

A Modified Jiles-Atherton Model for Estimating the Iron Loss of Electrical Steel Considering DC Bias

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Abstract—This paper proposed a novel approach to calculate the iron loss under DC bias condition using modified Jiles-Atherton (JA) model. The influence of DC bias on the JA model is analyzed. The parameters in the JA model are extracted from BH loops without DC bias but they are applicable for the one under DC bias condition. Thus the modified JA model is practically convenient for iron loss prediction. Moreover, the parameter identification is fast. In addition, the influence of different magnetic induction levels and frequency is incorporated in the model. An accurate measurement system is established to measure the BH loops and iron loss. The tested results are demonstrated and compared with the predicted iron loss to verify the modified JA model.

Keywords—Jiles-Atherton Model, iron loss, DC bias.

I. INTRODUCTION

Accurate prediction of iron loss of inductive components is difficult but has not been entirely solved yet [1]. Generally, there are two approaches to predict the iron loss. The first approach is based on the empirical equation, which is computationally fast and easy to use with relatively small errors at certain circumstances [2]. However, it ignores actual behavior of the ferromagnetic material and its accuracy becomes questionable when calculating the iron loss under DC bias. The second approach, based on mathematical models with physical insight, shows its advantage in predicting BH loops and iron loss. Among the physical models, Jiles-Atherton (JA) models is widely used. It is inspired by the physics of ferromagnetism and translates the physical phenomenon of hysteresis into a simple first order differential equation [3]. The equation is simple to implement and the parameters can be obtained theoretically [4] or using optimization algorithms [5] from the measured B-H loops.

Common situations, such as frequency dependence, have been incorporated in the physical model. D. C Jiles extended the model to consider the effects of classical and excess eddy current [6]. R. Du *et al.* made some modifications to the original dynamic JA model and estimated iron loss at high frequency [7]. K. Chwastek *et*

al. revised the dynamic model in the inverse form and built the hysteresis loops model with sine B-excitation [8]. In [9], M. Hamimid *et al.* introduced hybrid magnetic field in the dynamic inverse model, which gave better prediction of dynamic loops. A. P. S. Baghel *et al.* justified the energy equivalence between the JA approach and field separation theory and validated the usage of hybrid magnetic field [10].

Nevertheless, there is some inherent limitation in the JA model. The most disturbing one is that the parameters of the JA model (JA parameters) are not constant when the maximum magnetic induction changes. In order to overcome the problem, Lederer *et al.* proposed a scaling technique to model smaller loops. Compared to the optimizing parameters for every measured loop, scaling technique is easy, efficient and practical in numerical calculation [11]. In [12], Leite *et al.* introduced a dissipative factor to limit the irreversible magnetization variation rate and improved the inner loops representation. Based on this proposed model, S. Hussain *et al.* optimized one of the JA parameters at different magnetic induction levels to improve the accuracy of the modified model [13]. In order to extent the JA model to suit the DC bias condition, Y. Wang *et al.* analyzed the relationship between DC-biased magnetic induction and the dynamic JA model coefficient and predicted the iron loss under DC bias [14].

In this paper, a new approach for the JA parameter identification is adopted. Only two parameters which are physically related to the size of BH loops were updated with different magnetic induction level. All the other parameters are kept constant when the maximum magnetic induction level changed. From the physical meaning point of view, this new method with only two parameter varied with magnetic induction level is more close to the actual situation under DC bias. Thus, the modified JA model is more accurate and efficient. Comparisons of BH loops and iron loss between the traditional and modified JA model are performed to validate the proposed model.

