

Robust Anderson transition in non-Hermitian photonic quasicrystals

STEFANO LONGHI^{1,2,*}

¹ Dipartimento di Fisica, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy

² IFISC (UIB-CSIC), Instituto de Física Interdisciplinar y Sistemas Complejos - Palma de Mallorca, Spain

* stefano.longhi@polimi.it

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Anderson localization, i.e. the suppression of diffusion in lattices with random or incommensurate disorder, is a fragile interference phenomenon which is spoiled out in the presence of dephasing effects or fluctuating disorder. As a consequence, Anderson localization-delocalization phase transitions observed in Hermitian systems, such as in one-dimensional quasicrystals when the amplitude of the incommensurate potential is increased above a threshold, are washed out when dephasing effects are included. Here we consider localization-delocalization spectral phase transitions occurring in non-Hermitian quasicrystals with local incommensurate gain and loss, and show that, contrary to the Hermitian case, the non-Hermitian phase transition is robust against dephasing effects. The results are illustrated by considering synthetic quasicrystals in photonic mesh lattices.

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Anderson localization [1], i.e. the absence of diffusion in systems with static disorder, is ubiquitous in the physics of disordered systems [2]. Photonics has provided since more than three decades a fascinating laboratory platform for testing the localization properties of disordered systems, disclosing their rich physics [3–13] with potential applications to the design of the next generation of materials [14]. In low-dimensional systems, the Anderson localization-delocalization transition is observed in quasicrystals, i.e. in lattices displaying aperiodic order [15]. The most notably example is provided by the Aubry-André model [16]. This model describes a one-dimensional incommensurate potential on a lattice, in which all wave functions in the spectrum suddenly change from extended to localized when the strength of the quasiperiodic potential exceeds a threshold [15, 16]. Such a phase transition has been experimentally observed in several physical systems, including photonic waveguide arrays and Bose-Einstein condensates in optical lattices [17–19]. Recently, a non-Hermitian (NH) extension of the Aubry-André model, where the incommensurate potential is complex and accounts for aperiodic local gain and loss in the system, has been introduced [20–23], and the topological origin of the metal-insulator phase transition has been disclosed [20], with

experimental demonstrations reported in recent works [24, 25]. Anderson localization is rather generally a fragile phenomenon against dephasing, decoherence or frequent measurements [6, 26–31]: dephasing or fluctuating potentials destroy interference effects, Anderson localization is spoiled out and replaced by diffusive transport [6]. This implies that metal-insulator phase transitions, such as those observed in the Hermitian Aubry-André models, are washed out by dephasing effects. A natural and largely open question is whether Anderson localization is fragile or robust against dephasing when the underlying Hamiltonian is non-Hermitian.

In this Letter we address such a main question and show that the localization-delocalization phase transition in a specific quasicrystal model, described by the NH extension of the Aubry-André model with dissipative disorder [20], is robust against dephasing effects. The results are illustrated by considering synthetic quasicrystals realized in photonic mesh lattices [6, 25, 26, 36–39]. It is also pointed out that non-Hermiticity itself, neither the topological nature of Anderson transitions in non-Hermitian models, are not sufficient conditions to ensure robustness against dephasing.

To introduce the problem in a rather general framework, let us consider the coherent evolution of excitations in a tight-binding lattice, comprising N sites and described by the matrix Hamiltonian $H = H_{n,m}$ ($n, m = 1, 2, \dots, N$), and let us assume that at every time interval Δt the phase of the wave function ψ_n at any lattice site n is randomized. Such a randomized phase process basically emulates dephasing effects in the dynamics and has been used in photonic quantum walks to experimentally demonstrate transition from Anderson localization to diffusive transport for fluctuating disorder [6]. The time evolution of the wave function amplitudes $\psi_n(t)$ reads

$$i \frac{d\psi_n}{dt} = \sum_m H_{n,m} \psi_m + \psi_n \sum_{\rho=1,2,3,\dots} \varphi_n^{(\rho)} \delta(t - \rho \Delta t) \quad (1)$$

where the last term on the right hand sides of Eq.(1) describes the dephasing process. Here $\varphi_n^{(\rho)}$ are assumed to be uncorrelated stochastic phases, both in site index n and time step ρ , with a given probability density function. Fully coherent dynamics is obtained by letting $\varphi_n^{(\rho)} = 0$, whereas fully incoherent (classical) dynamics is obtained by assuming a uniform distribution in the range $(-\pi, \pi)$ for the probability density function. Under fully

