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STEERABLE FIRST-ORDER DIFFERENTIAL LOUDSPEAKER ARRAYS WITH MONOPOLE AND DIPOLE ELEMENTS

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ABSTRACT

Due to the recent trend toward miniaturization of acoustic transducers, differential loudspeaker arrays (DLA) of small size are becoming a popular approach to achieve directional sound radiation characterized by a near frequency invariant behavior. This makes DLAs suitable for broadband audio signal processing applications. So far, only arrays of omnidirectional loudspeakers have been used in the literature on DLAs. This work focuses on the design of steerable first-order DLAs characterized by a uniform linear geometry that alternates omnidirectional speakers with dipole speakers. The design is achieved by exploiting the theory of eigenbeams. First-order eigenbeam filters are formed and then combined to produce the desired steerable radiation response, making the implementation of the proposed beamformer very simple. Simulations show that the presented beamforming method is nearly frequency invariant for large frequency ranges. Moreover, we present an analysis of DLAs in terms of white noise gain and directivity index as the number of loudspeakers is varied.

Keywords: *loudspeaker arrays, differential beamforming, acoustic signal processing.*

1. INTRODUCTION

Over the last decades, there has been a significant trend toward the miniaturization of acoustic transducers and actuators, and among these MEMS-based technologies have gained popularity in many application scenarios [1, 2]. The reduced dimensions of these devices have also been usefully exploited in array configurations that combine multiple small transducers/actuators to perform advanced signal processing tasks. For example, small-sized arrays of microphones have been used in differential configurations, i.e., configurations in which differences between sensed signals are exploited for spatial filtering purposes. According to the theory of differential microphone arrays (DMAs), in fact, differences between sensor signals are combined to approximate derivatives of the acoustic pressure of arbitrary order [3–9]. DMAs have become popular because they allow to design beamformers with nearly frequency-invariant responses with high directivity and have been successfully implemented using different array geometries, including linear [6, 10], circular [8, 11, 12], or arbitrary planar geometries [13–17]. Moreover, several differential beamforming methods have been proposed that maximize different directivity or robustness metrics or attempt to match ideal directivity patterns [12, 14, 16]. Although the literature on DMA is extensive and well established, microphone elements are usually assumed to have omnidirectional sensitivity. An exception is found in [18], where a differential beamformer is developed for linear arrays consisting of both omnidirectional and dipole microphone elements.

More recently, taking advantage of design principles similar to those of DMAs, differential loudspeaker arrays (DLAs) have also been investigated to produce directional sound radiation with compact arrays of small-size loudspeakers [19]. Traditionally, loudspeaker arrays have been used in additive configurations which exploit the interference of the sound fields generated by the individual array elements to produce directional radiation

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patterns [19]. However, due to the diffraction limit [19], the low-frequency directivity of additive arrays is conditioned by the size of the array aperture, which should be significantly larger than the wavelength of the sound waves of interest. Therefore, a large array is required to achieve directional sound radiation at low frequencies. Instead, similarly to DMAs, DLAs can achieve good directivity and nearly frequency invariant beampatterns, provided that the distance between two adjacent loudspeakers is small enough to properly approximate the desired acoustic pressure differences. At present, there are only a limited number of publications in the literature that specifically focus on DLAs [20–22]. In particular, when it comes to linear arrays, in [20] the author introduces the notion of DLA, and he presents a beamforming method capable of emitting sound in the endfire direction while using an array of a size which is small if compared to the wavelength of the sound signals of interest, and the resulting directivity behavior is uniform across frequencies. In [21], instead, the authors propose a general approach to design DLAs that can produce broadside radiation patterns of any order. To enhance the steering capabilities of DLAs towards arbitrary directions in a plane, in [22] the authors propose a circular array design, similar to the method presented in [12] for circular DMAs. Nevertheless, the structure of circular arrays makes them unsuitable for scenarios where planar space is limited. In order to cope with this problem, in this manuscript we propose a design method to derive steerable first-order DLAs characterized by uniform linear geometries. In particular, we propose an array configuration composed of both monopole and dipole actuators, that are jointly exploited to generate any first-order spatial response. The proposed spatial filtering method allows us to produce an arbitrary first-order beam by linearly combining its three orthonormal eigenbeam components, namely an omnidirectional component and two orthogonal dipole components. The proposed method also enables to control the beam shape using a single parameter in a straightforward manner. Results show that the array beampattern can be steered without affecting its shape, and it is almost frequency-invariant in large frequency ranges. Moreover, an analysis of White Noise Gain (WNG) and Directivity Index (DI) as functions of the number of array elements is presented.