II. JILES-ATHERTON MODEL

A. Original JA Model including Dynamic Loss

The key mathematical model for hysteresis loop is the static JA model based on the domain wall motion theory. The model can simulate magnetization process and describe quasi-static or DC hysteresis loops of ferromagnetic materials [3]. The five original equations are listed below [7]

$$H_e = H + \alpha M \quad (1)$$

where H_e is the effective field, H is the applied field, M is the bulk magnetization, α is a mean field parameter representing interdomain coupling.

$$M_{an} = M_s \left(\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right) \quad (2)$$

where M_{an} is the anhysteretic magnetization, a quantifies domain wall density in the magnetic material and M_s represents saturation magnetization of material.

$$M = M_{irr} + M_{rev} \quad (3)$$

where M_{rev} is the reversible magnetization and M_{irr} is the irreversible magnetization.

$$M_{rev} = c(M_{an} - M_{irr}) \quad (4)$$

where c represents magnetization reversibility.

$$\begin{aligned} \mu_0 \int M_{an} dH_e &= \mu_0 \int M dH_e \\ &+ \mu_0 k \delta (1-c) \int \left(\frac{dM_{irr}}{dH_e} \right) dH_e \end{aligned} \quad (5)$$

where k quantifies average energy required to break pinning site in the magnetic material and δ is a directional parameter which ensures that energy is always lost through dissipation; which means $\delta=+1$ when $dH/dt \geq 0$ and $\delta=-1$ when $dH/dt < 0$. Thus the static JA model is derived from the five original equations in its inverse form as [8]

$$\frac{dM}{dB} = \frac{\delta_M (M_{an} - M) + kc \delta \frac{dM_{an}}{dH_e}}{\mu_0 (k\delta + (1-\alpha)(\delta_M (M_{an} - M) + kc \delta \frac{dM_{an}}{dH_e}))} \quad (6)$$

where δ_M is introduced to avoid non-physical negative susceptibilities dM/dH after a field reversal.

In order to take into account the influence of frequency in BH loops, the classical eddy current loss and excess loss are included in the original JA model, making it sufficient for iron loss calculation. The instantaneous classical eddy loss per unit volume is given as [6]

$$\frac{dW_{eddy}}{dt} = \frac{d^2}{2\rho\beta} \left(\frac{dB}{dt} \right)^2 = C_{eddy} \left(\frac{dB}{dt} \right)^2 \quad (7)$$

where d and ρ are the thickness and resistivity, respectively, β is a geometrical parameter. The excess loss

is attributed to discontinuous Barkhausen jump or bowing of active domain walls which induces magnetic flux around them, resulting in current in the magnetic materials [16]. This means the excess loss can be regarded as a special form of eddy current loss or excess eddy current loss. The instantaneous excess loss per unit volume is given as [6]

$$\frac{dW_{excess}}{dt} = \left(\frac{Gd\omega H_0}{\rho} \right)^{\frac{1}{2}} \cdot \left(\frac{dB}{dt} \right)^{\frac{3}{2}} = C_{excess} \cdot \left(\frac{dB}{dt} \right)^{\frac{3}{2}} \quad (8)$$

where G is a dimensionless coefficient, w is the width of laminations, and H_0 characterizes the statistical distribution of the internal domain wall field. Considering the influence of dynamic loss, the energy balance equation (5) is extended to

$$\begin{aligned} \mu_0 \int M_{an} dH_e &= \mu_0 \int M dH_e + \mu_0 k \delta (1-c) \int \left(\frac{dM_{irr}}{dH_e} \right) dH_e \\ &+ \int C_{eddy} \left(\frac{dB}{dt} \right)^2 dt + \int C_{excess} \cdot \left(\frac{dB}{dt} \right)^{\frac{3}{2}} dt \end{aligned} \quad (9)$$

Hence the original JA model can be directly obtained with (1), (2), (3), (4), (9) using numerical methods. According to the loss separation theory, the iron loss can be defined as the sum of the hysteresis loss, classical eddy current loss and the excess loss.

$$W_{total} = W_{hys} + W_{eddy} + W_{excess} \quad (10)$$

Hence, the loss separation equation can be equivalently represented by the following field separation equation, as given below [6].

$$H_{total} = H_{hys} + H_{eddy} + H_{excess} \quad (11)$$

where H_{total} is the total magnetic field, H_{hys} is the static hysteresis magnetic field, H_{eddy} is the classical eddy current magnetic field and H_{excess} is the excess magnetic field. H_{eddy} and H_{excess} can be manipulated from (7), (8)

$$H_{eddy} = C_{eddy} \frac{dB}{dt} \quad (12)$$

$$H_{excess} = C_{excess} \left| \frac{dB}{dt} \right|^{-1/2} \frac{dB}{dt} \quad (13)$$

Thus the two counteracting fields corresponding to the classical eddy loss and the excess loss can be included in the JA model via the effective field term as [10]

$$H_e = H + \alpha M - (H_{eddy} + H_{excess}) \quad (14)$$

It is clear from (14) that dynamic BH loops can be obtained by widening static hysteresis loops at each point by the classical eddy current field and the excess loss field. Hence the proposed JA model can be directly obtained with (2), (3), (4), (5), (12), (13), (14) using numerical methods.

