Journal of Intelligent & Robotic Systems (2024) 110:73
https://doi.org/10.1007/s10846-024-02110-6

REGULAR PAPER

Enhanced Traffic Light Guidance for Safe and Energy-Efficient Driving: A Study on Multiple Traffic Light Advisor (MTLA) and 5G Integration

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Received: 2 February 2023 / Accepted: 30 April 2024

Abstract

This paper presents Multiple Traffic Light Advisor (MTLA), a novel Green Light Optimal Speed Advisory (GLOSA) system that leverages 5G communication technology. GLOSA systems are emerging as a key component in intelligent transportation systems, thanks to the development of effective communication technologies. At its core, MTLA serves as a guidance system for drivers, providing real-time instructions to adjust vehicle speed to optimize the utilization of current and future states of traffic lights along their route.The work addresses several limitations in the current state-of-the-art approaches, including the use of an overly simplified velocity profile, the omission of potential grip and jerk in problem formulation, and the absence of a detailed description of the algorithm's implementation aspects. Initially, we comprehensively present an optimization-free implementation of the overall control architecture based on an unconventional speed profile. Subsequently, MTLA is improved within a non-linear Model Predictive Control (MPC) framework which uses the latter nonoptimal solution as an initial guess and considers potential grip and jerk in the problem formulation. The developed systems are numerically tested and compared within a high-fidelity simulation environment using the IPG CarMaker simulator. The results demonstrate promising performance in terms of energy savings, with a significant reduction of 37% in energy usage, as well as improved overall comfort with respect to the case where no guidance is given to the driver. These findings suggest a high potential for future developments in this domain.

Keywords GLOSA · ADAS · Traffic light advisor · Connected vehicles · 5G · V2X · MPC

1 Introduction

Green Light Optimal Speed Advisory (GLOSA) is an advisory system aimed at improving safety and sustainability in intelligent transportation systems (ITS) [1]. These systems are a subset of vehicle-to-infrastructure (V2I) applications that facilitate the transfer of signal information between vehicles and traffic lights. The primary objective of GLOSA systems is to provide drivers with speed recommendations that allow for a smoother approach to intersections, ideally passing through without the need to stop, thus reducing travel time [2–5] and improving fuel efficiency [6–8]. This is typically achieved through the utilization of road data and the integration of traffic light schedules into the architecture of the advisory system [9].

1.1 Literature Review

The current literature on GLOSA systems has demonstrated their efficiency and fostered their role as a key component within the realm of intelligent transportation systems. GLOSA systems can be categorized into two types based on the number of traffic lights they consider in real-time to provide recommended speeds: Single-segment GLOSA (S-GLOSA) and multiple-segment GLOSA (M-GLOSA). In [6], the performance and effectiveness of both types are compared, offering valuable insight into their respective

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Published online: 15 May 2024

advantages and limitations in optimizing traffic flow and improving overall transportation efficiency.

S-GLOSA systems analyze only the first traffic light encountered by the vehicle, while the M-GLOSA systems take into consideration multiple traffic lights along the vehicle route. S-GLOSA algorithms typically rely on modeling approaches, as demonstrated in [10], which incorporate velocity profiles upstream and downstream of the intersection. The determination of the speed profile takes into account various criteria. In [11], Barth et al. optimize the speed profile based on minimizing the total tractive power demand and idle time. In [12], the priority is to reduce driver annoyance by minimizing the difference between the suggested speed and the actual speed, or by aiming to pass the traffic light as quickly as possible [12].

In [13], GLOSA is optimized, taking into account considerations of both fuel efficiency and traffic efficiency. The approach to determine the target velocity involves calculating the time required for a given vehicle to reach the upcoming traffic light, assuming a uniformly accelerated motion profile. In cases where the vehicle approaches the traffic light while it is displaying a green signal, the driver is guided to maintain the maximum allowable speed on the road. Conversely, if the vehicle is anticipated to arrive during a red phase, the target speed is computed to facilitate the vehicle's arrival at the traffic light during the subsequent green phase, once again utilizing a uniformly accelerated motion profile. The simulation results demonstrate that, in scenarios with high traffic density, the benefits increase with a higher number of equipped vehicles. In [14], the performance of three velocity planning algorithms was evaluated, with the objective of minimizing the acceleration rates for a vehicle traversing an empty signalized 10-intersection corridor. The result of the stochastic simulations revealed a notable 12%-14% reduction in both fuel consumption and pollutant emissions.

Empirical studies, such as the experimental campaigns conducted in [15–17], have demonstrated the efficacy of GLOSA systems. In [15], a system designed specifically for buses (referred to as B-GLOSA), where a moving-horizon dynamic programming problem is designed and solved using an A-star search method. The proposed approach is limited to a single traffic light and does not take into account ground friction and comfort parameters, such as jerk. This system was developed and tested on a group of 30 participants. The results revealed significant savings in fuel and travel time, with an average reduction of 22.1% in fuel consumption and 6.1% in travel time compared to uninformed driving practices. These findings provide empirical evidence of the benefits and effectiveness of implementing GLOSA systems in real-world scenarios. In [16], Zhang et al. present a hierarchical GLOSA system. The simulation and field test evaluated the energy saving performance of the GLOSA system by considering queuing effects and driver tracking errors.

The effectiveness of M-GLOSA systems compared to S-GLOSA is demonstrated in [18], specifically under free-flow traffic conditions. It should be noted that the consideration of traffic light phase changes poses challenges in the optimization of M-GLOSA systems. The dynamic variability of traffic light phases, coupled with the complexity of objective functions, leads to nonconvex feasible solution spaces. As a result, a computationally expensive optimization problem arises, which may not converge to the global optimum [19]. To address this issue, various approaches have been proposed in the literature such as Genetic Algorithms (GAs) [3, 18], and search-based algorithms utilizing semi-heuristic or brute-force methods[10]. Furthermore, numerical optimization techniques, as demonstrated in [20], present a modified Discrete Differential Dynamic Programming (DDDP) approach capable of real-time execution on a vehicle onboard computer.

Model Predictive Control (MPC) is a widely adopted methodology for GLOSA systems [21]. In [19], a "Predictive Cruise Control (PCC)" is proposed where a set of logical rules are used to calculate a reference velocity for timely arrival at green lights considering a constant velocity profile. The calculated velocity profile serves as a reference input for the MPC, which then tracks this target velocity using a linearized vehicle model. The objective function is defined by a cost function that aims to minimize brake force and deviation from the target speed. The experimental results show significant benefits of adopting the PCC controller: 59% reduction of fuel consumption, 39% less CO_2 emission, and reduced travel time. Further simulations are performed in [22] to verify the effective reduction of fuel consumption and travel time of the developed system.

Several works, such as [23–25], employ simulations to assess the efficacy of developed GLOSA systems. In this work, IPG CarMaker is used to validate the performance of MTLA as it enables early Real Driving Emissions (RDE) tests with virtual test driving. The utilization of this high-fidelity simulator has demonstrated its superiority in generating reproducible real driving test scenarios, as demonstrated in previous work such as [26–29].

In [30], Kural et al. apply Model Predictive Control (MPC) to a road segment with multiple traffic lights. They use a two-level MPC approach to compute energy-efficient and time-saving trajectories. The "Fast MPC" generates an initial trajectory, which the "Main MPC" refines considering a linear kinematic vehicle dynamic model and acceleration limits. The goal is to minimize acceleration and trajectory deviation from a reference.

Data-driven approaches have emerged as a promising alternative, as highlighted in [31]. In this study, a reinforcement learning (RL) implementation that takes into account information from a single traffic light and minimal data from the preceding three vehicles is compared to a standard

S-GLOSA system resulting in an 11% improvement in terms of energy savings. Mousa et al. [32] introduces a deep reinforcement learning (DRL) agent designed to define driving strategies in signalized intersections, aiming at minimize fuel consumption. The DRL agent demonstrates a reduction in fuel consumption of approximately 13%, while ensuring adherence to the traffic light regulations. Notably, the agent operates without the need for pre-existing knowledge of the environment; instead, it autonomously acquires and utilizes data throughout the training phase to shape its decision-making and control strategies. In [33], passage of traffic lights in an autonomous driving context is considered. The algorithm provides the vehicle with information on whether to proceed through the intersection or come to a complete stop, effectively managing the interaction with the traffic signals. The work focuses on a hierarchical policy gradient method to train the algorithm, which is shown to be more efficient than a traditional flat reinforcement learning approach. In a more recent study, Ding et al. introduce *SpeedAdv* [34], a novel GLOSA system. This system employs a RL-based heterogeneous agent collaborative framework to deliver optimal driving speed advisories. Experimental findings demonstrate a significant 24.1% reduction in energy usage when compared to *GreenDrive*, a phone-based GLOSA system [35].

Advances in data-driven methodologies showcase the potential to improve the performance of model-based GLOSA systems and improve their energy efficiency. However, it is essential to recognize that the reliance on such approaches poses limitations in safety-critical applications. In such scenarios - where mathematical modeling and analysis are indispensable - a rule-based approach ensures a thorough fulfillment of safety constraints. Additionally, there is a notable challenge in these applications due to the considerable data requirements to train most of these systems effectively.

1.2 Contributions

In this paper, we propose a novel driving strategy advisory system formulation that also considers ground friction information as well as total energy consumption, travel time, and comfort. Connectivity plays a crucial role in obtaining the necessary information for the advisory system, including traffic light phases, road geometry, ground friction coefficient, and speed limits. In our simulations, we assume a connectivity infrastructure based on 5G technology, chosen for its high speed and reliability. The developed system, named Traffic Light Advisor (TLA), provides timely warnings on how to adjust vehicle velocity to achieve a green traffic light at multiple intersections. It is noteworthy that existing literature on GLOSA systems does not, to the best of our knowledge, incorporate the use of smart tires to account for the variability of the ground friction coefficient in their calculations. This novel aspect is a significant contribution of our paper, as it enhances the accuracy and effectiveness of the driving strategy advisory system, particularly in diverse road conditions with varying levels of friction as well as the main structure of the overall control system.

The contribution of this work is manifold. First, it provides a comprehensive description of a nonoptimal Multiple Traffic Light Advisor (MTLA) system, including detailed information about the models and algorithms utilized in its formulation. Second, an optimal MTLA is introduced, building upon the nonoptimal iteration by incorporating it as an initial guess within a Model Predictive Control (MPC) framework. The optimal MTLA seeks further improvements in terms of performance and efficiency while leveraging the strengths of the nonoptimal version. To assess and compare the performance of the MTLA versions, evaluations are conducted based on simulated data considering two key criteria: energy saving and comfort.

2 Problem Statement

This section provides a concise overview of the problem statement. It starts by presenting the data sources and communication principles utilized by the algorithms. Subsequently, we describe the expected behavior of the complete system under various driving conditions, including the assumptions and considerations incorporated during the system's development. Finally, we introduce the simulation setup, outlining the selected testing scenario for evaluating the overall performance of the system.

2.1 System Architecture

Conventional intelligent transportation systems (ITS) often rely on wireless data transmissions between vehicles (V2V) or vehicle and infrastructure (V2I). These systems commonly employ Dedicated Short-Range Communication (DSRC) frequencies to facilitate the transfer of data [36]. This work employs an intermediary dynamic layer to manage information flow, encompassing data reception, processing, analysis, and targeted data transmission. This approach leverages the advantages of 5G networks and edge computing, offering low latency, high bandwidth, and extensive device connectivity.