The article is organized as follows. Sec. 2 describes the signal model and the structure of the DLAs that we are proposing. Sec. 3 briefly defines the metrics used to evaluate the proposed beamformer. Sec. 4 describes the proposed spatial filtering method. Sec. 5 discusses some simulation results that confirm the validity of the proposed method and Sec. 6 draws the conclusions.

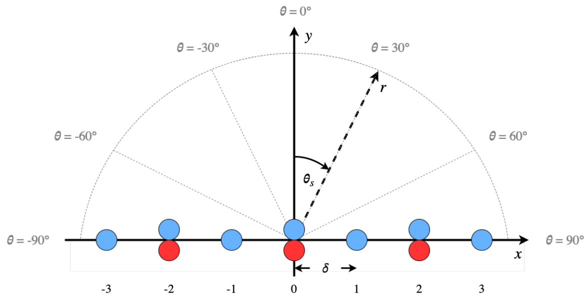

Figure 1: Linear array with $M = 4$ monopoles and $D = 3$ dipoles, i.e., $L = 7$ loudspeakers.

2. SIGNAL MODEL

Let us consider an uniform linear loudspeaker array composed of $L \geq 3$ elements lying on the x-axis, with the origin set as the reference point. Let us assume that the array is symmetrical with respect to the y-axis, and denote with δ the inter-element distance. We also assume δ to be much smaller than the wavelength of interest. The array is composed of an alternating series of M monopole elements and D dipole elements, i.e., $L = M + D$. Moreover, dipoles are aligned along the $\theta = 0$ direction, where θ denotes the azimuth angle. Without loss of generality, we always consider the case in which one vertical dipole element is placed at the origin, i.e., the reference point. Therefore, the resulting configuration is characterized by an odd number of loudspeakers L and dipoles D, while M, the number of monopoles, is always even. As an example, a system of the sort is depicted in Fig. 1.

If we assume that the array is compact, the resulting sound pressure generated by the array in far-field, at distance r and angle θ can be approximated as

$$p(r, \omega, \theta) \approx \frac{e^{j\frac{\omega}{c}r}}{4\pi r} \sum_{l=-L_0}^{L_0} w_l^*(\omega) T_l(\theta) e^{-j\frac{\omega}{c}l\delta \sin\theta}, \quad (1)$$

where j is the imaginary unit, ω is the angular frequency, c is the speed of sound, $L_0 = \frac{L-1}{2}$, w_l denotes the complex weight for the lth loudspeaker element, $(\cdot)^*$ denotes the complex-conjugate operator, and $T_l(\theta)$ is a term modeling the directivity pattern of the lth array loudspeaker, which in this context can be either a monopole or a dipole source.

Therefore, we can compactly write the resulting far-field pressure radiated by the loudspeaker array as

$$p(r,\omega,\theta) \approx \frac{e^{j\frac{\omega}{c}r}}{4\pi r}\mathbf{w}^H(\omega)\mathbf{T}(\theta)\mathbf{d}(\omega,\theta), \tag{2}$$

where

$$\mathbf{w}(\omega) = [w_{-L_0}(\omega),\dots,w_0(\omega),\dots,w_{L_0}(\omega)]^T, \tag{3}$$

$$\mathbf{d}(\omega,\theta) = [e^{j\frac{\omega}{c}L_0\delta\sin\theta},\dots,1,\dots,e^{-j\frac{\omega}{c}L_0\delta\sin\theta}]^T, \tag{4}$$

$$\mathbf{T}(\theta) = \text{diag}(T_{-L_0}(\theta),\dots,T_0(\theta),\dots,T_{L_0}(\theta)), \tag{5}$$

and where diag$(\cdot)$ denotes a diagonal matrix whose arguments correspond to its diagonal entries. As an example, considering the array configuration of Fig. 1,

$$\mathbf{T}(\theta) = \text{diag}(1,\cos(\theta),1,\cos(\theta),1,\cos(\theta),1).$$