B. Modified JA Parameters

Although the original JA model is able to represent the major hysteresis loops that reach the saturation, it cannot predict BH loops accurately at low magnetic induction which results in large errors in the predicted iron losses. To solve the problem, the most common approach is to change the four JA parameters (α , a , k , c) for every BH loop. It's to some degree accurate but not practical [11]. This paper proposed a phenomenological approach based on physical consideration to get better agreement between the measured and predicted BH loops. In the JA model, c represents the reversible component of magnetization due to reversible wall bending and reversible translation [3] while k is linked to the irreversible component of magnetization which result in the iron loss [18]. When the magnetic induction level changed, both irreversible and reversible components are modified. Thus k and c should be determined at different magnetic induction. Meanwhile the change of a and α is not necessary to fit the BH loops at low magnetic induction. If a and α stay constant, it means neither domain wall density nor interdomain coupling are significantly affected by the magnetic induction. This assumption can be validated if the measured and predicted BH loops using modified approach agree well at different magnetic induction level.

C. Parameters Identification

In the proposed JA model, there are seven parameters to be determined (M_s , α , a , k , c , C_{eddy} , C_{excess}). C_{eddy} is the classical eddy current loss coefficient which is calculated from equation (7) with the data provided by manufacturer. As H_0 is unknown for all materials, C_{excess} cannot be determined from the relations (8). This difficulty is overcome by solving equation (7), (8), (10) for two arbitrary frequencies and same induction level [9]. Besides on the assumption that the dc-biased magnetic induction influences hysteresis loss only but not the eddy current loss, dynamic loss coefficients C_{eddy} and C_{excess} are constant for arbitrary frequency or induction level because eddy current loss is only related to the change rate of flux density [17]. Subsequently, the five JA parameters (M_s , α , a , k , c) are obtained based on the largest measured BH loop using differential evolution (DE) optimization algorithm [5]. For smaller B-H loops, only k and c need to be optimized at different induction levels, due to the change of the reversible and irreversible magnetization. This process is fast and convenient because of the decreasing number of variables and same optimization algorithm. The objective function is set as

$$obj = \frac{\sum_{i=1}^n |H_{predicted}(i) - H_{measured}(i)|}{n * \max(H_{measured})} \quad (15)$$

It represents the error of the predicted BH loop. The goal of

DE optimization is to minimum the value of the function and determine the JA parameters.

As the BH loop is predicted using modified JA model, the total iron loss per unit volume can be calculated as [7]

$$P_{total} = A_{BH} \cdot f_m \quad (16)$$

where A_{BH} is the area within the BH loop at the magnetic induction frequency f_m .

D. Ring Frame Experiment System

As shown in Fig.1, the ring frame experiment system includes mainly a ring type iron core, a programmable power source and power analyzer. Toroidal iron core with exciting coils and B-coils are used for academic research under alternating excitation, which takes the advantages of the uniform magnetic field distribution and simple magnetic path [15]. The exciting current which is generated by the programmable power source flows in the exciting coil and induces voltage in the B-coil. By recording the waveform of the exciting current I and induced voltage u_2 , the flux density B and magnetic field strength H is calculated as

$$\phi(t_1) = \phi(0) + \int_0^{t_1} u_2 dt \quad (17)$$

$$B = \frac{\phi}{N_2 S} \quad (18)$$

$$H = \frac{N_1 I}{l} \quad (19)$$

where $\Phi(0)$ and $\Phi(t_1)$ is the magnetic flux at $t=0$ and $t=t_1$, N_1 and N_2 is the turn number of the exciting coil and B-coils, S is the effective cross section of the iron core, l is the equivalent length of the magnetic path.

