75th International Astronautical Congress (IAC), Milan, Italy, 14-18 October 2024.

IAC-24-B4.IPB.5.x91139

Relative GN&C solution to determine small-scale features and the internal structure of a 40 m diameter asteroid

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Abstract

Is performing a rendezvous with a 40 m diameter asteroid achievable? The Exploration of Near-Earth Asteroids (ENEA), a space mission concept developed by a team of 11 Master of Science students from Politecnico di Milano, addressed this inquiry. `2000 SG344`, an asteroid measuring only 40 meters in diameter, stands out as the most challenging target among the three envisioned by the project due to its limited size. It is recognized as a potentially hazardous object and is currently ranked fourth on the European Space Agency's risk list for potential Earth collisions. For this reason, the mission intends to map its small-scale features and internal structure. The main challenges in designing the mission come from the weak gravitational pull of the asteroid, and its unknown shape, which cannot be accurately predicted during the mission design. This paper highlights the studied strategies and algorithms to perform rendezvous operations with `2000 SG344`. A set of dynamic equations was developed in the LVLH reference frame to model the motion of the spacecraft around the asteroid. Due to the effects of perturbations, such as the solar radiation pressure, two different types of differential correction schemes were implemented to calculate the maneuvers needed to maintain the spacecraft on the desired trajectory. To tackle the challenges, the rendezvous was split into four distinct stages. The spacecraft would first observe the asteroid at a 100 km holding point, where the spin axis, rotational period, rough shape, and luminosity would be assessed. It would follow up with a second, closer, holding point at 15 km distance to map the surface. During this and the following phases, the spacecraft would be positioned between the Sun and the asteroid to achieve constant illumination conditions. Inspired by the Rosetta an OSIRIS-Rex missions, a stable rhombus-shaped orbit at 500 m distance was designed for high-resolution surface mapping of the target. Finally, a terminator orbit at 120 m distance was devised for the radio science experiment for gravity harmonics estimation. This study validates the feasibility of exploring very small celestial bodies with currently available technology, laying the groundwork for future space exploration and planetary defense investigations.

Keywords: Near-Earth Asteroid, 2000 SG344, smallsat, asteroid science, GN&C.

Acronyms

Dec Declination

ENEA Exploration of Near-Earth Asteroids

ESA European Space Agency

HPA Holding Point A

HPB Holding Point B

LTI Linear Time Invariant

LVLH Local-Vertical Local-Horizon

NEA Near-Earth Asteroid

RA Right Ascension

SA Solar Array

SAA Solar Aspect Angle

SMA Semi-Major Axis

SOI Sphere Of Influence

SRP Solar Radiation Pressure

1. Introduction

The Exploration of Near-Earth Asteroids (ENEA) mission aims to study the global physical properties of three Near-Earth Asteroid (NEA) in different size regimes. Specifically, ENEA shall determine the gravel size and surface composition of `2006 HZ51` and `2004 QD14` during flybys, and map small-scale features and gravitational properties of `2000 SG344` by rendezvous. The spacecraft shall stay below 500 kg of wet mass and provide images of the features with a resolution of 1 m/px during flybys and of 1 cm/px during rendezvous.

NEAs provide valuable insight on understanding the formation and evolution of the Solar system, and their study is important for the planning and execution of planetary defense missions. The focus asteroid of this paper, `2000 SG344`, is a potentially hazardous Aten class asteroid, and it is ranked fourth in the risk list for potential Earth collisions of the European Space Agency (ESA). Its known characteristics are reported in Table 1.

This preliminary mission design is conducted in the context of the "Applied Space Mission Analysis and Design" course, taught by Professor Michèle Lavagna at Politecnico di Milano during the academic year 2023/2024.
This paper focuses on the work done by the authors and other colleagues on the feasibility of the rendezvous with `2000 SG344`.