Figure 1 shows the overall architecture of the proposed system. The Active Map is populated with data from the infrastructure and the vehicles. Infrastructure data includes traffic light positions, timings, and phases, and the road speed limits. Vehicle-contributed data includes GPS coordinates

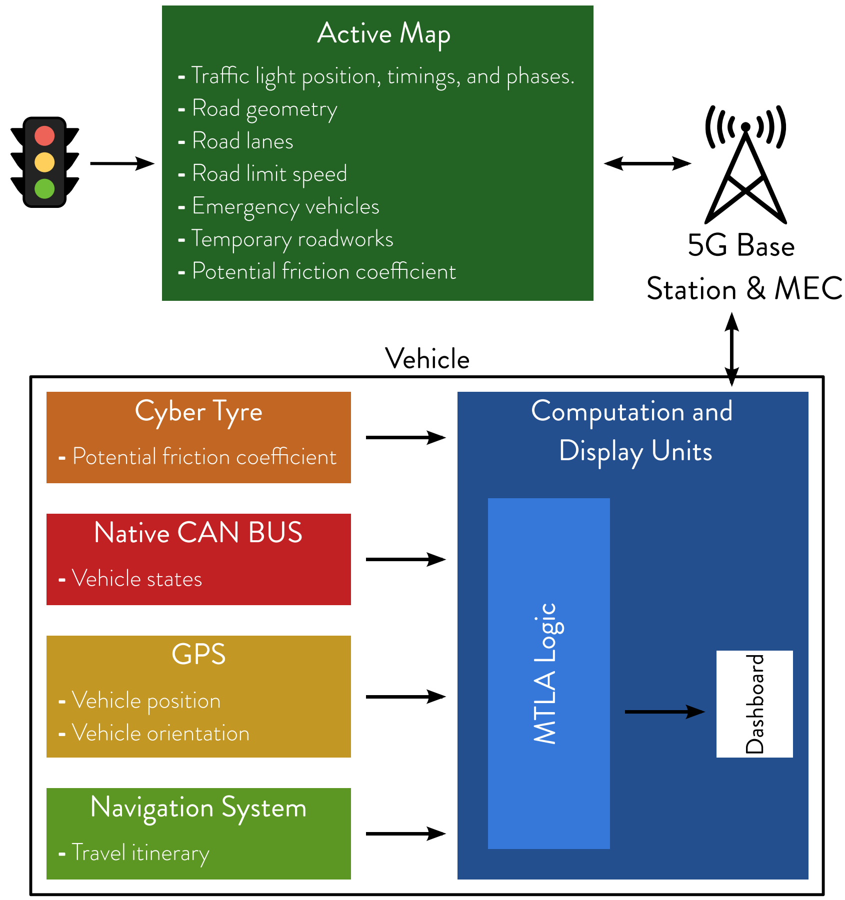

Fig. 1 Overall System Architecture: Intelligent Speed Adaptation & Control, Data Transmission Logic Scheme

and potential grip, with the latter being acquired through the utilization of smart tires [37]. Furthermore, the vehicle is equipped with a Computation and Display Unit. In an experimental setup, this is often a tablet device that performs the required computations and displays guidance for the driver.

2.2 Algorithm - Traffic Light Advisor Overview

In Fig. 2, a functional scheme of the MTLA system (MTLA block in Fig. 1) is presented. The data utilized by the system are classified into dynamic, static, and hybrid types, as presented in Table 1. The working principle is as follows:

- "Localization block" computes vehicles abscissa to locate the vehicle on the map;
- "Activation check block" triggers the activation of the warning system based on the data available (traffic lights, speed limits, and vehicle position);
- "Reference Generation" & "Warning Definition block" compute reference acceleration value and warning out for the driver.

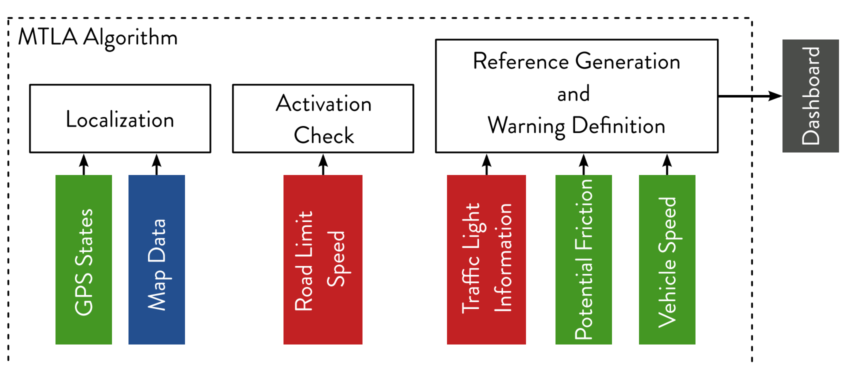

Fig. 2 Graphical overview of TLA algorithm scheme. Blocks colored in green, blue, and red correspond to incoming dynamic, hybrid, or static/dynamic data respectively

Table 1 Algorithm input parameter classification

Parameter	Variability	Source	Type
V	Dynamic	CAN BUS	Input
X	Dynamic	GPS	Input
Y	Dynamic	GPS	Input
Ψ	Dynamic	GPS	Input
Potential Grip μ	Dynamic	Active Map	Input/Output
Road Geometry	Static	Active Map	Input
Road Limit Speed	Static/Dynamic	Active Map	Input
Traffic Light Position	Static/Dynamic	Active Map	Input
Traffic Light Phases	Dynamic	Active Map	Input
Traffic Light Timing	Dynamic	Active Map	Input

The table specifies the inputs required for the developed TLA systems and their source. Information obtained through the active map is assumed to be received through communication from a central computing unit

The proposed MTLA system is expected to autonomously manage the following situations encountered at traffic lights:

- Stop&Go: the vehicle is approaching the traffic light when red. The algorithm should suggest a prior deceleration to avoid a complete stop when possible.
- Last-second braking: the algorithm should inform the driver to start braking when the green phase is close to end and an acceleration maneuver is not feasible.
- Unnecessary stop: the algorithm is expected to inform the driver to accelerate (respecting the road speed limit and guaranteeing vehicle safety) in order to pass the intersection with a green phase that is close to end.

MTLA performs the guidance by communicating with the driver through a graphical interface (tablet). The outputs are two:

1. Green Warning: it encourages the driver to increase the speed in order to take one or more green TLs without stopping while respecting speed limits of the road.
2. Red Warning: it suggests to the driver to reduce the speed of the vehicle.

It is important to note that when no modification of the speed is requested, no warning is issued.

2.3 Design Hypotheses

The assumptions used in the development of the algorithms are the following:

1. The abscissa of the traffic light corresponds to the abscissa value of the stop line.
2. The yellow phase of the traffic lights is considered part of the red phase.
3. The warning system is activated when the vehicle within a proximity of 500 m from one or more traffic lights.
4. Longitudinal behavior is modeled by means of a point mass model, where speed v is always tangent to the path, such that $v = \dot{s}$, and acceleration $a = \ddot{s}$. The reference point of the model is defined with respect to the front of the vehicle. Additional terms are considered in the dynamic model such as the aerodynamic drag as well as the rolling resistance [16]. The length of the vehicle is neglected: this can be done thanks to the presence of a yellow phase, which gives the vehicle the necessary time to vacate the intersection entirely.
5. No other vehicles on the lane are considered.

2.4 Testing Scenario

IPG CarMaker-Simulink environment is used to perform virtual testing and evaluate the performance of the systems. The main goal is to demonstrate that the MTLA system can help the driver take a green wave. To do so, a driver that receives no warning (denoted as driver 1 in the simulation study) is compared with a driver that receives guidance from the proposed systems and follows it correctly (denoted as driver 2 in the simulation study). The initial speed is kept constant for both drivers until action is required to deal with traffic lights. To simulate the behavior of drivers in the virtual environment, longitudinal speed control logic has been implemented based on a PID controller; this facilitates the tracking of a reference speed and performing braking maneuvers when necessary.

The test road scenario described in Fig. 3a is a 1500 m long straight road, where four traffic lights with a 75 s cycle are placed. Traffic light location, phases and timing are based on real TLs located in the city of Milan [38]. Their position and relative phases are reported respectively in Fig. 3b and c.

It is important to clarify that it is assumed that inputs are deterministic, and the proposed simulations are evaluated in ideal conditions: measurements are considered as ground truth. Communication uncertainty (delay and reliability are 10 ms and 99.9% respectively; latency and the typical lost

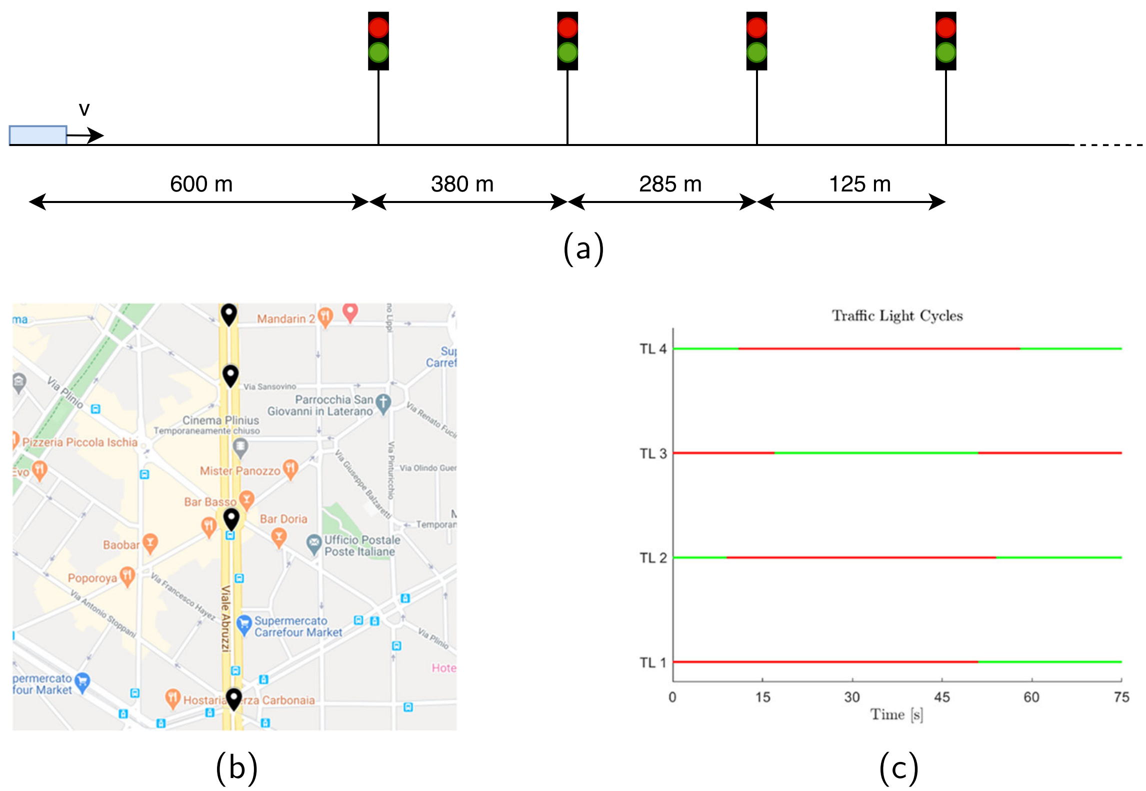

Fig. 3 (a) Sketch of test road showing the locations of the traffic lights, (b) Traffic lights positions on map, (c) Real phase cycles of the four traffic lights

packets ratio associated with 5G communication) as well as described in [37]. Additionally, the active map itself is simulated, as it is not the primary purpose of the simulation-based testing.

3 Nonoptimal MTLA

MTLA generates two key outputs. The first output is a reference acceleration profile, while the other is a graphical warning system designed to facilitate the efficient passage of green traffic lights. This warning system advises the driver on necessary velocity adjustments based on the calculated reference acceleration profile, thereby enhancing the vehicle's ability to navigate through traffic lights successfully.

Upon activation of the algorithm, it iteratively assesses the status of the four traffic lights located ahead of the vehicle. The algorithm then evaluates the possibility of getting one or more greens subject to the following conditions:

- a reference velocity that respects road limits and allows to pass at green phase has to be found for each TL;
- if an acceleration/deceleration maneuver is required by the driver, its safety, comfort, and feasibility need to be checked;
- a common reference velocity among the considered traffic lights needs to be found.

