# A data-driven approach for integral sliding mode control design

Giorgio Riva, Gian Paolo Incremona, Simone Formentin, and Antonella Ferrara

***Abstract*— In this paper, in light of the recent advances in direct data-driven control, a novel approach for the design of integral sliding mode control (ISMC) in the case of unknown model of the plant is devised. The proposed data-driven integral sliding mode control (DD-ISMC) relies on the so-called virtual reference feedback tuning (VRFT) approach to select the ideal control component of the classical ISMC law. The VRFT enables the selection of the ideal controller by exclusively exploiting data from experiments, through the solution of a global model-reference optimization problem. Then, the reference model exploited by the VRFT approach is employed in the design of the integral sliding variable, in place of the unknown nominal model of the plant. In the paper, the proposal is theoretically analyzed, and its effectiveness is illustrated in simulation.**

***Index Terms*— Integral sliding mode control, data-driven control, data-based tuning, model-reference control.**

## I. Introduction

Designing sliding mode controllers (SMCs) is of great interest in the research community due to their robustness in front of a wide class of uncertainty terms often affecting the plants [1]. In their classical versions, SMCs can be considered *model-independent*, only requiring the knowledge of the bound on the uncertainties. However, among many variants of SMCs, the so-called integral sliding mode control (ISMC) in [2] can be classified as a *model-based* approach. This strategy is indeeed characterized by a sliding variable with an integral term based on the nominal (i.e., without uncertainty) dynamics of the plant. Differently from [3], where an integral term arising from a decentralized structure of an interconnected system is designed, in [2], the presence also of the initial condition enhances the robustness property of classical SMCs since the initial time instant, thus removing the so-called reaching phase.

Many works have been published in the last two decades about ISMC, where the objective is to design a sliding variable with the so-called *transient function* depending on the nominal dynamics of the plant, stabilized by an *ideal control law*. In the ISMC law, such an ideal controller is added to a discontinuous control component aimed at rejecting the disturbance terms. This feature, together with the enforcement of a sliding motion since the initial time instant, provides a direct tool to simplify the design of robust control strategies. For instance, in [4]–[6] the ISMC has been used with a model predictive control (MPC) playing the role of the ideal law, designed on the basis of a disturbance-reduced (or, under certain assumption, even disturbance-free) plant model, giving rise to a new robust MPC scheme.

All these results can be classified as model-based ISMC laws. However, sometimes only partial or no knowledge of the plant model is available, and this substantial aspect calls upon alternative *model-independent* solutions. Among the possibilities, neural-networks (NNs) based ISMC laws have been proposed in [7]–[10], where NNs are adopted to identify the model dynamics in order to design the transient function. As downsides, these strategies require also the knowledge of the bounds of the weights of the NNs, and make use of the estimated model in the design of the ideal control law, generating further uncertainty. These problems require resorting to different data-based solutions to avoid the model estimation, while ensuring stability guarantees.

Spurred by these motivations, we propose a novel variation of the ISMC scheme in [2], where the ideal law is designed relying on input/output measurements (*data-based design*) without requiring a mathematical description of the plant. Specifically, the so-called virtual reference feedback tuning (VRFT) approach [11] is adopted to design the ideal control law and the integral sliding variable, exploiting the VRFT reference model in place of the unknown nominal model of the plant. The VRFT is a *direct* data-based method meaning that the controller is directly selected without preliminary using data to identify the plant model, differently from *indirect* NNs-based approaches. The controller parameters are achieved by solving an optimization procedure minimizing the 2-norm of the difference between the closed-loop transfer function and the reference model employed in the standard ISMC law to design the ideal controller. In the last decades, the VRFT method has been deeply applied and investigated [11]–[15], but to the best of our knowledge, never combined with SMC laws. In this paper, we deal with continuous-time single-input single-output (SISO) linear time invariant (LTI) systems with bounded matched disturbances, without any knowledge of the plant model.

The used variables and operators are mostly standard. The transpose of a matrix $T$ is denoted by $T'$. The sets of non-

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negative reals is denoted as $\mathbb{R}_+$. Signals in the time and Laplace domains are indicated by lower- and upper-case letters, respectively, like $x(t)$, or just $x$, and $X(s)$. Linear systems in the time domain are denoted by capital letters with no argument, like $G$, and $G(s)$ stands for the corresponding transfer function. We indicate the 2-norm of a signal $x(t)$ as $\|x\|_2 = \int_{-\infty}^{+\infty} x'(t)x(t)\mathrm{d}t$, and $\mathcal{L}_2$ to be the space of signals with finite 2-norm.