Table 1: Nominal orbital parameters of `2000 SG344` (Heliocentric frame, ecliptic J2000, epoch: 2023-Sep-13 [1]) and nominal diameter with range [2]

Parameter	Value
Semi-major axis a	0.977 44 AU
Eccentricity e	0.06696
Inclination i	0.11°
Longitude of ascending node Ω	191.8°
Argument of perihelion ω	275.5°
Diameter d	40 m (15 m to 70 m)

2. Assumptions

Because of the lack of accurate information on the rotation period and direction of `2000 SG344`, they are estimated based on the data gathered from similar asteroids visited by other missions. The North Pole Right Ascension (RA), Declination (Dec), and rotation period of `2000 SG344` are approximated by averaging the values of asteroids Bennu, Itokawa, Didymos, and Ryugu, reported in Table 2.

This method is justified for the rotation direction, since asteroids often spin in the same direction, while the rotation period is taken just as a working hypothesis, since it can vary greatly from asteroid to asteroid.
Given the scope of this preliminary analysis, and to simplify the model, `2000 SG344` is assumed to be a spherical object with uniform mass. The irregular shape and mass distribution of the asteroid will have to be assessed in further studies.

The environment of `2000 SG344` is challenging for the design of the rendezvous trajectory, which is driven by the following elements.

Gravity

Given the small size and mass of the asteroid, its gravitational pull is not the dominant force after just a few hundred meters of distance. Consequently, guidance schemes must focus on the relative dynamics between the spacecraft and the asteroid, rather than relying on a pure two-body problem model.

Solar Radiation Pressure

The Solar Radiation Pressure (SRP) significantly affects the trajectory due to the large exposed surface of the spacecraft, especially that of its Solar Array (SA).

Visibility

During the radio science experiment, the spacecraft must maintain continuous contact with the Deep Space Network of ESA. Continuous visibility is also essential to effectively manage and downlink the substantial volume of scientific data generated during the observation phase.

Robustness

Since the physical characteristics of `2000 SG344` are unknown, the operations during rendezvous must be robust to any type of possible scenario. This is achieved by designing maneuvers that are flexible and require minimal ΔV, so that by applying an appropriate margin to the propellant any (reasonable) unforeseen situation can be faced. This study deals only with verifying that it is feasible to rendezvous with `2000 SG344` and to give a preliminary quantification of the necessary ΔV. More work is necessary to devise really robust trajectories and operations.

Reference Frame

A Local-Vertical Local-Horizon (LVLH) reference frame is employed to model the motion of the spacecraft around `2000 SG344`. This frame is centered on the asteroid, with its axes defined as follows: the $\hat{\mathbf{x}}$ axis points radially outward from the Sun towards `2000 SG344` (R-bar), the $\hat{\mathbf{y}}$ axis is aligned with

Table 2: RA and Dec of the North Pole direction (in equatorial J2000) and rotation period of visited asteroids, and mean values assumed for `2000 SG344`

Asteroid	North Pole (RA)	North Pole (Dec)	Rotation period
Bennu [3]	85°	−60°	4.30 h
Itokawa [4]	90°	−66°	12.13 h
Didymos [5]	78°	−71°	2.26 h
Ryugu [6]	96°	−66°	7.63 h
SG344	87.25°	−65.75°	6.58 h

the in-track direction, along increasing true anomaly (V-bar), and the $\hat{\mathbf{z}}$ axis completes the right-handed frame, representing the cross-track direction.

3. Observation phase

The initial phase of the rendezvous aims at conducting scientific observations and approaching the asteroid. To achieve this, three trajectory elements are defined to gradually decrease the distance to `2000 SG344` while keeping the spacecraft in a safe region: Holding Point A (HPA), Holding Point B (HPB), and a rhomboidal trajectory close to the asteroid. This approach was inspired by the Rosetta and OSIRIS-REx missions [7, 8].
The spacecraft must reach a distance of 10 km to 16 km from the surface of the asteroid to use the narrow camera, and between 100 m to 400 m to use the wide camera. The upper bounds ensure compliance with the resolution requirements, while the lower bounds are to keep the asteroid in focus. The operations closest to the surface are scheduled for the final stages to ensure that the characteristics of the asteroid are well defined. Since a risk assessment is not performed, this process acts as margin to allow for a safer execution of the operations.

The detailed development and rationale of the guidance schemes can be found in appendix A.

