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A Markov chain-based approach to model the variance of times-to-failure and times-to-repair in manufacturing systems

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Abstract

The development of manufacturing systems with high level of automation is thrust by high-volume demand for heterogeneous engineered products. The paper focuses on the usage of Phase-type distributions in the description of reliability parameters, both times-to-failure (TTFs) and times-to-repair (TTRs), for a workstation with several failure modes. Differently from classical analytical models based on exponential distributions, the variance of reliability parameters can be exactly captured, allowing a sounder performance evaluation of the production system in which the workstation operates. While state-of-the-art research works adopt single-station models accounting for variance of TTR and/or TTF of a single failure mode, the presented model framework can capture the variance of TTRs and TTFs of all workstation failure modes, or only a portion of them. The formalized approach has been validated against a simulator replicating the workstation behavior, grounding on data acquired from the field. The application on an industrial case study showed the numerical impact of accounting for the actual variance on performance evaluation, exploiting an asynchronous continuous model of two machines-one buffer line, with finite buffer capacity and deterministic processing times. Further developments may concern the integration of the model in Markov chain-based analytical models of longer manufacturing lines.

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Keywords: Manufacturing systems; Analytical modeling; Markov chains; Performance evaluation; Phase-Type distributions.

1. Introduction

Modern manufacturing systems are updating with cutting-edge technologies, both in process execution and in process management and analysis. Industry 4.0 paradigm inspires engineers to increasingly employ data-driven techniques enabled by cloud computing tools and advanced sensoring of shopfloor area. The integration of corporate information with operations information is eased by the double-sided nature of emerging CPPS (Cyber-Physical Production Systems), flanking distributed real-time data acquisition and intelligent data management, based on historical databases and higher-level enterprise information [1]. This infrastructure, provided with high computational capabilities, often interacts with manufacturing systems which are reconfigurable or changeable, since dealing with variable and uncertain demand scenarios. Operations management aims at process optimization at different levels of detail, entailing the convenience of diversified modeling strategies. The main problems for performance of manufacturing operations are variability and randomness [2], which need to be tackled with proper techniques. Despite the better accuracy of discrete-event simulation, model building phase and transparency still remain major issues – in fact, simulation is mainly employed in testing manufacturing system configuration alternatives. Synthetic analytical modeling provides a clear and formal description of the system with good results in terms of precision and readability, with the possibility of updating the model with new modules or parameters, enhancing continuous improvement [3]. Focusing on Markov chain-based formulation, the related literature is huge and diversified [4], especially developed on performance evaluation. Models could be with continuous flow or discrete flow, with the former analytically depicted in [5], in the common case of a two-machine line with finite buffer

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capacity. This model allows to relax more assumptions with respect to reality than the discrete model. With its discrete approximation, authors in [6] reached a significantly better performance estimation of the average buffer level in the case of buffer sizes equal up to a few units. The approximation is based on the discretization of the limiting states, i.e., the ones corresponding to empty/full buffer, managed through a threshold-based mechanism [7]. Extending the view to longer manufacturing lines, the integration of quality control in the production stages and the evaluation of specific maintenance strategies are explored in [8] and [9], respectively. In [10], authors demonstrate the convenience of explicitly consider the different failure modes with their related distributions. The main issue of the larger part of models in literature is that they ground on exponential approximation, limiting the representation of the actual distribution. Basically, the variance is constrained to the mean of times. Phase-type distributions (PH) overcome this limitation thanks to their capability of combining more exponential distributions, and their natural compatibility with markovian representation [11].

The focus of this paper is on the Markov chain-based models for performance evaluation of production systems. In particular, the novel contribution of the proposed approach concerns the enlargement of the capability of capturing the variance of times-to-failure (TTFs) and times-to-repair (TTRs) in a workstation with multiple failure modes, modeled with a continuous-time Markov chain (CTMC). State-of-the-art modeling techniques adopt PH distributions for description of TTR and/or TTF of a single failure mode workstation, emphasizing the greater numerical impact of TTR variance on performance evaluation. The results coming from the application of this comprehensive method – able to describe the variance of parameters of multiple failure modes – highlight the numerical impact of variability in TTFs, as well.

