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Predictor-Based Adaptive Plant Augmentation Design with Application to Hierarchical Control

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***Abstract*— This paper presents a predictor-based adaptive augmentation scheme to recover the designed behavior of a baseline linear controller in presence of parametric uncertainty. Remarkably, the proposed scheme achieves the recovery of the baseline closed-loop performance without the need for explicit knowledge of the baseline controller states and structure; rather, the adaptive mechanism relies solely on the output of the baseline controller and plant states. We showcase how the proposed adaptive design seamlessly integrates into inner-outer loop control architectures, enhancing the overall performance and robustness while simultaneously reducing the control law complexity compared to available solutions.**

***Index Terms*— Robust adaptive control, hierarchical design, adaptive augmentation, UAV**

I. Introduction

IN flight control system design, research into adaptive control methodologies aims to enhance performance under uncertain conditions and failures. In this context, a consolidated approach involves designing a baseline controller for desired performance and robustness scheduled across the flight envelope. However, balancing performance improvement with robustness preservation is challenging in gain scheduling designs. Adaptive control strategies offer stability guarantees by dynamically augmenting the baseline controller to address discrepancies between nominal and actual responses, enhancing performance only when needed. The model reference adaptive control (MRAC) augmentation approach [7, Chapter 10] stands out as a prominent technique in this domain. By defining the closed-loop nominal system as the reference model, the control input is split in a baseline and adaptive component, with the latter handling system uncertainties to recover the baseline controller behavior. Advancements such as closed-loop MRAC [2] and observer-based MRAC [5], [9] have been proposed to enhance transient performance in MRAC schemes. This paper introduces a predictor-based plant augmentation scheme that relies solely on the controller output. This plug-in feature enables seamless integration into diverse control systems. The key idea behind the proposed design lies in leveraging the prediction error stemming from a replica of the ideal (uncertainty-free) plant dynamics forced by the baseline controller to update the adaptive parameters, enhancing the scheme effectiveness in learning uncertainties, which are estimated only when there is a mismatch with respect to the baseline behavior. Compared to the closest works in the literature, where proportional-integral (PI) baseline controllers are typically considered for augmentation [8], [5], our solution applies to a completely general class of linear baseline controllers. The proposed design results in lower controller complexity and computational burden compared to full augmentation designs [7], where a replica of the baseline controller is needed within the adaptive law together with real-time values of its states. In contrast, the order of our adaptive scheme corresponds to the sum of the plant states and the uncertain parameters to be estimated. Furthermore, the proposed adaptive design provides a beneficial anti-windup effect for saturating control inputs, and seamlessly integrates into hierarchical control architectures for kinodynamic systems, enhancing the overall performance and robustness while reducing control law complexity compared to monolithic solutions. Simulation results comparing the performance and differences between augmenting a baseline cascaded P/PI controller for unmanned aerial vehicles (UAVs) tracking control with the proposed design and a full augmentation version of a predictor-based design [6] are reported.

Notation. Given $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, $(x,y) := [x^\top \ y^\top]^\top$ and $\mathrm{diag}(x) := \mathrm{diag}\{x_1,\ldots,x_n\} \in \mathbb{R}^{n\times n}$. For $x \in \mathbb{R}^n$, $|x| = \sqrt{x^\top x}$. For a piecewise continuous function $u(\cdot): \mathcal{I} \to \mathbb{R}^n$, $\mathcal{I} \subseteq \mathbb{R}$, we let $\|u(\cdot)\|_{\mathcal{I}} := \sup_{t\in\mathcal{I}} |u(\cdot)|$. For $\mathcal{I} = [0,\infty)$, $\|u\|_a := \limsup_{t\to\infty} |u(t)|$. The set of bounded and locally-bounded piecewise-continuous functions are denoted respectively by $\mathcal{L}_\infty := \{u(\cdot) \in \mathcal{PC}^0_{[0,\infty)} : \|u(\cdot)\|_{[0,\infty)} < \infty\}$ and $\mathcal{L}_{\infty,e} := \{u(\cdot) \in \mathcal{PC}^0_{[0,\infty)} : \|u(\cdot)\|_{[0,\tau)} < \infty, \text{ for all } \tau > 0\}$.

II. Background and Problem Statement

In this work, we consider the class of nonlinear time-varying systems with matched parametric uncertainty of the form

$$\Sigma_p : \begin{cases} \dot{x}_p = A_p x_p + B_p\big(\Lambda u + \varphi_p(t,x_p)\theta_p\big), \ x_p(t_0) \in \mathbb{R}^{n_p} \\ y_p = C_p x_p, \end{cases} \tag{1}$$

where $x_p \in \mathbb{R}^{n_p}$ is the plant state, $A_p \in \mathbb{R}^{n_p\times n_p}$ is the state matrix, $u \in \mathbb{R}^{n_u}$ is the control input, $B_p \in \mathbb{R}^{n_p\times n_u}$ is the input

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matrix, $\Lambda\in\mathbb{R}^{n_u\times n_u}$ is the control effectiveness matrix, $y_p\in\mathbb{R}^{n_y}$ is the output, $C_p\in\mathbb{R}^{n_y\times n_p}$ is the output matrix, $\theta_p\in\mathbb{R}^{n_\theta}$ is a vector of uncertain parameters, and $\varphi_p(\cdot,\cdot):\mathbb{R}_{t\ge 0}\times\mathbb{R}^{n_p}\to\mathbb{R}^{n_u\times n_\theta}$ is a regressor. As the whole state x_p is available for feedback, y_p serves as a performance output.