3.1. Holding Point A

The HPA is situated at a y_A of −100 km on the V-bar to ensure passive safety, since from this point the spacecraft will naturally drift away from the asteroid due to the SRP, as can be seen from Fig. 1. Upon reaching this point, the condition of the spacecraft following the interplanetary trajectory is assessed. Using the payload cameras, an initial estimate of the rotation period and spin axis of `2000 SG344` is calculated.

The spacecraft will remain at the HPA for 14 days, performing station-keeping maneuvers every 2 days. These maneuvers require little ΔV, as can be seen from Fig. 2. The total estimated ΔV for the phase is 0.474 m/s.

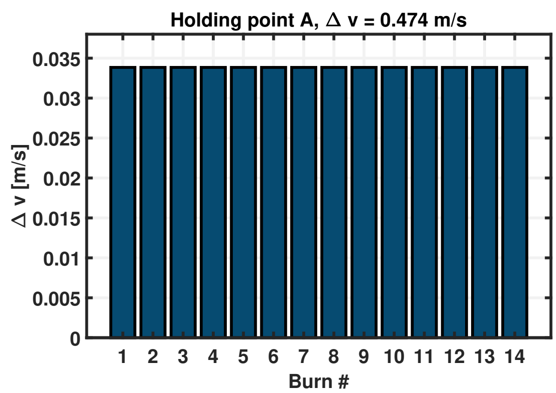

Figure 2: HPA ΔV breakdown

3.2. Holding Point B

The HPB is situated at an x_B of −15 km on the R-bar, Fig. 1. From this position, the image resolution requirements are met, with the Sun directly behind the cameras providing maximum illumination and the asteroid entirely within the field of view of the cameras. At HPB, images of the asteroid are used to establish a latitude-longitude reference frame of the asteroid, required for the development of a non-uniform gravitational model. From this position, data on the surface composition is also collected using the scientific payload.

The spacecraft will move from HPA to HPB with a transfer time of 7 days and then will remain on HPB for 5 days performing station-keeping maneuvers every 6 hours.

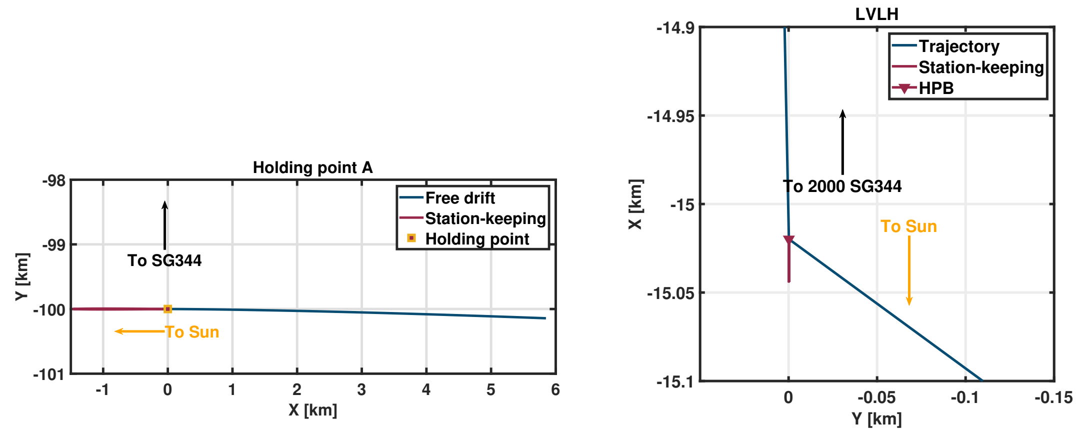

Figure 1: Left: Comparison between a 2-days free drift trajectory and the station-keeping trajectory at HPA. Right: Incoming and outgoing trajectory from HPB. The station-keeping trajectory is highlighted in red. LVLH centered on `2000 SG344`