coherent dynamics, the wave function amplitudes evolve according to $\psi_n(t) = \sum_m U_{n,m}(t) \psi_m(0)$, where $U(t) = \exp(-iHt)$ is the coherent propagator. On the other hand, for fully incoherent dynamics indicated by $P_n(t_\rho) = |\psi_n(t_\rho)|^2$ the occupation probabilities (populations) at various sites of the lattices, where $t_\rho = \rho \Delta t$ and the overbar denotes statistical average, the time evolution is described by the classical map

$$P_n(t_{\rho+1}) = \sum_m \mathcal{U}_{n,m} P_m(t_\rho) \quad (2)$$

where $\mathcal{U}_{n,m} = |U_{n,m}(\Delta t)|^2$ in the incoherent propagator. Equation (2) can be readily obtained by solving Eq.(1) in each time interval Δt and then taking the statistical average of $|\psi_n|^2$. It is instructive to unveil the fate of Anderson localization under dephasing. Let us assume that H displays spectral Anderson localization. This means that the eigenstates of H are exponentially localized, which corresponds generally (but not universally) to suppression of wave spreading in the lattice (dynamical localization). Under fully incoherent dynamics, provided that the Hamiltonian H is Hermitian it can be demonstrated that, as expected, Anderson localization is spoiled out and excitation spreads in the lattice, finally reaching a stationary state with equal distributions of populations in the lattice, i.e. $P_n(t_\rho \rightarrow \infty) = 1/N$. The proof is given in Sec.1 of the Supplemental document. For example, in the Aubry-André model describing a one-dimensional quasicrystal, the Hamiltonian H reads explicitly

$$H_{n,m} = J(\delta_{n,m+1} + \delta_{n,m-1}) + 2V_0 \delta_{n,m} \sin(2\pi \alpha n + ih) \quad (3)$$

where J is the hopping amplitude between adjacent sites in the lattice, $2V_0$ is the amplitude of the incommensurate sinusoidal potential, α is irrational Diophantine, and ih a complex phase term [20]. The Hermitian case is obtained by letting $h = 0$. In this case under coherent dynamics the model displays a metal-insulator phase transition when V_0 is increased above the critical value $V_0 = J$. Clearly, such a phase transition is not observed anymore in the incoherent regime (see Fig.S1 in the Supplemental Material). However, in the NH case $h \neq 0$ a different metal-insulator phase transition can be observed under coherent dynamics [20], where the control parameter is provided by the complex phase h . Assuming $V_0 < J$, for $h = 0$ the system is in the delocalized phase. As h is increased above the critical value

$$h_c = \log(J/V_0), \quad (4)$$

the system undergoes a delocalization-localization phase transition, with all eigenstates being exponentially localized for $h > h_c$ [20]. Remarkably, and this is the central result of this work, such a NH phase transition *is not* washed out by dephasing effects, contrary to what happens for Hermitian phase transitions. This point is discussed further in Sec.2 of the Supplemental document. Why is localization robust in this case? Basically in the Hermitian quasicrystal, where the incommensurate on-site potential V_n is real, dephasing effects spoil the delicate phase interference of waves scattered off by the potential disorder, which is at the heart of Anderson localization. However, when the on-site potential is a complex function, i.e. when we have lattice disorder in local gain or loss, phase randomization does not affect the local damping or amplification of the wave function, and such a dissipative disorder can provide a route toward localization robust against dephasing effects. Specifically, assuming a small potential amplitude $V_0/J \sim \epsilon$, with ϵ a small parameter, and a short time interval $\Delta t \sim \epsilon/J$, under incoherent dynamics a

delocalization-localization phase transition is observed as h is increased above the critical value