The velocity and acceleration profiles are selected from a velocity and an acceleration range, respectively. Thus, for each traffic light a minimum and maximum velocity that permits the driver to get a green are defined, along the corresponding accelerations. Through a comparison between the calculated reference velocity and the actual vehicle velocity, the warning is issued.

In order to calculate the velocity and acceleration reference profiles, a certain type of motion needs to be determined. For instance, in literature, the velocity range is computed considering a constant speed profile, then the maximum velocity from this interval is considered as the reference one [19, 22]. The drawback of the constant velocity profile is that the time needed for the driver to reach the reference velocity is not taken into account; hence, it is not possible to use it directly as a reference for the warning. In fact, in [19, 22], the reference speed is used in a cost function which is then optimized considering the constraint of vehicle dynamics to calculate the control action.

In this work, the driver acceleration phase is considered in the computation of the velocity range: it is represented utilizing a uniformly accelerated motion (UAM) profile. This chosen profile serves as an approximation of the dynamic behavior of the vehicle, as it facilitates a balance between accuracy and simplicity.

The following hypotheses are assumed in the computation of the velocity range required to reach a green traffic light:

1. if the phase of the traffic light under analysis is green, the feasibility of getting either the actual or the following green phase is analyzed;
2. if the phase of the traffic light under analysis is red, the feasibility of getting only the first green is analyzed;
3. if the first traffic light in front of the vehicle is analyzed, a UAM until the traffic light is supposed;
4. if a traffic light different from the first one is analyzed, the following motion profile is assumed: a uniformly accelerated motion up to the first traffic light is considered, then a constant speed motion (CSM) is adopted (UAM+CSM).

Henceforth, to distinguish between UAM and UAM combined with CSM, the initial traffic light is denoted as the "first traffic light," while subsequent ones are denoted as the "i^{th} traffic lights."

3.1 Reference Generation

3.1.1 First Traffic Light

The UAM in Eq. 1 is adopted to generate the speed interval for the first traffic light:

$$\begin{cases} d = vt + \frac{1}{2}at^2 \\ v_{\text{target}} = v + at \end{cases} \tag{1}$$

where v is the actual speed of the vehicle, v_{target} is the target speed, d is the acceleration distance which is set equal to the distance between the front of the vehicle and the traffic light ($d = l_1$).

It is important to note that Eq. 1 is a set of two equations with three unknowns: v_{target}, a and t, so a parameter still needs to be fixed in order to obtain a unique solution. To do so, two cases can be distinguished according to the phase of the traffic light: green or red.

Green Phase

If the first green of the TL is taken into consideration, the minimum velocity is the one that allows crossing the traffic light when the phase is just starting to shift to red. The time at which the phase shift occurs is the remaining time of the first green phase ($t_{1,g1}$), where the first index refers to the number of the TL and the second to the number of the green phase. By substituting $t = t_{1,g1}$ in Eq. 1, minimum velocity and acceleration can be computed as follows:

$$\begin{cases} a_{\min} = 2\frac{l_1 - vt_{1,g1}}{t_{1,g1}^2} \\ v_{\min} = v + a_{\min}t_{1,g1} \end{cases} \tag{2}$$

The maximum velocity is the one that allows getting the green state in the shortest time possible. So, in order to respect the rules of the road, the maximum velocity is set equal to $v_t = v_{\text{lim,road}}$. By substituting $v_t = v_{\text{lim,road}}$ in Eq. 1, maximum velocity and acceleration can be computed as follows:

$$\begin{cases} a_{\max} = \frac{(v_{\max} - v)^2}{2l_1} + v\frac{(v_{\max} - v)}{l_1} \\ v_{\max} = v_{\text{lim,road}} \end{cases} \tag{3}$$

An example of the two profiles is represented in Fig. 4a. t_{lim} is the time needed to reach the traffic light with a speed equal to $v_{\text{lim,road}}$, i.e. the minimum possible time assuming a UAM to the first traffic light.

If the second green phase is under analysis, the minimum time needed to reach the traffic light corresponds to the time of the red to green phase shift $t_{1,r1}$. The maximum one coincides with the time of the second green to red phase shift $t_{1,g2}$. In this way, the possibility of getting either the first or the second red phase is avoided. By substituting $t_{1,g2}$ and $t_{1,r1}$ in Eq. 1, the minimum and maximum velocity and acceleration

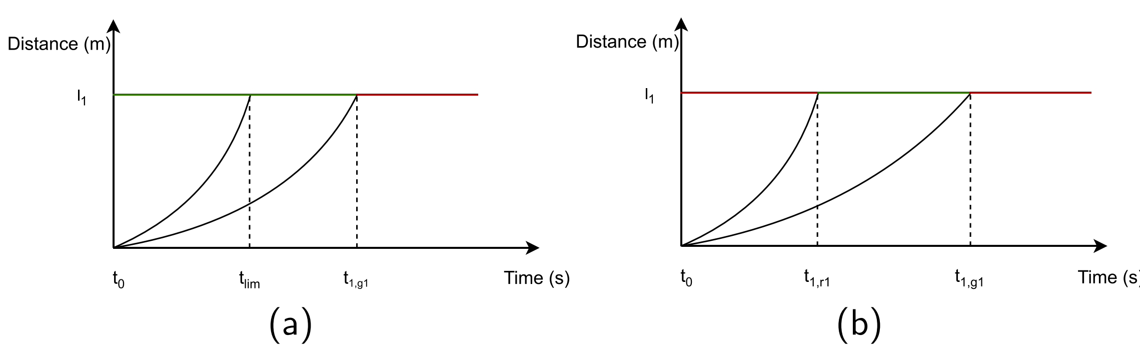

Fig. 4 (a) Uniformly accelerated motion to reach the first traffic light during its first green phase when the green is on. (b) Uniformly accelerated motion to reach the first traffic light during its first green phase when the red is on. t_0 is the algorithm triggering time

profiles can be computed as follows:

$$\begin{cases} a_{\min} = 2\frac{(l_1 - vt_{1,g2})}{t_{1,g2}^2} \\ v_{\min} = v + a_{\min} t_{1,g2} \end{cases} \tag{4}$$

$$\begin{cases} a_{\max} = 2\frac{(l_1 - vt_{1,r1})}{t_{1,r1}^2} \\ v_{\max} = v + a_{\max} t_{1,r1} \end{cases} \tag{5}$$

Red Phase

The minimum time needed for the driver to reach the first available green phase is the one of the red to green phase shift $t_{1,r1}$. The maximum one corresponds to the end of the green phase and so to the time of the green to red phase shift $t_{1,g1}$. The profiles are shown in Fig. 4b. By substituting the previously mentioned times $t_{1,g1}$ and $t_{1,r1}$ in Eq. 1, the final formulation can be obtained as follows:

$$\begin{cases} a_{\min} = 2\frac{(l_1 - vt_{\max})}{t_{1,g1}^2} \\ v_{\min} = v + a_{\min} t_{1,g1} \end{cases} \tag{6}$$

$$\begin{cases} a_{\max} = 2\frac{(l_1 - vt_{1,r1})}{t_{1,r1}^2} \\ v_{\max} = v + a_{\max} t_{1,r1} \end{cases} \tag{7}$$

3.1.2 ith Traffic Light

The motion profile is based upon an acceleration up to the first traffic light, followed by a constant velocity motion up to the i^{th} traffic light. Such motion profile is described by:

$$\begin{cases} d_1 = vt_1 + \frac{1}{2}at_1^2 \\ v_t = v + at_1 \\ d_2 = v_t t_2 \end{cases} \tag{8}$$

where: v is the actual speed of the vehicle, v_{target} is the target speed reached after the acceleration phase and d_1 is the distance traveled during the acceleration phase, i.e. the distance l_1 between the vehicle and the first traffic light. Then, d_2 is the space driven at a constant speed, equal to the distance between the i^{th} traffic light and the first one, and t_2 is the difference between the duration of the overall maneuver t_{tot} and that of acceleration phase t_1.

Note that Eq. 8 comprises a set of three equations with four unknowns: t_1, a, v_t, t_{tot}. In order to find a solution a parameter still needs to be fixed. Again, two cases need to be distinguished according to the actual traffic light phase: green or red.

Green Phase

No constraints on the minimum time to reach the traffic light during its first green exists, thus the maximum velocity is set equal to the road limit speed. By imposing $v_{\text{target}} = v_{\text{lim,road}}$ in Eq. 8, maximum velocity and acceleration are computed as follows:

$$\begin{cases} a_{\max} = \frac{(v_{\max} - v)^2}{2l_1} + v\frac{(v_{\max} - v)}{l_1} \\ v_{\max} = v_{\text{lim,road}} \end{cases} \tag{9}$$

The maximum time to get the first green of the i^{th} TL is the time of the green to red phase change of that TL. Given the remaining time of the green phase $t_{i,g1}$, the total maneuver time is equal to the latter. By imposing $t_{\text{tot}} = t_{i,g1}$ in Eq. 8 minimum velocity and acceleration can be computed. The solution is in Eq. 10:

$$\begin{cases} a_{\min} = 2\frac{l_1}{t_{1,\min}^2} - 2\frac{v}{t_{1,\min}} \\ v_{\min} = \dfrac{l_i + l_1 - t_{i,g1}v + \sqrt{(-l_i - l_1 + t_{i,g1}v)^2 + 4t_{i,g1}v(l_i - l_1)}}{2t_{i,g1}} \\ t_{1,\min} = t_{i,g1} - \dfrac{l_i - l_1}{v_{\min}} \end{cases} \tag{10}$$

The profiles obtained with the minimum and maximum velocities are shown in Fig. 5a, respectively on the right and left. If the second green phase of the i^{th} TL is considered for passing the it, the motion profile is represented in Fig. 5b. The minimum time needed to reach the i^{th} traffic light is the time of the red to green phase shift $t_{i,r1}$. The maximum time is the time of the second green to red phase shift $t_{i,g2}$. Minimum and maximum velocity and acceleration are computed respectively in Eqs. 11 and 12:

$$\begin{cases} a_{\min} = 2\frac{l_1}{t_{1,\min}^2} - 2\frac{v}{t_{1,\min}} \\ v_{\min} = \frac{(l_i + l_1 - t_{i,g2}v) + \sqrt{(-l_i - l_1 + t_{i,g2}v)^2 + 4t_{i,g2}v(l_i - l_1)}}{2t_{i,g2}} \\ t_{1,\min} = t_{i,g2} - \frac{l_i - l_1}{v_{\min}} \end{cases} \tag{11}$$

$$\begin{cases} a_{\max} = 2\frac{l_1}{t_{1,\max}^2} - 2\frac{v}{t_{1,\max}} \\ v_{\max} = \frac{(l_i + l_1 - t_{i,r1}v) + \sqrt{(-l_i - l_1 + t_{i,r1}v)^2 + 4t_{i,r1}v(l_i - l_1)}}{2t_{i,r1}} \\ t_{1,\max} = t_{i,r1} - \frac{l_i - l_1}{v_{\max}} \end{cases} \tag{12}$$

Red Phase

The minimum time needed for the driver to reach the first green phase of the i^{th} TL is the one of the red to green phase shift $t_{i,r1}$. The maximum one is the time of the end of the green phase $t_{i,g1}$. This scenario is represented in Fig. 6, respectively on the left (minimum time) and right (maximum