With this array setup, our main goal is to derive the filter coefficients $w_l(\omega)$ to obtain a steerable first-order radiation pattern.

3. METRICS

In order to characterize the spatial response of the proposed beamformer, we exploit three common metrics in loudspeaker array literature; namely, the beampattern, the White Noise Gain (WNG) and the Directivity Index (DI).

The beampattern can be defined as the spatial response of the beamforming filter $\mathbf{w}(\omega)$, and, according to (2) it is formally expressed as

$$\begin{aligned}\mathcal{B}[\mathbf{w}(\omega),\theta] &= \mathbf{w}^H(\omega)\mathbf{T}(\theta)\mathbf{d}(\omega,\theta)\\ &\approx p(r,\omega,\theta)\frac{4\pi r}{e^{j\frac{\omega}{c}r}}.\end{aligned} \tag{6}$$

Practical implementations of beamformers for loudspeaker arrays can include nonidealities. In this regard, the WNG can be used as a robustness measure against loudspeaker gain or phase mismatches, or misplacements of array elements. The WNG can be formally expressed as

$$\text{WNG}(\omega) = \frac{|\mathcal{B}[\mathbf{w}(\omega),\theta_s]|^2}{\mathbf{w}^H(\omega)\mathbf{w}(\omega)}. \tag{7}$$

Finally, the DI is the ratio, on a *log* scale, between the intensity of a given sound source and that of an omnidirectional sound source that emits the same acoustic power as the source of interest [19]. It is mathematically defined as

$$\text{DI}(\omega) = 10\log_{10}\left(\frac{|\mathcal{B}[\mathbf{w}(\omega),\theta]|^2}{I_{\text{avg}}}\right). \tag{8}$$

where I_{avg} denotes the spatial intensity averaged over the plane.

4. FILTER DESIGN

In this section, we propose a method to design the beamforming filter $\mathbf{w}(\omega)$ such that the resulting beampattern $\mathcal{B}[\mathbf{w}(\omega),\theta]$ in (6) approximates any first-order ideal beampattern $\mathcal{B}(\theta)$. The target beampattern is frequency-invariant and it is expressed as

$$\mathcal{B}(\theta) = (1-q) + q\cos(\theta-\theta_s), \tag{9}$$

where θ_s is the steering direction and $0 \le q \le 1$ is a parameter controlling the beam shape [23]. In the two limit cases, in which $q = 0$ and $q = 1$, the beampattern of a monopole and of a dipole steered toward θ_s are obtained, respectively. Any first-order beampattern can be decomposed into the linear combination of three spatially orthonormal components (also known as eigenbeams), i.e., an omnidirectional component and two orthogonal dipole components. This fact becomes evident by rewriting $\mathcal{B}(\theta)$ as

$$\mathcal{B}(\theta) = (1-q) + q\left[\cos(\theta_s)\cos(\theta) + \sin(\theta_s)\sin(\theta)\right], \tag{10}$$

which is the linear combination of a monopole component, weighted by $(1-q)$, and two orthogonal dipole components, i.e., $\cos(\theta)$ and $\sin(\theta)$, which are weighted by $q\cos(\theta_s)$ and $q\sin(\theta_s)$, respectively.