The power analyzer is used to measure the iron loss and harmonic content of u_2 in order to generate pure sinusoidal waveform of flux density. Feedback algorithm is developed to analyze the harmonic content of u_2 and adjust the exciting voltage to make the harmonic content

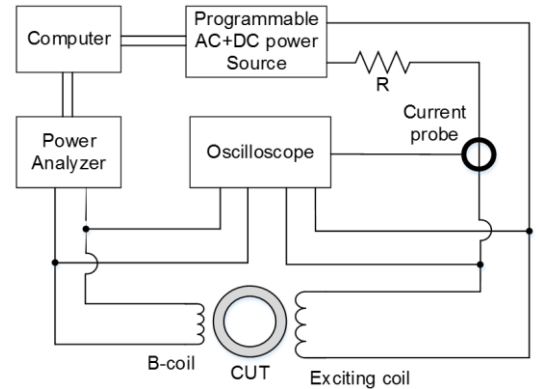


Fig. 1. Overview of the measurement system.

less than 1%.

The resistance R is used to decrease the time constant in the circuit and make the flux density steady in a short time. Another advantage is that a slight change of the DC bias voltage will have negligible influence on the current, which guarantees the accuracy. However it also brings some inconvenience because the exciting voltage in the iron core is more non-sinusoidal and it takes longer time to obtain the flux density that is sinusoidal enough via the feedback algorithm. The authors think the inconvenience is worthwhile considering the improved accuracy. Meanwhile, it is necessary and important to degauss the iron core before measurement to make $\Phi(0)=0$ and simplify (17). The method to eliminate remnant magnetic field in this paper is to apply low frequency AC voltage to the iron core and decrease the amplitude to zero slowly.

III. VERIFICATION OF THE MODIFIED JA MODEL

A. Parameters Fitted Results

In order to explore the proposed model, the parameters k and c are fitted and analyzed, as shown in Fig. 2. When the BH loops reach saturation, k stays constant because there is no further movement of domain walls [13]. Reversible magnetization also keeps unchanged in the saturated BH loops and c stays constant, which is clearly seen from Fig. 2. For smaller BH loops, both reversible magnetization and irreversible magnetization decreased, as k and c decreased. However, when the magnetic induction B_{ac} is below 0.5T, k is nearly unchanged but c is still decreasing, which means only the ratio between the reversible magnetization and irreversible magnetization influence the BH loops at low induction level. In Table I, the errors between the predicted and measured BH loops are calculated using (15). In Figs. 3&4, BH loops obtained using the original and modified JA model are compared with measured loops. The

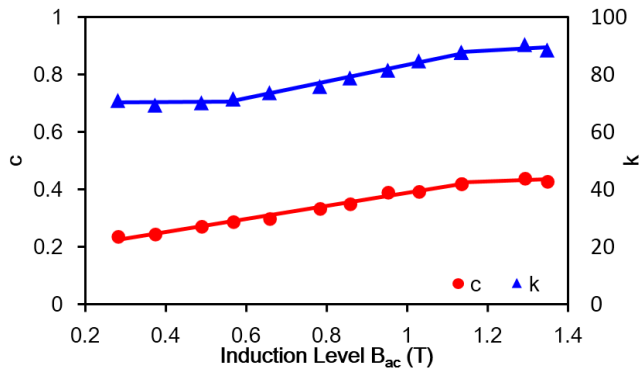


Fig. 2 Optimized JA parameters c and k using modified JA model at different magnetic induction levels.

predicted BH loops using modified JA model are in good agreement with the measured BH loops that indicated that the assumption of keeping a and α unchanged is convincing and reasonable. Compared to the original JA model, the modified JA model has slightly worse performance to fit BH loops without DC bias but it cost much less computation time and resource. It is recorded that the average time to obtain all parameters in the modified JA model is 1.7 hours while it is 6.8 hours in the original JA model using the same computer. It is also seen from Table I that there is no significant difference in the prediction of iron loss between the original and modified JA model.

