

Mitigation of unmeasured environmental and operational variability in long-term bridge health monitoring by unsupervised statistical learning

Alireza ENTEZAMI¹, Stefano MARIANI¹

¹Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza L. da Vinci 32, 20133 Milano, Italy, alireza.entezami@polimi.it, stefano.mariani@polimi.it

Abstract. Environmental and operational variations are significant challenges in long-term bridge health monitoring due to their potential to cause serious confounding influences and wrong decisions emerging as false alarms and mis-warnings. To mitigate these negative consequences, this paper aims at proposing an innovative unsupervised statistical learning method involving three steps of data clustering, unsupervised feature selection, and novelty detection. Firstly, the k-means clustering is applied to categorize dynamic features (bridge modal frequencies) into five types of clusters, reflecting the influences of five key factors that can alter bridge dynamic responses; that is, temperature, humidity, wind, traffic, and damage. Secondly, the clustered features are then processed through an unsupervised feature selection algorithm based on reconstruction independent component analysis to extract reduced features. Thirdly, a novelty detection model that leverages the Mahalanobis-squared distance is used to generate anomaly indices essential for decision-making. A steel arch bridge is utilized to verify the proposed method. Results demonstrate that this method can effectively handle the demanding issue stemming from unmeasured environmental and operational variability conditions.

Keywords: Structural Health Monitoring, Environmental and Operational Variability, Unsupervised Learning, Statistical Learning, Modal Data.

1. Introduction

Structural health monitoring (SHM) brings a practical and necessary tool for assessing the integrity and reliable performance of civil structures [1], especially bridges [2]. The great advantage of this process is to prevent dramatic events caused by failure and collapse [3]. Thanks to progress in sensing technologies, computational algorithms, and computer systems, most of the SHM applications concentrate on data-driven approaches that only use measured data from various sensors without any finite element modeling and updating [4]. Due to such a great opportunity, one can take advantage of various feature extractor and



feature analyzer for the evaluation of health and integrity of civil structures. Operational modal analysis (OMA) and machine learning are the practical and promising techniques for feature extraction and feature analysis within the paradigm of pattern recognition [5].

Considering the OMA, structural natural frequencies, modal vectors, and damping values are the main dynamic features suitable for SHM, especially early warning of damage occurrence. Machine learning is the main computational algorithm for SHM in a data-driven manner. This algorithm consists of two main categories as supervised learning and unsupervised learning. In SHM, a supervised learning method needs the features of both undamaged and damaged conditions (i.e., fully labeled training data) to train a supervised model (i.e., classifier or regressor) [6], whereas an unsupervised learning method is only dependent on the features of the undamaged condition (i.e., unlabeled training data) to train an unsupervised model (i.e., novelty detector) [7]. Because the preparation of fully labeled data in civil engineering is not a trivial task and it is not logical to damage vital civil structures to provide such data, unsupervised learning is often the main machine learning algorithm for SHM, particularly early damage detection.

The combination of an unsupervised learning model with modal frequencies is an effective approach to SHM of bridge structures. For this purpose, a set of modal frequencies of the undamaged structural state is taken as the main unlabeled training data to train an unsupervised learning model. When damage occurs, a new set of structural frequencies concerning the damaged structure is applied to the trained model and the output of this model exhibits deviation, which is an indicator of damage occurrence. Although this premise is correct, damage is not the only factor contributing to structural changes, especially when the civil structure remains in its undamaged condition. More precisely, environmental and operational variability is inevitable in real-world SHM. These conditions originate from changes in inherent physical properties (i.e., mass and stiffness) and then bridge responses caused by fluctuations in daily and seasonal temperature, humidity/rain/snow, wind speed/direction, and traffic loads [8]. The challenging issue for this event is that the aforementioned fluctuating conditions may cause confounding influences on the final decision. This implies that variations in these conditions may modify the structural characteristics, causing the trained model to incorrectly indicate damage, even though the bridge is functioning properly. In contrast, the bridge may really suffer from damage, which the environmental and operational variability mask damage and lead to neglect of correctly warning of damage occurrence. Economic and human losses are the critical consequences of the confounding influences. Thus, it is indispensable to consider this important challenge in all long-term SHM projects and make an attempt to mitigate the negative effects of environmental and operational variability. Unsupervised data normalization is among the most effective solutions to the aforementioned problem, especially when no information about environmental/operational data is needed. However, the focus on a single unsupervised data normalizer for mitigating the environmental and operational effects may not be sufficiently effective. In the light of advanced machine learning algorithms, many valuable research studies have been conducted to address this limitation [9-14].