Inspired by (10), we approach the beamforming design by deriving three separate filters, each responsible for approximating a different eigenbeam. In particular, the filter $\mathbf{w}_o \in \mathbb{R}^L$ approximates the omnidirectional eigenbeam, by computing the mean response of all monopole actuators of the array as

$$\mathbf{w}_{\text{o}} = [w_{\text{o},-L_0},\dots,w_{\text{o},0},\dots,w_{\text{o},L_0}]^T \tag{11}$$

where $w_{\text{o},l} = \frac{1}{M}I_{\text{o}}(l)$, $l \in [-L_0, L_0]$ and I_{o} is an indicator function defined as

$$I_{\text{o}}(l) = \begin{cases}1, & \text{if } l \in \mathcal{M}\\ 0, & \text{otherwise,}\end{cases} \tag{12}$$

where $\mathcal{M} \subset [-L_0, L_0]$ is the set of indices referring to omnidirectional loudspeakers, which in the considered array configuration corresponds to the set of odd indices. Similarly, filter $\mathbf{w}_{\text{d}}^0 \in \mathbb{R}^L$ approximates the dipole eigenbeam steered towards direction $\theta_s = 0$, by computing the mean response of all dipole elements of the array as

$$\mathbf{w}_{\text{d}}^0 = [w_{\text{d},-L_0}^0,\dots,w_{\text{d},0}^0,\dots,w_{\text{d},L_0}^0]^T \tag{13}$$