$$h'_c = \text{asinh} \left(\frac{\Delta t J^2}{2V_0} \right). \quad (5)$$

An example of the incoherent NH phase transition is shown

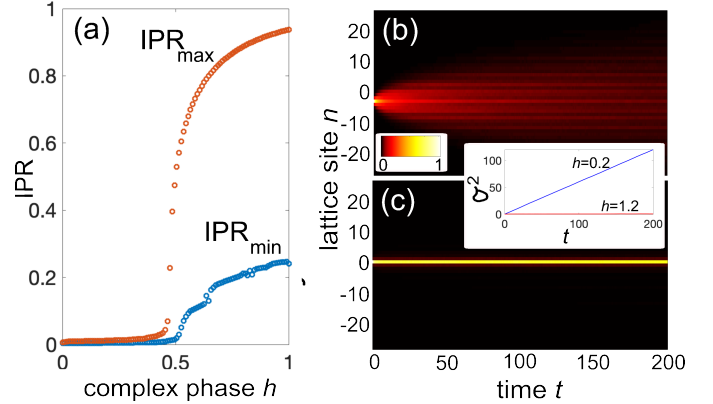


Fig. 1. (a) Behavior of the maximum and minimum inverse participation ratio (IPR) of eigenstates of the incoherent propagator \mathcal{U} versus the complex phase h in the NH Aubry-André model for parameter values $J = 1$, $V_0 = 0.3$, $\Delta t = 0.3$ and $\alpha = (\sqrt{5} - 1)/2$. (b) Temporal evolution of the normalized occupation probability $P_n(t)$ versus time t in the delocalized phase ($h = 0.2$). The excitation spreads diffusively in the lattice. (c) Same as (b) but for $h = 1.2$, corresponding to localization. The inset in panels (b,c) depicts the corresponding temporal evolution of the second moment $\sigma^2(t)$ in the two cases.

in Fig.1. The localization properties of \mathcal{U} are characterized by the inverse participation ratio (IPR) of any eigenvector Θ_n of \mathcal{U} , which for a normalized eigenstate is defined by $\text{IPR} = \sum_n |\Theta_n|^4$. For a tightly-confined eigenstate, the IPR is independent of N and takes a finite value close to 1, whereas for an extended state the IPR is small and vanishes as $\sim 1/N$ as $N \rightarrow \infty$. The largest and smallest values of the IPR for any eigenvector of \mathcal{U} are indicated by IPR_{\max} and IPR_{\min} , respectively. Figure 1(a) shows the behavior IPR_{\max} and IPR_{\min} versus h for parameter values $\alpha = (\sqrt{5} - 1)/2$, $J = 1$, $V_0 = \Delta t = 0.3$, clearly indicating a delocalization-localization phase transition near the critical point $h'_c \simeq 0.48$ as predicted by Eq.(5). The eigenstates of \mathcal{U} have been numerically computed for a lattice size $N = 233$ under periodic boundary conditions, and using the rational approximant $\alpha = 144/233$ of the inverse of the golden ratio [20]. The distinct transport properties, in the delocalized ($h < h'_c$) and localized ($h > h'_c$) phases, are shown in Figs.1(b) and (c). The figure panels illustrate the temporal evolution of occupation probabilities $P_n(t) = |\psi_n(t)|^2$, normalized at each time step such that $\sum_n P_n(t) = 1$, as obtained by numerical integration of the Schrödinger equation (1), after a statistical average over 1000 realizations of stochastic phase randomization. The lattice is initially excited in site $n = 0$. The spreading dynamics is characterized by the second moment $\sigma^2(t) = \sum_n P_n(t) n^2 / \sum_n P_n(t)$, which is depicted in the inset of Figs.1(b,c). Note that in the delocalized phase [Fig.1(b)] the second moment increases linearly with time t , corresponding to diffusive spreading, whereas in the localized phase [Fig.1(c)] wave spreading is halted. It should be mentioned that the robustness of Anderson transition against dephasing strictly requires some dissipative disorder, and thus it is not observed in other NH models. For

example, in a quasicrystal with real on-site potential and asymmetric hopping rates, like in the Hatano-Nelson model, the localization-delocalization transition of topological nature is not robust against dephasing (see Sec.3 of the Supplemental material).

To illustrate the predicted phenomenon in an experimentally accessible platform, let us consider a photonic implementation of a quasicrystal based on light pulse dynamics in synthetic mesh lattices [6, 25, 26, 40], where decoherence can be introduced and controlled by random dynamic phase changes [6]. The system consists of two fiber loops of slightly different lengths that are connected by a fiber coupler with a coupling angle β . Phase and amplitude modulators are placed in one of the two loops, which provide a desired control of the phase and amplitude of the traveling pulses. Light dynamics is described by the set of discrete-time equations [25, 37, 38, 40]

$$u_n^{(m+1)} = (\cos \beta u_{n+1}^{(m)} + i \sin \beta v_{n+1}^{(m)}) \exp(-2i\phi_n^{(m)}) \quad (6)$$

$$v_n^{(m+1)} = (\cos \beta v_{n-1}^{(m)} + i \sin \beta u_{n-1}^{(m)}) \quad (7)$$

where $u_n^{(m)}$ and $v_n^{(m)}$ are the pulse amplitudes at discrete time step m and lattice site n in the two fiber loops, and $2\phi_n^{(m)}$ comprises the phase and amplitude changes impressed by the modulators. The modulators are driven such that