## II. Preliminaries

In this section, some preliminaries on ISMC and VRFT strategies will be recalled for the readers' convenience.

### A. ISMC law

According to [2], the following perturbed LTI system is considered:

$$\mathcal{P}\,:\,\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) + Bd_u(t)\\ y(t) = Cx(t)\end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, the pair $(A, B)$ is controllable, and $CB > 0$. The signal $d_u(t) \in \mathbb{R}$ represents the whole load (i.e., matched) disturbance, and such that the following assumption holds.

$\boldsymbol{\mathcal{A}_1}$: The disturbance $d_u$ is such that $|d_u| < k$, where $k$ is a known positive scalar.

Let $P(s)$ the transfer function from $u$ to $y$ associated with the plant $\mathcal{P}$. Given the reference signal $r \in \mathbb{R}$ such that $e = r-y$, suppose that there exists a feedback control law $u = u_0(e)$, as output of a controller $\mathcal{R}_0^\circ : e \mapsto u_0$, capable of stabilizing in a desired way (e.g., tracking the reference signal with a required accuracy) system (1) when the term $d_u \in \mathbb{R}^n$ is not present. Note that, in the following, the dependence of all the variables on time will be omitted, when obvious, for the sake of compactness.

Now, let $R_0^\circ(s)$ be the transfer function of the controller $\mathcal{R}_0^\circ$ and $(A_{R_0^\circ}, B_{R_0^\circ}, C_{R_0^\circ}, D_{R_0^\circ})$ its state-space representation. Then, the *ideal* closed-loop system can be written as

$$\mathcal{M}:\begin{cases}\begin{bmatrix}\dot{x}_\mathrm{o}\\ \dot{x}_{R_0^\circ}\end{bmatrix} = \underbrace{\left[\begin{array}{c|c} A-BD_{R_0^\circ}C & BC_{R_0^\circ}\\ \hline -B_{R_0^\circ}C & A_{R_0^\circ}\end{array}\right]}_{A_\mathrm{m}}\begin{bmatrix}x_\mathrm{o}\\ x_{R_0^\circ}\end{bmatrix} + \underbrace{\begin{bmatrix}BD_{R_0^\circ}\\ B_{R_0^\circ}\end{bmatrix}}_{B_\mathrm{m}} r\\ y_\mathrm{o} = \underbrace{\begin{bmatrix}C & 0\end{bmatrix}}_{C_\mathrm{m}}\begin{bmatrix}x_\mathrm{o}\\ x_{R_0^\circ}\end{bmatrix}\end{cases} \tag{2}$$

where $x_\mathrm{o}$ and $y_\mathrm{o}$ are the corresponding state and the output trajectories, $x_{R_0^\circ}$ is the controller state, while $M(s) = C_\mathrm{m}(sI - A_\mathrm{m})^{-1}B_\mathrm{m}$ is the transfer function of the closed-loop system $\mathcal{M}$ from $r$ to $y_\mathrm{o}$.

For plant $\mathcal{P}$ in (1), following [2], the ISMC input is designed as $u = u_0 + u_1$, where $u_0$ is the *ideal* component, previously indicated as the output of the controller $\mathcal{R}_0^\circ$, while $u_1$ is a discontinuous component aimed at rejecting the disturbance term $d_u$. Secondly, the so-called *sliding variable* is

$$\sigma = \sigma_0(x, r) + z \tag{3}$$

with $\sigma$, $\sigma_0(x, r)$, $z \in \mathbb{R}$, consisting of two components, $\sigma_0$ designed as the linear combination of states and reference, and $z$, called *transient function*, which induces the integral term. In the following, without loss of generality, we select $\sigma_0 = e$, similar to conventional SMC, with the associated *sliding manifold* defined as $\sigma_0 = 0$.

According to [2], the ISMC paradigm consists of defining an integral control feedback such that the so-called equivalent control [1] is $u_{1\mathrm{eq}}(t) = -d_u(t)$, $\forall\, t \geq 0$. Such a condition holds if

$$\mathcal{Z}\,:\,\dot{z} = -(\dot{r} - CAx - CBu_0(x, r)),\ z(0) = -\sigma_0(x(0), r(0)), \tag{4}$$

where $z(0)$ is determined so as to have $\sigma(0) = 0$. Specifically, designing the discontinuous component of the controller, namely $\mathcal{R}_1$, as

$$u_1 = \gamma \cdot \mathrm{sign}(\sigma), \tag{5}$$

with $\gamma \in \mathbb{R}_+$ large enough to dominate the worst realization of the disturbance $d_u$ (see $\boldsymbol{\mathcal{A}_1}$), by using the Lyapunov second method with $V = 0.5\sigma^2$ as the Lyapunov candidate, then the enforcement of a sliding mode since the initial time instant is proved.