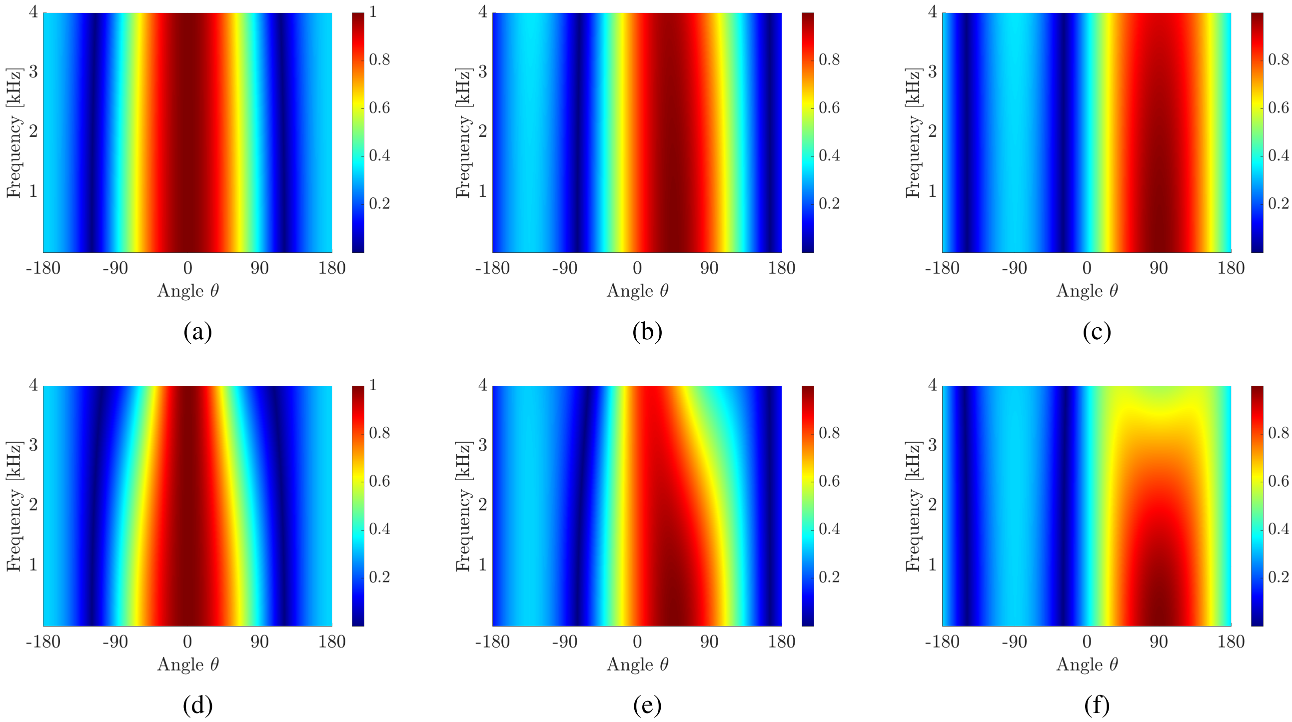

Figure 2: Hypercardioid beampatterns generated by an array of $L = 3$ loudspeakers (Fig. 2a, Fig. 2b, Fig. 2c), or $L = 7$ loudspeakers (Fig. 2d, Fig. 2e, Fig. 2f), steered at angles $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

where $w^0_{\mathrm{d},l} = \frac{1}{D} I_\mathrm{d}(l)$ and I_d is and indicator function defined as:

$$I_\mathrm{d}(l) = \begin{cases} 1, & \text{if } l \in \mathcal{D} \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

where $\mathcal{D} \subset [-L_0, L_0]$ is the set of indices referring to dipole loudspeakers, which in the considered array configuration corresponds to the set of even indices. Finally, filter $\mathbf{w}^{\pi/2}_\mathrm{d}(\omega) \in \mathbb{R}^L$ approximates the dipole eigenbeam steered towards direction $\theta_s = \pi/2$, by computing the mean response of all differences between consecutive monopole elements of the array along the x-axis, as

$$\mathbf{w}^{\pi/2}_\mathrm{d}(\omega) = [w^{\pi/2}_{\mathrm{d},-L_0}, \ldots, w^{\pi/2}_{\mathrm{d},0}, \ldots, w^{\pi/2}_{\mathrm{d},L_0}]^T \tag{15}$$

where

$$w^{\pi/2}_{\mathrm{d},l} = \begin{cases} \frac{c}{j\omega 2\delta}\left(\frac{I_{\mathrm{od}_1}}{M} + \frac{I_{\mathrm{od}_2}}{M-2}\right), & \text{if } M > 2 \\ \frac{c}{j\omega 2\delta}\left(\frac{2I_{\mathrm{od}_1}}{M}\right), & \text{otherwise} \end{cases} \tag{16}$$

with I_{od_1} and I_{od_2} two indicator functions defined as

$$I_{\mathrm{od}_1}(l) = \begin{cases} -\cos(\bar{l}\frac{\pi}{2}), & \text{if } l \in \mathcal{M} \text{ and } L_0 \text{ is odd} \\ -\sin(\bar{l}\frac{\pi}{2}), & \text{if } l \in \mathcal{M} \text{ and } L_0 \text{ is even} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

$$I_{\mathrm{od}_2}(l) = \begin{cases} \cos(\bar{l}\frac{\pi}{2}), & \text{if } l \in \mathcal{M} \setminus \{\inf(\mathcal{M}) \cup \sup(\mathcal{M})\} \\ & \text{and } L_0 \text{ is odd} \\ \sin(\bar{l}\frac{\pi}{2}), & \text{if } l \in \mathcal{M} \setminus \{\inf(\mathcal{M}) \cup \sup(\mathcal{M})\} \\ & \text{and } L_0 \text{ is even} \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

having $\bar{l} = l + L_0$.

Once the three eigenbeam filters are available, according to (10), it is possible to compute the total first-order beamforming filter $\mathbf{w}(\omega)$ in (6) as

$$\mathbf{w}(\omega) = (1-q)\mathbf{w}_\mathrm{o} + q\left[\cos(\theta_s)\mathbf{w}^0_\mathrm{d} + \sin(\theta_s)\mathbf{w}^{\pi/2}_\mathrm{d}(\omega)\right]. \tag{19}$$

It is worth noting that, as designed, filters $\mathbf{w}_\mathrm{o}$ and $\mathbf{w}^0_\mathrm{d}$ do not depend on ω, since they simply compute the average of