$$\phi_n^{(m)} = 2V_0 \sin(2\pi\alpha n + ih) + \phi_n^{(m)} \quad (8)$$

where the first term on the right hand side of Eq.(8) describes

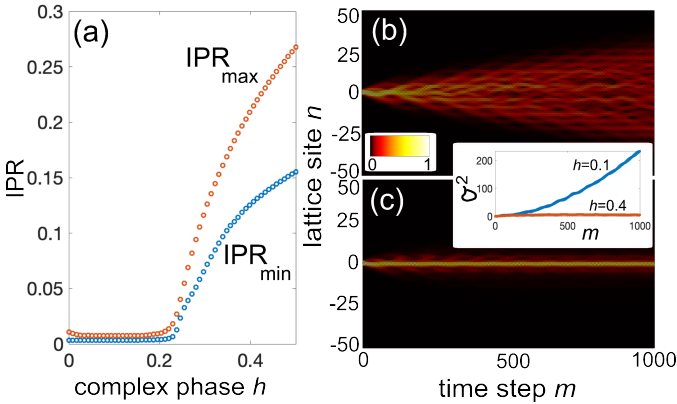


Fig. 2. Non-Hermitian localization-delocalization phase transition in a photonic quasicrystal under coherent dynamics. (a) Behavior of IPR_{\min} and IPR_{\max} of eigenstates of the coherent propagator versus the complex phase h for parameter values $\beta = 0.96 \times \pi/2$, $V_0 = 0.025$, and $\alpha = (\sqrt{5} - 1)/2$. Note the localization-delocalization transition at the critical value $h_c \simeq 0.228$, according to the theoretical prediction [Eq.(9)]. (b) Temporal evolution of the normalized occupation probability $P_n^{(m)}$ versus time step m under coherent dynamics in the delocalized phase ($h = 0.1$). The excitation spreads ballistically in the lattice, corresponding to a quadratic increase of σ^2 with time step m . (c) Same as (b) but in the localized phase ($h = 0.4$). The inset in panels (b,c) depicts the temporal evolution of the second moment $\sigma^2(m)$ versus time step in the two cases.

the static incommensurate complex on-site potential of the quasicrystal [40] whereas the additional stochastic phases $\phi_n^{(m)}$, when applied, introduce decoherence in the system [6]. For a coupling angle β close to $\pi/2$ and for a small potential amplitude such that $V_0 \exp(h) \ll 1$, under coherent dynamics, i.e. for $\phi_n^{(m)} = 0$, the model described by Eqs.(6) and

(7) reproduces the NH Aubry-André model [Eq.(3)] with an hopping amplitude $J = \pm(1/2) \cos \beta$ and on-site potential $V_n = \phi_n = 2V_0 \sin(2\pi\alpha n + ih)$ (see Sec.3 of the Supplemental document; see also [40]). Assuming $V_0 < J$, a phase transition should be therefore observable as h is varied to cross the critical point $h = h_c$, where

$$h_c = \log \left(\frac{\cos \beta}{2V_0} \right). \quad (9)$$

The theoretical prediction is confirmed by full numerical simulations of Eqs.(6) and (7). Typical numerical results, clearly showing a delocalization-localization phase transition as the complex phase h is increased above the critical value h_c , are shown in Fig.2. Figure 2(a) shows the behavior of maximum and minimum IPR of eigenstates of the one-step coherent propagator versus the complex phase h . A dynamical fingerprint of the NH phase transition is the dynamical delocalization of the wave packet for $h < h_c$ and the dynamical localization for $h > h_c$, as shown in Figs.2(b) and (c). The two panels illustrate the discrete-time evolution of light intensity distribution in the lattice, $P_n^{(m)} = |u_n^{(m)}|^2 + |v_n^{(m)}|^2$, normalized at each time step ($P_n^{(m)} \rightarrow P_n^{(m)} / \sum_n P_n^{(m)}$), when a single pulse is injected into the system at lattice site $n = 0$, namely for the initial condition $u_n^{(0)} = v_n^{(0)} = (1/\sqrt{2})\delta_{n,0}$. The spreading of the light wave packet in the lattice at successive time steps is measured by the second moment $\sigma^2(m) = \sum_n n^2 P_n^{(m)} / \sum_n P_n^{(m)}$. The numerical results clearly demonstrate dynamical delocalization for $h < h_c$ with ballistic transport (σ^2 increases quadratically with time step m) and dynamical localization for $h > h_c$.

