Dephasing-induced mobility edges in quasicrystals

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Mobility edges (ME), separating Anderson-localized states from extended states, are known to arise in the single-particle energy spectrum of certain one-dimensional lattices with aperiodic order. Dephasing and decoherence effects are widely acknowledged to spoil Anderson localization and to enhance transport, suggesting that ME and localization are unlikely to be observable in the presence of dephasing. Here it is shown that, contrary to such a wisdom, ME can be created by pure dephasing effects in quasicrystals in which all states are delocalized under coherent dynamics. Since the lifetimes of localized states induced by dephasing effects can be extremely long, rather counter-intuitively decoherence can enhance localization of excitation in the lattice. The results are illustrated by considering photonic quantum walks in synthetic mesh lattices.

Introduction. Anderson localization and mobility edges[1–4] are fundamental concepts in the physics of disordered systems. They concern the localization behavior of quantum or classical waves in systems with disorder, and play a crucial role in different areas of physics, ranging from condensed matter physics[1–4] to ultracold atoms[5–12] and disordered photonics[13–19]. Mobility edges (ME) generally refer to points or thresholds in the energy spectrum where the localization features of the wave functions change, from being exponentially localized to being spatially extended[2,4]. The ME leads to various fundamental phenomena, such as metal-insulator transition by varying the particle number density or disorder strength[4], and can survive under perturbations and interactions[20,21]. While in non-interacting low-dimensional systems ME are prevented and the wave functions are localized with arbitrarily small disorder strength[3,22], it is well known that ME can exist in certain one-dimensional (1D) systems with quasiperiodic order, i.e. in quasicrystals[23–40].

Anderson localization and ME are generally observable in systems with non-fluctuating disorder. Besides, decoherence and dephasing effects are known to spoil localization and to enhance transport[41–52]. This evidence would suggest that dephasing effects are detrimental to enhance localization or to create ME, especially when the system does not display Anderson localization under coherent evolution.

In this Letter it is shown that, contrary to such a wisdom, ME can be created and localization can be enhanced by dephasing effects, even in systems which do not display Anderson localization under coherent dynamics. This unexpected result is illustrated by considering the off-diagonal Aubry-André model[53–59] in the delocalized phase, where dephasing effects can create ME and slow down delocalization in the lattice. A photonic quantum walk setup in a synthetic quasicrystal is suggested as an experimentally accessible platform for the observation of dephasing-induced ME.

Model. We consider a tight-binding one-dimensional lattice with aperiodic order described by the Hamiltonian

$$\hat{H} = -\sum_n \left(J_n \hat{a}^\dagger_{n+1}\hat{a}_n + \text{H.c.}\right) + \sum_n V_n \hat{a}^\dagger_n \hat{a}_n \qquad (1)$$

where J_n is the hopping amplitude between adjacent sites n and $(n+1)$, V_n is the on-site potential, and $\hat{a}^\dagger_n$, $\hat{a}_n$ are the creation and annihilation operators of bosonic particles at lattice site n, satisfying the usual commutation relations. Aperiodic order can be introduced in either or both the hopping amplitudes or on-site potential. We will focus our attention to two paradigmatic models, namely (i) the generalized diagonal Aubry-André model[27], corresponding to $J_n = J$, $V_n = 2A\cos(2\pi\alpha n + \theta)/[1 - B\cos(2\pi\alpha n + \theta)]$, with $J, A > 0$, $0 \leq B < 1$ and α irrational Diophantine (this model reduces to the ordinary Aubry-André model[53] for $B = 0$); and (ii) the off-diagonal Aubry-André model[54–58], corresponding to $J_n = A + B\cos(2\pi\alpha n + \theta)$, $V_n = 0$, with $A, B > 0$. The first model displays ME in certain regions of parameter space at the energy $E_m = (2/B)(J - A)$[27] [Fig.1(a)], whereas the second model does not display ME and for $B < A$ all single-particle eigenstates of H are extended [Fig.1(c)]. In such regimes an initial excitation in the lattice spreads ballistically, as illustrated in Figs.1(b) and (d). In the numerical analysis we assume $\alpha = (\sqrt{5}-1)/2$ (the golden mean) and take a lattice of finite (but large) size L with periodic boundary conditions, where L is a Fibonacci number and the golden mean is approximated by a rational. The localization properties of the eigenstates $\psi_n^{(l)}$ of the matrix Hamiltonian H are captured by the inverse participation ratio (IPR), which for a normalized wave function is defined as[60] $\text{IPR}_l = \sum_{n=1}^{L} |\psi_n^{(l)}|^4$. For an extended state, the IPR scales as $1/L$, hence vanishing in the thermodynamic limit $L \rightarrow \infty$, while it remains finite for a localized state. The spreading dynamics of initial single-site excitation of the lattice is captured by the time behavior of the second moment $\sigma^2(t) = \sum_n n^2 |\psi_n(t)|^2$, where $\psi_n(t)$ are the amplitude probabilities at various lattice sites at time t. Dynamical delocalization corresponds to a secular growth of $\sigma^2(t)$ in time, with $\sigma^2(t) \sim t^2$ for ballistic spreading and $\sigma^2(t) \sim t$ for diffusive spreading[61].