Standing assumptions on the model (1), customary in adaptive control [7], are set as follows:

Assumption 1: The matrices A_p, B_p, C_p are constant and known. The regressor $\varphi_p(t,x_p)$ is a known piecewise continuous function of t, and a locally Lipschitz function in x_p, uniformly in t. The control effectiveness matrix $\Lambda=\operatorname{diag}(\lambda)$ is comprised of unknown constant entries ranging over a compact set, $\lambda\in\mathcal{X}_\lambda\subset\mathbb{R}^{n_u}$. It is assumed that there exists a known compact hypercube $\Theta_\lambda\subset\mathbb{R}^{n_u}$ such that $\Theta_\lambda\supset\mathcal{X}_\lambda$, and $\lambda_i\ge\lambda_{\min}$, $i=1,\dots,n_u$, for some $\lambda_{\min}>0$ and all $\lambda\in\Theta_\lambda$. The entries of θ_p are constant and unknown, but range over a compact set $\mathcal{X}_\theta\subset\mathbb{R}^{n_\theta}$ contained in a known compact convex set $\Theta_\theta\supset\mathcal{X}_\theta$.[1] The nominal plant model is defined as system (1) with $\Lambda=I_{n_u}$ and $\theta_p=0$.

The point of departure is the availability for the nominal plant model of a dynamic *baseline controller* of the form

$$\Sigma_c:\begin{cases}\dot{x}_c=A_cx_c+B_{cy}y_p+B_{cr}r, & x_c(t_0)\in\mathbb{R}^{n_c}\\ u_b=C_cx_c+D_{cy}y_p+D_{cr}r\end{cases}\tag{2}$$

where $x_c\in\mathbb{R}^{n_c}$ is the controller state, $u_b\in\mathbb{R}^{n_u}$ the controller output, and $r(\cdot):\mathbb{R}_{\ge 0}\to\mathbb{R}^{n_r}$ is a piecewise-continuous reference signal. The matrices $A_c,B_{cy},B_{cr},C_c,D_{cy},D_{cr}$ are constant and have suitable dimension. When $\Lambda=I_{n_u}$ and $\theta=0$, the feedback interconnection $u=u_b$ of the plant (1) with the controller (2), that is, the system

$$\begin{bmatrix}\dot{x}_p\\ \dot{x}_c\end{bmatrix}=\underbrace{\begin{bmatrix}A_p+B_pD_{cy}C_p & B_pC_c\\ B_{cy}C_p & A_c\end{bmatrix}}_{A_m}\begin{bmatrix}x_p\\ x_c\end{bmatrix}+\underbrace{\begin{bmatrix}B_pD_{cr}\\ B_{cr}\end{bmatrix}}_{B_m}r,$$
$$y_p=\underbrace{\begin{bmatrix}C_p & 0\end{bmatrix}}_{C_m}\begin{bmatrix}x_p\\ x_c\end{bmatrix}\tag{3}$$

takes the role of a reference model

$$\Sigma_m:\begin{cases}\dot{x}_m=A_mx_m+B_mr, & x_m(t_0)\in\mathbb{R}^{n_p+n_c}\\ y_m=C_mx_m\end{cases}\tag{4}$$

with state $x_m:=(x_{pm},x_{cm})\in\mathbb{R}^{n_p+n_c}$. The baseline controller is designed to ensure internal stability and nominal performance guarantees when applied to the nominal plant model:

Assumption 2: System (4) is internally stable. Furthermore, letting $y_r(t):=C_rr(t)\in\mathbb{R}^{n_y}$ be a reference trajectory for y_m, there exists[2] $\kappa>0$ such that the *reference model tracking error* $e_r:=y_m-y_r$ satisfies $\|e_r(\cdot)\|_a\le\kappa\|r(\cdot)\|_a$ for all $r(\cdot)\in\mathcal{L}_{\infty,e}$.

System (4) defines the desired dynamic behavior of the uncertain plant (1), to be attained by a control policy to be defined. Consequently, define the *model mismatch error* e_m as

$$e_m:=x-x_m,\tag{5}$$

where $x:=(x_p,x_c)\in\mathbb{R}^{n_p+n_c}$ is the augmented state of the uncertain plant (1) and the baseline controller (2). The problem addressed in this work is then formally stated as follows:

Problem 1: Given the uncertain plant model (1) and the baseline controller (2), find a dynamic feedback controller

$$\Sigma_a:\begin{cases}\dot{x}_a=f_a(t,x_a,x_p,u_b), & x_a(t_0)\in\mathcal{X}_a\\ u_a=h_a(x_a,x_p,u_b)\end{cases}\tag{6}$$

and a set of initial conditions $\mathcal{X}_a\subset\mathbb{R}^{n_a}$ such that after setting $u=u_a+u_b$, all forward solutions $(x(t),x_a(t))$, $t\ge t_0$, of (1), (2) and (6) originating from initial conditions $(x(t_0),x_a(t_0))\in\mathbb{R}^{n_p+n_c}\times\mathcal{X}_a$, $t_0\ge 0$, are bounded and satisfy $\|e_m(\cdot)\|_a=0$.

Remark 1: Problem 1 readily falls within the paradigm of Model Reference Adaptive Control (MRAC), in particular to the class of the so-called *control augmentation* approaches [7]. It is worth noting, however, that in the particular formulation considered herein, the controller (6) does not require knowledge of the state of the baseline controller (2). As shown in the sequel, this feature is advantageous over standard solutions. ⌟