Under incoherent dynamics, i.e. when $\phi_n^{(m)}$ are uncorrelated stochastic phases with uniform distribution in the range $(-\pi, \pi)$, the incoherent light evolution is described by the following map for the light pulse intensities $X_n^{(m)} = \overline{|u_n^{(m)}|^2}$ and $Y_n^{(m)} = \overline{|v_n^{(m)}|^2}$ in the two fiber loops

$$X_n^{(m+1)} = (\cos^2 \beta X_{n+1}^{(m)} + \sin^2 \beta Y_{n+1}^{(m)}) \exp(g_n) \quad (10)$$

$$Y_n^{(m+1)} = \sin^2 \beta X_{n-1}^{(m)} + \cos^2 \beta Y_{n-1}^{(m)} \quad (11)$$

where the overline denotes statistical average and where we have set

$$g_n = 4 \text{Im}(\phi_n^{(m)}) = 8V_0 \sinh h \cos(2\pi\alpha n). \quad (12)$$

The above equations describing incoherent light dynamics are readily obtained after taking the modulus square of both sides in Eqs.(6) and (7) and making the statistical average, using the property that $\overline{u_n^{(m)} v_n^{(m)*}} = 0$ owing to phase randomization. In the Hermitian limit $h = 0$, i.e. for $g_n = 0$, Eqs.(10) and (11) describe a classical random walk, Anderson localization is washed out and transport in the lattice is diffusive [6]. In this case for a finite lattice of size N with open boundaries, the dynamics is finally attracted toward the state with equal populations in the various sites, i.e. $X_n = Y_n = 1/(2N)$. As h is increased, remarkably a delocalization-localization phase transition can be observed and transport in the lattice is suppressed. The critical value h'_c of the imaginary phase at which the phase transition occurs can be calculated analytically for a coupling angle β close to $\pi/2$ and reads (see Sec.5 of the Supplemental document for technical details)

$$h'_c = \text{asinh} \left(\frac{\cos^2 \beta}{4V_0} \right). \quad (13)$$

Typical numerical results, showing the delocalization-

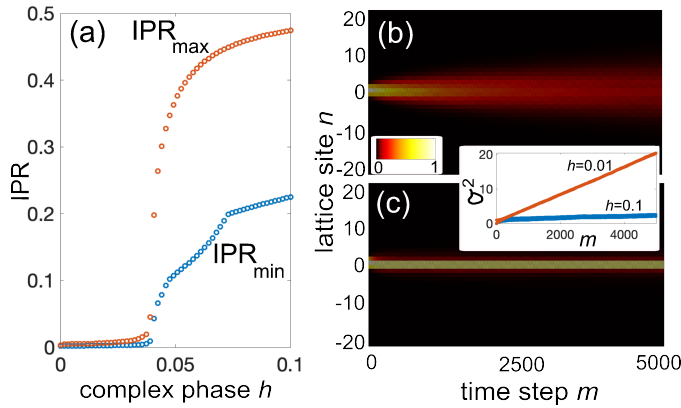


Fig. 3. Non-Hermitian localization-delocalization phase transition in a photonic quasicrystal under incoherent dynamics for the same parameter values as in Fig.2. (a) Behavior IPR_{min} and IPR_{max} of eigenstates of the incoherent propagator versus the complex phase h for the same parameter values as in Fig.2. Note the localization-delocalization transition at the critical value $h'_c \simeq 0.0394$, according to the theoretical prediction of Eq.(13). (b) Temporal evolution of the normalized occupation probability $P_n^{(m)}$ versus time step m under incoherent dynamics in the delocalized phase ($h = 0.01$). The excitation spreads diffusively in the lattice, corresponding to a linear increase of the second moment σ^2 with time step m . (c) Same as (b) but in the localized phase ($h = 0.1$). The inset in panels (b,c) depicts the temporal evolution of the second moment $\sigma^2(m)$ versus time step m in the two cases.

localization phase transition with dephasing effects as h is increased above the critical value h'_c are depicted in Fig.3. Note that for $h < h'_c$ the system is in the delocalized phase and dynamical delocalization is observed, corresponding to diffusive wave spreading (contrary to ballistic spreading as in the coherent regime). On the other hand, spectral and dynamical localization are observed for $h > h'_c$.

In conclusion, we predicted that, while Anderson localization in Hermitian models is generally a fragile effect and is washed out in the presence of dephasing or fluctuating potentials, in non-Hermitian systems with dissipative disorder Anderson localization can survive against dephasing effects. We illustrated such a remarkable result by showing robustness of localization-delocalization phase transitions in non-Hermitian quasicrystals with incommensurate local gain and loss, which should be observable in synthetic quasicrystals based on light pulse dynamics in coupled fiber loops. Our results unravel new physical insights onto Anderson localization and indicate that photonic quantum walks could provide an experimentally accessible platform for the observation of persistent Anderson localization in NH systems against dephasing effects.

Disclosures. The author declares no conflicts of interest.

Data availability. No data were generated or analyzed in the presented research.

Supplemental document. See Supplement 1 for supporting content.

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