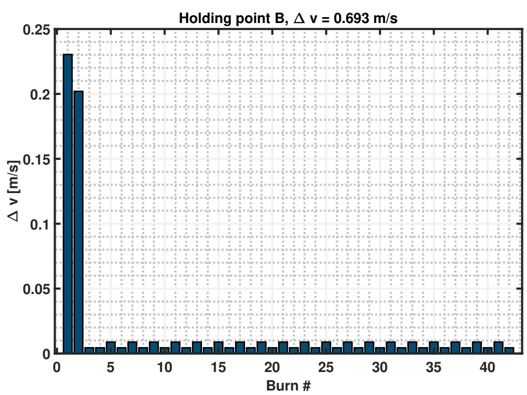

Figure 3: HPB ΔV breakdown. The first burn is to start the transfer from the HPA, and the second one is to stop at HPB

3.3. Rhombus

The rhomboidal orbit is the last element of the observational phase. Its four vertices are placed in the locations specified in Table 3.

Table 3: Vertices of the rhombus

Vertex	x [m]	y [m]	z [m]
r_1	-520	-300	0
r_2	-520	0	300
r_3	-520	300	0
r_4	-520	0	-300

The vertices are selected to ensure that the wide camera can focus properly and to provide a ± 30° latitude ground track, allowing the observation of high-latitude features near the poles. With this setup it is possible to capture images of the surface features from different viewing angles. Each leg of the polygon is traversed in an integer multiple of the rotation period of `2000 SG344`, enabling observation of the entire surface from four different directions. In this case the spacecraft will transfer from HPB to the rhomboidal orbit in 2 days and will complete the necessary number of orbits to gather the required data. For this study, 3 orbits are used as a reference, with each edge traversed in one period of `2000 SG344`.

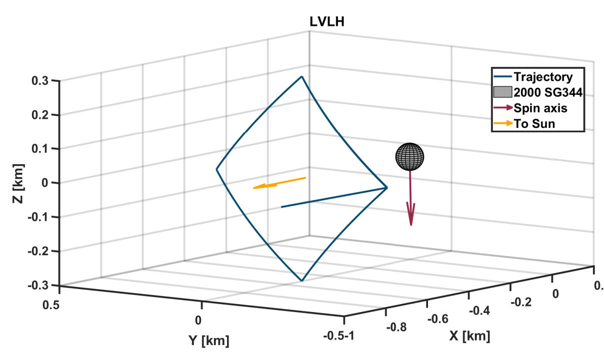

Figure 4: Rhomboidal orbit. LVLH, centered on 2000 SG344

While on the rhomboidal orbit, the spacecraft will continue to capture images of the surface, to improve the latitude-longitude mapping, and to conduct spectrometer surveys.

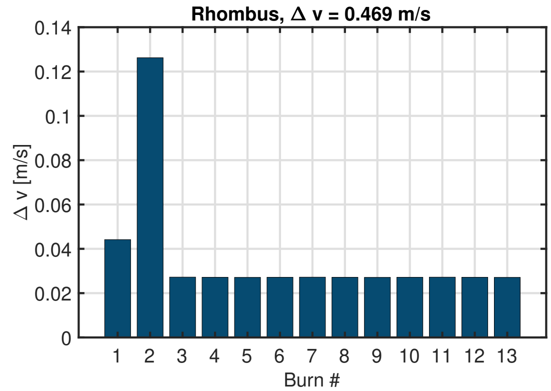

Figure 5: Rhombus ΔV breakdown. The first burn is to start the transfer from the HPB, and the second one is to stop at the first vertex of the rhombus

Table 4: ΔV budget for the proposed observation phase

Phase	ΔV [m/s]
HPA	0.474
HPB	0.693
Rhombus	0.469
Total	1.636

4. Radio science

A radio science experiment is used to reconstruct the gravity field of a celestial body. A spacecraft is put into a quiescent orbit (an orbit with no maneuvers) around the body, and measurements, usually in the form of Doppler ranging from ground antennas, are continuously taken. With these measurements the state of the spacecraft can be estimated using common estimation techniques (e.g. least-square fitting). It also possible to estimate the gravitational parameters of the body (standard gravitational parameter, harmonics coefficients or others) by using an augmented state vector that includes both the state of the spacecraft, and the gravitational parameters. This is a so-called radio science experiment.
This study investigates only the feasibility of a quiescent orbit suitable for radio science, and its visibility windows with the ESA Deep Space Network. The assessment of the accuracy of the estimated gravitational parameters obtained with the experiment is left for future work.