III. FULL ADAPTIVE AUGMENTATION

Following [7, Chapter 10], we introduce an established solution addressing the MRAC augmentation problem, which relies on the knowledge of both the baseline controller structure and its states for its implementation. Let u be selected as $u=u_a+u_b$, and collect the plant parameters into the vector $\theta_a=(\lambda,\theta_p)\in\mathcal{X}_\lambda\times\mathcal{X}_\theta$. Correspondingly, introduce the parameter estimates $\hat{\theta}_a=(\hat{\lambda},\hat{\theta}_p)\in\mathbb{R}^{n_\theta+n_u}$, and rewrite (1) as

$$\dot{x}_p=A_px_p+B_p\big(\hat{\Lambda}u+\varphi_p(t,x_p)\hat{\theta}_p\big)+B_p\varphi_a(t,x_p,u)e_\theta\tag{7}$$

where $\varphi_a(t,x_p,u):=\big[\operatorname{diag}(u)\ \ \varphi_p(t,x_p)\big]$, $\hat{\Lambda}:=\operatorname{diag}(\hat{\lambda})$, and $e_\theta:=\theta_a-\hat{\theta}_a$ denotes the parameter estimation error. The adaptive control law is designed according to the robust MRAC paradigm

$$\dot{\hat{\theta}}_a=\operatorname*{Proj}_{\hat{\theta}_a\in\Theta_a}\big(\hat{\theta}_a,\Gamma_a\varphi_a(t,x_p,u)^\top B_a^\top Pe_m\big),\quad \hat{\theta}_a(t_0)\in\Theta_a\tag{8}$$
$$u_a=(\hat{\Lambda}^{-1}-I_{n_u})u_b-\hat{\Lambda}^{-1}\varphi_p(t,x_p)\hat{\theta}_p\tag{9}$$

where $P=P^\top>0$ is the unique solution of the Lyapunov equation $PA_m+A_m^\top P=-Q$ for a given $Q=Q^\top>0$, $B_a:=\begin{bmatrix}B_p\\ 0_{n_c\times n_u}\end{bmatrix}$, $\Gamma_a\in\mathbb{R}^{(n_u+n_\theta)\times(n_u+n_\theta)}$ is a positive definite gain matrix, and $\operatorname{Proj}(\cdot,\cdot)$ is a locally Lipschitz projection operator [p. 329][7], which constrains the evolution of $\hat{\theta}_a(t)$ within the compact and convex set $\Theta_a:=\Theta_\lambda\times\Theta_\theta\supset\mathcal{X}_\lambda\times\mathcal{X}_\theta$.

The closed-loop error dynamics is written as the interconnection of (4) and the system

$$\begin{aligned}\dot{e}_m&=A_me_m+B_a\varphi_a(t,x_p,u)e_\theta\\ \dot{e}_\theta&=-\operatorname*{Proj}_{\hat{\theta}_a\in\Theta_a}\big(\hat{\theta}_a,\Gamma_a\varphi_a(t,x_p,u)^\top B_a^\top Pe_m\big)\end{aligned}\tag{10}$$

where, for notational convenience, we have kept the original coordinate $x_p=[I_{n_p}\,0_{n_c}](e_m+x_m)$ and the baseline control input $u_b=u_b(t,e_m,x_m)$ in the expression of the regressor. Unsurprisingly, the control law (8)–(9) provides the following classical result:

[1]In standard MRAC, knowledge of the bounds on uncertain parameters is not necessary. Conversely, robust implementations of MRAC using projection methods, as the one used in this work, do require knowledge of these bounds.

[2]The given nominal performance requirement does not exclude the possibility of asymptotic regulation for specific classes of reference signals.

Theorem 1: For any $r(\cdot) \in \mathcal{L}_\infty$, all forward solutions $x_m(t)$, $e_m(t), e_\theta(t)$, $t \geq t_0$, of the closed-loop system (4)–(10) originating from initial conditions $x_m(t_0) \in \mathbb{R}^{n_p+n_c}$, $e_m(t_0) \in \mathbb{R}^{n_p+n_c}$, $\hat{\theta}_a(t_0) \in \Theta_a$, $t_0 \geq 0$, are bounded and satisfy $\|e_m(\cdot)\|_a = 0$.

Proof: The proof is standard, but is sketched here for completeness and future use. Fix, arbitrarily, $t_0 \geq 0$, $r(\cdot) \in \mathcal{L}_\infty$, and initial conditions $x_m(t_0), e_m(t_0), \hat{\theta}_a(t_0)$ in the given sets. Denote with $\mathcal{I}_{\max} = [t_0, t_0 + T_{\max})$ the maximal interval of existence and uniqueness of the corresponding solution of (4)–(10). Owing to the properties of the projection operator, the Lie derivative of the Lyapunov function candidate $V(e_m, e_\theta) := e_m^\top P e_m + e_\theta^\top \Gamma_a^{-1} e_\theta$ along the vector field of (10) satisfies $\dot{V}(e_m, e_\theta) = -e_m^\top Q e_m \leq 0$. As the function $V(e_m(t), e_\theta(t))$ evaluated along the given solution of the closed-loop system is non-increasing over the interval $\mathcal{I}_{\max}$, it follows that $\|e_m(\cdot)\|_{\mathcal{I}_{\max}} < \infty$ and $\|e_\theta(\cdot)\|_{\mathcal{I}_{\max}} < \infty$ by positive definiteness of $V(e_m, e_\theta)$. As the reference model (4) is an internally stable system driven by a bounded signal, $\|x_m(\cdot)\|_{\mathcal{I}_{\max}} < \infty$ as well, which implies that $x(\cdot)$ and $u_b(\cdot)$ are bounded over $\mathcal{I}_{\max}$. As a consequence, $u_a(\cdot)$ is also bounded over $\mathcal{I}_{\max}$. Boundedness of all closed-loop signals over $\mathcal{I}_{\max}$ implies that $\mathcal{I}_{\max} = [t_0, +\infty)$. Convergence of $e_m(\cdot)$ then follows from application of LaSalle-Yoshizawa theorem [4, Thm. 8.4]. ∎

While the adaptive controller (8)–(9) meets the requirements of Problem 1, its implementation relies on a duplicate of the baseline controller driven by the nominal plant model, as the update law (8) depends on the reference model error e_m. This results in a controller of dimension $N_c = n_p + 2n_c + n_u + n_\theta$.