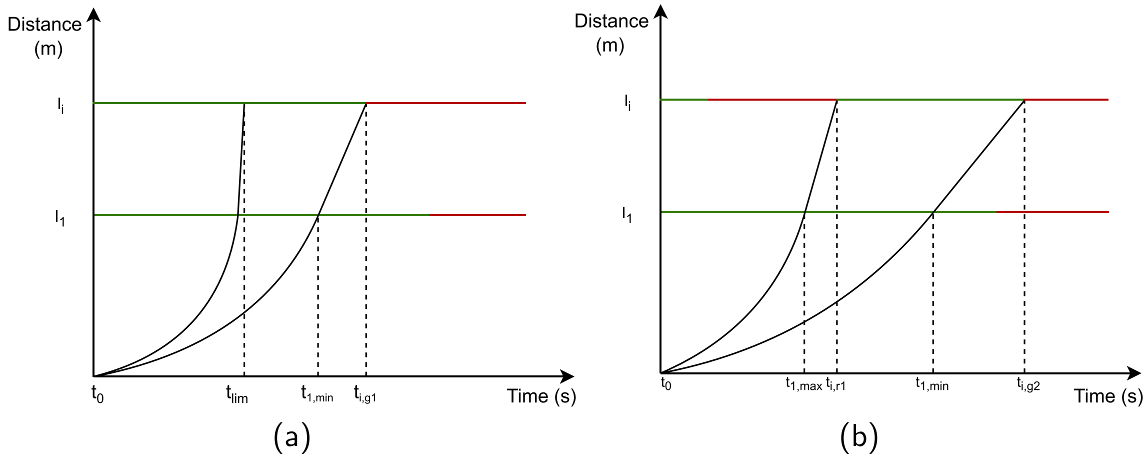

Fig. 5 (a) Motion profile to reach the i^{th} traffic light during its first green phase when the green is on. (b) Motion profile to reach the i^{th} traffic light during its second green phase when the green is on

time). The expressions are given as follows:

$$\begin{cases} a_{\min} = 2\frac{l_1}{t_{1,\min}^2} - 2\frac{v}{t_{1,\min}} \\ v_{\min} = \frac{(l_i+l_1-t_{i,g1}v)+\sqrt{(-l_i-l_1+t_{i,g1}v)^2+4t_{i,g1}v(l_i-l_1)}}{2t_{i,g1}} \\ t_{1,\min} = t_{i,g1} - \frac{l_i-l_1}{v_{\min}} \end{cases} \tag{13}$$

$$\begin{cases} a_{\max} = 2\frac{l_1}{t_{1,\min}^2} - 2\frac{v}{t_{1,\min}} \\ v_{\max} = \frac{(l_i+l_1-t_{i,r1}v)+\sqrt{(-l_i-l_1+t_{i,r1}v)^2+4t_{i,r1}v(l_i-l_1)}}{2t_{i,r1}} \\ t_{1,\min} = t_{i,r1} - \frac{l_i-l_1}{v_{\max}} \end{cases} \tag{14}$$

After defining the speed profile for each TL, the working principle of the nonoptimal MTLA algorithm that finds the final profile is shown in the flowcharts of Fig. 7a, where the "Green Check" and "Red Check" blocks are further

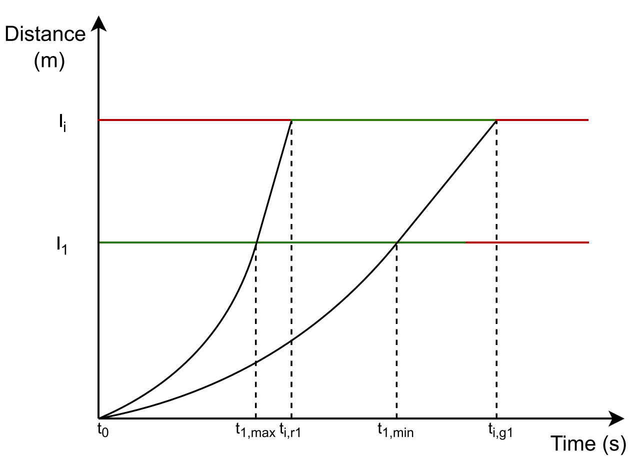

Fig. 6 Motion profile to reach the i^{th} traffic light during its first green phase when the red is on

illustrated in Figs. 7b and c respectively. Index i represents the number of traffic light in analysis, while index j refers to the green phase analyzed. The maximum value of j has been set to 2 so as not to perform too many iterations each time the algorithm is run.

The first two steps of Fig. 7a are "Localization and the Activation Check". These aim tolocalize the vehicle and check the presence of a TL within a certain distance, called planning horizon. Once the algorithm finds the first of the four traffic lights within the planning horizon, the system should be always active until the last traffic light is passed. If the distance between two subsequent TLs is greater than the planning horizon, the system is deactivated. To avoid so, the planning horizon for this application is set to 500 m. This value has been decided considering distances between traffic lights in [38]. Once the algorithm is activated, an iterative cycle that analyzes the four traffic lights ahead of the vehicle starts.

The feasibility to get a green phase is examined and whenever a feasible maneuver to reach the i^{th} traffic light is found, the following variables are stored inside the algorithm: reference velocity (v_{ref}), reference acceleration (a_{ref}), warning to be issued and interval of admissible velocities ($v_{\text{adm},i} = [v_{\min,i}, v_{\max,i}]$).

If it is not possible to pass the traffic light under examination during a green phase, the algorithm terminates and the last stored warning is issued. The "Green Check" block, (Fig. 7b) iterates according to the index j and analyzes the possibility of getting either the first ($j = 1$) or second ($j = 2$) green phase of the i^{th} traffic light. If neither of them can be utilized, the algorithm terminates and a warning is issued to the driver. The first step of the Green Check is the "Velocity Range Definition", this step computes the required velocity

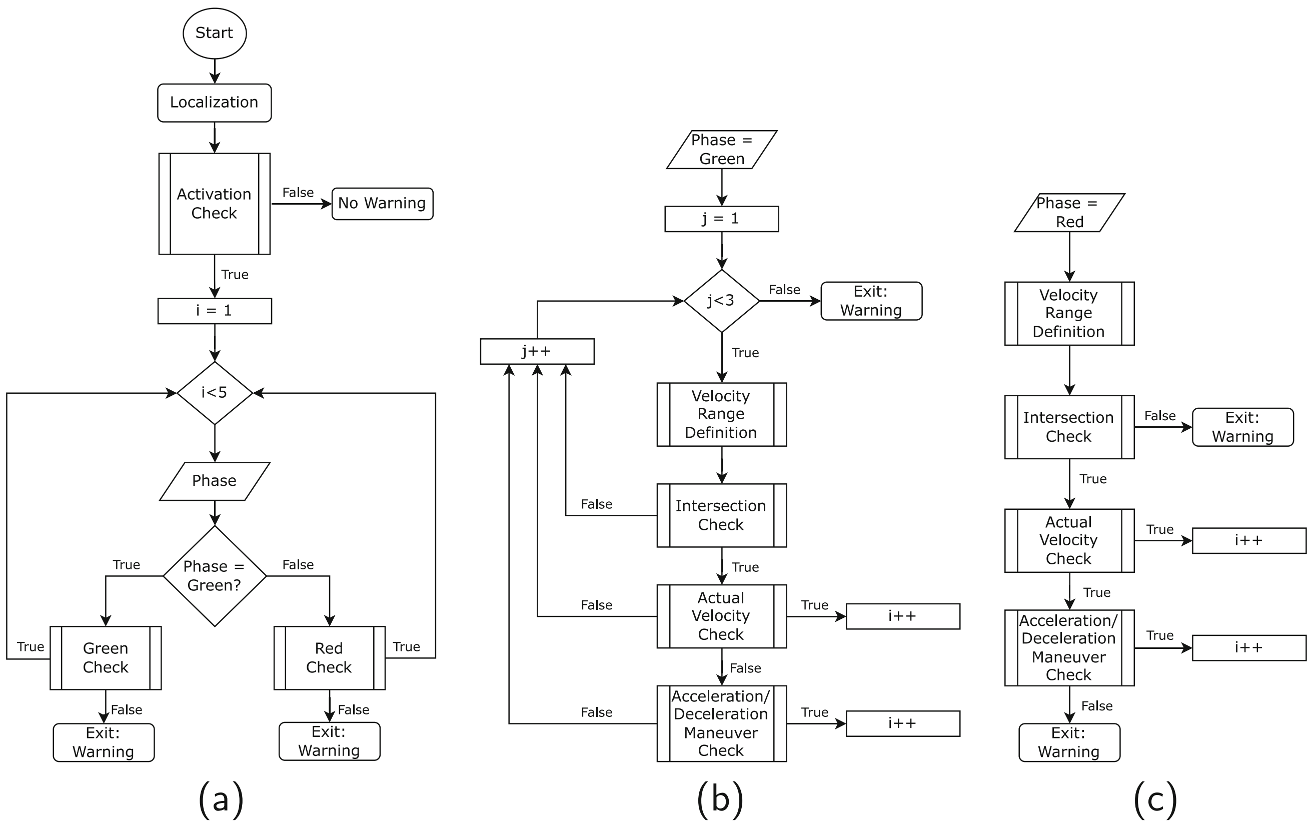

Fig. 7 (a) Nonoptimal Multiple Traffic Light Advisor algorithm flowchart. Nonoptimal Multiple Traffic Light Advisor algorithm flowchart - Green Check. (c) Nonoptimal Multiple Traffic Light Advisor algorithm flowchart - Red Check. Index j refers to the green phase analyzed

range to get the actual green phase $v_{\text{req,i}}$ according to the indexes i and j. The second step is the "Intersection Check". In this block, the required velocity range to get the i^{th} green with the one needed to get the $i - 1^{th}$ green are intersected as in Eq. 15. The aim is to find a velocity range that allows passing both the i^{th} and $i - 1^{th}$ traffic lights.

$$v_{\text{adm},i} = v_{\text{req},i} \cap v_{\text{adm},i-1} \tag{15}$$

If the first traffic light is studied ($i = 1$), the admissible velocity range is defined by the minimum and maximum admissible road velocities, (20 kmh^{-1}) and (20 kmh^{-1}) respectively. On the other hand, when $i > 1$ the admissible interval is already defined from the $i - 1^{th}$ iteration. Now, if Eq. 15 results in an empty interval, it is not possible to reach the i^{th} traffic light when its j^{th} green phase is on, so the possibility of getting the next one is then analyzed ($j + 1$). On the contrary, if the interval is non-empty, $v_{\text{adm},i}$ is defined.

Then, the "Actual Velocity Check" (AVC) block analyzes the possibility for the driver to keep their current speed rather than perform an acceleration or deceleration maneuver. If the last check is satisfied, the next traffic light is analyzed ($i + 1$), if not, the "Acceleration/Deceleration Maneuver" Check (AMC) is performed. Given that the diver cannot keep its actual speed constant, the safeness of the acceleration or deceleration maneuver is checked. If the AMC block result is positive the driver can get the j^{th} green of the i^{th} traffic light. The index i is increased by one unit and the next traffic light is studied. If the result is negative, the next green phase is analyzed.

The steps performed by the "Red Check" block are the same as for the green case. The only difference is that, if the actual phase is red, only the possibility of getting the first green is evaluated Fig. 7c.