For this reason, this paper aims to propose a new mixed learning method in an unsupervised statistical learning strategy. The method follows different tasks in three stages: (i) data segmentation by the k-means clustering, (ii) unsupervised feature selection using reconstruction independent component analysis (RICA), and (iii) novelty detection by the Mahalanobis-squared distance (MSD). The first step intends to split the unlabeled training features into five clusters in the view of five influential variability factors for changes in bridge structures. These factors include temperature, humidity, wind, traffic, and damage. The key innovation of this step is the elimination of the need to determine the number of data clusters. This requirement is addressed through engineering justification that considers the influential variability factors affecting bridge structures. The second step utilizes the

clustered features obtained from the data clustering to select the most informative and new features with smaller sizes. Such new feature sets are finally applied to the MSD to train a non-parametric novelty detection model. A set of modal frequencies related to an undamaged state of a steel arch bridge is considered to validate the technique presented in this paper. Results indicate that it can effectively address the major challenges related to the environmental and operational variability conditions and the removal of their effects.

2. Proposed Method

2.1 Data Segmentation by Clustering

The k -means clustering is a partitioning method for dividing data points into k mutually exclusive clusters and returns the index of the cluster label or index to each data point. The fundamental principle of the k -means clustering is to find similar data points, which are as close to each other as possible and as far from other points, and insert them into a cluster. Simply speaking, the k -means clustering starts by choosing k representative points as the initial centroids or clusters. Each point is then assigned to the closest centroid based on a particular distance metric. Once the clusters have been formed, the centroids for each cluster are updated. These two scenarios are repeated until the centroids do not change or any other alternative relaxed convergence criterion is met.

A wide range of distance metrics can be applied within the algorithm of the k -mean clustering. The choice can significantly affect the centroid assignment and the quality of the final solution. The various kinds of measures which can be used here are Euclidean distance, Manhattan distance, and Cosine similarity. Totally, the Euclidean distance is the most popular choice in this clustering method. Once the distance metric has been selected, it is necessary to determine the number of clusters for data partitioning. Although this number is the main hyperparameter of any partitioning clustering techniques [18] and it is possible to take advantage of different cluster number selection approaches [19], this paper disregards this challenge and utilizes five clusters based on an engineering justification. In the problem of SHM of a bridge structure, the variability in structural responses and features (modal frequencies) can generally be caused by five factors including air temperature, humidity/rain/snow, wind speed/direction, traffic, and damage.

The final step of the k -means clustering is to assign similar data points to their centroids and update this process. Given the training data $\mathbf{X} \in \mathbb{R}^{p \times n}$ including n feature vectors (i.e., data points). The algorithm of the clustering should make five clusters $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$, each of which is a matrix of p row and different columns. The objective function for data clustering in the k -means clustering is based on the function of sum-of-squared-errors (SSE), which is expressed as follows:

$$SSE = \sum_{j=1}^k \sum_{\mathbf{x}_i \in \mathbf{C}_k} \|\mathbf{x}_i - \mathbf{c}_k\|_2^2 \quad (1)$$

where $k = 1 \dots 5$ and these numbers are equivalent to $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$. Moreover, \mathbf{c}_k is the centroid of the k th cluster, which is defined as:

$$\mathbf{c}_k = \frac{\sum_{\mathbf{x}_i \in \mathbf{C}_k} \mathbf{x}_i}{k} \quad (2)$$

The goal is to find a clustering that minimizes the SSE score. The iterative assignment and update steps of the k -means algorithm aim to minimize the SSE score for the given set of centroids. At the end of this algorithm, the training data \mathbf{X} is divided into the pre-determined clusters $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$ and the training features find their clusters and get their labels.