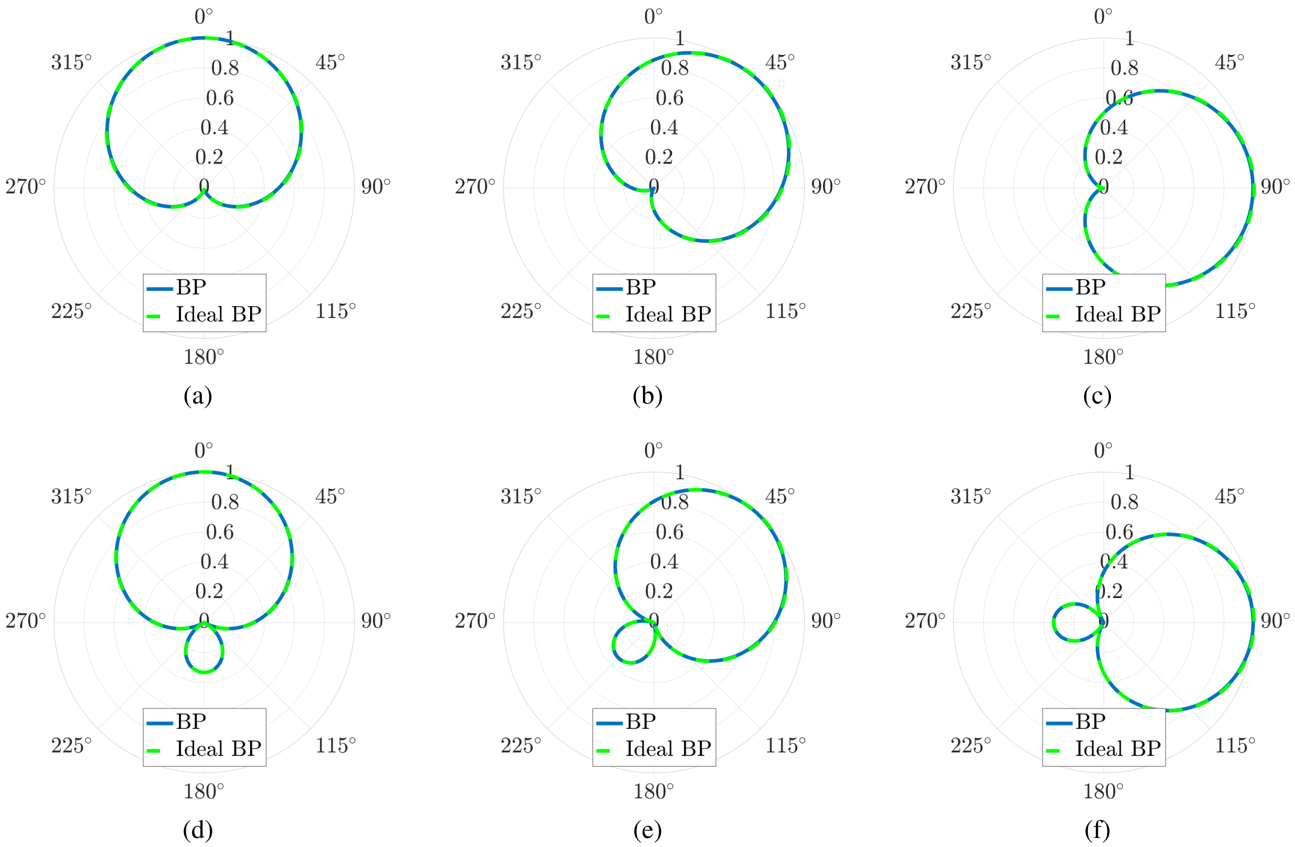

Figure 3: Cardioid (Fig. 3a, Fig. 3b, Fig. 3c) and hypercardioid (Fig. 3d, Fig. 3e, Fig. 3f) beampatterns generated by an array of $L = 3$ loudspeakers, computed at frequency $f = 1\,\mathrm{kHz}$ and steered at angles $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

loudspeaker responses. The only frequency dependence appears in differential filter $\mathbf{w}_{\mathrm{d}}^{\pi/2}(\omega)$.