2. Adopted phase-type distributions

This section reports the details about the employed PH distributions, with the advantages and limitations they carry.

The considered family of distributions is the one of acyclic continuous PH distributions (APH). They describe the time-to-absorption of a CTMC and are characterized by an upper triangular transition rate matrix, thus excluding the possibility of loops among phases. The order of the APH distribution is n and it is lower limited by the squared coefficient of variation (SCV) of the variable described.

$$n \geq \frac{1}{SCV} \tag{1}$$

For PH fitting, the method applied was the moment matching, giving the first two central moments as input. Thus, the distributions used in the analysis have the same first two moments of the dataset, without aiming at approximating the shape of the actual distribution - in manufacturing applications, approximation to the second order produces negligible errors in performance evaluation. For the purpose, functions illustrated in [12] have been adopted. Figure 1 depicts two cases of APH distributions fitted with moment matching method, depending on SCV. As stated in (1), lower variance requires a larger number of phases, which exactly correspond to the underlying CTMC states.

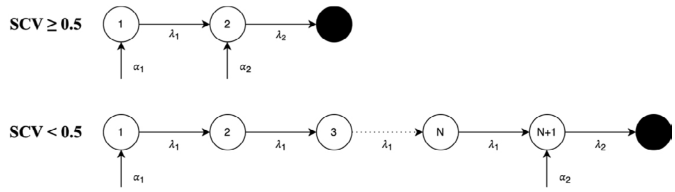

Fig. 1. Number of phases depending on SCV.

The output of the fitting is a set of branch probabilities and transition rate matrices describing APH distributions having at maximum two branches. In most manufacturing systems applications, e.g., for automated assembly lines, TTRs have lower variance than the exponential one, suiting fine to Erlang model and finding many industrial applications. Despite MTTR having major impact on transfer lines performance [2], as well as TTR variability, very large variance in TTFs could affect throughput as well – this is the main point to be demonstrated by the paper. The choice of continuous APH distributions, rather than discrete APH ones, is more convenient in heavy-tailed distributions, i.e., distributions with greater variance than the exponential one [13]. In fact, the model outlined in the paper will allow to account for two phases for the modeling of TTFs of each failure mode, at maximum. The next section describes the assumptions, the logic and the main features of the CTMC mimicking the reliability behavior of a single workstation with multiple failure modes.

3. Model features

The single non-deteriorating workstation model proposed in this paper introduce the possibility to account for generally distributed TTFs of multiple failure modes. The main advantage is to model a manufacturing system with tens of failure mechanisms, and almost all the variability included in failure and repair times. Performance evaluation of systems with a huge set of mechanical and electrical equipment involved in the technological process may be biased without the consideration of the actual variance in times registered by on-field sensors. Starting from the CMTC of a single-up multiple-down machine model, accounting for exponential TTFs and TTRs, the proposed approach increases the number of operative states of the Markov chain, injecting memory in the manufacturing system operations by means of APH distributions. The main assumption of this approach is that the failure modes are independent one of another. Table 1 outlines the admitted input for the model usage, which possesses a multi-dimensional structure for the "up" portion, i.e., the one carrying memory of the TTFs.

Table 1. Input distributions accepted by the model formalism.

Parameter	Allowed distributions	SCV range
TTF	APH(2) Exponential	SCV ≥ 0.5
TTR	APH(n) Erlang Exponential	Unlimited

Starting from a simple single-up single-down machine, elementary cases with APH(2) distributed TTF are illustrated in Figure 2. To extend to the case of a multiple-down machine with *ns* failure modes having APH(2) distributed TTFs, is necessary to define ordered and structured rules for relationships among elementary states, whose number is tending to explode as *ns* increase. To visualize the elaborated structure, Figure 3 depicts a single machine with three failure modes, each of them with exponentially distributed TTR and APH(2) distributed TTF.