IV. Plant Augmentation with Predictor

Predictor- and observer-based adaptive augmentation schemes, which still rely on the knowledge of the baseline controller states and structure, have been devised in the literature to improve transient performance of the classic approach outlined in the previous section (see [2], [6]). Inspired by [1], a predictor-based strategy is proposed in this section to provide a more efficient solution to Problem 1. To this end, define the adaptive predictor of the plant dynamics (7)

$$\dot{\hat{x}}_p = A_p x_p + B_p\big(\hat{\Lambda} u + \varphi_p(t, x_p)\hat{\theta}_p\big) + L\hat{e}_p, \tag{11}$$

with initial condition $\hat{x}_p(t_0) \in \mathbb{R}^{n_p}$ and gain matrix $L = L^\top > 0$, and let the corresponding prediction error be

$$\hat{e}_p := x_p - \hat{x}_p. \tag{12}$$

When $\hat{e}_p \equiv 0$, the predictor dynamics (11) replicates the ideal behavior of the nominal plant, as it can be inferred by substituting u_a from (9) into (11):

$$\dot{\hat{x}}_p = A_p x_p + B_p u_b + L\hat{e}_p. \tag{13}$$

Owing to (13), the change of coordinates $x = (x_p, x_c) \mapsto \hat{x} := (\hat{x}_p, x_c)$ transforms the interconnection of the baseline controller (2) and the uncertain plant (1) into the system

$$\dot{\hat{x}} = A_m \hat{x} + B_m r + \tilde{B}_a \hat{e}_p, \quad \hat{x}(t_0) = (\hat{x}_p(t_0), x_c(t_0)) \tag{14}$$

where

$$\tilde{B}_a := \begin{bmatrix} A_p + B_p D_{cy} C_p + L \\ B_{cy} C_p \end{bmatrix}.$$

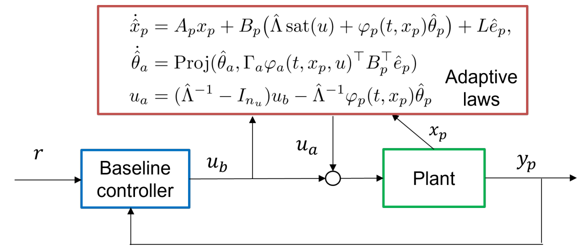

Fig. 1. Proposed augmentation scheme (anti-windup implementation).

Finally, consider the controller (9) with modified update law

$$\dot{\hat{\theta}}_a = \underset{\hat{\theta}_a \in \Theta_a}{\mathrm{Proj}}\,(\hat{\theta}_a, \Gamma_a \varphi_a(t, x_p, u)^\top B_p^\top \hat{e}_p), \quad \hat{\theta}_a(t_0) \in \Theta_a \tag{15}$$

and corresponding parameter estimation error $e_\theta := \theta_a - \hat{\theta}_a$. In preparation for the forthcoming stability analysis, let us introduce the predicted model mismatch error (compare with (5))

$$\hat{e}_m := \hat{x} - x_m \tag{16}$$

where $x_m \in \mathbb{R}^{n_p+n_c}$ is the state of the (now, fictitious) reference model (4) with same initial condition of (14), $x_m(t_0) = \hat{x}(t_0)$.

Remark 2: It is stressed that system (4) is now only used to define a desired behavior for the trajectories of (14), and it is not implemented in the overall controller, as opposed to the previous section. This is due to the fact that the update law (15) no longer depends on the model mismatch (5) or (16), but only on the prediction error (12). Consequently, the dimension of the controller has been reduced to $N_c' = n_p + n_c + n_u + n_\theta$. ⌟

Using the error coordinates $(\hat{e}_p, e_\theta, \hat{e}_m)$ in lieu of the original coordinates $(\hat{x}_p, \hat{\theta}_a, x)$, the complete closed-loop error dynamics is described by the feedback interconnection (see Figure 1)

$$\dot{\hat{e}}_p = -L\hat{e}_p + B_p \varphi_a(t, x_p, u) e_\theta \tag{17}$$

$$\dot{e}_\theta = -\underset{\hat{\theta}_a \in \Theta_a}{\mathrm{Proj}}\,(\hat{\theta}_a, \Gamma_a \varphi_a(t, x_p, u)^\top B_p^\top \hat{e}_p) \tag{18}$$

$$\dot{\hat{e}}_m = A_m \hat{e}_m + \tilde{B}_a \hat{e}_p, \tag{19}$$

driven by the reference model (4). Once again, we have kept the original coordinate $x_p = [I_{n_p}\ 0_{n_c}](\hat{e}_m + x_m) + \hat{e}_p$ and the baseline control input $u_b = u_b(t, \hat{e}_m, x_m)$ in the expression of the regressor $\varphi_a(\cdot)$. The main result is stated next.

Theorem 2: The controller (9),(11),(15) solves Problem 1.