In a nutshell, the ISMC algorithm allows to robustly achieve exact closed-loop reference model matching in presence of matched disturbances. As a downside, it requires the full knowledge of a plant model state-space representation $(A, B, C)$ to design both the ideal controller $\mathcal{R}_0^\circ$ and the transient function $z(t)$.

### B. VRFT

Before introducing the proposed DD-ISMC, some preliminaries on the adopted data-based approach, the so-called VRFT strategy, are recalled. Given the continuous-time setting of our work, we refer to [14] adapting the notation to be consistent with the previous sections.

VRFT is a prominent direct data-based control design technique, dating back to [11], where a parametric controller in a specified class $\mathcal{R}_{0,\rho}$ is tuned directly from open-loop input-output data, matching a desired closed-loop reference model without resorting to the identification of a model of the plant. As in [14], we consider the class of linearly parametrized controllers $\mathcal{R}_{0,\rho}$ defined as

$$R_0(s, \rho) = \rho'\beta(s), \tag{6}$$

where $\rho$ is the vector of tunable parameters, and $\beta(s)$ the vector of specified basis of transfer functions.

The starting point of VRFT is represented by the model-reference control problem defined in [14], where the optimal model-reference controller that minimizes the cost function

$$J_{mr}(\rho) = \int_{-\infty}^{\infty}\left|\left(\frac{P(j\omega)R_0(j\omega,\rho)}{1+P(j\omega)R_0(j\omega,\rho)} - M(j\omega)\right)W_{mr}(j\omega)\right|^2 \mathrm{d}\omega \tag{7}$$

is defined by

$$\rho_{mr} = \arg\min_{\rho} J_{mr}(\rho). \tag{8}$$

In (7), $P(j\omega)$, $M(j\omega)$, and $R_0(j\omega,\rho)$ represent the frequency behavior respectively of the plant model, the desired reference model, and the controller parameterized by $\rho$, namely the vector of optimization variables. As for $W_{mr}(j\omega)$, instead, it is a frequency shaping function, which can be freely tuned to improve the identification of the controller. To streamline the discussion, we recall here the definition of the ideal model-reference controller $\mathcal{R}_0^{\rm o}$ achieving perfect reference model matching, i.e., minimizing (7),

$$R_0^{\rm o}(j\omega) = \frac{M(j\omega)}{P(j\omega)(1-M(j\omega))}. \tag{9}$$

In the scenario where the plant model is unknown, (7) cannot be directly solved. To cope with this limitation, VRFT recasts (7) into a data-based controller identification problem, minimizing the cost

$$J_{vr}(\rho) = \int_{-\infty}^{\infty} \left(u_{\rm F}(t) - R_0(\rho)e_{\rm F}(t)\right)^2 {\rm d}t. \tag{10}$$

The optimal controller obtained by the VRFT approach is defined as

$$R_0(s,\rho_{vr}) = \rho_{vr}'\beta(s), \tag{11}$$

where

$$\rho_{vr} = \arg\min_{\rho} J_{vr}(\rho). \tag{12}$$

In (10), $u_{\rm F}$ and $e_{\rm F}$ are the filtered versions, through a filter $\mathcal{F}$, respectively of the input $u(t)$ of the open-loop data, and the so-called virtual error $e_{\rm v}(t)$. The latter signal is obtained from the output $y(t)$ of the open-loop data as

$$e_{\rm v}(t) = r_{\rm v}(t) - y(t), \tag{13}$$

where $r_{\rm v}(t) = M^{-1}y(t)$ is called virtual reference, and it is computed feeding the inverse of the desired reference model $\mathcal{M}$, whose transfer function representation is $M(s)$, with the measured output $y(t)$.

The main theoretical results of the VRFT strategy proposed in [14] can be summarized as follows. In the *noise-free setting*: (i) when the ideal model-reference controller (9) belongs to the chosen class $\mathcal{R}_{0,\rho}$, i.e., $R_0^{\rm o} \in \mathcal{R}_{0,\rho}$, it holds that $\rho_{vr} = \rho_{mr}$ and $R_0(s,\rho_{vr}) = R_0^{\rm o}(s)$; (ii) when the ideal model-reference controller (9) does not belong to the chosen class $\mathcal{R}_{0,\rho}$, i.e., $R_0^{\rm o} \notin \mathcal{R}_{0,\rho}$, still $\rho_{vr} \approx \rho_{mr}$ provided that the filter $\mathcal{F}$ is chosen so that $F(j\omega) = M(j\omega)(1 - M(j\omega))\frac{W_{mr}(j\omega)}{U(j\omega)}$, where $U(j\omega)$ is the spectrum of the signal $u(t)$. In the *noisy setting*, i.e., when the output data are corrupted by an additive noise $\eta(t) \in \mathcal{L}_2$, the *noise-free* solutions can be recovered using the instrumental variable (IV) $\xi(t)$ [14].