The weak gravity of `2000 SG344` poses many challenges to the development of a quiescent orbit. The radius of the Sphere Of Influence (SOI) is $R_{SOI} = 154_{99}^{239}$ m, while the radius of `2000 SG344` is $R = 20_{9}^{40}$ m. It is clear, then, that the orbit can exist only in a tight range, to avoid hitting the asteroid or leaving its gravity field. Moreover, it could be that `2000 SG344` is of irregular shape, making staying close to the asteroid even more difficult. In any case, for this study the body is assumed to be spherical.

Since the gravitation pull is so weak, SRP is the main disturbance and will tend to push away the spacecraft. It is even more relevant for ENEA, since the spacecraft has very large solar panels and a relatively low mass, making the SRP disturbance particularly strong. To mitigate this effect the Solar Aspect Angle (SAA) of the solar panels should be kept as high as possible (ideally 90°), but at the same time low enough to guarantee enough power to the spacecraft.

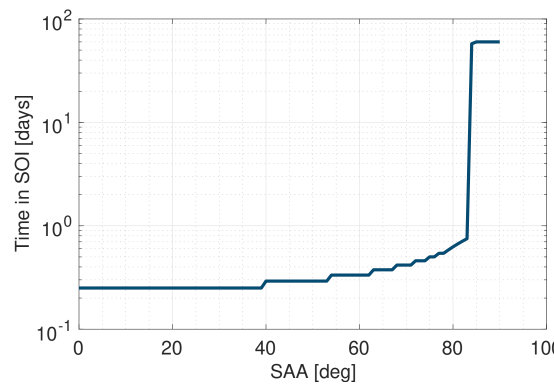

Figure 6: Time inside the SOI as a function of the SAA

4.1. Orbit design

The chosen orbit is a terminator orbit of radius $r = 120$ m, to both mitigate the effects of SRP and to remain in a safe zone inside the SOI.
As can be seen from Fig. 6, the time inside the SOI (with no correction maneuvers) is linked to the SAA of the SA. A SAA of 85° is chosen as the best compromise, since it is the highest one that will ensure enough electrical power, while keeping the spacecraft inside the SOI for at least 10 days. The 10 days threshold is chosen based on the radio science experiment of Osiris-REx [8] which had similar objectives. Since the gravity of `2000 SG344` is much weaker than that of `Bennu`, a longer radio science experiment might be needed to obtain accurate gravitational parameters.

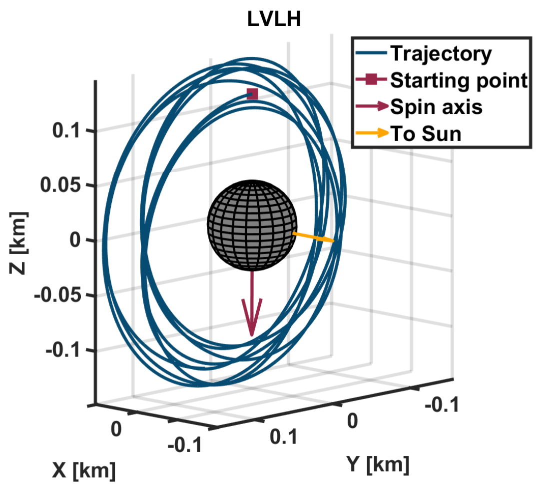

Figure 7: The 10 days terminator orbit

As can be seen from figures Figs. 6, 8 and 9, a radio science experiment that satisfies the requirements is feasible. The spacecraft remains in the SOI for the required time while being continuously visible from ESA's Deep Space Network.