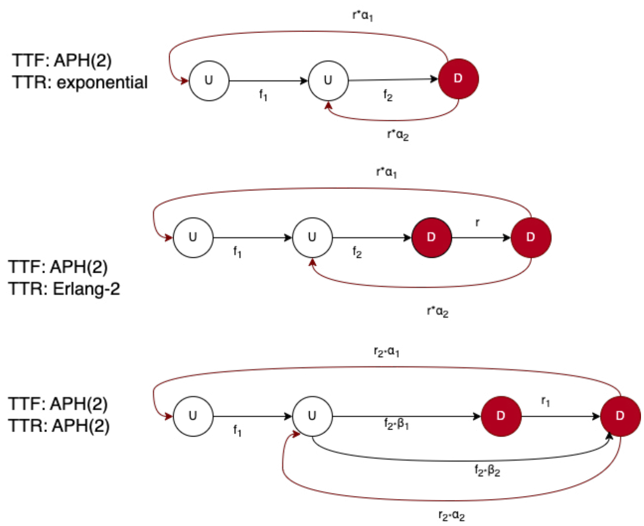

Fig. 2. Elementary cases for a single-down machine with APH(2) distributed TTFs.

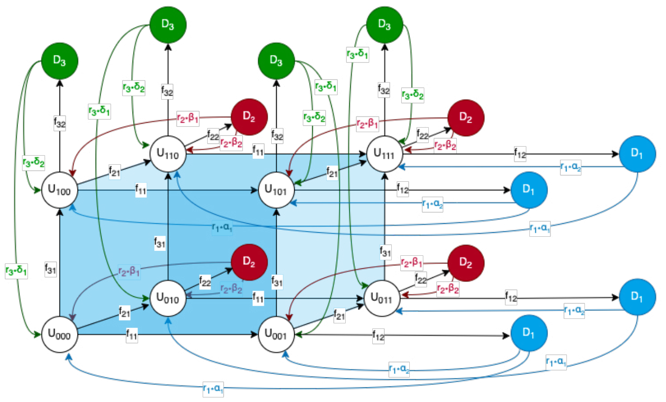

Fig. 3. Intuitive graphical example of model proposed.

The representation clearly shows the substantial complexity, even for a simple behavior. In the case of Figure 3 ($ns = 3$), the "up" portion of the Markov chain exhibits a cubic structure – as *ns* increases, the dimensionality proportionally does, making graphical representation too cumbersome. The formalization of the rules for states connections is carried out by means of a binary indexing system for "up" elementary states (since the size of APH distributions modeling TTFs is equal to two). Referring to Figure 3:

- U_{011} is the "up" state referring to first stage according to the third failure mode, and to second stage according to the first and second failure modes.
- U_{101} is the "up" state referring to first stage according to second failure mode, and to second stage according to the first and third failure modes.

The description of the markovian structure of the drum line allows the modeler to build the overall transition rate matrix **Q** algorithmically, eventually enabling the solution of the steady state problem $\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}$, where $\boldsymbol{\pi}$ is the row vector of steady state probabilities of the elementary states. The size of the problem is determined as follows.

$$\#UP = 2^{ns}$$
$$\#DOWN = \sum_{j=1}^{N_{cat}} N_j \cdot R_j \qquad (2)$$

According to (2), the total number of up and down states determines the size of the **Q** matrix, being:

- N_{cat} the number of failure modes,
- R_j the number of elementary down states of failure mode *j*,
- N_j a coefficient depending on the TTF distribution of failure mode *j*: it is 2^{ns-1} if TTF_j distribution is APH, whereas it is 2^{ns} if TTF_j distribution is exponential.

The number of failure modes and the parameters' distributions is intuitively restricted by the complexity of the line model in which the proposed single-station model is implemented, and further depends on the computational capability of the calculator. This modeling approach has been validated against a simulator reproducing the behavior of a single machine with multiple failure modes characterized by empirical time distributions with variance different from the exponential. Results are summarized in Figure 4, where two characterizations of the failures have been evaluated separately.