2.2 Unsupervised Feature Selection

Unsupervised feature selection is an important methodology in machine learning that intends to choose or extract relevant and useful features from unlabeled data and discard irrelevant and useless ones. One of the great advantages of unsupervised feature selection techniques is their dimensionality reduction characteristics and removal of variability sources [20]. Reconstruction independent component analysis (RICA) is one of the unsupervised feature selection methods that get the original data (features), select some relevant features, and retrieve new reduced data (features) [21]. In this paper, we use a new application of RICA. Accordingly, it receives the clusters from the previous step and makes an attempt to yield reduced formats; that is, $\{\hat{\mathbf{C}}_{Te} \hat{\mathbf{C}}_H \hat{\mathbf{C}}_W \hat{\mathbf{C}}_{Tr} \hat{\mathbf{C}}_D\}$ with smaller column numbers than $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$. Note that both sets have the same row numbers equal to p .

Given the original training data \mathbf{X} , the main target of the RICA is to minimize the following objective function:

$$\min_{\mathbf{W}} \lambda \|\mathbf{W}\mathbf{x}_i\|_1 + \frac{1}{2} \|\mathbf{W}^T \mathbf{W}\mathbf{x}_i - \mathbf{x}_i\|_2^2 \quad (3)$$

where \mathbf{x}_i is the i^{th} feature of \mathbf{X} and $i = 1 \dots n$; λ is a regularization parameter; and $\mathbf{W} \in \mathbb{R}^{n \times q}$ is the weight matrix by considering q independent components. When $\lambda \rightarrow \infty$, the RICA algorithm returns to its classical version; that is, independent component analysis (ICA). To avoid applying complex techniques for estimating this regularization parameter, this paper sets λ as 1. Moreover, limited memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) quasi-Newton optimizer is considered to minimize Eq. (3). Once the weight matrix \mathbf{W} has been optimized, it is possible to obtain a reduced reconstructed feature set as $\hat{\mathbf{X}} = \mathbf{X}\mathbf{W}$, where $\hat{\mathbf{X}} \in \mathbb{R}^{p \times q}$ is the reduced reconstructed feature set from \mathbf{X} . In other words, the RICA allows us to select q samples from the initial n data points. Accordingly, one can consider that the selected q features are insensitive to any source of variability. The other note is that the aforementioned strategy is based on the whole training data \mathbf{X} . For the proposed method, each of the clustered sets $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$ is replaced with \mathbf{X} to obtain the reduced clustered sets $\{\hat{\mathbf{C}}_{Te} \hat{\mathbf{C}}_H \hat{\mathbf{C}}_W \hat{\mathbf{C}}_{Tr} \hat{\mathbf{C}}_D\}$. In fact, these reduced sets are the main outputs of the unsupervised feature selection of the proposed method.

2.3 Novelty Detection

Feature analysis in unsupervised learning heavily relies on novelty detection. This technique trains a detector using unlabeled data to assess subsequent test data. It involves training a novelty detector with unlabeled data to evaluate new test data. If the test data indicates an abnormal scenario, such as damage within a civil structure, the detector is designed to identify such anomalies. This study utilizes a non-parametric approach based on the Mahalanobis-squared distance (MSD) to monitor changes in the outputs of the model, specifically the novelty scores, and to determine if environmental and operational impacts have been successfully reduced.

Suppose that reconstructed features $\{\hat{\mathbf{C}}_{Te} \hat{\mathbf{C}}_H \hat{\mathbf{C}}_W \hat{\mathbf{C}}_{Tr} \hat{\mathbf{C}}_D\}$ have been obtained from the unsupervised feature selection strategy based on the RICA. The main parameters for developing the detector are the mean vectors $\{\boldsymbol{\mu}_{Te} \boldsymbol{\mu}_H \boldsymbol{\mu}_W \boldsymbol{\mu}_{Tr} \boldsymbol{\mu}_D\}$, each of which includes p elements, and covariance matrices $\{\boldsymbol{\Sigma}_{Te} \boldsymbol{\Sigma}_H \boldsymbol{\Sigma}_W \boldsymbol{\Sigma}_{Tr} \boldsymbol{\Sigma}_D\}$, which are $(p \times p)$ matrices, of the mentioned feature sets. To determine the novelty scores or distance values, it is necessary to utilize each actual training feature in the following equation:

$$D_m(\mathbf{x}_i) = \min \left((\mathbf{x}_i - \boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_j^{-1} (\mathbf{x}_i - \boldsymbol{\mu}_j) \right) \quad (4)$$

where $i = 1 \dots n$ and the subscript j denotes one of the five variability factors. For any testing point, one requires to replace it with \mathbf{x}_i in accordance with Eq. (4). If there are m testing points, hence, it can be derived m distance values, each of which is written as follows:

$$D_m(\mathbf{z}_k) = \min \left((\mathbf{z}_k - \boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_j^{-1} (\mathbf{z}_k - \boldsymbol{\mu}_j) \right) \quad (5)$$

where $k = 1 \dots m$.