It is worth to highlight that the noisy scenario previously described is not restricted to the output case, and the results can be seamlessly applied for input disturbances, as in the ISMC framework described in §II-A. Indeed, any matched disturbance $d_u(t) \in \mathcal{L}_2$ can be equivalently represented by an output disturbance $\eta(t) \in \mathcal{L}_2$. This result holds both for stable and unstable plants. For the latter, the identification experiments must be performed in closed-loop with a preliminary stabilizing controller [11].

## III. Data-driven Integral Sliding Mode Control

We are now in a position to introduce the proposed DD-ISMC strategy. Under the assumption that the plant model is not available, the standard ISMC law described in §II-A cannot be employed for two reasons: (i) the ideal model-reference regulator $\mathcal{R}_0^{\rm o}$ achieving exact reference model matching cannot be computed, and (ii) the expression of the transient function $z(t)$ in (4), which depends explicitly on matrices $(A,B,C)$, is not applicable. In this section, we describe how these two limitations can be overcome in a data-based framework, leading to the formulation of the proposed DD-ISMC law.

For the design of the controller $\mathcal{R}_0 : e \to u_0$, we leverage the VRFT approach described in §II-B, which approximately solves the model-reference control problem in (7) given the transfer function representation $M(s)$ of the *ideal*, desired, closed-loop model in (2), and based only on a set of input-output data from the plant. Regarding the formulation of the transient function $z(t)$, instead, we show that the expression (4) can be exactly rephrased, in an *equivalent sense*, solely in terms of the desired *ideal* closed-loop system (2). Indeed, when the discontinuous component $u_1$ of the standard ISMC law is designed according to (5), two main results hold: (i) the closed-loop system affected by the matched disturbance $d_u$ behaves, in an *equivalent sense*, exactly as the *ideal* closed-loop system (2), i.e., $y(t) = y_{\rm o}(t)\ \forall t$, and (ii) $\sigma(t) = 0\ \forall t \geq 0$. As a direct consequence, from the definitions of $\sigma(t)$ in (3) and knowing that $\sigma_0(t) = e(t)$, we can write

$$z(t) = -e(t) = -(r - y(t)) = -(r - y_{\rm o}(t)), \tag{14}$$

meaning that the transient function can be expressed in *equivalent sense* as

$$z(t) = -(r(t) - Mr(t)), \tag{15}$$

which only depends on the *ideal* closed-loop dynamics and the reference signal. Since the formulation in (15) is independent of the plant state-space matrices, the proposed design philosophy becomes that of designing an integral control feedback such that the motion of the transient dynamics is exactly as in (15). However, the closed-loop realization $(A_{\rm m}, B_{\rm m}, C_{\rm m})$ defined as in (2) cannot be directly selected by the designer, but only the closed-loop input-output map $M(s)$ can be defined according to the VRFT strategy in §II-B. This means that one can define any possible *minimal* realization of $M(s)$, possibly of a lower order than that in (2), given by $(A_{\rm M}, B_{\rm M}, C_{\rm M})$, that is $M(s) = C_{\rm M}(sI - A_{\rm M})^{-1}B_{\rm M}$, such that condition (4) becomes

$$\mathcal{Z}_{\rm M} : \dot{z} \triangleq C_{\rm M}A_{\rm M}x_{\rm M} + C_{\rm M}B_{\rm M}r - \dot{r}, \quad z(0) = -e(0), \tag{16}$$

where $x_{\rm M}$ is the corresponding state variable.

Eventually, the proposed DD-ISMC scheme is reported in Fig. 1, highlighting on the top the VRFT loop to design the controller $\mathcal{R}_0$ based on input-output data, and on the bottom the ISMC scheme with the novel definition of the transient variable $z$ in (16). The whole control input is obtained as the