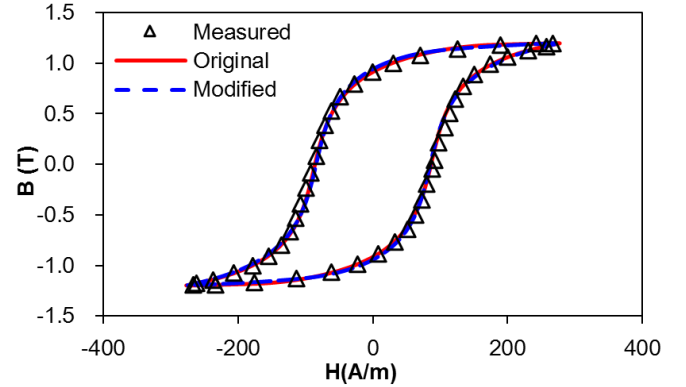


Fig. 3 Measured and computed BH loops using the original JA model and modified JA model at $B_{ac}=1.2T$, $B_{dc}=0$.

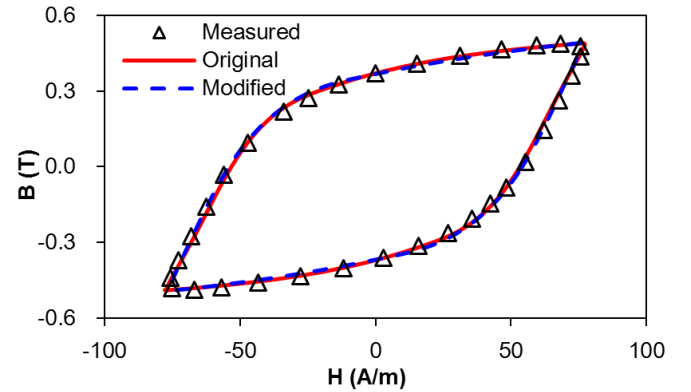


Fig. 4 Measured and computed BH loops using the original JA model and modified JA model at $B_{ac}=0.5T$, $B_{dc}=0$.

B. Frequency Dependency

One of the advantages of the proposed JA model is the inclusion of dynamic losses, making it convenient to calculate the iron loss at arbitrary frequency. The parameters of the model are the same as those extracted from the measured BH loops at 50Hz. Using the modified JA model, the predicted BH loops match well with the measured ones at a certain range of frequency, as shown in Fig 5. For iron loss prediction, the measured and

predicted results agree well in Fig. 6. The differences of iron loss prediction using the original and modified JA model are negligible. However, we do see the rise of error at higher frequencies, which may be attributed to the change of C_{excess} that was determined at two lower frequencies. Thus it is better to choose two higher frequencies of iron loss for parameter C_{excess} extraction when calculating iron loss at high frequency [9].

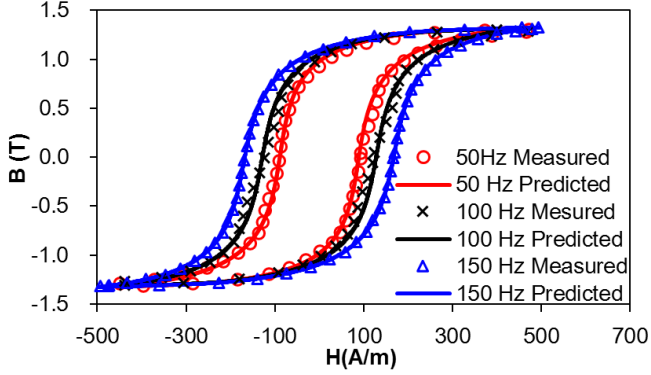


Fig. 5 Measured and computed BH loops using the modified JA model at different frequency when $B_{ac} = 1.3T$.