Proof: We begin by noticing that system (9)–(11)–(15) has indeed the structure of (6), with $x_a := (\hat{x}_p, \hat{\theta}_a)$. The proof then follows from arguments similar to those in Theorem 1. Specifically, for arbitrary initial conditions $(\hat{e}_p(t_0), \hat{\theta}_a(t_0), \hat{e}_m(t_0), x_m(t_0)) \in \mathbb{R}^{n_p} \times \Theta_a \times \mathbb{R}^{n_p+n_c} \times \mathbb{R}^{n_p+n_c}$, $t_0 \geq 0$, and arbitrary reference signals $r(\cdot) \in \mathcal{L}_\infty$, the forward trajectory $(\hat{e}_p(t), e_\theta(t))$ is bounded over its maximal interval of existence and uniqueness $\mathcal{I}_{\max} = [t_0, t_0 + T_{\max})$ owing to the fact that the Lie derivative of $V(\hat{e}_p, e_\theta) := \hat{e}_p^\top \hat{e}_p + e_\theta^\top \Gamma_a^{-1} e_\theta$ satisfies $\dot{V}(\hat{e}_p, e_\theta) = -\hat{e}_p^\top L \hat{e}_p \leq 0$. Note that the $\hat{e}_m$-subsystem (19) is input-to-state stable (ISS) with respect to the input $\hat{e}_p(\cdot)$, hence the forward trajectory $\hat{e}_m(t)$ satisfies $\|\hat{e}_m(\cdot)\|_{\mathcal{I}_{\max}} < \infty$. Since $\|x_m(\cdot)\|_{\mathcal{I}_{\max}} < \infty$ as well, all closed-loop signals are bounded and $T_{\max} = +\infty$. Convergence of both $\hat{e}_p(t)$ and $\hat{e}_m(t)$ is established by LaSalle-Yoshizawa theorem [4, Thm. 8.4], and by the converging input–converging state (CICS) property of ISS systems, respectively. Since $e_m = x - x_m = x - \hat{x} + \hat{x} - x_m = (\hat{e}_p, 0) + \hat{e}_m$, it follows that $\|e_m(\cdot)\|_a \leq \|\hat{e}_p(\cdot)\|_a + \|\hat{e}_m(\cdot)\|_a = 0$, as required. ∎

A. Anti-windup Property in Presence of Saturating Inputs

Apart from yielding a reduction of the overall controller order, the proposed predictor-based design is beneficial in dealing with saturating inputs, as it provides an anti-windup mechanism similar to that proposed originally in [3]. To see why this is the case, let the control input in (1) be subject to saturation, namely let the plant dynamics be replaced with

$$\dot{x}_p = A_p x_p + B_p\big(\Lambda \operatorname{sat}(u) + \varphi_p(t, x_p)\theta_p\big), \; x_p(t_0) \in \mathbb{R}^{n_p} \quad (20)$$

The corresponding adaptive predictor becomes

$$\dot{\hat{x}}_p = A_p x_p + B_p\big(\hat{\Lambda} \operatorname{sat}(u) + \varphi_p(t, x_p)\hat{\theta}_p\big) + L\hat{e}_p, \quad (21)$$

which, upon substitution of the control law $u = u_b + u_a$, with u_b as in (2) and u_a given in (9), yields

$$\dot{\hat{x}}_p = A_p x_p + B_p u_b + L\hat{e}_p - B_p \hat{\Lambda} \operatorname{dz}(u) \quad (22)$$

where $\operatorname{dz}(\cdot) : u \mapsto u - \operatorname{sat}(u)$ is the dead-zone function corresponding to $\operatorname{sat}(\cdot)$. At this point, the design proceeds exactly along the exact same lines presented in this section, with the regressor $\varphi_a(t, x_p, \operatorname{sat}(u))$ replacing the one employed in the update law (15). This leads to the closed-loop error dynamics

$$\dot{\hat{e}}_p = -L\hat{e}_p + B_p \varphi_a(t, x_p, \operatorname{sat}(u)) e_\theta \quad (23)$$

$$\dot{e}_\theta = -\operatorname*{Proj}_{\hat{\theta}_a \in \Theta_a}\big(\hat{\theta}_a, \Gamma_a \varphi_a(t, x_p, \operatorname{sat}(u))^\top B_p^\top \hat{e}_p\big) \quad (24)$$

$$\dot{\hat{e}}_m = A_m \hat{e}_m + \tilde{B}_a \hat{e}_p - \tilde{B}_a \hat{\Lambda} \operatorname{dz}(u). \quad (25)$$

It is apparent that in the adaptation loop (23)–(24) the regressor contains the correct information about the behavior of the control input, hence the adaptation mechanism does not react abnormally to the occurrence of saturation. The following result follows directly from the proof of Theorem 2:

Proposition 1: The modified predictor-based adaptive controller ensures the following properties:

1) For all initial conditions $(\hat{e}_p(t_0), \hat{\theta}_a(t_0), \hat{e}_m(t_0), x_m(t_0)) \in \mathbb{R}^{n_p} \times \Theta_a \times \mathbb{R}^{n_p+n_c} \times \mathbb{R}^{n_p+n_c}$, $t_0 \geq 0$, and reference signals $r(\cdot) \in \mathcal{L}_\infty$ such that the forward trajectory $\hat{e}_m(t)$ is bounded, the trajectory $(\hat{e}_p(t), \hat{\theta}_a(t))$, $t \geq t_0$, is bounded and satisfies $\|\hat{e}_p(\cdot)\|_a = 0$.
2) Under the same assumptions as in 1), $\|\operatorname{dz}(u(\cdot))\|_a = 0$ implies $\|\hat{e}_m(\cdot)\|_a = 0$, hence $\|e_m(\cdot)\|_a = 0$.

V. Application to Hierarchical Control Design

In this section, we address a particular instance of the dynamical system (1), which often arises when implementing hierarchical control strategies, such as inner-outer loop control laws. Such a strategy is particularly relevant when dealing with kinodynamic systems, where parametric uncertainties lie entirely within the dynamic part and the control design complexity can be reduced by focusing only on this part of the system for the development of adaptive laws. Specifically, we refer to the class of systems described by

$$\dot{p} = v, \qquad p(t_0) \in \mathbb{R}^{n_p} \quad (26)$$

$$\dot{v} = A_p v + B_p(\Lambda u + \varphi_p(t, v)\theta_p), \qquad v(t_0) \in \mathbb{R}^{n_p} \quad (27)$$

with state $x_p = (p, v) \in \mathbb{R}^{2n_p}$ representing position and velocity, respectively. The objective is to ensure that the state of the system asymptotically tracks a given a desired continuously differentiable trajectory $t \mapsto x_d(t) := (p_d(t), v_d(t))$, where $v_d(t) := \dot{p}_d(t)$. For future use, we let $a_d(t) := \dot{v}_d(t)$ denote the desired acceleration, and define $r_d := (v_d, a_d)$.