3. Case Study: A Real-World Bridge

The proposed method is evaluated by benchmarking it with the dynamic features of a steel arch bridge named KW51 [22]. Serving as a railway bridge, this structure connects Leuven with Brussels in Belgium on the L36N railway line. It spans a total length of 115 meters and a width of 12.4 meters. Fig. 1 illustrates an actual image of this bridge. From 02 October 2018, this structure has been installed with SHM systems, including accelerometers and environmental sensors, to acquire data on acceleration time histories and different environmental parameters. This paper uses the dynamic features, specifically the structural modal frequencies from 02 October 2018 to 15 May 2019, to assess the effectiveness of the proposed method. During this period, the structure stayed at its normal condition; therefore, one should evaluate the effects of environmental factors in the modal frequencies.



Fig. 1. The KW51 Bridge

According to the work by Maes et al. [22], bridge natural frequencies were extracted from acceleration responses by an automated OMA technique yielding fourteen modes. However, some of these modes are faced with significant missing data points, presenting challenges for their use in any learning model due to inadequate training data. For this reason, we exploit the only four modes, which exhibit fewer missing samples comparing to others. Accordingly, the total count of modal frequencies for the bridge undamaged state reached 2688 By addressing the remaining missing values within the selected modes. Evolution of the final natural frequencies of the bridge in the modes 6, 10, 12, and 13 is illustrated in Fig. 2. It is evident that the primary environmental impact is reflected in the abrupt changes between samples 1345 and 2017. Even though the air temperature below 0°C may be the main reason for such variability, the other variation patterns can affect the bridge modal frequencies. In particular, Maes et al. [22] conducted a correlation analysis between the identified frequencies and air temperature. As there were low correlations between these variables, one can conclude that the other variability factors were influential in the bridge natural frequencies. Accordingly, this has been the main motivation for conducting this research. Hence, it is attempted to eliminate the confounding impacts visible in the modal frequencies and obtain normalized features.