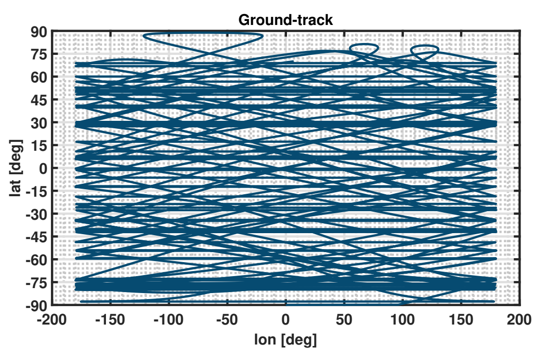

Figure 8: Groundtrack of the terminator orbit

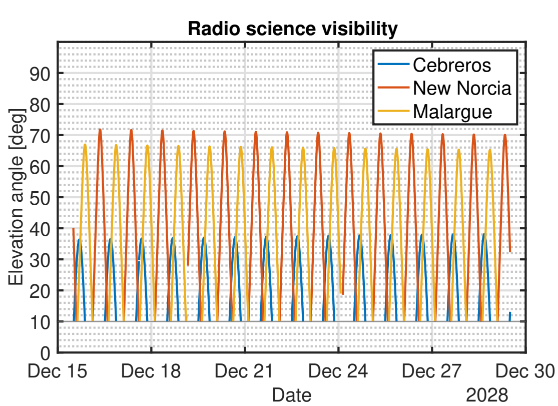

Figure 9: Elevation of the spacecraft as seen from ESA ground stations

5. Conclusions

This preliminary study shows that it is possible to operate in close proximity of `2000 SG344` using current technology and that there are the conditions to achieve the scientific requirements. Each of the four stages, even though challenging, is cheap in terms of ΔV and can be easily modified, suggesting that there are ample margins for complex trajectories and safety maneuvers. Based on this, it is reasonable to assume that a more refined design and a precise plan of operations will prove feasible as well.

More work needs to be done to achieve a fully satisfactorily and robust design, in particular the elements that need more attention are:
Reducing the size of the solar arrays to mitigate the effect of the SRP, validating the proposed trajectories with a non-spherical body and gravity field, estimating the risks of close proximity operations, estimating the accuracy of the gravitational parameters derived from the radio science experiment, developing an autonomous navigation scheme that accounts for an irregularly shaped body.

Acknowledgments

We would like to thank our colleagues in the Applied Space Mission Analysis and Design course: Sarah Aiani, Alice Balest, Carlos Albiñana Burdiel, Giulio Morandi, Dario Scimone and Amanda Sanchez Magris.

A. Appendix: Guidance schemes

To model the motion of the spacecraft around `2000 SG344`, a LVLH reference frame (described in section 2) is used. The relative dynamics are modeled using the equations for formation flying by [9] with the gravity of `2000 SG344` and the SRP added as external disturbances.

Because the equations account for the eccentricity of the heliocentric orbit of the center of formation (`2000 SG344`), its true anomaly θ is part of the equations of motion. Let $\mathbf{s} = (\mathbf{x}, \theta)$ be the expanded state, the equations of motion $\dot{\mathbf{s}} = \mathbf{h}(\mathbf{s}, t)$ are:

$$\dot{\mathbf{s}} = \mathbf{h}(\mathbf{s}, t) = \begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \\ \dot{\theta} = g(\theta, t) \end{cases} \tag{1}$$

With:

$$\mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} \mathbb{O}_{3\times 3} & \mathbb{I}_{3\times 3} \\ \mathbf{A} & \mathbf{B} \end{bmatrix} \mathbf{x} - \frac{\mu_{SG344}}{\|r\|^3} \begin{pmatrix} \mathbf{0}_3 \\ \mathbf{r} \end{pmatrix} + \frac{J_0}{c} \frac{A_c}{m_{sc}} C_r \begin{pmatrix} \mathbf{0}_3 \\ \hat{\mathbf{i}} \end{pmatrix} \tag{2}$$