Assumption 3: The reference signal $r_d(\cdot)$ is bounded.

The trajectory tracking problem is addressed via a hierarchical approach based on an inner-outer loop control paradigm. At the outer loop level (26), v is assumed to be a control variable, and a control law is developed to make the tracking error $e_p := p - p_d$ asymptotically vanish. Then, at the inner loop level (27), the plant input u is designed following the predictor-based design proposed in Section IV to make the actual velocity track the reference v_c provided by the outer loop, while compensating for the parametric uncertainty. Differently from backstepping approaches, we do not resort on the construction of a Lyapunov function for the whole system, thus avoiding the explosion of terms typical of iterative procedures. Closed-loop stability is achieved by requiring the individual controllers to possess certain structural properties detailed in the sequel.

Outer-loop Design: The outer loop design is in charge of computing a virtual velocity v_c that lets $p(t)$ track $p_d(t)$. Given (26), we introduce the virtual input v_c to obtain

$$\dot{e}_p = v_c - v_d + e_v, \quad (28)$$

where $e_v := v - v_c$ is the virtual velocity error. Due to linearity of (28), a host of linear techniques can be employed to achieve the tracking control objective. However, more elaborate solutions can be envisioned to enforce additional performance requirements, for example to limit the commanded velocity when operating far from the nominal trajectory. To this end, let v_c be defined as

$$v_c := -k(e_p) + v_d, \quad (29)$$

where $k(\cdot) : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$ is a smooth control law.

Inner-loop Design: Select the structure of the *baseline controller* as in (2), with $y_p = v$ and reference input $r_c := (v_c, a_c)$, where the virtual acceleration command

$$a_c := \dot{v}_c = k'(e_p)k(e_p) - k'(e_p)e_v + a_d \quad (30)$$

has been included in the command r_c to improve tracking performance. In (30), $k'(\cdot) : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p \times n_p}$ denotes the Jacobian of $k(\cdot)$. The *reference model* is defined as system (4) with state $x_m := (v_m, x_{cm})$, and reference input defined as

$$r := (v_c, a_{cm}), \quad a_{cm} := k'(e_p)k(e_p) - k'(e_p)e_r + a_d \quad (31)$$

where e_r denotes the reference model tracking error $e_r := y_m - y_r = v_m - v_c$ (that is, $v_c = C_r r$ in Assumption 2). Let $\delta :=$

$e_v - e_r$, and consider the interconnection of the outer-loop and the reference model

$$\begin{aligned} \dot{e}_p &= -k(e_p) + e_r + \delta, & e_p(t_0) &\in \mathbb{R}^{n_p} \\ \dot{x}_m &= A_m x_m + B_m r + \bar{B}_m(e_p)\delta, & x_m(t_0) &\in \mathbb{R}^{n_p+n_c} \end{aligned} \tag{32}$$

where δ is regarded as a disturbance, and the term $\bar{B}_m(e_p)\delta$ is due to the difference between a_c and a_{cm}.

The next assumption characterizes the class of control laws $k(\cdot)$ and baseline controllers employed in the design:

Assumption 4: For all initial conditions $(e_p(t_0), x_m(t_0)) \in \mathbb{R}^{n_p} \times \mathbb{R}^{n_p+n_c}$, all reference signals $r_d(\cdot) \in \mathcal{L}_\infty$ and all $\delta(\cdot) \in \mathcal{L}_{\infty,e}$, the forward trajectories $(e_p(t), x_m(t))$, $t \geq t_0$, of (32) are forward complete and satisfy:

(a) $\|x_m(\cdot)\|_{[t_0,\tau)} \leq \mu_0 + \mu_1\|\delta(\cdot)\|_{[t_0,\tau)}$ for some constants $\mu_0, \mu_1 > 0$ and all $\tau \geq t_0$.

(b) There exist constants $\mu_3 > 0$, $\mu_4 \geq 0$ such that $\|\delta(\cdot)\|_a = 0$ and $\|r_d(\cdot)\|_a \leq \mu_3$ implies $\|e_p(\cdot)\|_a \leq \mu_4\|r_d(\cdot)\|_a$.

The above assumption encompasses several outer-loop designs, from linear to saturated stabilizers with no (or only slight) restrictions on the inner loop baseline controller beyond the requirements stated in Assumption 2.

For the adaptive controller, the predictor is defined as

$$\dot{\hat{v}} = A_p v + B_p u_b + L\hat{e}_v \tag{33}$$

where $\hat{e}_v := v - \hat{v}$ is the prediction error, whereas the adaptive controller is given by (9)–(15) with $x_p = v$. To keep consistency with the previous analysis, we define the following variables for the interconnection of the velocity subsystem (and its predictor) with the baseline controller

$$x := (v, x_c), \quad e_m := x - x_m, \quad \hat{x} := (\hat{v}, x_c), \quad \hat{e}_m := \hat{x} - x_m \tag{34}$$

where the latter is further specialized for the predictor as

$$\tilde{e}_v := \hat{v} - v_m \tag{35}$$

Consequently, the tracking error e_v is decomposed as follows:

$$e_v = v - v_c = v - \hat{v} + \hat{v} - v_m + v_m - v_c = \hat{e}_v + \tilde{e}_v + e_r \tag{36}$$

from which it follows that $\delta = \hat{e}_v + \tilde{e}_v$ in system (32).