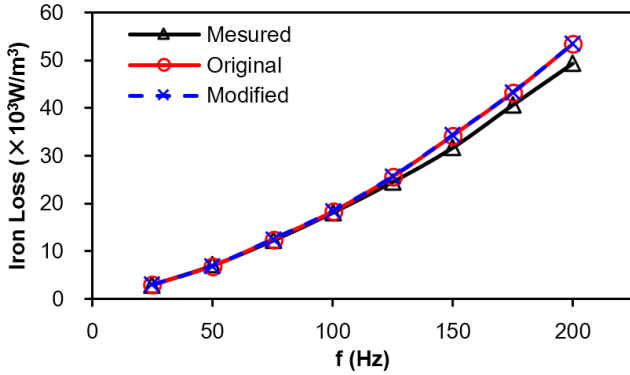


Fig. 6 Comparison of the measured and calculated iron loss using the original and modified JA model at different frequency when $B_{ac} = 0.7T$

C. Verification the Model under DC Bias

In the proposed JA model, the same parameters extracted from the measured BH loops are used to calculate the BH loops and iron loss under DC bias. Compared with the original JA model, the modified JA model predicts more accurate results, which indicated that the JA parameters are not affected by the DC bias [14] while k and c is dependent on the AC component of the magnetic induction (B_{ac}). As shown in Figs. 7&8, the measured and calculated BH loop agree well using the modified JA model, except a slight of difference at the bottom of the BH loops. Such phenomena are common in

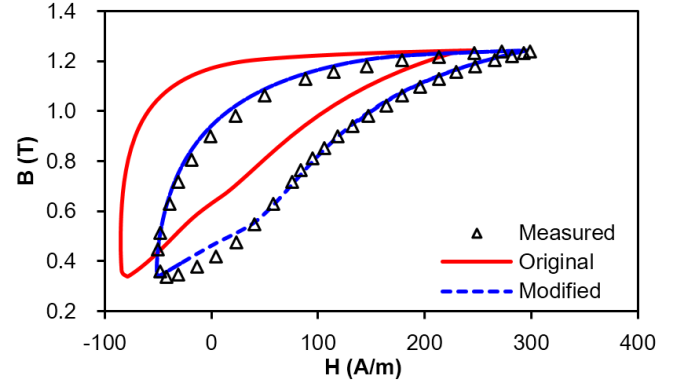


Fig. 7 Measured and computed BH loops using original JA model and modified JA model at $B_{ac}=0.5T$, $B_{dc}=0.8T$.

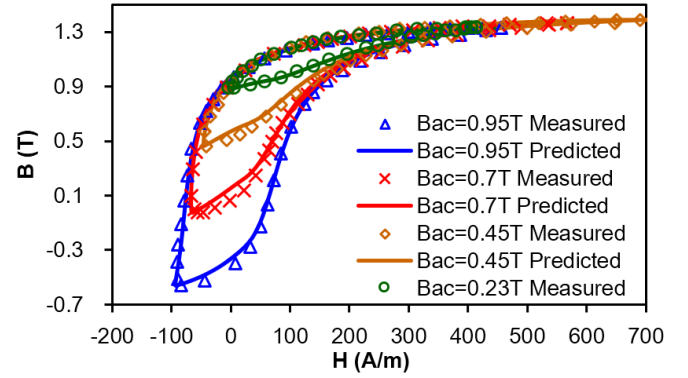


Fig. 8 Measured and computed BH loops using modified JA model when $B_{max}=1.35T$.

all BH loops under DC bias. The reason is the presence of DC bias which changed the magnetization process at the bottom of the BH loop and resulted in the change of k and c . However, for engineering simplicity with acceptable accuracy, the difference at the bottom of BH loops is ignored in the modified JA model. As for original JA model, the predicted BH loop is far from the measured BH loop, as shown in Fig. 7. It is clearly seen that using modified JA model to calculate the BH loop and iron loss is much more accurate, especially when B_{ac} of the predicted BH loop is small, as shown in Figs. 9-12.