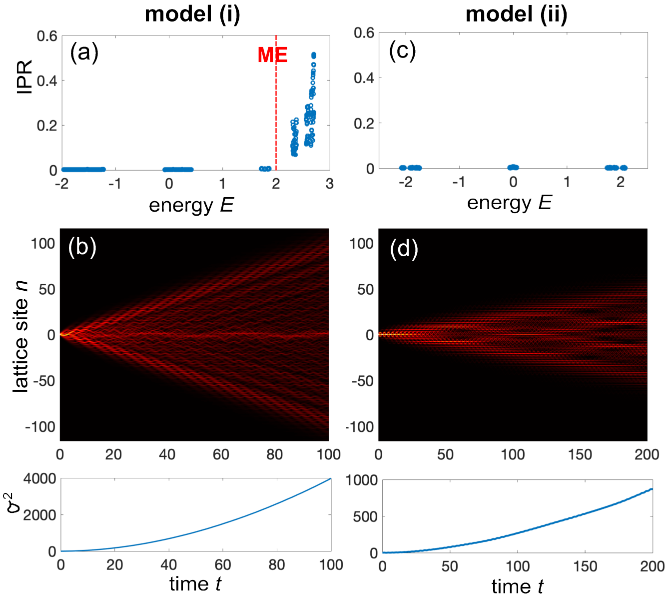

FIG. 1: (a) IPR of the eigenstates of the Hamiltonian H for the generalized diagonal Aubry-André model [model (i)] for parameter values $\alpha = (\sqrt{5}-1)/2$, $J = 1$, $A = 0.6$, and $B = 0.4$. Note the existence of a ME at the energy $E_m = 2$, with extended states for $E < E_m$ and localized states for $E > E_m$. (b) Numerically computed wave spreading dynamics in the lattice (snapshot of $|\psi_n(t)|$ on a pseudocolor map) for initial excitation of site $n = 0$. The lower panel in (b) depicts the corresponding behavior of the second moment $\sigma^2(t)$. (c,d) Same as (a,b), but for the off-diagonal Aubry-André model [model (ii)] with parameter values $A = 1$, $B = 0.9$. Note that in this case there are not ME and all wave functions are extended. In both models the spreading in the lattice is ballistic. Lattice size is $L = 987$.

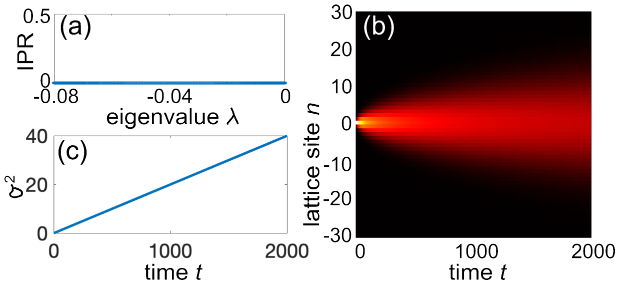

FIG. 2: Suppression of ME and diffusive spreading in the generalized diagonal Aubry-André model induced by dephasing. (a) IPR of the eigenstates of the Markov matrix W versus the eigenvalues λ. Dephasing rate $\gamma = 100$; other parameter values are as in Fig.1(a,b). (b,c) Spreading of an initial single-site excitation of the lattice (snapshot of $P_n(t)$ and temporal evolution of second moment σ^2). Note that the spreading is diffusive, $\sigma^2(t) = Dt$, with diffusion coefficient $D = 2J^2/\gamma$.