5. RESULTS

In this section we asses the performance of the proposed beamforming method, in terms of the metrics discussed in Sec. 3. Throughout this section, we set the array inter-element distance to $\delta = 8\,\mathrm{mm}$, the speed of sound to $c = 340\,\frac{\mathrm{m}}{\mathrm{s}}$ and we consider frequencies in the range $0\,\mathrm{Hz}$ - $4\,\mathrm{kHz}$.

First, we evaluate how well the proposed beamformer approximates an ideal beampattern. Fig. 2 shows the beampatterns generated by the DLA, computed as in (6), as a function of frequency. In this experiment, the target beampattern is set as a first-order hypercardioid (with $q = \frac{2}{3}$ [23]) and steered towards directions $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$. Fig. 2a, Fig. 2b, and Fig. 2c show the beampattern when the number of loudspeakers is set as $L = 3$. Similarly, Fig. 2d, Fig. 2e, and Fig. 2f show the same beampatterns when an array of $L = 7$ loudspeakers is used instead. We can notice that these configurations maintain a quite frequency-invariant behaviour, especially when a lower number of loudspeakers ($L = 3$) is considered. In fact, with the increase of the number of loudspeakers L, the aperture of the array increases as well, and the frequency invariance condition does not hold anymore.

Fig. 3 shows both cardioid ($q = \frac{1}{2}$ [23]) and hypercardioid beampatterns generated by a DLA of $L = 3$ loudspeakers, at frequency $f = 1\,\mathrm{kHz}$ and steering angles $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$. Along with each beampattern we also plot also the corresponding target ideal version, computed as in (9). We notice that, at $1\,\mathrm{kHz}$ there is an almost complete overlap between the ideal and computed beampatterns, meaning that the proposed method is able to provide a good approximation of first-order ideal beampat-

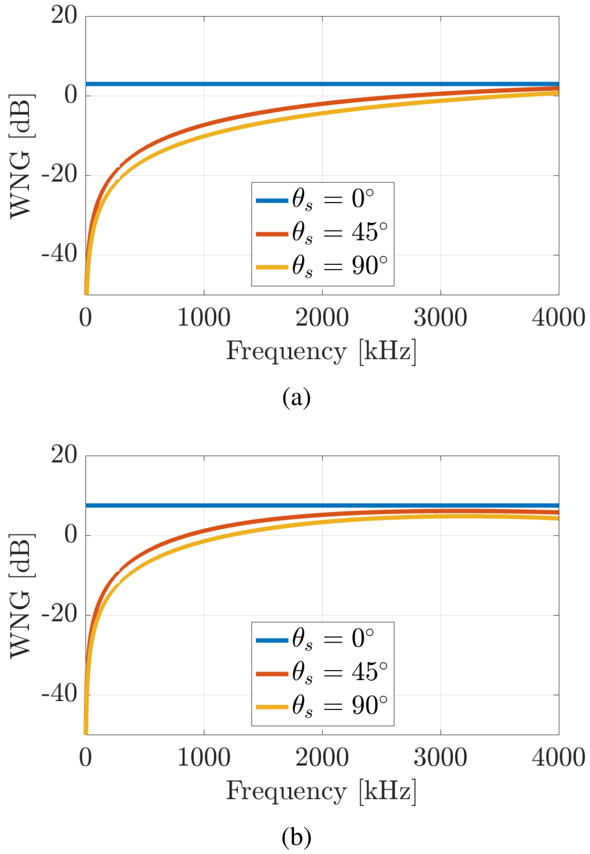

Figure 4: WNG as a function of frequency, measured for an array of $L = 3$ elements (Fig. 4a) and $L = 7$ elements (Fig. 4b), producing an hypercardioid beampattern.

terns steered toward different directions.