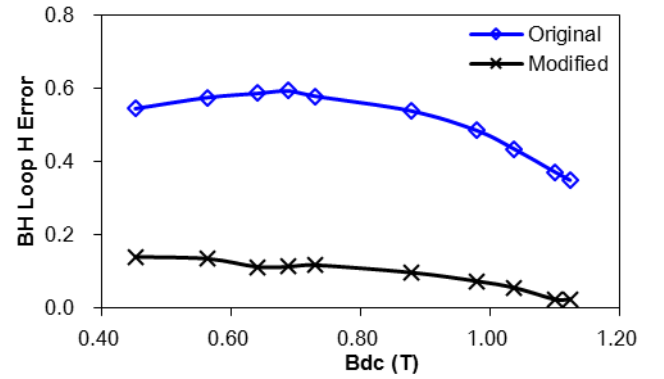


Fig. 9 Comparison of the BH loop errors between the original and modified JA model at $B_{ac} = 0.23T$

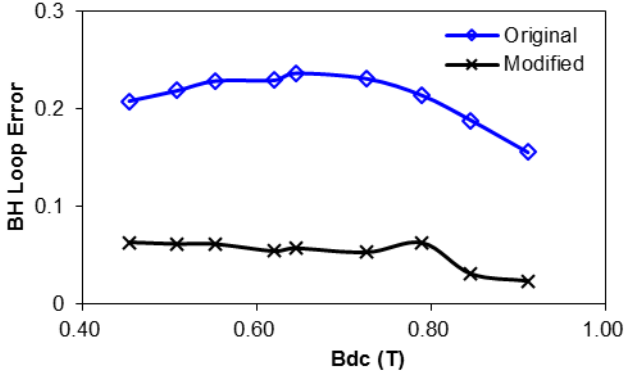


Fig. 10 Comparison of the BH loop errors between the original and modified JA model at $B_{ac} = 0.45T$

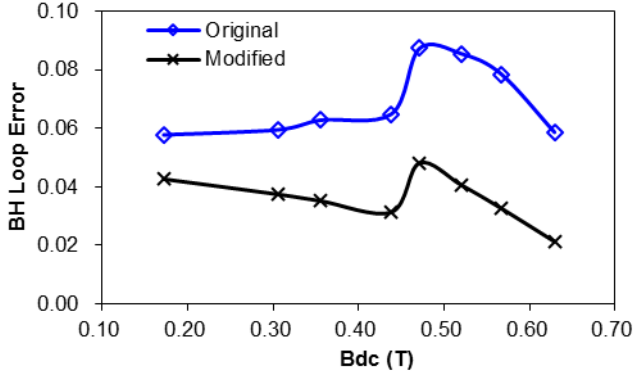


Fig. 11 Comparison of the BH loop errors between the original and modified JA model at $B_{ac} = 0.7T$

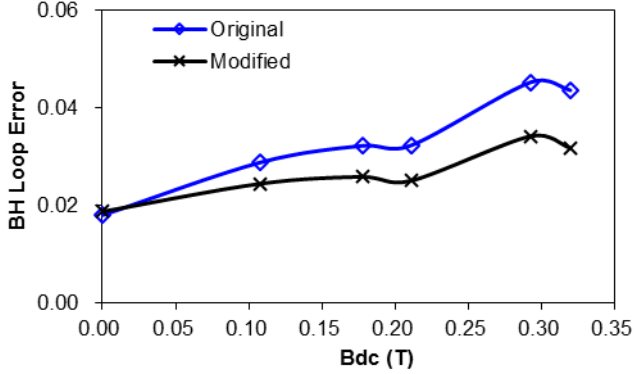


Fig. 12 Comparison of the BH loop errors between the original and modified JA model at $B_{ac} = 0.95T$

It should be noticed that the error using original JA model to predict iron loss is rather large when the B_{ac} is small (see Figs. 13&14). As the B_{ac} of the predicted BH loop becomes bigger, the parameters in the original and modified JA model are nearly the same, resulting in the similar accuracy of iron loss prediction under DC bias (see Figs. 15&16).