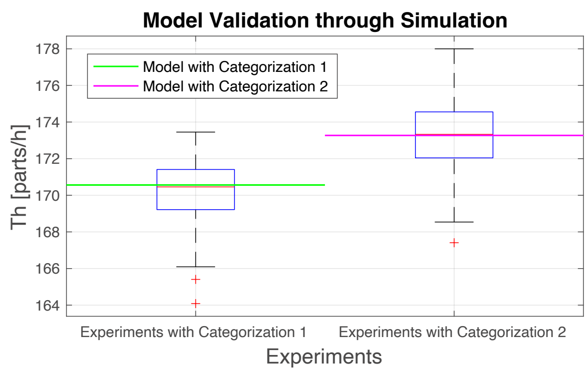

Fig. 4. Model validation against a simulator.

The reference performance indicator adopted for validation is the average hourly throughput (considering constant nominal processing time). Categorization 1 shows a lower average throughput since it presents more failure modes and empirical distributions of each failure mode are not as rich in data points as the second categorization. The maximum registered percentage error is below .2%, validating the modeling approach.

4. Numerical evaluation

The formalization of the modeling approach has been followed by an application on a real industrial case. In particular, the addressed manufacturing system is located in a production facility of a multinational operating in the white goods industry. The system is an automated mixed forming-assembly line fabricating drums for washing machines, namely drum line, as depicted in Figure 5.

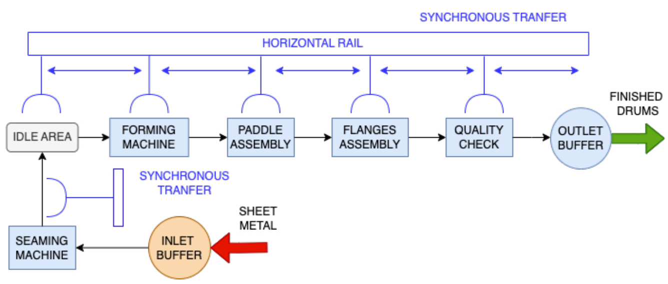

Fig. 5. Drum forming-assembly line schematics.

Welding, forming, assembly, labeling and quality checking operations are automatically performed by robotized stations. The non-tracked work-in-process is moved forward by means of an in-line synchronous transfer system, which permits to have each single semi-finished product processing to a station, so the WIP of the line is constant and equal to 6 parts. The transfer arms cannot move independently, so a stop at a station implies the whole drum line to stop. Each stage of the drum line has its own deterministic cycle time, and differences in cycle times between stages are absorbed by the transfer system, which provides a deterministic cycle time to the line, making the whole drum line an asynchronous transfer line without intermediate buffer between stages. These features permit to concentrate the drum line stations in a single-workstation model with constant cycle time, accounting for all failure modes of actual production stages. Failure modes have been identified, TTFs and TTRs average value and variance have been estimated from three months of net production data, and depending on parameter SCV the choice of the distribution and the fitting have been carried out.
The proposed modeling approach applied at the drum line can capture the effect of the variance of parameters when implemented in a line model – for the assessment, the continuous two-machine line model with multiple up and down states formalized in [14] has been adopted. The drum line has been coupled with the activity of a human operator (or robot) unloading the finished workpiece, modeled as a single-up single-down server, as depicted in Figure 6. The intermediate buffer size is varied in the analysis.

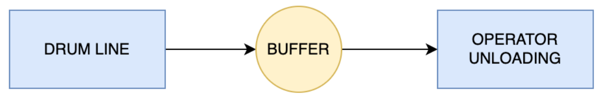

Fig. 6. Analyzed continuous two-machine line.

An extensive experimental campaign has been performed, eventually confirming the results found in research literature: increasing variance implies lower line average throughput. While research has widely demonstrated this for TTRs variance, the approach presented allows to isolate the contribution of TTFs variance, in a flexible modeling framework able to capture variance coming from multiple failure modes. The impact of high-frequency micro stops (MS), having relatively short repair times, has been showed. Being the SCV of their TTFs greater than 1, the negative effect of high variability in TTFs has been highlighted. Experimental setup is described in Table 2. All failure modes considered have higher variance than the exponential, but only MS variance has a tangible impact on average throughput estimation, due to small MTTF.

Table 2. Model setup to show the effect of TTFs variance effect on performance evaluation.