Stability Analysis: Since system (32) has the required stability properties when $\delta(t)$ vanishes, we look at the error system

$$\dot{\hat{e}}_v = -L\hat{e}_p + B_p\varphi_a(t, v, u)e_\theta \tag{37}$$

$$\dot{e}_\theta = -\operatorname*{Proj}_{\hat{\theta}_a \in \Theta_a}(\hat{\theta}_a, \Gamma_a \varphi_a(t, v, u)^\top B_p^\top \hat{e}_v) \tag{38}$$

$$\dot{\hat{e}}_m = A_m\hat{e}_m + \tilde{B}_a\hat{e}_v, \tag{39}$$

which is driven by the outer loop and the reference model (4) via $v = [I_{n_p}\, 0_{n_c}](\hat{e}_m + x_m) + \hat{e}_v$ and $u_b = u_b(t, \hat{e}_m, x_m)$. The goal is to prove that all trajectories of (37)–(38)–(39) are bounded, and satisfy $\|\hat{e}_v(\cdot)\|_a = \|\hat{e}_m(\cdot)\|_a = 0$, so that $\|\delta(\cdot)\|_a = 0$. To this end, note that by Assumption 4, the forward solutions of (32) exist as long as the solutions of (37)–(38)–(39), hence $\delta(t)$, exist. Following the proof of Theorem 2, the forward trajectory $(\hat{e}_v(t), e_\theta(t), \hat{e}_m(t))$ is bounded over the maximal interval of existence, $\mathcal{I}_{\max}$, of the closed-loop system. Owing to Assumption 4.(a), $x_m(\cdot)$ is bounded over $\mathcal{I}_{\max}$ (notice that $|\delta| = |\hat{e}_v + \tilde{e}_v| \leq |\hat{e}_v| + |\hat{e}_m|$), hence $\mathcal{I}_{\max} = [t_0, +\infty)$, and convergence of $\|\hat{e}_v(\cdot)\|_a$ and $\|\hat{e}_m(\cdot)\|_a$ follows by application of LaSalle-Yoshizawa theorem.

VI. Example: Control of a Multirotor UAV

Consider the position dynamics of a multirotor UAV at moderate speed and small angles, which can be described by

$$\dot{p} = v, \quad \dot{v} = ge_3 + \tfrac{1}{m}(\lambda \operatorname{sat}(u) - \varphi_v(v)\theta_v) \tag{40}$$

where $\varphi_v(v) = \operatorname{diag}(v)$ and $\theta_v := (C_{\ell,11}, C_{\ell,22}, C_{\ell,33})$ is a vector of uncertain aerodynamic parameters, $\lambda > 0$ is the thrust efficiency, and $\operatorname{sat}(u)$ takes into account actuator limits. To ensure (global) asymptotic tracking of a desired position trajectory $p_d(t)$, $t \geq 0$, for the nominal plant model, the following P/PI baseline controller is employed:

$$\dot{x}_i = v - v_c, \quad u_b = -K_v(v - v_c) - K_i x_i + m(\dot{v}_c - ge_3). \tag{41}$$

The velocity command is obtained from the saturated outer-loop proportional controller

$$v_c = -v_M \sigma\left(\frac{K_p}{v_M}(p - p_d)\right) + v_d. \tag{42}$$

where $\sigma(\cdot)$ is a vector whose components are $\sigma_i(\cdot) = \tanh(\cdot)$. The predictor-based controller

$$\begin{aligned} \dot{\hat{v}} &= ge_3 + \frac{1}{m}\left(\hat{\lambda}\operatorname{sat}(u) - \varphi_v(v)\hat{\theta}_v\right) + L\hat{e} \\ \dot{\hat{\theta}}_a &= \operatorname*{Proj}_{\hat{\theta}_a \in \Theta_a}\left(\hat{\theta}_a, \frac{\Gamma_{\theta_a}}{m}\varphi_a(v, u_b)^\top \hat{e}_v\right) \end{aligned} \tag{43}$$

where $\varphi_a(v, u_b) := [u_b\ \varphi_v(v)] \in \mathbb{R}^{3\times 4}$, aims at compensating for the parametric model uncertainties. The closed-loop system is thus recast into the form studied in the previous section.

We compare our results with an augmentation version of the predictor-based MRAC design (P-MRAC) proposed in [6]:

$$\begin{aligned} \dot{\hat{x}} &= A_m x + B_m r + L_p\hat{e}, \quad \dot{x}_m = A_m x_m + B_m r \\ \dot{\hat{\theta}}_a &= \operatorname*{Proj}_{\hat{\theta}_a \in \Theta_a}\left(\hat{\theta}_a, \frac{\Gamma_{\theta_a}}{m}\varphi_a(v, u_b)B_a^\top(P_p\hat{e} + P_m e_m)\right) \end{aligned} \tag{44}$$

where $x = (v, x_i) \in \mathbb{R}^6$, $\hat{e} := x - \hat{x}$, $e_m := x - x_m$, $A_m, B_m \in \mathbb{R}^{6\times 6}$ are the closed-loop reference model matrices (dependent on the baseline controller and nominal plant matrices), $L_p \in \mathbb{R}^{6\times 6}$ is the predictor gain matrix, while $P_p = P_p^\top > 0$ and $P_m = P_m^\top > 0$ must satisfy $A_p^\top P_p + P_p A_p = -Q_p$ and $A_m^\top P_m + P_m A_m = -Q_m$ for some $Q_m = Q_m^\top > 0$ and $Q_p = Q_p^\top > 0$. Beyond requiring knowledge of the baseline controller structure and states, the controller (44) require the integration of 16 differential equations and the tuning of $\Gamma_{\theta_a} \in \mathbb{R}^{4\times 4}$ and three 6×6 matrices (L_p, Q_p, Q_m), for which no systematic method exists.