IV. CONCLUSION

In this paper, the modified JA model is proposed to predict the iron loss at different magnetic induction under

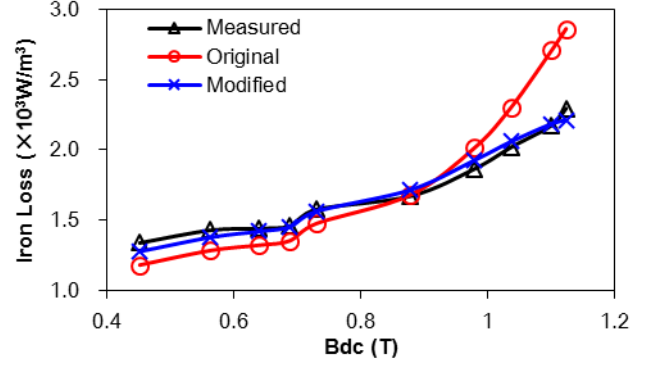


Fig. 13 Comparison of the measured iron losses and calculated iron loss using original JA model and modified JA model at $B_{ac} = 0.23T$

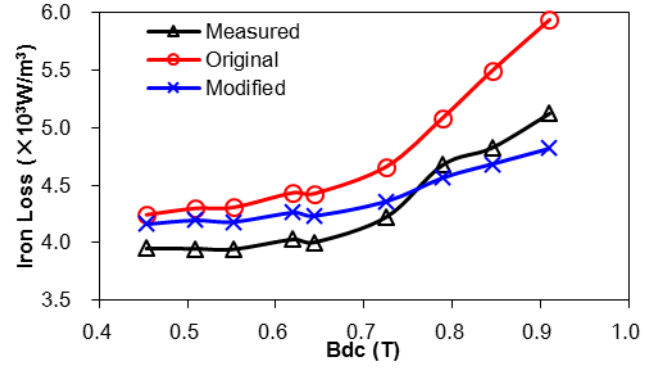


Fig. 14 Comparison of the measured iron losses and calculated iron loss using original JA model and modified JA model at $B_{ac} = 0.45T$

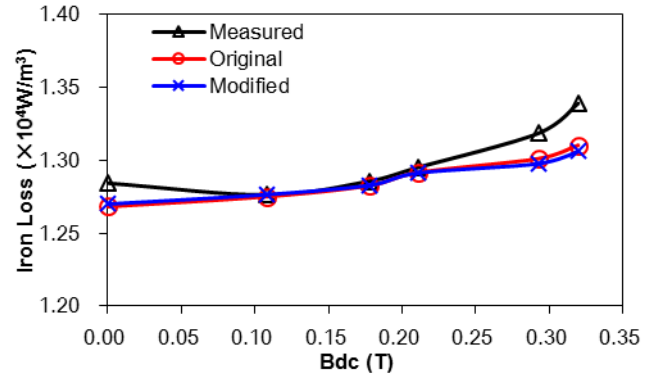


Fig. 15 Comparison of the measured iron losses and calculated iron loss using original JA model and modified JA model at $B_{ac} = 0.7T$

DC bias condition. The modified JA model consumed much less computation time to extract JA parameters as the magnetic induction level only affected the two JA parameters (k and c). The dependence of the JA parameters on the magnetic induction level is analyzed. The variation of frequency and DC-biased magnetization are included in the modification of JA model. As the DC-biased magnetization in the proposed model has negligible influence on the JA parameters and dynamic loss coefficients, it is practical and convenient to use the proposed JA model to calculate the BH loops and iron

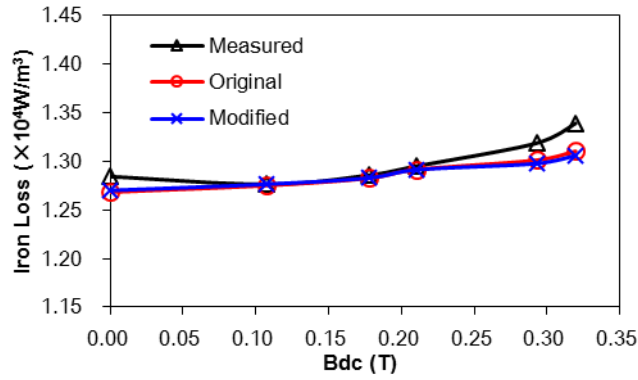


Fig. 16 Comparison of the measured iron losses and calculated iron loss using original JA model and modified JA model at $B_{ac}=0.95T$

loss under DC bias condition. The experiment results verify the great accuracy of the modified JA model, especially when the B_{ac} is low.

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