Machine	Processing rate μ[p/h]	Isolation efficiency e[-]	
Upstream	1/CT	0.8011	
Downstream	1.05/CT	0.8311	
Failure Mode	MTTF [hh:mm:ss]	SCV of TTF	MTTR [hh:mm:ss]
1	02:10:51	1.5621	00:03:33
2	12:43:55	4.5746	00:22:29
3	21:40:25	1.6932	00:16:26
MS	00:17:29	3.2250	00:02:03
4	06:58:51	2.8370	00:25:40

Two set of experiments have been performed separately, to isolate the contribution of TTFs(MS) variance on throughput estimation (in both sets all TTRs have been described with the same distributions):

- *First set*: TTFs of failure modes from 1 to 4 are modeled with APH(2) distributions, whereas TTFs(MS) are considered exponential.
- *Second set*: TTFs of failure modes from 1 to 4 and TTFs(MS) are taken as APH(2) distributions.

Figure 7 shows the results of such analysis. Throughput curve (vs buffer size) of the first set is higher than the throughput curve of the second set, since variance of TTFs(MS) is not included in the model.

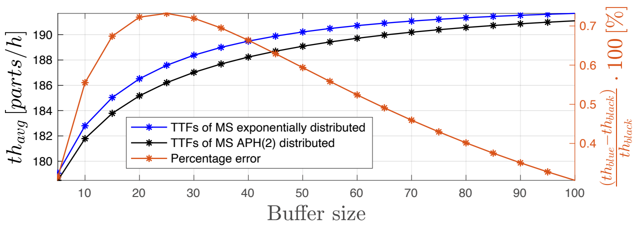

Fig. 7. Isolated effect of TTFs(MS) variance on throughput.

For medium-sized buffers, not to consider this variability can produce an overestimation error higher than .7%. To further deploy the capabilities of the novel modeling approach, the drum line has been evaluated accounting for all failure modes present in the actual dataset, comparing it to the simple multiple-down exponential approach. Figure 8 reports the percentage error of the average line throughput between the two aforementioned approaches (the pure exponential model overestimates throughput).

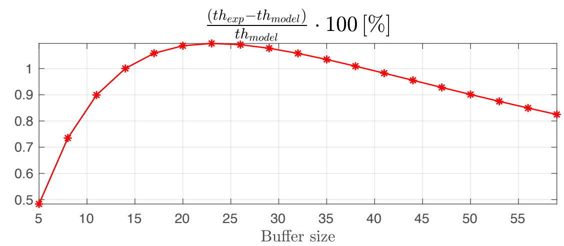

Fig. 8. Percentage deviation in throughput estimation between proposed approach and exponential approach applied to the drum line.

Both TTFs and TTRs have been modeled with mixed APH(2)-exponential distributions, depending on SCV deviation with respect to one. The total number of reconstructed failure modes is 18 (using APH(2) distributions for TTFs of 9 failure modes, and for TTRs of 17 failure modes), with a total number of elementary states exceeding $2 \cdot 10^4$. For the specific case study, within a reasonable buffer range, the exponential approximation produces an overestimation of more than 1% of the steady-state throughput.

5. Conclusions

A Markov chain-based modeling approach able to capture the actual variance of the failure and repair times' distributions has been presented. Exploiting the flexibility of APH distributions, a multiple-down continuous-time Markov model of a single station can consider the actual data variability to better estimate the performance of the overall manufacturing system.

An industrial case of relevance showed that the error in using the exponential variance approximation in both TTFs and TTRs for a workstation operating in a two-machine line depends on the buffer size, and on the variance compared to the average value. With the application of the presented method, it is possible to numerically quantify the effect on performance of failure modes parameters' variance, and to better evaluate the performance indicators in case a single machine with multiple failure modes is operating in the system. It has been confirmed that parameters with SCV > 1 negatively affect the throughput of the line, whereas parameters with SCV < 1 positively affect the throughput of the line. Complex automated stations producing non-tracked parts can benefit from this approach by improving soundness in performance evaluation. Future developments could concern the employment of the proposed single-station model in other approximate analytical methods, as methods treating the discrete approximation of continuous model for serial lines composed by more than two servers.

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