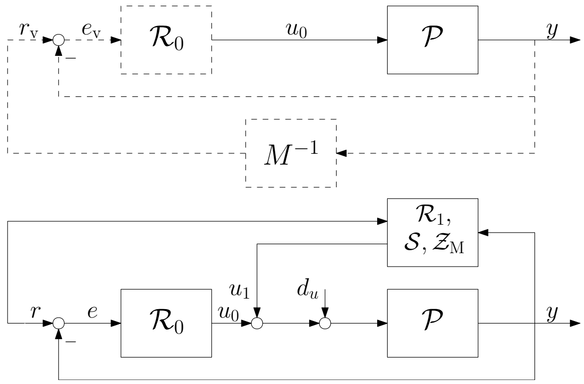


Fig. 1. The *virtual loop* with the real plant (top) and the proposed DD-ISMC scheme (bottom).

sum of the output of the regulator in (11) and (5), i.e.,

$$u = R_0(\rho_{vr})e + \gamma \cdot \text{sign}(\sigma), \tag{17}$$

which does not require any knowledge of the plant dynamics, apart from $\boldsymbol{\mathcal{A}_1}$. In analogy with the conventional ISMC, under specific conditions detailed in the next section, the closed-loop is stabilized by (17) following the desired reference model $\mathcal{M}$.

## IV. Stability Analysis

In this section, the main theoretical results associated with the proposed DD-ISMC approach are presented. In particular, the following lemmas are instrumental for the discussion.

*Lemma 1 (VRFT reference model matching [14]):* When the class of parametric controllers $\mathcal{R}_{0,\rho}$ is selected such that $R_0^{\rm o} \in \mathcal{R}_{0,\rho}$, the VRFT approach achieves perfect reference model matching, namely that $R_0(\rho_{vr}) = R_0^{\rm o}$, also in the presence of noise or disturbances in the data.

*Lemma 2 (VRFT reference model mismatch [14]):* When the controller $R_0^{\rm o}$ does not belong to the parametric class $\mathcal{R}_{0,\rho}$, the VRFT cannot achieve perfect model-matching, but still, it guarantees to return the optimal model-reference controller.

Now let us discuss the enforcement of the sliding motion since the initial time instant, according to the original philosophy in [2], even if the proposed DD-ISMC is totally blind to the plant dynamics. We start considering the general situation of Lemma 2, which represents the common situation in practice. In this case, perfect reference model matching cannot be achieved, meaning that the actual closed-loop will be different from the desired one. To streamline the discussion, we label as $(\widetilde{A}_m, \widetilde{B}_m, \widetilde{C}_m)$, with state variables $\widetilde{x}_m$, the closed-loop state-space realization obtained by coupling the unknown plant with the VRFT ideal controller. Note that, the related closed-loop transfer function representation generally differs from the reference one, i.e., $\widetilde{M}(s) \neq M(s)$.

*Lemma 3 (Integral sliding motion):* Given the plant $\mathcal{P}$ in (1) with $d_u \neq 0$, controlled via $u = u_0 + u_1$ as in (17), and sliding variable selected as $\sigma = \sigma_0 + z$, with $z$ as in (16), if $\boldsymbol{\mathcal{A}_1}$ holds and the control gain $\gamma \in \mathbb{R}_+$ is such that

$$\gamma > k + (CB)^{-1}|C_{\rm M}A_{\rm M}x_{\rm M} - \widetilde{C}_m\widetilde{A}_m\widetilde{x}_m + (C_{\rm M}B_{\rm M} - \widetilde{C}_m\widetilde{B}_m)r|, \tag{18}$$

then a sliding mode $\sigma(t) = 0$ is enforced $\forall\, t \geq 0$.

*Proof:* The proof follows by selecting the Lyapunov candidate $V = 0.5\sigma^2$. Given $\boldsymbol{\mathcal{A}_1}$, and that $CB > 0$, the first time derivative of $V$ is

$$\begin{aligned}
\sigma\dot{\sigma} &= \sigma(\dot{e} + \dot{z}) \qquad (19)\\
&= \sigma(\dot{r} - \widetilde{C}_m\widetilde{A}_m\widetilde{x}_m - \widetilde{C}_m\widetilde{B}_m r - CB(\gamma\text{sign}(\sigma) + d_u)\\
&\quad + C_{\rm M}A_{\rm M}x_{\rm M} + C_{\rm M}B_{\rm M}r - \dot{r})\\
&\leq CB(-\gamma + k + (CB)^{-1}|C_{\rm M}A_{\rm M}x_{\rm M} - \widetilde{C}_m\widetilde{A}_m\widetilde{x}_m\\
&\quad + (C_{\rm M}B_{\rm M} - \widetilde{C}_m\widetilde{B}_m)r|)|\sigma|.
\end{aligned}$$

Then, according to the *reaching condition* [1], selecting the gain as in (18), there exists $\bar{t} \geq 0$ such that $\sigma(t) = 0$ for any $t \geq \bar{t}$. Since $\sigma(0) = 0$, then $\sigma(t) = 0$, $\forall\, t \geq 0$. ■