4 Optimal MTLA

The reference acceleration profile calculated by the nonoptimal MTLA is further optimized by means of MPC and then the new optimal acceleration profile is used to define the warning produced for the driver. An overview of the system is presented in Fig. 8, where the profile generated by the nonoptimal MTLA algorithm ("MTLA Algorithm" block) is fed as "reference profile" to the model predictive controller (MPC

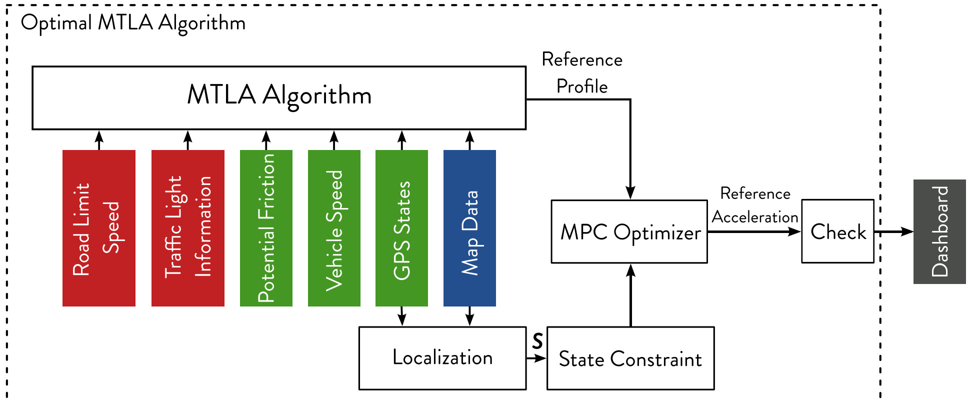

Fig. 8 Optimal Traffic Light Advisor algorithm scheme

Optimizer block) along with state constraints (computed in the "State Constraints" block). With respect to state-of-the-art work, some noteworthy differences include the following:

1. the proposed cost function penalizes both jerk and deviations from the nonoptimal reference profile. In this way, it is possible to increase driving comfort [39]. Moreover, the potential grip is considered by means of the constraints on longitudinal forces available;
2. the dynamic variability of traffic light phases makes the solution space of the optimal control problem non-convex and a constant speed reference motion profile is used in literature to tackle this problem. Here, through the use of a more accurate profile (UAM+CSM), a solution can be found faster and is more likely to be a global-optimum [19];

Since the literature lacks a detailed explanation of how to consider TL phases into state constraints, an algorithm for generating the position constraints according to TL color is proposed and discussed.

4.1 Motivation

The previously introduced algorithm, denoted as nonoptimal MTLA, generates a UAM+CSM profile that serves as the basis for defining the output warning. However, this approach exhibits certain weaknesses. The UAM+CSM reference profile lacks continuity in acceleration and does not account for vehicle dynamics. Consequently, following the suggested velocity profile may prove challenging for the driver or result in discomfort due to the high jerk values required. To address these issues, we enhance the reference profile through further optimization using an MPC (Model Predictive Control) approach.

4.2 Problem Formulation

The resulting Optimal Control Problem (OCP), shown in Eq. 17, is defined by a cost function, a vehicle dynamic model, and constraints. To obtain a smooth acceleration profile the problem has been defined to minimize the predicted jerk while still following the reference trajectory. Concerning the dynamic model, a simplified one that is suitable for our applications is considered: a one-degree-of-freedom plane model is used, thus neglecting road inclination and lateral dynamics. The input constraints limit the maximum net braking and traction force. In addition, state constraints are used to avoid passage of the vehicle during the red phase of the traffic light. To do so, the vehicle position is limited to some region of the time-space plane as shown in Fig. 9a, where s_0 is the abscissa of the vehicle on the path, s_1 and s_2 are the abscissa of the first and second traffic light respectively, the lines' color represents the phase (green or red), and t_1 and t_2 are the times of the phase change for the first and second TLs, respectively.

It is noteworthy that only two traffic lights and one phase shift for each of them are considered. This is due to the fact that, having chosen the length of the prediction horizon t_f equal to 6 s, more than two TLs cannot be encountered in this interval and more than one phase shift cannot occur. The length of the prediction horizon is chosen as a trade-off between small enough discretization and computational effort while still guaranteeing good optimization results.

In order to calculate the admissible region in Fig. 9a, the algorithm represented by the flowcharts in Fig. 9b and c is used. The flowchart of the "Red Check" block is shown in Fig. 10a. If the time of the phase shift is larger or equal to the planning horizon, the vehicle should be ahead of the TL for the whole horizon. Otherwise, it can be beyond the TL only after the phase shift. The flowchart of the "Green Check" block is illustrated in Fig. 10b. If the time of the phase shift is larger than the planning horizon, the vehicle can be ahead

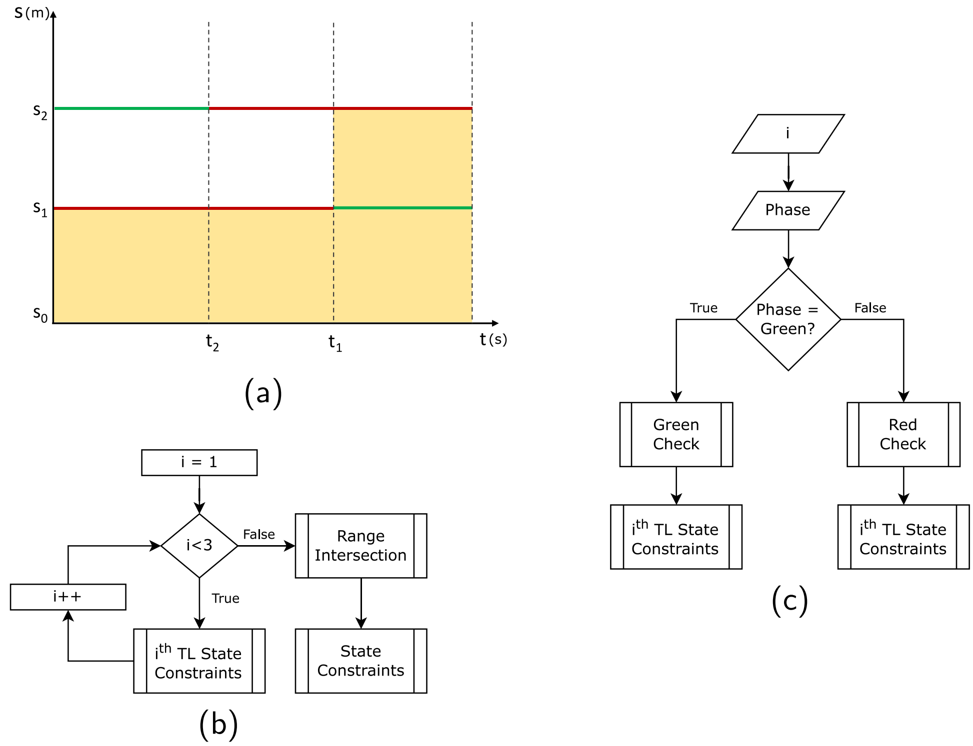

Fig. 9 (a) Graphical explanation of the algorithm for state constraints formulation. The highlighted areas represent the feasible space-time solution domain. (b) Flowchart of the algorithm for the definition of state constraints. (c) i^{th} TL State Constraints block - flowchart of the algorithm for the definition of the i^{th} traffic light state constraints

or beyond the TL. Contrary, if the time is lower or equal, a further analysis is performed on the possibility of passing the TL during the actual green phase. This is done by checking if the MTLA algorithm calculates that it is not possible to pass the i^{th} traffic light ($N_{\text{green}} < i$) or to pass it during the second green phase ($N_{\text{pass}} > 1$). If one of these two conditions is verified the vehicle should be ahead of the TL after the phase shift, otherwise it should be beyond it.

Once the admissible regions are obtained for the two TLs, the "Range Intersection" block (Fig. 9b) is executed. It consists of making the intersection of the state constraints of the single TLs as in Eq. 16:

$$[s_{\min}(t), s_{\max}(t)] = [s_{\min,1}(t), s_{\max,1}(t)] \cap [s_{\min,2}(t), s_{\max,2}(t)] \tag{16}$$

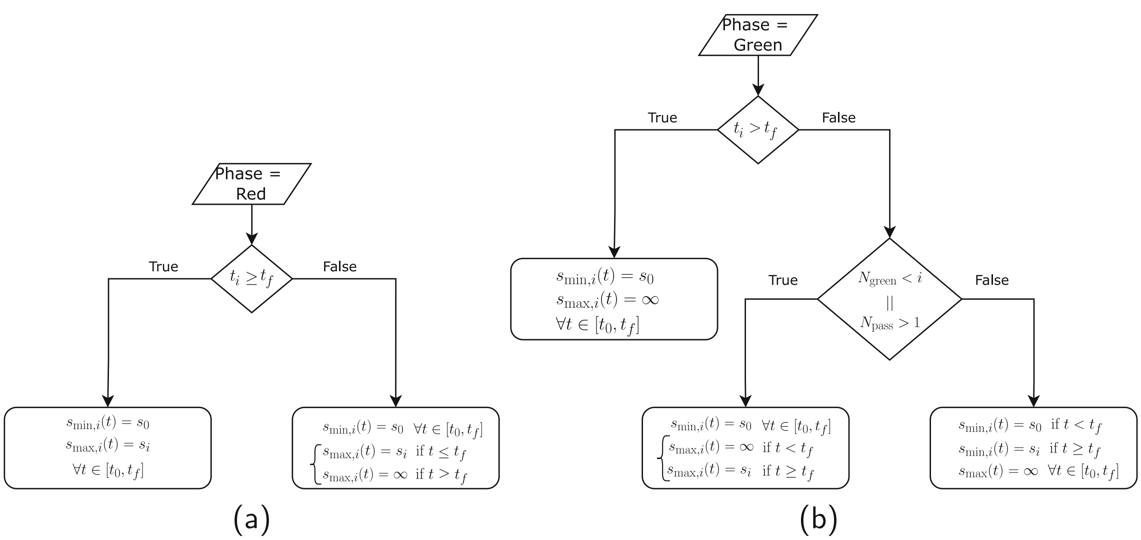

Fig. 10 (a) Red Check block - flowchart of the algorithm for the definition of the i^{th} traffic light state constraints (b) Green Check block - flowchart of the algorithm for the definition of the i^{th} traffic light state constraints

In addition to the position state constraint, also velocity, acceleration jerk, and max longitudinal forces are constrained. Hence, the optimal control problem (OCP) to be solved is formulated as follows:

$$\begin{cases} \min\limits_{v(t),a(t),j(t)} \int\limits_{0}^{t_f} w_v(v(t)-v_{\text{ref}}(t))^2 + w_a(a(t)-a_{\text{ref}}(t))^2 \\ \qquad + w_j j(t)^2 \mathrm{d}t \\ \text{s.t} \quad \text{m}a(t)+\frac{1}{2}A_f\rho C_d v^2(t)+\text{mg}C_r = F(t) & \forall t \in [0,t_f] \\ \quad j(t)=\dot{a}(t) & \forall t \in [0,t_f] \\ \quad F_{\min} \leq F(t) \leq F_{\max} & \forall t \in [0,t_f] \\ \quad s_{\min}(t) \leq s(t) \leq s_{\max}(t) & \forall t \in [0,t_f] \\ \quad v(t) \geq 0 & \forall t \in [0,t_f] \\ \quad a(t) \leq a_{\max} & \forall t \in [0,t_f] \\ \quad j_{\min} \leq j(t) \leq j_{\max} & \forall t \in [0,t_f] \end{cases} \tag{17}$$

Where $j(t)$ is the jerk, $a_{\text{ref}}(t)$ and $v_{\text{ref}}(t)$ are the reference acceleration and velocity from the nonoptimal MTLA algorithm, and w_a, w_v and w_j are the weights of acceleration, velocity, and jerk cost terms, respectively. A_{f} is the frontal area of the vehicle, ρ is the air density, C_{d} is the aerodynamic drag coefficient, m is the vehicle mass, C_{r} is the rolling resistance coefficient and F is the generalized longitudinal tire force. $F_{\min}(<0)$ and $F_{\max}(>0)$ represent the maximum braking and traction forces, respectively.

The numerical solution of the OCP problem is calculated using open-source ACADO Toolkit software [40, 41]. By solving the OCP (17) in a receding horizon fashion, the planned trajectory of the vehicle is adjusted such that velocity and acceleration minimally deviate from the reference ones while satisfying the physical friction constraints as well as abiding by the dynamics governing the longitudinal motion. Moreover, an additional penalty term is added to the jerk for added comfort.

