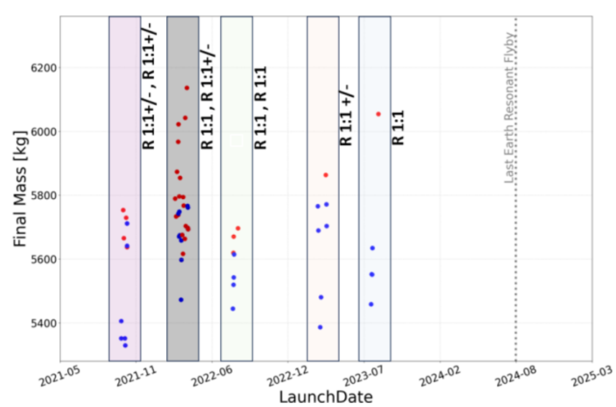

Figure 13 - Retrieved alternatives for the 15 family: Launch possibilities

This property is of particular interest to mission analysts, as each column represents a different set of launch opportunities. A bigger variance in launch date is present in the bands containing a pseudo resonance and, in theory, two very nearby bands for the "+" and "-" alternatives could be highlighted. Considering a LEGA or different v_∞ leveraging manoeuvres also naturally makes the transfer duration deviate from that of the original resonance. Getting back to Figure 12, the two highlighted solutions #1 and #2 appear to be among the most promising, as they grant the highest final masses for times of flight shorter than one and a half years.

These alternatives correspond to the reference sequences 150la and 150lb as identified for the JUICE mission. Table 2 and Table 3 compare the retrieved results with the reference:

Table 2 - Comparison between the retrieved solution and JUICE's baseline trajectory

	#1	**150lb**
Launch	10/04/2023	11/04/2023
Pericentre manoeuvre	236 m/s	198 m/s
Apocentre manoeuvre	-	20 m/s
Moon swing-by	19/08/2024	19/08/2024
Earth swing-by	20/08/2024	20/08/2024
Total Δv	236 m/s	228 m/s

Table 3 - Comparison between the retrieved solution and JUICE's backup trajectory

	#2	**150la**
Launch	10/08/2023	23/08/2023
Pericentre manoeuvre	240 m/s	164 m/s
Apocentre manoeuvre	-	68 m/s
Moon swing-by	19/08/2024	19/08/2024
Earth swing-by	20/08/2024	20/08/2024
Total Δv	240 m/s	232 m/s

While the results show good correspondence to the reference cases, a remark can be made on this case, highlighting a limitation of the proposed approach. Looking at the 150la case, it can be seen how by employing two v_∞ leveraging manoeuvres, the trajectory duration is brought back to almost exactly one year (i.e. a perfect 1:1 resonance). As the discussed approach only considers one VILT per arc, this is not a possibility that is investigated in the initial guess, and a slight deviation is generated. This reduces the freedom on the crank angle, which is not an issue in this case, but could exclude some interesting trajectories if considerable changes in inclination were needed. During the optimisation process this effect is partially mitigated by considering a zero Δv manoeuvre at the non-leveraging apse point.

Good correspondence is also obtained on the Moon-Earth flyby geometry. Figure 14 shows the process of retrieving the solution for 150lb's LEGA: starting from the corresponding EGA, a first guess is generated for the double swing-by's geometry. The guess is then optimised and compared with the reference solution:

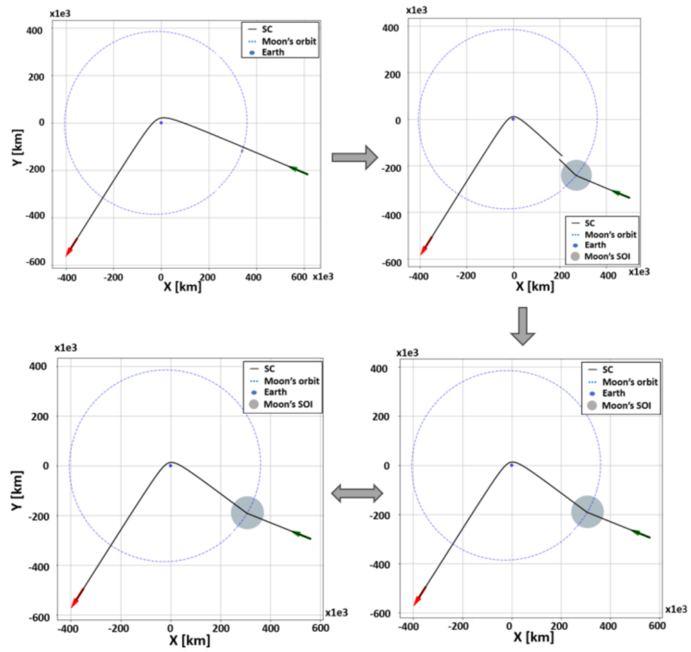

Figure 14 – tool validation on 150lb's LEGA

The sequence 150l0a is however not found in this first search. The circled alternative in Figure 12 represents its Earth-only equivalent, but when adding a Lunar Gravity Assist, the tool prioritized the closest alternative, as discussed in section 3.3.1. Consequently, starting from an Earth Gravity Assist on 14/08/2023, the tool explored the suboptimal possibility of performing a LEGA on 04/08/2023, overlooking a better option available on 01/09/2023. Given the difficulty in predicting a priori which alternative will yield better results, the algorithm is re-applied to account for both encounters. Through this approach, 150l0a was retrieved with a similar degree of accuracy. It is worth noting that, as was previously anticipated, the Earth-only sequence from which 150l0a stems features a high-altitude flyby, with perigee significantly higher than the Earth-Moon distance. Figure 15 shows the retrieved flyby geometry.

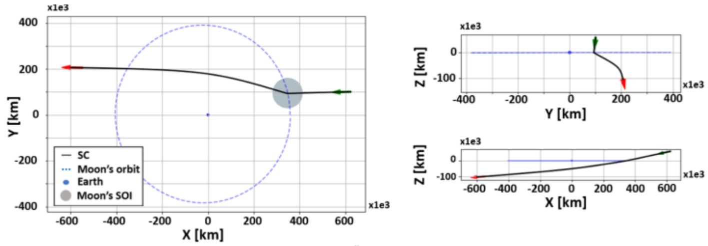

Figure 15 - 150l0a, retrieved LEGA

In accordance with the assumptions, the flyby has limited effect on the direction of the v_∞ vector, as the effects of the Earth and the Moon counteract each other. Although not considered in the initial guess, small out of plane components are introduced by the optimiser.
One last remark shall be made for this case, as it 150l0a has slightly different Patching Point conditions than 150la and 150lb, in particular the epoch, which is shifted by almost one week. This shows another limit of the method: considering the Patching Point as immutable, the problem is made stiffer, penalising some solutions. While this is still appropriate for the purpose of the presented work, the quality of the solutions could benefit by optimising the resonant trajectory up to the first encounter with a different body (Venus, in this case). The cost would be in computational time for the optimisation.

5. Conclusions

An algorithmic solution is proposed for the identification of promising Earth-resonant sequences that, starting from Launch, lead to the starting point of a given interplanetary transfer. These sequences include full resonant transfers, pseudo resonances, pi transfers and v_∞ leveraging manoeuvres.
By using a crank and pump parametrisation, the tool takes advantage of the combinatorial nature of resonance problems (the pump angle) and reduces the continuous components (the crank angle) to discrete. This enables a systematic investigation of the solution space in which resonant solutions exists. The problem can be further simplified by taking into account the specific application case, cutting out higher resonance ratios.
The possibility of Moon-Earth gravity assists within the retrieved solutions is considered, and a preliminary design is provided for the double flyby. The main design assumption for this process is that the LEGA's geometry is "similar" to that of the originating EGA for an observer outside of the Earth's sphere of influence, exception made for the increase in infinite velocity with respect to the planet. Special care is taken to handle boundary cases for high altitude EGAs. The generated trajectories are numerically optimised in full ephemeris through ESA's flight dynamics software Godot.
The proposed approach was tested on the interplanetary sequence for JUICE and proved to be able to retrieve the considered alternatives with a good degree of approximation, and to provide the trajectory designer with an overview of feasible and promising solutions, that can then be further analysed individually.

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