*Remark 4.1 (ISMC gain):* Condition (18) means that the gain $\gamma$ has to dominate the worst disturbance realization, as in conventional ISMC, and also the discrepancy between the actual evolution and the one of the nominal reference model. Nevertheless, it is worth noticing that, although a price is paid in terms of control effort, the controller is totally blind to the plant model and adaptive approaches could be possibly exploited to reduce such conservativeness. Eventually, we highlight that, when the condition of Lemma 1 is satisfied, (18) simplifies in

$$\gamma > k + (CB)^{-1}|C_{\rm M}A_{\rm M}x_{\rm M} - \widetilde{C}_m\widetilde{A}_m\widetilde{x}_m| \tag{20}$$

since the perfect reference model matching is achieved, i.e., $\widetilde{M}(s) = M(s)$, meaning that $C_{\rm M}B_{\rm M} = \widetilde{C}_m\widetilde{B}_m$. This result implies that the term, which $\gamma$ has to dominate, does not depend on the reference $r$, but only on the possible discrepancy introduced by the minimal state-space realization $(A_{\rm M}, B_{\rm M}, C_{\rm M})$ employed in (16). ▽

We are now in a position to introduce the main result.

*Theorem 4 (Model-reference matching):* Given the plant $\mathcal{P}$ in (1) with $d_u \neq 0$ such that $\boldsymbol{\mathcal{A}_1}$ holds, controlled via $u = u_0 + u_1$ as in (17) with (18), and sliding variable selected as $\sigma = \sigma_0 + z$, with $z$ as in (16), then it holds that

$$y(t) = \widetilde{C}_m\widetilde{x}_m(t) = C_{\rm M}x_{\rm M}(t) = y_{\rm o}(t),\ \forall\, t \geq 0, \tag{21}$$

that is a perfect reference model matching of the plant output is ensured.

*Proof:* The proof follows from Lemma 3. Indeed, from the definitions of both the sliding variable $\sigma(t)$ in (3) and the transient variable $z(t)$ in (15), when the sliding mode is enforced, i.e., when $\sigma(t) = 0\ \forall t \geq 0$, it holds $\forall t \geq 0$ that

$$e(t) = -z(t) = r - Mr = r - C_{\rm M}x_{\rm M}(t), \tag{22}$$

which, given that $e(t) = r(t) - y(t) = r(t) - \widetilde{C}_m\widetilde{x}_m(t)$, implies that

$$y(t) = \widetilde{C}_m\widetilde{x}_m(t) = C_{\rm M}x_{\rm M}(t) = y_{\rm o}(t), \tag{23}$$

which completes the proof. ■

## V. Illustrative Example

In this section, the proposed DD-ISMC scheme is evaluated on a numerical example relying on the yaw-rate tracking problem of a linear bicycle vehicle model. The employed model can be expressed in the following LTI second-order state-space representation

$$\dot{x} = \begin{bmatrix} -\frac{C_\mathrm{f}+C_\mathrm{r}}{Mv} & -1-\frac{C_\mathrm{f}L_\mathrm{f}-C_\mathrm{r}L_\mathrm{r}}{Mv^2} \\ -\frac{C_\mathrm{f}L_\mathrm{f}-C_\mathrm{r}L_\mathrm{r}}{J_z} & -\frac{C_\mathrm{f}^2L_\mathrm{f}+C_\mathrm{r}^2L_\mathrm{r}}{J_z v} \end{bmatrix} x + \begin{bmatrix} \frac{C_\mathrm{f}}{Mv\mu_\mathrm{sw}} \\ \frac{C_\mathrm{f}L_\mathrm{f}}{J_z\mu_\mathrm{sw}} \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x \tag{24}$$

where the state vector is composed by the side-slip angle $\beta$ and the yaw-rate $\dot{\psi}$, i.e., $x(t) = \begin{bmatrix} \beta(t) \\ \dot{\psi}(t) \end{bmatrix} \in \mathbb{R}^2$, the latter being the output variable, $y(t) = \dot{\psi}(t) \in \mathbb{R}$, while the input variable is the steering-wheel angle $\delta_\mathrm{sw}(t)$. The employed physical parameters, which are representative of a real sport car, are reported in Table I. We consider a matched disturbance acting on the input steering wheel angle with the following shape,

$$d_u(t) = A_{d,1}\sin(2\pi f_1 t - \varphi_1) - A_{d,2}\cos(2\pi f_2 t - \varphi_2) \tag{25}$$

where $A_{d,1} = A_{d,2} = 1$, while $f_1$, $f_2$ are independent uniformly distributed random variables inside the range $[2,5]$, and $\varphi_1$, $\varphi_2$ are independent uniformly distributed random variables inside the range $[0, 2\pi]$. In this way, we achieve different realizations of $d_u$ between identification and validation, while maintaining the same bound,

$$|d_u| \leq A_{d,1} + A_{d,2} = 2 \tag{26}$$

Finally, we select a sensible reference model for the application, namely the first-order low-pass filter with unitary gain

$$M(s) = \frac{2\pi f_M}{s + 2\pi f_M} \tag{27}$$

with $f_M = 2$ Hz.