5 Simulation Results

In this section, the results of test cases are showcased and discussed. The main analysis parameters are position, speed, acceleration, and energy consumption. Moreover, to prove that the developed MTLA can effectively reduce vehicle consumption, a simple energetic analysis is proposed considering instantaneous and average energy consumption.

Instantaneous energy consumption is computed starting from the instantaneous power P ($P = T\omega$ with T being the motor torque and ω the motor angular velocity) as in Eq. 18:

$$I.E.C. = P\frac{\text{travelled time}}{\text{travelled distance}} \tag{18}$$

Average energy consumption is computed starting from the instantaneous one, an in Eq. 19:

$$\begin{aligned} A.E.C._{\cdot n} &= \frac{I.E.C._{\cdot 1} + \ldots + I.E.C._{\cdot n}}{n} \\ A.E.C._{\cdot n+1} &= \frac{I.E.C._{\cdot n+1} + \ldots + nA.E.C._{\cdot n}}{n+1} \end{aligned} \tag{19}$$

It is important to specify that the sampling frequency for the simulations is set to 20 Hz, while MPC formulation is implemented with a time horizon of 6 s and a time discretization equal to 0.1 s. The system is implemented in MATLAB/Simulink on a commercial laptop with an Intel i7-8750H processor. The working frequency of both algorithms is around 10 Hz, so a real-time implementation would require further development of the compiled code.

5.1 Non Optimal MTLA

This test case highlights the advantages of MTLA in terms of stops, travel time, and vehicle consumption. Figure 11a reports traveled distance with respect to simulated time of traveling, while the red and green lines represent traffic light phases accordingly. It is visible how driver 1 is subject to three stops, thus increasing the total travel time to cross all the traffic lights. With the initial velocity of 40 kmh^{-1} driver 1 is able to reach the first traffic light in its green phase, while it is not able to get the second one. On the contrary, driver 2 given an acceleration warning is able to avoid such stop. From Fig. 11b and c it can be seen that driver 1 increases its velocity as soon as the first traffic light is in the horizon of activation of the algorithm. This acceleration allows also to get the third traffic light, but not the fourth. It is to be noticed that the vehicle stops accelerating before reaching the first traffic light (Fig. 11b and c) because a velocity that allows reaching all the previously analyzed three green lights is reached.

When the second traffic light phase shifts to green again driver 1 accelerates and reaches the speed of 40 kmh^{-1}. While approaching the third traffic light, driver 1 starts decelerating to stop the vehicle again as it is red. The same deceleration/acceleration maneuver occurs while reaching the fourth traffic light. With a deceleration warning, driver 2 is able to avoid these two stops. As soon as driver 2 overcomes the second traffic light, a new profile can be calculated which allows the vehicle to pass also the fourth traffic light without stopping. Comparing the speed and longitudinal acceleration profiles of drivers 1 and 2, it could be seen that driver 1 performs multiple hard-braking maneuvers (which are completely avoided by driver 2) resulting in a less comfortable and more time-consuming trajectory.

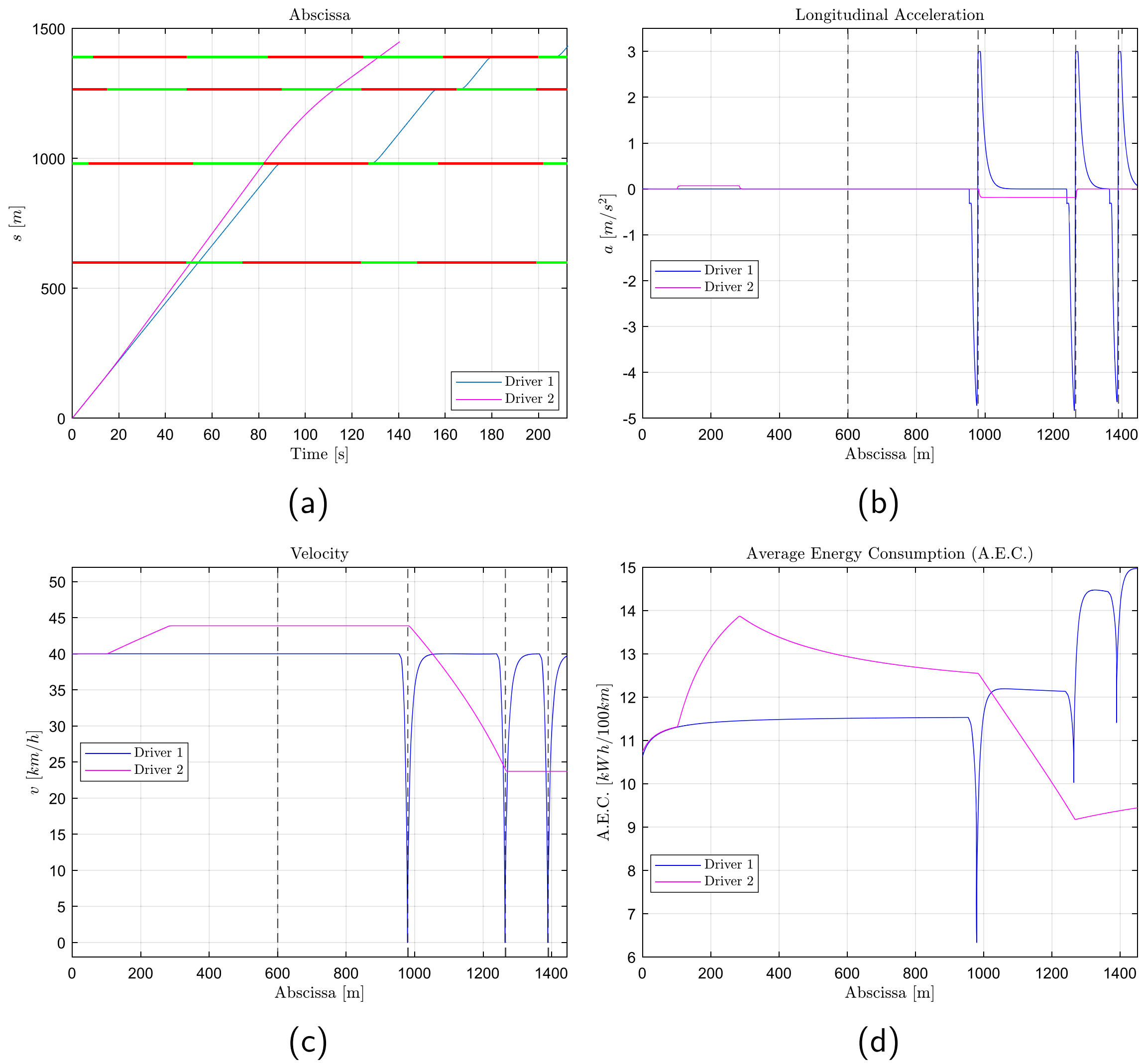

Fig. 11 MTLA Test Case. (a) abscissa of driver 1 and driver 2, (b) velocity profiles of driver 1 and driver 2, (c) acceleration profiles of driver 1 and driver 2, (d) average energy consumption of driver 1 and driver 2

It is noteworthy to note that when the algorithm is triggered at first, no feasible solution could be found to reach the fourth traffic light; however, as the vehicle passes across the TLs, new profiles are calculated and a feasible UAM+CVM profile to cross the fourth traffic light is found. This is due to the algorithm's continuous update.

Moreover, it is clear from Fig. 11c that the acceleration values of driver 2 are lower than driver 1, meaning higher maneuver comfort. Figure 11d shows the average energy consumption. It can be noticed that the final average consumption (after passing the four traffic lights) of driver 2 is lower than that of driver 1: 9.4 kWh 100 km^{-1} versus 15 kWh 100 km^{-1} for drivers 2 and 1, respectively. Therefore, MTLA shows a 37.3% reduction in energy consumption. It is important to note that this reduction is significant, as similar works obtained values in the $14\% - 18\%$ range [42, 43].

5.2 Optimal MTLA

First, the comparison between the vehicle equipped with the optimal MTLA system (driver 2) and the one without any kind of warning system is reported (driver 1). Looking at Fig. 12a it can be noticed how similarly to nonoptimal results, driver 1 has to stop at three traffic lights, while driver 2 is able to take a green wave and thus reduce the total travel time. The comments done for the nonoptimal MTLA are also valid for this case since driver 1 is the same and driver 2 has a similar behavior to the nonoptimal one but its acceleration

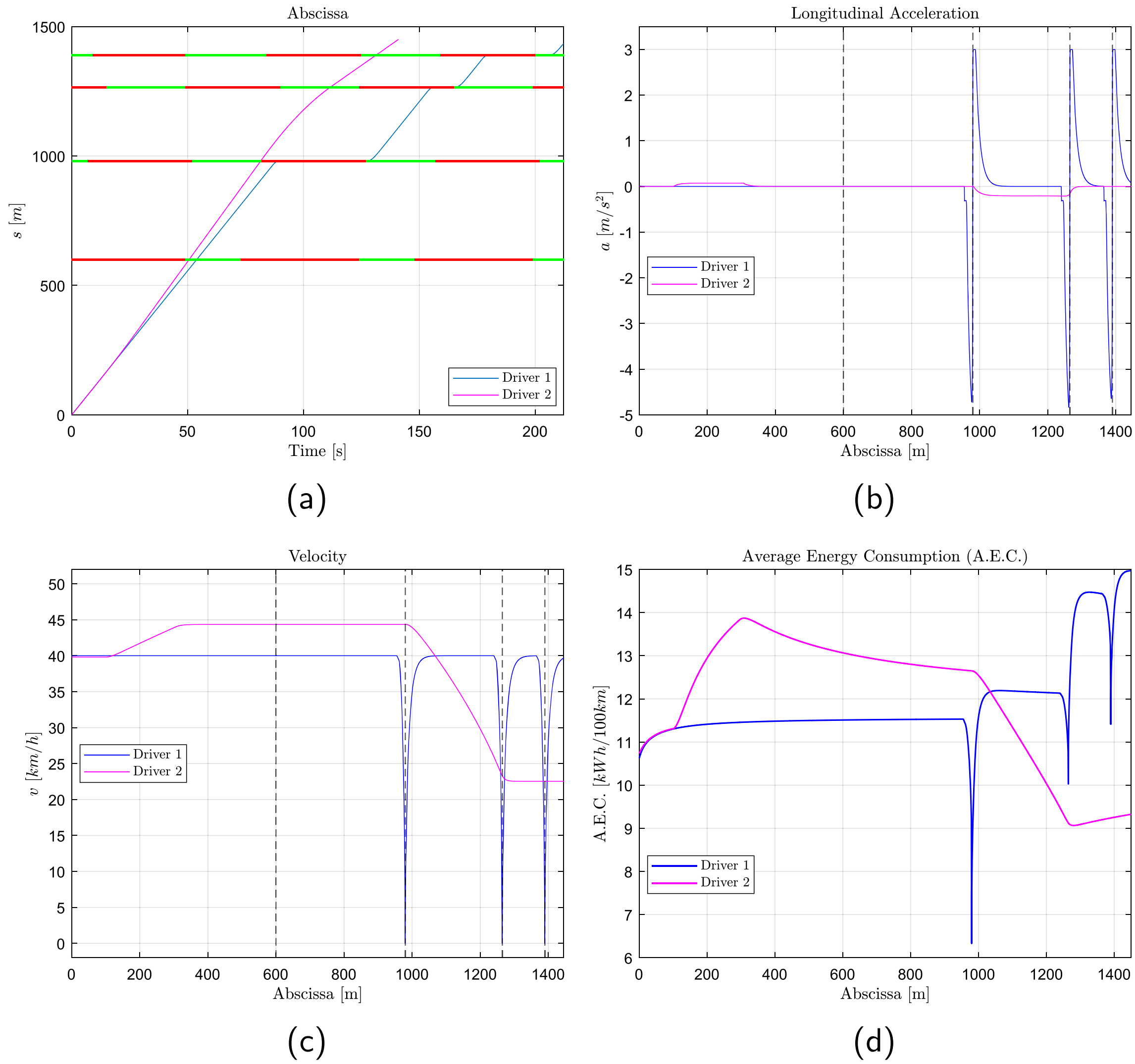

Fig. 12 Optimal MTLA Test Case. (a) abscissa of driver 1 and driver 2, (b) acceleration profiles of driver 1 and driver 2, (c) velocity profiles of driver 1 and driver 2, (d) average energy consumption of driver 1 and driver 2

profile is smoother. Acceleration and velocity of drivers 1 and 2 are plotted with respect to abscissa in Fig. 12b and c. Figure 12d shows the average consumption (computed as in Eq. 19) for driver 1 and driver 2. As for the previous plots, comments done previously are valid also for the energetic analysis. It is noteworthy that the final average consumption of driver 1 (15 kWh100 km^{-1}) is higher than that of driver 2 (9.3 kWh100 km^{-1}), resulting in a 38% reduction.