Dephasing effects can be modeled using different methods, such as stochastic Schrödinger equations or quantum master equations in the Lindblad form (see e.g.[42,44,46,48–52,62,63]). The Lindblad master equation for the density matrix $\hat{\rho}$ describing dephasing effects reads[49,52,62,63]

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] + \gamma \sum_{n=1}^{L} \left(\hat{L}_n \hat{\rho} \hat{L}_n^\dagger - \frac{1}{2} \left\{ \hat{L}_n^\dagger \hat{L}_n, \hat{\rho} \right\} \right) \quad (2)$$

where $\hat{L}_n = \hat{a}_n^\dagger \hat{a}_n$ is the dissipator describing pure dephasing at lattice site n and $\gamma > 0$ is the dephasing rate. This model conserves the number of particles. In the single-particle sector the Hilbert space, spanned by the set of states $|n\rangle = \hat{a}_n^\dagger |vac\rangle$, the evolution equations for the density matrix elements $\rho_{n,m}(t) = \langle n|\hat{\rho}|m\rangle$ read[62,63]

$$\begin{aligned} \frac{d\rho_{n,m}}{dt} &= i(J_{n-1}\rho_{n-1,m} + J_n \rho_{n+1,m}) \\ &- i(J_m \rho_{n,m+1} + J_{m-1}\rho_{n,m-1}) \\ &+ i(V_m - V_n)\rho_{n,m} - \gamma(1-\delta_{n,m})\rho_{n,m}. \end{aligned} \quad (3)$$

Dephasing-induced mobility edges and localization. The main question is whether dephasing effects can create ME and induce some kind of localization. To answer this question, we focus our analysis to the strong dephasing regime ($\gamma \gg |J_n|, |V_n|$), such that the coherences $\rho_{n,m}$ $(n \neq m)$ are small and the particle dynamics can be described by a classical one-dimensional hopping model[49,50]. Basically, the time evolution of the populations $P_n(t) = \rho_{n,n}(t)$ is given by a classical master equation, where the quantum jumps generate the transition rates between classical configurations[49]. The classical master equation is obtained from Eq.(3) after adiabatic elimination of coherences and reads

$$\frac{dP_n}{dt} = \sum_{m=1}^{N} W_{n,m} P_m(t) \quad (4)$$

where the elements of the Markov transition matrix W are given by (Sec.S1 of[64])

$$W_{n,m} = -\frac{2}{\gamma}(J_n^2 + J_{n-1}^2)\delta_{n,m} + \frac{2J_n^2}{\gamma}\delta_{n,m-1} + \frac{2J_{n-1}^2}{\gamma}\delta_{n,m+1}. \quad (5)$$

The same classical master equation can be derived from the single-particle Schrödinger equation with periodic phase randomization of the wave function at time intervals $\Delta t = 2/\gamma$ (Sec.S2 of[64]). Note that, since W is an Hermitian matrix, the classical master equation (4) can be viewed as a Schrödinger equation with a Wick-rotated anti-Hermitian Hamiltonian $H' = iW$. Let us indicate by λ_l and $\Theta_n^{(l)}$ the eigenvalues and corresponding eigenvectors of W $(l = 1, 2, ..., L)$. Since W is Hermitian, the eigenvalues are real and can be ordered such that $\lambda_L \leq ... \leq \lambda_2 \leq \lambda_1$, with the constraint $\lambda_l \leq 0$. Moreover, there is always a vanishing eigenvalue, $\lambda_1 = 0$, with eigenvector $\Theta_n^{(1)} = (1/L)(...1, 1, 1, ...)^T$. This state does not decay in time and corresponds to an extended (maximally-mixed) state. Since any initial state of the system has a non-vanishing projection onto such an eigenstate, in the long-time limit any initial localized excitation tends to spread and homogenize