Fig. 4a plots the WNG, computed as in (7), as a function of frequency, for a loudspeaker array of $L = 3$ loudspeakers producing an hypercardioid beampattern steered at angles $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$. Fig. 4b, instead, show the WNG of an array composed of $L = 7$ loudspeakers. We notice that, as expected from differential beamforming theory, an increase in the number of loudspeakers corresponds to an increase of the WNG, hence to an increase of the robustness of the DLA. The flat WNG in the case $\theta_s = 0$ is due to the fact that, with such steering angle, eigenbeam filter $\mathbf{w}_\mathrm{d}^{\pi/2}(\omega)$ is neglected, and thus only non-differential filters $\mathbf{w}_\mathrm{o}$ and $\mathbf{w}_\mathrm{d}^0$ are used by the proposed beamformer.

As far as DI is concerned, we computed it as in (8). In particular, we computed the I_avg term as the average of the squared absolute value of the beampattern over all the considered angles. Fig. 5a plots DI as a function of frequency, for a loudspeaker array of $L = 3$ loudspeakers producing an hypercardioid beampattern at steering angles $\theta_s = 0, \frac{\pi}{4}, \frac{\pi}{2}$, while Fig. 5b, plots the same metric when the considered array is composed by $L = 7$ loudspeakers. It can be seen that, at low frequencies, both configurations present an almost flat response. Instead, when higher frequencies are considered, the configuration having $L = 7$ loudspeakers worsens in terms of DI.

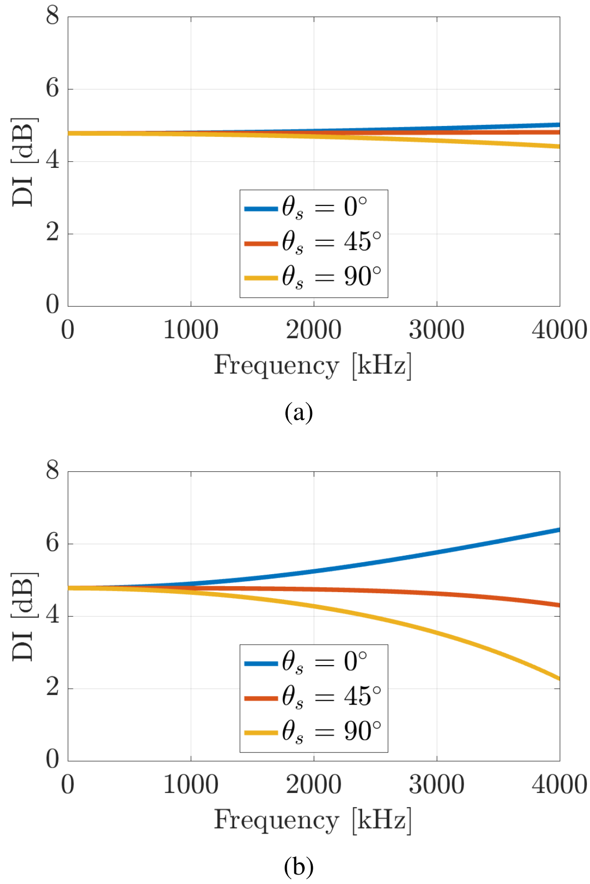

Figure 5: DI as a function of frequency, measured for an array of $L = 3$ elements (Fig. 4a) and $L = 7$ elements (Fig. 4b), producing an hypercardioid beampattern.

6. CONCLUSIONS

In this manuscript, we proposed an approach to design steerable first-order differential loudspeaker arrays, characterized by an uniform linear geometry in which monopole and dipole actuators are alternated. The beamforming filter is obtained by combining three different filters, each one accounting for the formation of an eigenbeam. The metrics used in this study suggest that this method performs well under the assumed conditions and exhibits almost frequency-invariant responses for large ranges of frequencies.

Overall, these findings suggest that differential loudspeaker arrays can be a promising approach for achieving directional sound reproduction in various applications, where small-size transducers are considered. Further research may be necessary to explore the full potential of this technology, including investigating the effects of different array configurations, the use of physical actuators with different directional characteristics and exploring the application of such methods in real-world scenarios.

7. ACKNOWLEDGMENTS

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