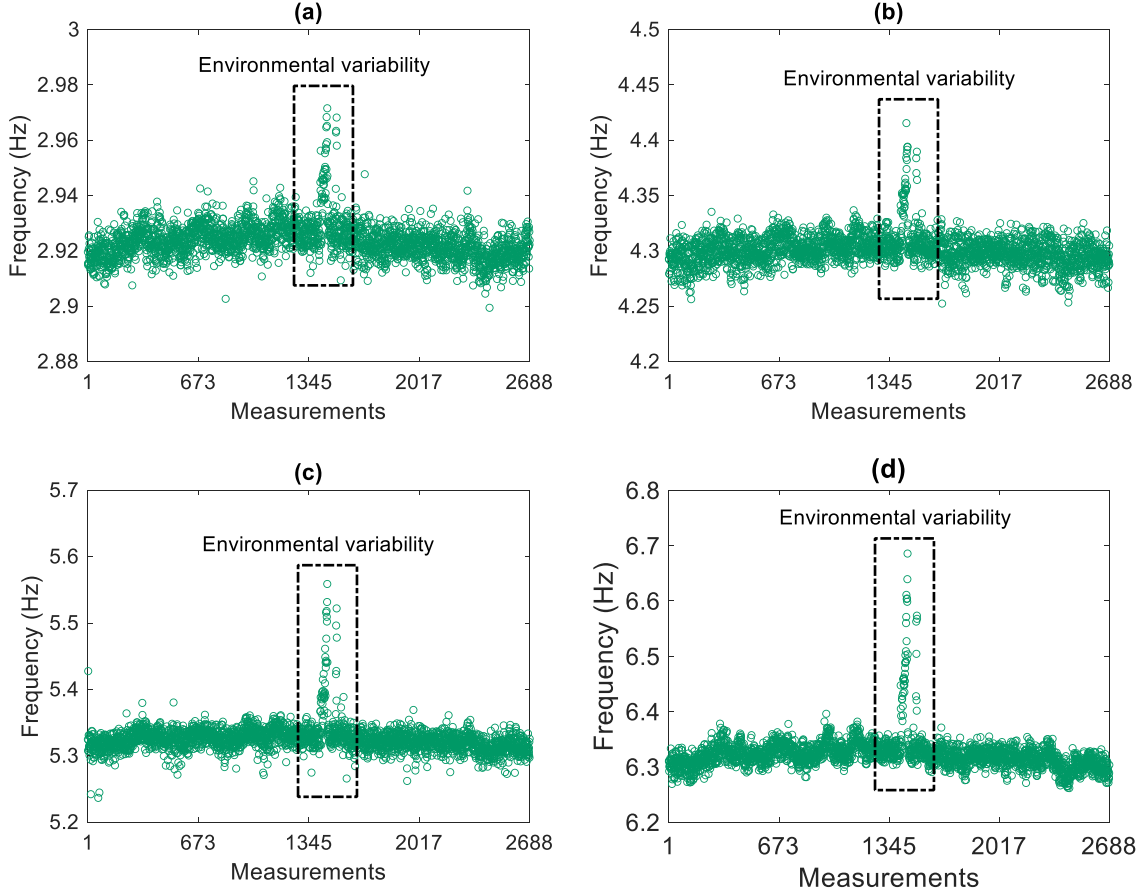


Fig. 2. (a)-(d) Final sets of the bridge natural frequencies in the modes 6, 10, 12, and 13

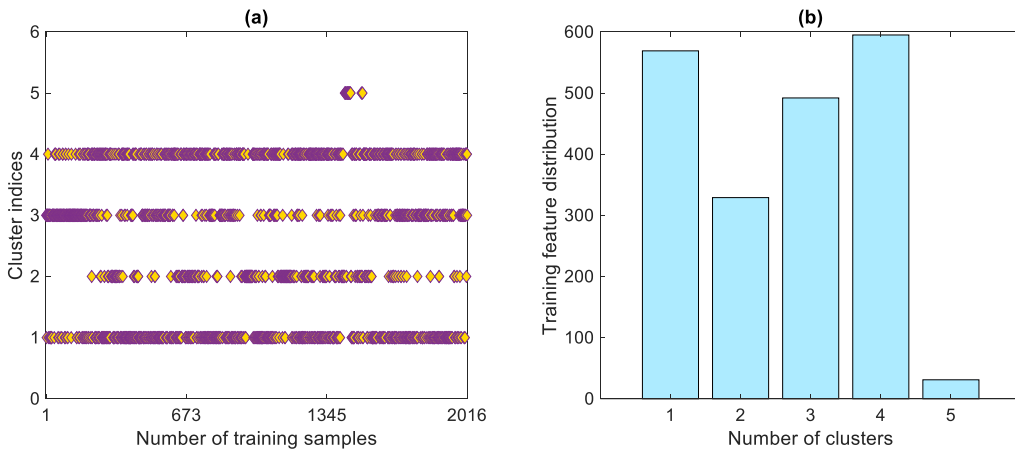


Fig. 3. Outputs of data partitioning via the k -means clustering when $k=5$: (a) the distribution of clustered training features and their cluster indices, (b) the number of features within each cluster

To implement the proposed method, all features are divided into the training and testing datasets in the ratios of 75% and 25%. Based on the proposed method, the training data $\mathbf{X} \in \mathbb{R}^{4 \times 2016}$ should be clustered by the k -means clustering by considering $k = 5$. The outputs of this step are shown in Fig. 3, where Fig. 3 (a) displays the distribution of the clustered training features and their cluster indices and Fig. 3 (b) illustrates the number of features within each cluster. It is essential to clarify that the assignment of some features to a specific cluster does not mean that those features exactly relate to that cluster. To put it another way, in the light of five influential variability factors in the modal frequencies of bridge structures, we

consider five clusters to partition the available data. Hence, the final goal is to remove any variability effect from such data and provide normalize them.