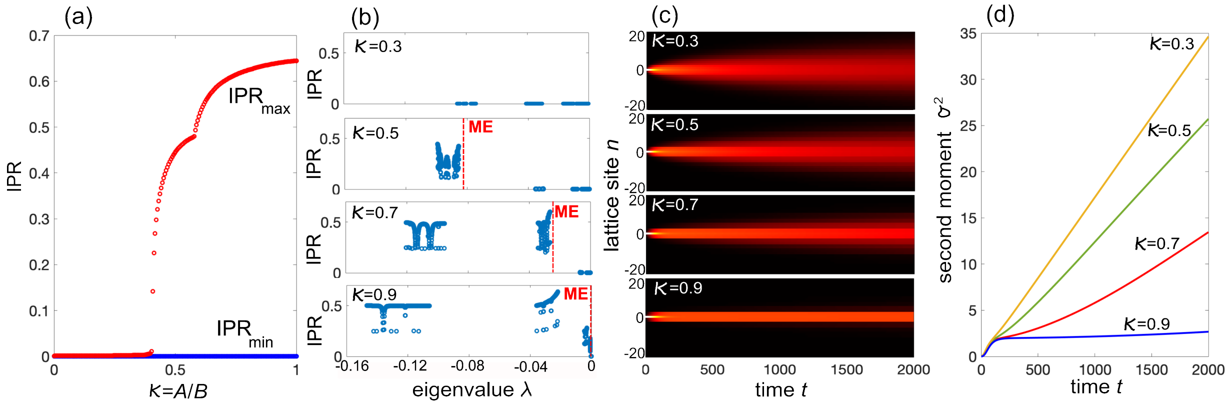

FIG. 3: Dephasing-induced ME in the off-diagonal Aubry-André model. (a) Behavior of $\mathrm{IPR}_{\mathrm{min}}$ and IPR_{max} versus the ratio $\kappa = B/A$. ME are created for $\kappa > \kappa_c \simeq 0.4$. (b) Behavior of IPR of all eigenvectors of W versus the eigenvalues λ for a few increasing values of κ. Note that the ME $\lambda = \lambda_m$, separating extended ($\lambda > \lambda_m$) from localized ($\lambda < \lambda_m$) states approaches zero as κ is increased toward $\kappa = 1$; parameter values are $A = 1$ and $\gamma = 100$. (c,d) Spreading dynamics of initial single-site excitation of the lattice (snapshots of $P_n(t)$ and temporal evolution of second moment σ^2). Lattice size $L = 987$.

in all sites of the lattice. However, the relaxation dynamics can be significantly slowed down when there are localized eigenstates of W with extremely long lifetimes. The localization features of the eigenstates of W are characterized by their IPR, and the appearance of ME is signaled by the coexistence of states with small (vanishing as $\sim 1/L$) and finite (i.e. independent of L) IPR values.

An interesting property of the matrix W is that its elements do not depend on the on-site potential V_n. Therefore, when the disorder is introduced in the on-site potential V_n solely and the hopping rates $J_n = J$ are homogeneous, such as in model (i), the incoherent hopping terms are homogeneous, all eigenvectors of W are extended and thus there are not mobility edges. This means that, as expected, dephasing destroys ME and leads to delocalization. The spreading in the lattice is diffusive, i.e. $\sigma^2(t) = Dt$, with a diffusion coefficient $D = 2J^2/\gamma$. Washing out of ME in model (i) due dephasing and diffusive spreading of excitation in the lattice is illustrated in Fig.2.