Simulation results consider a small-scale UAV with mass $m = 0.3$kg, aerodynamic coefficients $C_{\ell,11} = C_{\ell,22} = 0.05$, $C_{\ell,33} = 0.1$kg/s, maximum thrust $M = 6N$ (roughly double the weight) that is commanded to track a circular reference trajectory in the xy plane at 2m of altitude starting from a hovering position at the origin. From time $t = 0$s to $t = 30$s, the reference is a circle of radius 1m traveled with angular frequency $\Omega = 1$rad/s, which corresponds to a 1m/s tangent velocity. From time $t = 30$s onward, the radius of the circle is increased to 5m, which corresponds to a 5m/s tangent velocity. Finally, at time $t = 60$s a sudden drop of 60% in control efficiency is considered ($\lambda = 1$ for $t \leq 60$s, $\lambda = 0.6$ for

$t > 60$s), mimicking a failure in the UAV propulsive system. The baseline controller gains are selected as $(K_p, K_v, K_i) = (I_3, \frac{2}{3}I_3, \frac{1}{4}I_3)$ with the maximum speed $v_M := 5$m/s. We manually tuned the gains of both the proposed adaptive controller (43) and the P-MRAC (44) to achieve a similar response and avoid fast oscillations in the control signal. The following values are used: $L_v = 10$ and $\Gamma_{\theta_a} = 1I_4$ for (43), whereas $\Gamma_{\theta_a} = 0.7I_4$, $Q_m = 0.1I_6$, $L_p = -5A_m$, and $Q_p = 5Q_n$ for (44).

The results obtained with the proposed design outperform those of the baseline controller alone in terms of tracking performance, as it is particularly evident when the reference speed is increased at time $t = 30$s (larger circle in Figure 2). One notable feature of the proposed design is that it does not apply adaptation when not necessary. Figure 3 (top) shows that the parameter estimates $\hat{\lambda}$ and $\hat{C}_{\ell,3}$, when initialized at their exact values, remain unchanged until the fault occurs at time $t = 60s$. This stands in contrast to the P-MRAC, whose parameters react both at the start of the mission and when the reference changes (see Figure 4). While the settling time of the reference model mismatch is similar for both controllers, a larger overshoot is noticed for the P-MRAC (see Figure 3 and Figure 4 (bottom plot)). Due to windup effects (see the input norm in Figure 5), the P-MRAC fails to stabilize the system when the efficiency drop at 60% of its nominal value (see the norm of the reference model error diverging in Figure 4).

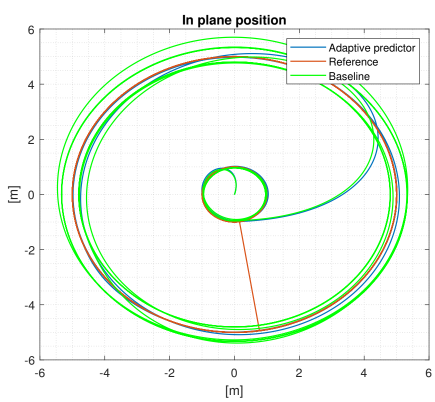

Fig. 2. In plane position, $p(t)$, vs desired trajectory, $p_d(t)$.

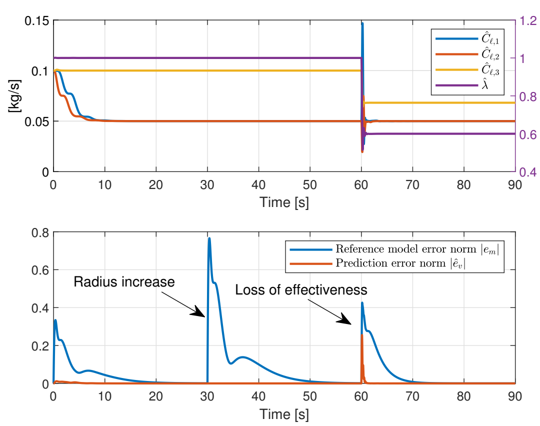

Fig. 3. Proposed controller (43). Parameter estimates, $\hat{\theta}_a(t)$ (top); Norms of the reference model and prediction error, $e_m(t)$, $\hat{e}_v(t)$ (bottom).

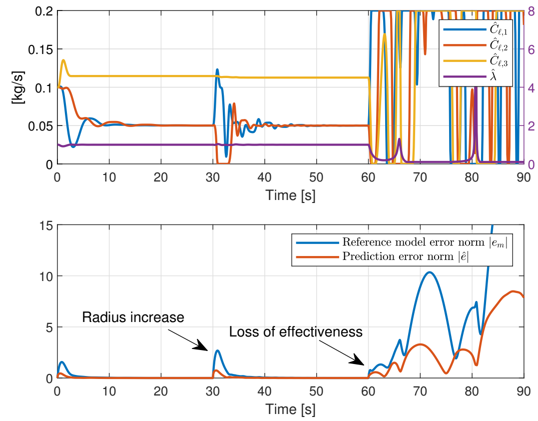

Fig. 4. P-MRAC. Parameter estimates, $\hat{\theta}_a(t)$ (top); Norms of the reference model and prediction error, $e_m(t)$, $\hat{e}_v(t)$ (bottom).

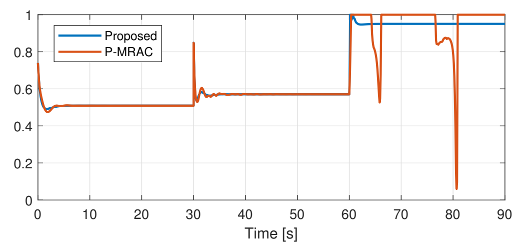

Fig. 5. Comparison of the normalized norm of the control input, $|u|/M$.

VII. Conclusions

This paper presents a novel predictor-based plant augmentation scheme that seamlessly integrates into an existing baseline linear control architecture using only the baseline controller output. The approach maintains low complexity, provides an anti-windup effect, and can be exploited in hierarchical control architectures. Simulation results show its efficacy in improving performance over full prediction-based MRAC schemes.

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