TABLE I

Bicycle model parameters.

| | | | |
|---|---|---|---|
| vehicle mass | $M$ | 1728.6 | kg |
| vehicle yaw moment of inertia | $J_z$ | 2297 | $\mathrm{kg\,m^2}$ |
| front tires cornering stiffness | $C_\mathrm{f}$ | $17.9890 \times 10^4$ | $\mathrm{N\,rad^{-1}}$ |
| rear tires cornering stiffness | $C_\mathrm{r}$ | $31.9810 \times 10^4$ | $\mathrm{N\,rad^{-1}}$ |
| front tires distance from c.o.m. | $L_\mathrm{f}$ | 1.4542 | m |
| rear tires distance from c.o.m. | $L_\mathrm{r}$ | 1.1943 | m |
| steering ratio | $\mu_\mathrm{sw}$ | 11.9 | - |
| vehicle speed | $v$ | 100 | $\mathrm{km\,h^{-1}}$ |

Under the assumption of unknown plant, to solve the model-reference control problem, we select the class of PI controllers $\mathcal{R}_{0,\rho}$, which can be expressed in the form (6) as

$$R_{0,\rho}(s) = \rho_1 + \rho_2 \frac{1}{s}. \tag{28}$$

We highlight that, in this numerical example, the ideal model-reference controller $R_0^\mathrm{o}$ can be analytically computed resulting in a second order transfer function, meaning that $R_0^\mathrm{o} \notin \mathcal{R}_{0,\rho}$, as in Lemma 2.

As discussed in §III, in the DD-ISMC scheme we replace the model-based design of the controller $\mathcal{R}_0$ by the direct data-driven VRFT method. To this aim, we performed a suitable open-loop experiment on (24) exploiting the established pseudorandom binary sequence (PRBS) input, limiting the bandwidth at 20 Hz for the specific application. The results of the identification experiment are shown in Fig. 2, where the employed PRBS input, together with the related realization of the matched disturbance in (25), is reported on the top, while the obtained yaw-rate profile on the bottom.

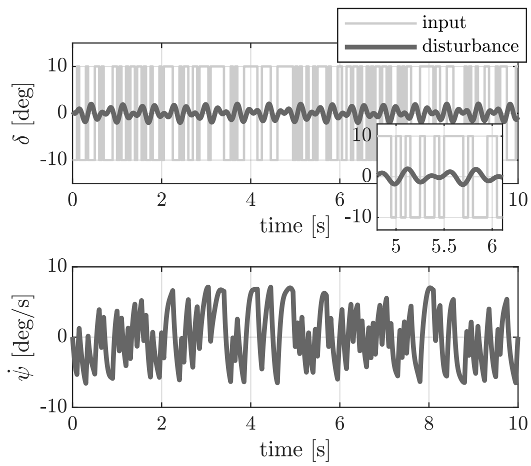


Fig. 2. Open-loop experiment: input steering wheel angle PRBS and disturbance on the top, and yaw-rate output on the bottom.

Given the presence of the matched disturbance in the identification experiment, we leverage the IV approach discussed in [14] to obtain an unbiased estimation of the optimal model-reference controller $R_0(\rho_{vr})$, which is parametrized by