And:

$$g(\theta,t) = \frac{n(1+e\cos\theta(t))^2}{(1-e^2)^{\frac{3}{2}}} \tag{3}$$

The first right-hand side term of Eq. (2) models the relative dynamics, while the second and third are, respectively, the disturbances due to the gravity of `2000 SG344` and the SRP, where $\mu_{SG344} = 4.6719 \times 10^{-12}\,\mathrm{km^3/s^2}$, e and n are, respectively, the assumed gravitational parameter, the eccentricity and the mean motion of `2000 SG344`, $\mathrm{J}_0 = 1436\,\mathrm{W/m^2}$ is the solar irradiance at $0.977\,\mathrm{AU}$[1], c is the speed of light in vacuum, A_c is the area of the spacecraft exposed to the solar radiation, $\mathrm{m}_{sc} = 363\,\mathrm{kg}$ is the mass of the spacecraft at arrival at `2000 SG344`, and $\mathrm{C}_r = 1.28$ is the reflectivity coefficient. The choice of C_r is based on common values in the literature for preliminary designs.
The **A** and **B** matrices are:

$$A = \begin{bmatrix} \dot\theta^2 + 2\Gamma & \ddot\theta & 0 \\ -\ddot\theta & \dot\theta^2 - \Gamma & 0 \\ 0 & 0 & -\Gamma \end{bmatrix} \tag{4}$$

$$B = \begin{bmatrix} 0 & 2\dot\theta & 0 \\ -2\dot\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

Where:

$$\Gamma = n^2 \left(\frac{1+e\cos\theta}{1-e^2}\right)^3 \tag{6}$$

It should be noted that the unperturbed dynamics allow for periodic orbits in the relative frame of reference, but the period of such orbits is equal to the orbital revolution period of the center of formation. Since `2000 SG344` has a SMA $\approx 1\,\mathrm{AU}$, the periodic orbit would have a period of about one year. For the purposes of `ENEA`, such orbits are not useful.
Moreover, in a LVLH frame the SRP acts as a constant acceleration with fixed direction $\hat{\mathbf{i}}$, meaning that these periodic orbits cannot be achieved anyways.

Therefore, it is necessary to use forced orbits, whose characteristics can be designed to suit the needs of the mission. The secondary thrusters are used to perform maneuvers of station-keeping or to target specific trajectories. To calculate the maneuvers, differential correction schemes are implemented.

Differential correction schemes

Two differential correction schemes are implemented, one for the station keeping at holding points and the other one to target trajectories. The equations of motion Eq. (1) need to be solved together; however, since θ affects $\mathbf{f}(\mathbf{x},t)$ but not vice-versa, the equations of motion $\mathbf{f}(\mathbf{x},t)$ can be treated in isolation to simplify the notation. It is clear that any solution $\mathbf{x}(t)$ is a sub-set of the solution $\mathbf{s}(t)$, where both $\mathbf{h}(\mathbf{x},t)$ and $g(\theta,t)$ have been integrated together.

Holding point station-keeping

A solution to $\mathbf{f}(\mathbf{x},t)$ is:

$$\mathbf{x}(t) = (\mathbf{r}(t), \mathbf{v}(t))^\intercal = \boldsymbol{\varphi}(\mathbf{x}_0, t_0; t) \tag{7}$$

where $\boldsymbol{\varphi}(\mathbf{x}_0, t_0; t)$ is the flow of $\mathbf{f}(\mathbf{x},t)$, with initial conditions $\mathbf{x}_0$ and t_0.
Let $\mathbf{x}_c = (\mathbf{c}, \mathbf{0}_3)^\intercal$ be the desired state at an holding point located in $\mathbf{c}$, where $\mathbf{c}$ is the position in the relative frame of `2000 SG344`. By linearizing Eq. (7) around $\mathbf{x}_c$, it becomes:

$$\mathbf{x}(t) \approx \boldsymbol{\varphi}(\mathbf{x}_c, t_0; t) + \Phi(t_0,t)(\mathbf{x}_0 - \mathbf{x}_c) \tag{8}$$