In the following, the nonoptimal and optimal MTLA are directly compared for the test case considered. In Fig. 13a the vehicle acceleration of both algorithms is plotted with respect to abscissa. Also from this plot, it can be seen how the optimal strategy allows to have a more continuous acceleration profile that leads to a more comfortable vehicle motion. It is noteworthy that the optimal deceleration profile has a smoother profile and consequently a higher peak deceleration.

In Fig. 13b, an example of the nonoptimal and optimal acceleration references along the prediction horizon is shown, the instant to which it refers is the beginning of the first acceleration phase (100 m from the starting point of the simulation). It can be noticed how the predicted acceleration (optimal reference) gradually reaches the nonoptimal reference acceleration from the current acceleration (null) resulting in a more realistic and jerk-optimized profile. Also, the velocity profiles are reported in Fig. 13c. nonoptimal and optimal average consumption profiles are depicted in Fig. 13d where it can be seen how, from an energy point of view, the optimal approach does not introduce significant advantages. However, a final reduction of 2% of the average energy consumption is visible at the end of the simulation.

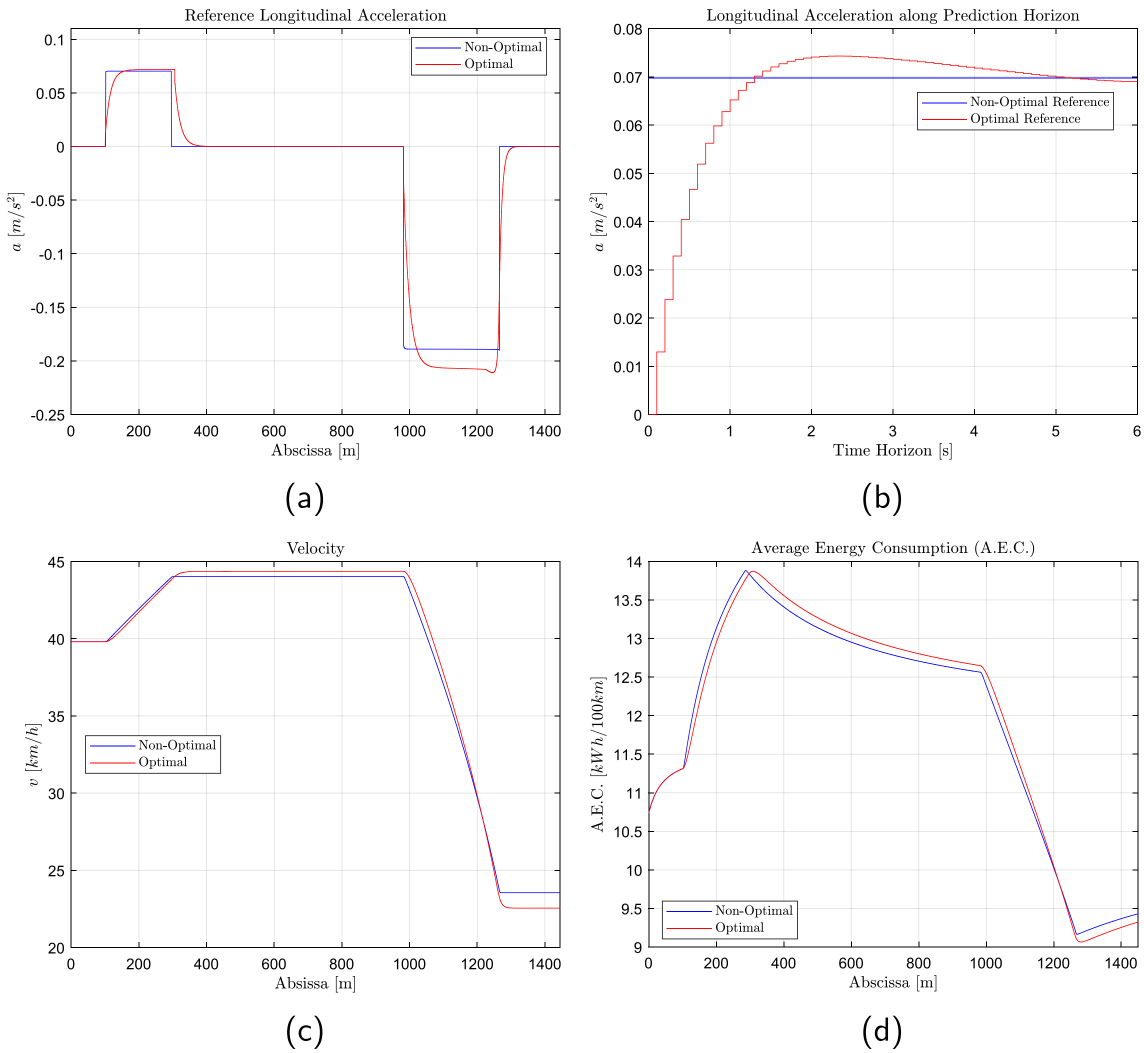

Fig. 13 Optimal vs Nonoptimal comparison. (a) nonoptimal and optimal acceleration profiles, (b) nonoptimal and optimal acceleration profiles along the planning horizon at a given instant of simulation, (c) nonoptimal and optimal velocity profiles, (d) nonoptimal and optimal Average Energy Consumption

6 Conclusions

In this work, we have introduced a novel formulation for a driving strategy advisory system, named Multiple Traffic Light Advisor (MTLA). Initially, we introduced a nonoptimal formulation that, unlike existing state-of-the-art approaches, integrates integrates uniformly accelerated motion and constant velocity profiles, alongside a consideration of ground potential grip, to compute an optimal suggested acceleration profile. This approach aims to enhance intersection safety and reduce power consumption.

Subsequently, we present a novel optimal algorithm that refines the nonoptimal solution, named optimal MTLA. The algorithm optimizes the reference output to generate a continuous, smooth, and comfortable acceleration profile that guides the driver. In particular, optimal MTLA further enhances comfort by considering jerk in the problem formulation.

Simulation-based testing indicated an effective reduction in energy consumption as well as a generalized reduction of acceleration peaks that reflect a comfort increase for both systems. A comparative analysis between the proposed systems highlights a minor improvement (2%) in average energy consumption for the optimal MTLA, but with more realistic and jerk-optimized acceleration profiles.

We consider this work as a stepping stone towards a comprehensive MTLA framework. Thus, it is important to note that the approach described is not without limitations. To improve the algorithm's real-world applicability, future work could extend its formulation to account for traffic density and the presence of other vehicles on the road. Furthermore, comprehensive simulations, which take into account the not-deterministic knowledge of data, especially in addressing the challenge of self-localization in urban areas where GNSS solutions may be absent or unreliable, and incorporating measurement and localization error estimates into the framework, could yield more accurate performance assessments. Currently, ongoing efforts involve an experimental campaign in which the MTLA system serves as an informative Advanced Driver Assistance System (ADAS) for the driver. This real-world testing will offer valuable insights, as deviations from the recommended velocity profile are likely to be more prevalent, and can serve as a basis for further developments and refinements. Moreover, this work establishes a foundation for research into an intuitive and non-distracting user interface, incorporating user feedback, which will enhance user adoption and satisfaction. The primary focus will involve an in-depth examination of the effects on the human driver in two key aspects. Firstly, an exploration of the implications for user interface design, assessing its influence on user adoption and satisfaction within the context of driving. Secondly, a comprehensive analysis of the change in driving characteristics resulting from the implementation of the MTLA system, as demonstrated in [7, 44, 45]. Future experimental campaigns involve exploring the integration of the algorithm within an autonomous driving framework and performing an environmental impact analysis.

Alternatively, a learning-based model-free approach could be carefully explored, where the algorithm's behavior is learned from diverse real-world scenarios, providing a more adaptable solution to the complexities of urban environments. However, such an approach requires extensive testing to validate its performance and a special emphasis on ensuring that safety is guaranteed.

Appendix A In-depth Explanation of Flowcharts

A.1 Flowchart in Fig. 7a

The algorithm for the nonoptimal MTLA system is explained in detail. Its working principle is shown in the flowchart of Fig. 7a, the Green Check and Red Check blocks are further illustrated in Figs. 7b and c respectively. Index i represents the number of the traffic light in analysis, meaning that if $i = 2$ the second TL in front of the vehicle is considered. On the other hand, index j refers to the green phase analyzed, $j = 1$ is relative to the first green phase and $j = 2$ refers to the second green phase. The maximum value of j has been set to 2 in order not to perform to many iterations each time the algorithm is run. Moreover, if $j = 3$ it would mean to pass the third green phase, in case the actual phase is green, or the second green, in case it is red, and to do so such a low speed would be required that it would result annoying for the driver.

The first two blocks are the "Localization" and the "Activation Check". "Localization" block consists of the localization of the vehicle on a map by means of its abscissa. Moreover, all traffic light positions are reported on the same map.

"Activation" block defines if further calculations (and output warning for the driver) are required or not. It is based on the geometrical distance and speed evaluation between vehicle and TLs.

In the case of multiple traffic lights, once the algorithm finds the first of the four traffic lights within the horizon, the system should always be active until the last traffic light is passed. If the distance between two subsequent TLs is greater than the horizon, the following situation may occur: after passing the first traffic light, the system is deactivated as it does not find a TL within the horizon until the vehicle gets close enough to the second TL.

Once the algorithm is activated, an iterative cycle that analyzes the four traffic lights ahead of the vehicle starts. The first thing that is checked is the actual phase ("Phase" block) of the TL considered, if green the "Green Check" block is executed, otherwise the "Red Check" one. Inside these blocks, the feasibility to get a green phase is examined and whenever a feasible maneuver to reach the i^{th} traffic light is found, the following variables are stored inside the algorithm:

- reference velocity v_{ref} that the driver should reach to get the green traffic light under analysis;
- reference acceleration a_{ref} that the driver should perform to get the green traffic light under analysis;
- the warning to be issued to the driver;
- the interval of admissible velocities $v_{\text{adm},i} = [v_{\min,i}, v_{\max,i}]$ to get the green traffic light under analysis.

If it is not possible to pass the traffic light under examination during a green phase, the algorithm stops and the last stored warning is issued.

A.2 Flowcharts in Figs. 7b and c

Once the "Green Check" block is activated, a whole cycle starts. It iterates according to the index j and analyzes the possibility to get either the first ($j = 1$) or second ($j = 2$) green phase of the i^{th} traffic light. If neither can be reached, the algorithm stops and a warning is issued to the driver. More details on the "Exit" block are given in A.2.3. The first step of the "Green Check" is the Velocity Range Definition, this step computes the required velocity range to get the green phase $v_{\text{req},i}$. Concerning the $j = 1$ case, Eqs. 2 and 3 are used if $i = 1$, else Eqs. 9 and 10 are adopted. Otherwise, when $j = 2$, Eqs. 4 and 5 are used if $i = 1$ and Eqs. 11 and 12 if $i > 1$. The second step is the Intersection Check. In this block, the required velocity range to get the i^{th} green with the one needed to get the $i-1^{th}$ green are intersected as in Eq. 15. The aim is to find a velocity range that allows passing both the i^{th} and $i-1^{th}$ traffic lights.