$$\rho_{vr} = [\rho_{1,vr}, \rho_{2,vr}]' = [1.27, 16.75]'. \tag{29}$$

Fig. 3 shows a comparison between the actual and desired behaviors in the frequency domain. More precisely, the actual controller $R_0(\rho_{vr})$ and the ideal model-reference one $R_0^\mathrm{o}$ are compared on the top, while the closed-loop transfer functions on the bottom. Once the controller $\mathcal{R}_0$ has been designed, to test the overall DD-ISMC scheme in Fig. 1, we choose a double lane change maneuver as a validation closed-loop test, where a realization of the matched disturbance in (25) affects the plant. As discussed in §III, the evolution of the transient function $z(t)$ is based on (16), where $(A_\mathrm{M}, B_\mathrm{M}, C_\mathrm{M})$ are selected as a minimal realization of $M(s)$ in (27), namely with $A_\mathrm{M} = -2\pi f_M$, $B_\mathrm{M} = 2\pi f_M$, and $C_\mathrm{M} = 1$. Finally, the ISMC gain $\gamma$ in $\mathcal{R}_1$ is chosen large enough to satisfy condition (18) in the selected double lane change test, i.e., $\gamma = 6$, which, as expected, is higher than the bound of $d_u$ in (26). Fig. 4 shows the closed-loop yaw-rate tracking performance, highlighting the perfect tracking of the desired reference model (27) guaranteed by Theorem 4. To conclude the analysis, we compare the proposed DD-ISMC scheme with the standard one described in §II-A, assuming perfect knowledge of the plant in (24). In Fig. 5 we compare the

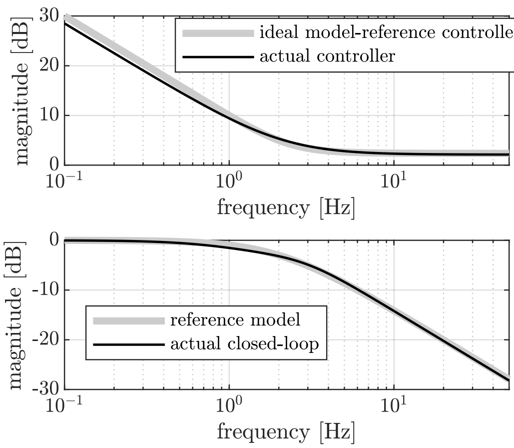


Fig. 3. VRFT results. Top: ideal model-reference $(R_0^o)$ vs. actual controller $(R_0(\rho_{vr}))$ transfer function frequency response. Bottom: reference model $(M)$ vs. actual closed-loop $(\widetilde{M})$ transfer function frequency response.

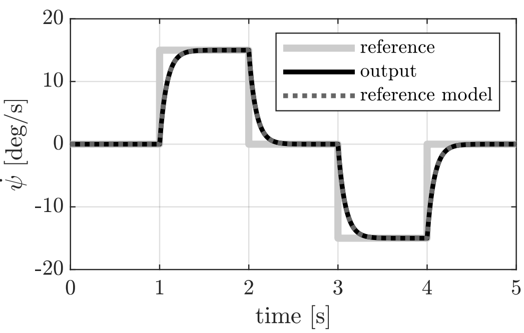


Fig. 4. Yaw-rate tracking in the double lane change maneuver: comparison between the reference model and the DD-ISMC scheme.

equivalent control action $u_{1,eq}$, computed via a suitable low-pass filter on the actual discontinuous signal $u_1$, between two schemes. As expected, in the standard ISMC scheme, $\mathcal{R}_1$ has to compensate only the matched disturbance $d_u$. On the other side, in the proposed DD-ISMC scheme, $\mathcal{R}_1$ has also to compensate a term which depends on the reference signal $r$, thus motivating the choice of $\gamma = 6$ for this specific test. As a final comment, looking at (18), since simulations are

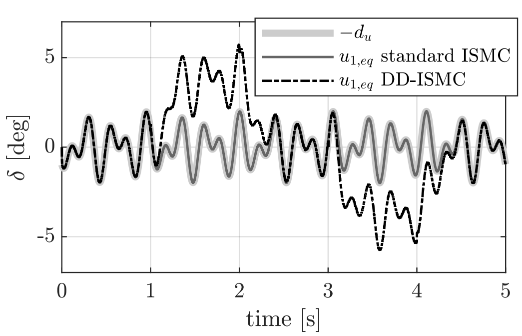


Fig. 5. Comparison of the equivalent ISMC actions between the standard and data-driven algorithm on the double lane change test.

performed in the framework of Lemma 2, under assumption $\boldsymbol{\mathcal{A}_1}$, the choice of $\gamma$ requires a trial and error tuning to dominate the residual uncertainty terms.

## VI. Conclusions

This paper contributes to the literature of ISMC by proposing a novel data-driven methodology for the design of integral sliding mode control. Under the assumption that the model of the plant is unknown, which would prevent the design of a classical ISMC, two changes are introduced: (i) the so-called VRFT direct data-based method is adopted to design the ideal control component of the ISMC law using only experimental data, and (ii) the sliding variable is re-designed exploiting the desired reference model. We provide specific conditions on the design of the proposed DD-ISMC, assessing stability results in simulation on a benchmark example. Future works will be devoted to the extension of the proposal to more complex settings (e.g., multi-input multi-output case, unmatched disturbances, and nonlinear systems).

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