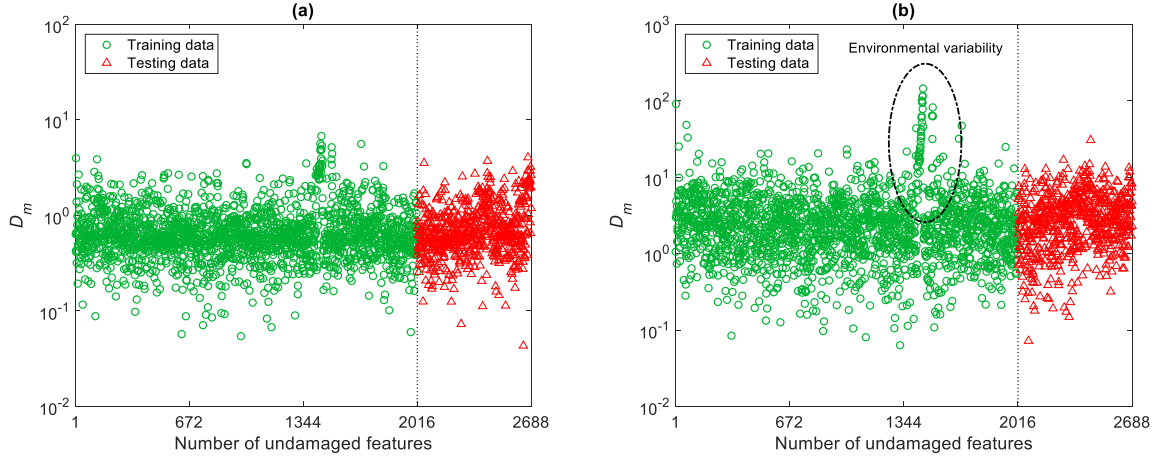


Fig. 4. Novelty values of the training and testing features regarding the proposed method (a) and comparing it with the conventional MSD without data segmentation and feature selection (b)

For the unsupervised feature selection via the RICA, the clustered feature sets $\{\mathbf{C}_{Te} \mathbf{C}_H \mathbf{C}_W \mathbf{C}_{Tr} \mathbf{C}_D\}$ are considered to extract their reduced versions $\{\hat{\mathbf{C}}_{Te} \hat{\mathbf{C}}_H \hat{\mathbf{C}}_W \hat{\mathbf{C}}_{Tr} \hat{\mathbf{C}}_D\}$. Note that the train crossing is equivalent to the traffic case in the reduced feature sets related to the problem of the KW51 Bridge. The main requirement for the RICA is the variable q . Although it needs to determine it before the implementation of the unsupervised feature selection, one can consider 5% of all samples as the final number for q [23]. It should be mentioned that $q > p$ to satisfy the condition of well-established and invertible covariance matrix for the MSD-based novelty detector. If this condition does not hold, one should select the lower bound of q equal to $p + 1$. Accordingly, the first five reduced feature sets (matrices) are comprised of four rows and 28, 16, 24, 29, and 5 columns, where the last one is the lower bound of q .

Finally, the reduced feature sets are incorporated into the MSD to compute the novelty scores of the training and testing data. Fig. 4 (a) shows the evolution of these scores gained by the proposed method. For a comparison, the same novelty detection is carried out by the only MSD metric without the data clustering and unsupervised feature selection as illustrated in Fig. 4 (b). As can be observed in Fig. 4 (a), the proposed is able to appropriately mitigate the environmental variability emerged as the sudden jumps. In contrast, as Fig. 4 (b) appears, the direct use of the MSD could not yield such a proper result. This conclusion not only demonstrates the superiority of the proposed method over the classical MSD-based novelty detection technique but also confirms the positive influences of the main steps of the proposed method, especially local data preparation by the k -means clustering and relevant feature selection based on the RICA.

4. Conclusions

To mitigate the confounding influences of unmeasured environmental variability on bridge modal frequencies, this study proposed a new unsupervised statistical learning method in accordance with the concept of local mixed learning. The proposed method contained three main steps of data partitioning via the k -mean clustering, unsupervised feature selection based on the RICA, and novelty detection through the MSD. Instead of determining the number of clusters needed for the k -means clustering, it was considered five influential variability factors in bridge modal frequencies including temperature, humidity/rain/snow, wind speed/direction, traffic, and damage. Accordingly, the whole training data was divided

into five clusters. These sets were then moved to the second stage of the proposed method regarding the unsupervised feature selection. Hence, the RICA method provided reduced clusters to use in the MSD for novelty score calculation. Some of the natural frequencies of the KW51 Bridge with fewer missing values were utilized to benchmark the suggested approach. The findings of this study showed that the solution presented here can successfully eliminate environmental variability in the natural frequencies, which were visible as the sudden jumps. It was also observed that the proposed method outperformed the classical novelty detection based on the direct use of the MSD metric.

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