If the first traffic light is studied ($i = 1$), the admissible velocity range is the road admissible one as in Eq. A1

$$v_{\text{adm},i-1} = [v_{\text{min,road}}, v_{\text{max,road}}] \tag{A1}$$

where $v_{\text{max,road}}$ is equal to the road maximum limit speed $v_{\text{lim,road}}$, which is 50 kmh^{-1} in urban environment, and $v_{\text{min,road}}$ is an arbitrary value set to 20 kmh^{-1} so to avoid travelling at too low speed. On the other hand, when $i > 1$ the admissible interval is already defined from the $i-1^{th}$ iteration. As a matter of fact, whenever there is the possibility to get the green of the i^{th} TL, an admissible velocity interval is defined. This will become the $i-1$ interval used in the Intersection Check for the next traffic light. Furthermore, it is noteworthy that, once a velocity range to pass the first traffic light is defined, the road limits are respected. For instance, if an intersection between $v_{\text{req},2}$ and $v_{\text{adm},1}$ exists, this respects the road limits too. Now, if Eq. 15 results in an empty interval, it is not possible to reach the i^{th} traffic light when its j^{th} green phase is on, so the possibility to get the next one is then analyzed ($j = j+1$). Contrary if the interval is non-empty, $v_{\text{adm},i}$ is defined as in Eq. A2:

$$v_{\text{adm},i} = [v_{\text{min},i}, v_{\text{max},i}] \tag{A2}$$

Then, the "Actual Velocity Check" (AVC) block is executed, it analyzes the possibility for the driver to keep the current speed rather than performing an acceleration or deceleration maneuver. Further explanation is given in the following section, A.2.1. If the last check is satisfied, the next traffic light is analyzed ($i = i+1$), if not, the "Acceleration/Deceleration Maneuver Check" (AMC) is performed. Given that the diver cannot keep the current speed constant, the safeness of the acceleration or deceleration maneuver is checked. A more comprehensive analysis of this block is in A.2.2. If the "AMC" block result is positive the driver can get the j^{th} green of the i^{th} traffic light through an acceleration or deceleration maneuver. The index i is increased by one unit ($i = i+1$) and the next traffic light is studied. If the result is negative, the next green phase is analyzed ($j = j+1$).

A.2.1 Actual Velocity Check

The "Actual Velocity Check" analyzes the possibility for the driver to keep its actual speed constant and to get the green phase under analysis. In this way, a useless acceleration/deceleration maneuver is avoided. If the actual speed belongs to the admissible velocity range ($v_{\text{adm},i}$), it will allow the driver to get the green. Therefore, there is no necessity to either accelerate or decelerate and the driver can keep its speed constant. This condition occurs when Eq. A3 is satisfied:

$$v \in v_{adm,i} \tag{A3}$$

When Eq. A3 is satisfied, the following variables are stored:

- $v_{\text{ref}} = v$
- $a_{\text{ref}} = 0$
- No warning
- $v_{\text{adm},i} = [v_{\text{min},i}, v_{\text{max},i}]$

A.2.2 Acceleration/Deceleration Maneuver Check

The "Acceleration/Deceleration Maneuver Check" analyzes the possibility to perform an acceleration/deceleration maneuver to get the green phase of the i^{th} TL. In order to reach the traffic light as fast as possible, the target velocity $v_{t,i}$ is set equal to the maximum velocity of the admissible interval. The corresponding target acceleration $a_{t,i}$ is computed considering a UAM up to the first traffic light as in Eq. A4.

$$\begin{cases} v_{t,i} = \max(v_{\text{adm},i}) \\ a_{t,i} = \frac{(v_{t,i}-v)^2}{2l_1} + \frac{v(v_{t,i}-v)}{l_1} \end{cases} \tag{A4}$$

When $a_{t,i} > 0$, an acceleration maneuver is required, otherwise, if $a_{t,i} < 0$, a deceleration maneuver has to be performed. Now, it is verified if such acceleration/deceleration maneuver is safe for the driver. To this aim, safety acceleration and deceleration parameters are defined. The safety acceleration value a_{safety} is set to 3 ms^{-2}, while the safety deceleration value d_{safety} is set to 1 ms^{-2}. The latter has been chosen so to have soft braking maneuvers, indeed d_{safety} is the 25% of the harsh braking limit of 4 ms^{-2}.

The acceleration maneuver feasibility analysis is hereby described. The required acceleration to get the green of the i^{th} TL is compared with the safety one, as in Eq. A5:

$$a_{t,i} < a_{\text{safety}} \tag{A5}$$

When the target acceleration is smaller than the safety one, the maneuver can be performed by the driver without risks. Then, the following variables are defined and stored:

- $v_{\text{ref}} = v_{t,i}$
- $a_{\text{ref}} = a_{t,i}$

- Green Warning
- $v_{\mathrm{adm},i} = [v_{\min,i}, v_{\max,i}]$

If the safety acceleration check result is negative, an alternative way to reach the traffic light is analyzed. Instead of trying to reach the green with the acceleration corresponding to the maximum velocity of $v_{\mathrm{adm},i}$ (A4), the acceleration value is a_{safety}. Then the feasibility of this acceleration is checked, if the speed that the vehicle reaches at the end of the acceleration phase is within the admissible velocity range $v_{\mathrm{adm},i}$, this maneuver can be performed. If not, the a_{safety} acceleration will not allow the driver to reach the traffic light when the green phase is on, meaning that the result of the "Acceleration/Deceleration Maneuver Check" block is negative. The velocity reached by the vehicle at the end of the acceleration phase is computed starting from Eq. 1, where the acceleration distance d is the distance to the first traffic light l_1 and acceleration a is equal to a_{safety}. Then, acceleration time t_{acc} and target velocity $v_{t,i}$ are computed as in Eq. A6:

$$\begin{cases} t_{\mathrm{acc}} = \frac{-v+\sqrt{v^2+2a_{\mathrm{safety}}l_1}}{a_{\mathrm{safety}}} \\ v_{t,i} = v + a_{\mathrm{safety}}t_{\mathrm{acc}} \end{cases} \tag{A6}$$

If the target velocity belongs to the interval of admissible speeds, the following variables are stored:

- $v_{\mathrm{ref}} = V_{t,i}$
- $a_{\mathrm{ref}} = a_{\mathrm{safety}}$
- Green Warning
- $v_{\mathrm{adm},i} = [v_{\min,i}, v_{t,i}]$

The deceleration maneuver feasibility analysis is now explained. The required deceleration to get the green of the i^{th} TL is compared with the safety one, as in Eq. A7:

$$\mathrm{abs}(a_{t,i}) < d_{\mathrm{safety}} \tag{A7}$$

When the absolute value of the target acceleration is smaller than the safety one, the maneuver can be performed by the driver without risks. The following variables are stored:

- $v_{\mathrm{ref}} = v_{t,i}$
- $a_{\mathrm{ref}} = a_{t,i}$
- Red Warning
- $v_{\mathrm{adm},i} = [v_{\min,i}, v_{\max,i}]$

If Eq. A7 is not satisfied, the deceleration maneuver to get the green phase of the i^{th} TL is not acceptable. Moreover, it is not possible to restrict the deceleration range, as done for the acceleration case. Indeed, the acceleration corresponding to the maximum velocity of the interval of admissible speeds $v_{\mathrm{adm},i}$ is the smallest in absolute value among all the admissible accelerations. Hence, by imposing the value of deceleration equal to the safety one, the respective speed is not within the interval $v_{\mathrm{adm},i}$.

A.2.3 Exit

The "Exit" block is performed whenever a traffic light cannot be passed during one of its green phases. Therefore, an analysis of the following ones is deemed useless. The warning stored during the previous iteration ($i-1$) is issued to the driver. When $i = 1$ and the Exit block is executed, the vehicle needs to be stopped. In fact, it is not feasible to reach the first traffic light when its green phase is on, so the vehicle needs to stop. The deceleration is computed as in Eq. A8:

$$d_{\mathrm{stop}} = -\frac{1}{2}\frac{v^2}{l_1} \tag{A8}$$

If such deceleration is smaller than the harsh braking limit value of 4 ms^{-2}, the Red Warning is issued to the driver. On the contrary, if a hard deceleration is required the visual and acoustic warning (Red +Sound Warning) is issued.

It is noteworthy that the deceleration required to stop the vehicle at the i_{th} traffic light is calculated only once the TL has become the first one ahead of the vehicle (the vehicle passed the traffic light $i-1$). It can be objected that suggesting a stop in this way could make the deceleration maneuver dangerous or even not feasible. However, traffic lights are designed so to always make a safe arrest maneuver feasible if the speed limit is respected.

A.3 Flowcharts in Fig. 9b, Fig. 9c

In order to calculate the admissible region in Fig. 9a, the algorithm represented by the flowchart in Fig. 9b is used. The algorithm analyzes the first two traffic lights ahead of the vehicle (conditional block of the flowchart in Fig. 9b) and for each one the state constraints are defined (i^{th} TL State Constraints block of Fig. 9b). The state constraints are representative of the limited time-space region the vehicle can occupy so to pass the i^{th} traffic light only during green phases and they depend on the actual phase of the traffic light under analysis, as explained next.

The algorithm of the i^{th} TL State Constraint block is represented in the flowchart of Fig. 9c, where according to the actual phase the "Green Check" block or "Red Check" block is executed. The output of these blocks is the i^{th} TL State Constraints". Lastly, the state constraints identified for each traffic light are intersected (Range Intersection block of

Fig. 9b) to get the final range of admissible positions (State Constraints block of Fig. 9b).

A.4 Flowcharts in Figs. 10a and b

The flowchart of the "Red Check" block is shown in Fig. 10a. If the time of the phase shift is larger or equal to the prediction horizon, the vehicle should be ahead of the TL for the whole horizon. Otherwise, it can be beyond the TL only after the phase shift. The flowchart of the "Green Check" block is illustrated in Fig. 10b. If the time of the phase shift is larger than the prediction horizon, the vehicle can be ahead or beyond the TL. Contrary, if the time is lower or equal, a further analysis is performed on the possibility to pass the traffic light during the actual green phase. This is done by checking if the MTLA algorithm in Fig. 7a calculates that it is not possible to pass the i^{th} traffic light ($N_{\text{green}} < i$) or to pass it during the second green phase ($N_{\text{pass}} > 1$). If one of these two conditions is verified the vehicle should be ahead of the traffic light after the phase shift, otherwise it should be beyond it.

Once the admissible regions are obtained for the two TLs, the "Range Intersection" Block is executed. It consists of making the intersection of the state constraints of the single traffic lights as in Eq. 16. Finally, the position state constraint can be formalized as in Eq. A9:

$$s_{\min}(t) \leq s(t) \leq s_{\max}(t)\ \forall t \in [0, t_f] \tag{A9}$$

Acknowledgements The authors would like to thank all their partners in the Vodafone 5G Trial in Milan: Vodafone Automotive, Marelli, Pirelli, and Stellantis. A special acknowledgment is extended to IPG for their generous provision of student licenses that were necessary for the development and testing of the developed algorithms.

Author Contributions All authors contributed equally to the development and testing of algorithms in the manuscript. The manuscript was written and revised by Michael Khayyat, Alberto Gabriele, Francesca Mancini, and Stefano Arrigoni and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding Open access funding provided by Politecnico di Milano within the CRUI-CARE Agreement. The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Availability of data and materials Data is subject to third-party restrictions.

Code Availability Code is unavailable due to privacy and third-party restrictions.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Competing Interests The authors have no relevant financial or non-financial interests to disclose.

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