Where $\Phi(t_0,t)$ is the state transition matrix of Eq. (7) evaluated in $\mathbf{x}_c$. By forcing the trajectory to begin and end on $\mathbf{c}$:

$$\mathbf{x}(t) = (\mathbf{c}, \mathbf{v}(t))^\intercal \tag{9}$$

$$\mathbf{x}_0 = (\mathbf{c}, \mathbf{v}_0)^\intercal \tag{10}$$

the required $\mathbf{v}_0$ to end up in $\mathbf{c}$ after $\Delta t = t - t_0$ is calculated as:

$$\mathbf{v}_0 = \Phi_{rv}^{-1}(\mathbf{c} - \boldsymbol{\varphi}_r(\mathbf{x}_c, t_0; t)) \tag{11}$$

where $\boldsymbol{\varphi}_r(\mathbf{x}_c, t_0; t)$ is the position subset of the flow, and Φ_{rv} is the 3×3 sub-matrix of $\Phi(t_0,t)$:

$$\Phi(t_0,t) = \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix} \tag{12}$$

The velocity needed to stop the spacecraft once it reaches again $\mathbf{c}$ is:

$$\mathbf{v}(t) = \boldsymbol{\varphi}_v(\mathbf{x}_0, t_0; t)$$

Where $\boldsymbol{\varphi}_v(\mathbf{x}_0, t_0; t)$ is the velocity subset of the flow. Summing the velocity needed to start the maneuver and the velocity needed to stop it gives the total ΔV of the maneuver:

$$\Delta V = \|\mathbf{v}_0\| + \|\mathbf{v}(t)\| \tag{13}$$

[1] $0.977\,\mathrm{AU}$ is the value of the Semi-Major Axis (SMA) of `2000 SG344`. The irradiance at this distance is used to simplify the calculations, since `2000 SG344` has a relatively low eccentricity.

General trajectories

For general trajectories, a point around which to linearize the flow is not as easily found as for holding point maneuvers. A possible solution is to use the Clohessy-Wiltshire [10] equations for circular formation flying, to find a suitable initial condition. Using the same reference frame, the Clohessy-Wiltshire equations can be written as:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} \tag{14}$$

The Clohessy-Wiltshire equations are Linear Time Invariant (LTI), meaning that the solution can be written as:

$$\mathbf{x}(t) = \exp A(t - t_0)\mathbf{x}_0 = \Psi(t_0, t)\mathbf{x} \tag{15}$$

Therefore, given a desired end state $\hat{\mathbf{x}}(t)$, the corresponding initial conditions can be found by solving the linear system:

$$\mathbf{x}_0 = \Psi^{-1}\hat{\mathbf{x}}(t) \tag{16}$$

With Eq. (16) a linearization point for Eq. (7) can now be identified.

For a trajectory that begins on $\mathbf{r}_k$ and ends on $\mathbf{r}_{k+1}$, the initial velocity $\tilde{\mathbf{v}}_k$ in the CW dynamics can be found using Eq. (16):

$$\tilde{\mathbf{v}}_k = \Psi_{rv}^{-1}(\mathbf{r}_{k+1} - \Psi_{rr}\mathbf{r}_k) \tag{17}$$

where Ψ_{rv} and Ψ_{rr} are the sub-matrices of $\Psi(t_0, t)$ in Eq. (15):

$$\Psi(t_0, t) = \begin{bmatrix} \Psi_{rr} & \Psi_{rv} \\ \Psi_{vr} & \Psi_{vv} \end{bmatrix}$$

With the state $\tilde{\mathbf{x}}_k = (\mathbf{r}_k, \tilde{\mathbf{v}}_k)$ identified, the same scheme used for the holding point can be used. Given the linearized flow Eq. (8) starting from $\tilde{\mathbf{x}}_k$, the required initial velocity $\mathbf{v}_k$ is:

$$\mathbf{v}_k = \Phi_{rv}^{-1}(\mathbf{r}_{k+1} - \boldsymbol{\varphi}_r(\tilde{\mathbf{x}}, t_0; t)) + \tilde{\mathbf{v}}_k \tag{18}$$

For a trajectory starting from an holding point and passing through n points, the total ΔV is:

$$\Delta V = \|\mathbf{v}_0\| + \sum_{k=1}^{n} \|\mathbf{v}_k - \boldsymbol{\varphi}_v(\mathbf{x}_{k-1}, t_0; t)\|$$

where $\boldsymbol{\varphi}_v(\mathbf{x}_{k-1}, t_0; t)$ is the velocity subset of the flow.

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