A very different behavior can be found in model (ii), where incommensurate disorder is introduced in the hopping rates J_n. In this case, W displays *both* diagonal and off-diagonal disorder, and numerical computation of IPR of the eigenstates of W clearly shows that ME are created when the ratio $\kappa = B/A$ becomes larger than the critical value $\kappa_c \simeq 0.4$; see Fig.3(a). The figure depicts the values of the smallest (IPR_{min}) and largest (IPR_{max}) values of the IPR of all eigenstates of W, as κ is increased from 0 to 1. The $\kappa > 1$ regime is not considered since in this case J_n can vanish at some values of n and there is trivial localization in the system. Remarkably, when κ approaches 1 from below, the position of the ME λ_m moves toward zero, indicating that a large portion of localized eigenstates displays an extremely long lifetime, as shown in Figs.3(b). The appearance of such strongly-localized eigenstates of W with long lifetimes have a great impact on the dynamical spreading of excitations in the lattice, since they can trap the excitation for long times and can be thus largely slow down thermalization toward the maximally-mixed state. The transient trapping of excitation as κ is increased toward 1 is clearly illustrated in Figs.3(c) and (d).

Dephasing-induced mobility edges in photonic quantum walks. Discrete-time quantum walks provide experimentally-accessible models to test disorder and decoherence phenomena[65–74]. In particular, photonic quantum walks realize synthetic lattices with controllable disorder and decoherence[69–72], which could provide a platform for the observation of dephasing-induced ME. We consider a photonic implementation of a quasicrystal based on light pulse dynamics in synthetic mesh lattices[75–80], where dephasing is emulated by random dynamic phase changes[69–71]. The system consists of two fiber loops of slightly different lengths that are connected by a fiber coupler with a time-dependent coupling angle β. A phase modulator is placed in one of the two loops, which impresses stochastic phases to the traveling pulses emulating dephasing effects. Off-diagonal disorder is introduced by proper temporal modulation of the coupling angle β between the two fiber loops[78]. Light dynamics is described by the set of discrete-time equations[78]

$$u_n^{(m+1)} = \left(\cos\beta_{n+1}u_{n+1}^{(m)} + i\sin\beta_{n+1}v_{n+1}^{(m)}\right)\exp(i\phi_n^{(m)}) \quad (6)$$

$$v_n^{(m+1)} = i\sin\beta_{n-1}u_{n-1}^{(m)} + \cos\beta_{n-1}v_{n-1}^{(m)} \quad (7)$$

where $u_n^{(m)}$ and $v_n^{(m)}$ are the pulse amplitudes at discrete time step m and lattice site n in the two fiber loops, β_n is the site-dependent coupling angle, and $\phi_n^{(m)}$ are uncorrelated stochastic phases with uniform distribution in the range $(-\pi, \pi)$. For coupling angles β_n close to $\pi/2$ and under coherent dynamics, i.e. for $\phi_n^{(m)} =$

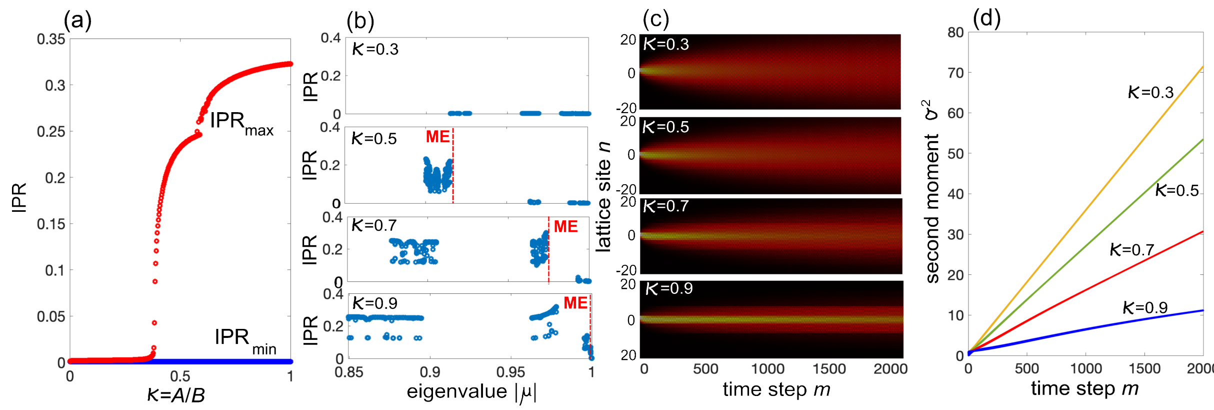

FIG. 4: Dephasing-induced ME in photonic quantum walks. (a) Behavior of $\mathrm{IPR}_{\mathrm{min}}$ and IPR_{max} versus the ratio $\kappa = B/A$ for $A = 0.1$ and $\alpha = (\sqrt{5}-1)/2$. The coupling angle is $\beta_n = \pi/2 - 2A - 2B\cos(2\pi\alpha n)$. (b) Behavior of IPR of all eigenvectors of the incoherent matrix $\mathcal{U}$ versus $|\mu|$ for a few increasing values of κ. (c,d) Spreading dynamics of initial single-site excitation of the lattice. Panels in (c) show snapshots of $P_n^{(m)} = X_n^{(m)} + Y_n^{(m)}$ versus discrete time step m for initial excitation $X_n^{(0)} = \delta_{n,0}$, $Y_n^{(0)} = 0$. Panel (d) depicts the corresponding evolution of second moment σ^2. Lattice size $L = 377$.

0, the model described by Eqs.(6) and (7) reproduces the off-diagonal Aubry-André model with hopping rates $J_n = \pm(1/2)\cos\beta_{n+1}$ (Sec.S3 of[64]),; the model (ii) is thus retrieved after letting $\beta_n = \pi/2 - 2A - 2B\cos(2\pi\alpha n)$ $(A, B \ll 1)$. When the stochastic phases are applied at every time step, the incoherent dynamics is described by the probability-conserving map (Sec.S4 of[64])

$$X_n^{(m+1)} = \cos^2\beta_{n+1} X_{n+1}^{(m)} + \sin^2\beta_{n+1} Y_n^{(m)} \quad (8)$$

$$Y_n^{(m+1)} = \sin^2\beta_n X_n^{(m)} + \cos^2\beta_n Y_{n-1}^{(m)} \quad (9)$$

for the light pulse intensities $X_n^{(m)} = \overline{|u_n^{(m)}|^2}$ and $Y_n^{(m)} = \overline{|v_{n+1}^{(m)}|^2}$, where the overline denotes statistical average. The incoherent pulse dynamics is thus fully captured by the eigenvalues μ and corresponding eigenstates of the incoherent propagation matrix $\mathcal{U}$, defined by Eqs.(8) and (9). The eigenvalues μ satisfy the constraint $|\mu| \leq 1$, and there is always the eigenvalue $\mu_1 = 1$ corresponding to the non-decaying extended eigenstate $X_n = Y_n = 1/(2L)$, where the photon is found with equal probability in each site of the two loops. This means that asymptotically any initially localized excitation in the lattice spreads to reach a uniform distribution. However, if there are localized eigenstates of the matrix $\mathcal{U}$ with extremely long lifetimes, i.e. with $|\mu|$ very close to 1, the spreading can be greatly slowed down and excitation can be transiently trapped in the lattice. The appearance of ME induced by dephasing effects and corresponding slowing-down of light spreading in the quantum walk is illustrated in Fig.4. This behavior is clearly similar to the one shown in Fig.3, given that for $\beta_n \sim \pi/2$ the incoherent photon dynamics can be mapped into the master equation (4) (Sec.S4 of[64]). A similar behavior is nevertheless observed even when the angles β_n are not close to $\pi/2$.

Conclusions. In summary, we predicted that in certain one-dimensional lattices with aperiodic order ME can be created (rather than destroyed) by dephasing effects. While Anderson localization and ME arising from diagonal (on-site potential) disorder is always spoiled out by dephasing, in models where disorder is off-diagonal (i.e. in the coupling constants) ME, separating localized from extended states of the Markov transition matrix, can be created by dephasing effects, even when the coherent Hamiltonian does not display any Anderson localization and all states are extended. While the fate of dephasing is always to drive the system toward an incoherent state with excitation uniformly distributed in the lattice via a diffusive process, the spreading dynamics can be greatly slowed down when the Anderson-localized states of the Markov matrix have extremely long lifetimes. The present study unveils a distinctive physical mechanics of ME formation and indicates that, contrary to a common wisdom, dephasing effects can slow down (rather than enhance) transport in a disordered lattice, a result that could be of relevance in different physical, chemical or biological systems where disorder and dephasing noise play a crucial role.

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