Rendiconti Lincei. Scienze Fisiche e Naturali (2024) 35:351–358
https://doi.org/10.1007/s12210-024-01234-z

GeoMed2,the geoid of the Mediterranean: work in progress

Riccardo Barzaghi[1] · Daniela Carrion[1]

Received: 3 December 2023 / Accepted: 27 February 2024 / Published online: 18 March 2024

Abstract

Geodesy can provide valuable information on marine current estimation based on the combination of gravity and altimetry. Gravity is standardly used to estimate the geoid undulation, i.e. the height of the geoid over a given reference ellipsoid. As it is well known, the geoid undulation over the oceans is closely related to the Mean Sea Surface (MSS) with discrepancies that can reach 1–2 m at global scale. By satellite altimetry, one can get the MSS and then estimate the Mean Dynamic Topography (MDT) as the difference between the MSS and the geoid undulation. As the MDT is related to the ocean circulation, information on the ocean circulation to be compared with oceanographic estimates can be provided using these geodetic measurements. In this context, the GeoMed2 project aims at estimating a high-accuracy and high-resolution geoid model for the Mediterranean Sea based on land and marine gravity data and on recent Global Geopotential Models. In this paper, the processing methodology based on the well-known remove–compute–restore approach for the determination of the geoid in the Mediterranean area is presented. In a pre-processing step, all available gravity observations for the wider Mediterranean basin have been collected, validated, homogenized, and unified in terms of their horizontal and gravity system. In this way, a reliable gravity database to be used for the determination of the geoid has been prepared. This data set has been used in computing a gravimetric geoid estimate based on which the MDT over the Mediterranean Sea was obtained. The results of this computation were then revised, commented and compared with other existing MDT solutions. By these comparisons, it can be concluded that the geodetic computed MDT is not yet satisfactory since it is too noisy. This is possibly due to some inconsistencies still present in the gravity data used for estimating the geoid undulation and to the adopted MSS which seems to be too smooth over the Mediterranean area.

Keywords Geoid · Mediterranean Sea · Mean Sea Surface · Mean Dynamic Topography · Altimetry

1 Introduction

The geoid is the so-called mathematical surface of the Earth (Heiskanen and Moritz 1967). By definition, it is the equipotential surface of the Earth gravity field that can be approximated by the Mean Sea Level (MSL). Nowadays, the geoid is estimated using gravity data that are collected on ground or via airborne/marine observation campaigns. Over the ocean, gravity data can be obtained from altimetry data that are densely known. Furthermore, satellite dedicated missions have been performed in the last decades, e.g. the GOCE gradiometric mission (Drinkwater et al. 2003), that have remarkably improved the knowledge of the long-wavelength components of the Earth gravity field and of the geoid.

Global estimates of the geoid are provided either from satellite data only or in combination with ground-based data. Satellite-based global models are provided in spherical harmonic expansion up to degree and order (d/o) 300 while high-frequency models can be at d/o 2190.

Methods for estimating the gravimetric geoid, i.e. the geoid estimated from the gravity data, have been refined in the years. Particularly, the remove–compute–restore approach (Barzaghi 2016) has been proposed and applied for estimating high-precision geoid undulations. This approach is applied as a standard in combination with the Stokes formula (Heiskanen and Moritz 1967; Sansò and

This peer-reviewed paper belongs to the Topical Collection motivated by the Conference "The Mediterranean System: a Hotspot for Climate Change and Adaptation" organized in Rome at the Accademia Nazionale dei Lincei on March 21-22, 2023

✉ Riccardo Barzaghi
riccardo.barzaghi@polimi.it

[1] DICA, Politecnico di Milano, Piazza L. da Vinci 32, 20133 Milan, Italy

Sideris 2013) or with the collocation method (Tscherning 2015; Sansò and Sideris 2013).

The geoid undulations are commonly used for getting the orthometric height from Global Navigation Satellite System (GNSS) observations. As it is well known, using GNSS the ellipsoidal heights h are obtained relative to a given ellipsoid. So, in all the applications where the orthometric heights H are needed, a conversion is required. This is based on the knowledge of the geoid undulation N that is related to the ellipsoidal height h and to the orthometric height H by the simple formula $h = H + N$. Thus, by knowing h and the undulation N, one can estimate H in the so-called GNSS/levelling. This is the method that is applied in many nations all over the world (e.g. Smith et al. 2013) replacing, in technical applications, the time consuming and expensive spirit leveling.

Over the ocean, precise geoid estimates are required to estimate the Mean Dynamic Topography (MDT) that is related to the ocean circulation. This is done in combination with altimetry that provides the instantaneous Sea Surface Height (SSH) with respect to the ellipsoid. In this context, several projects have been carried out all over the world (e.g. Rio and Mulet 2014; Jahanmard et al. 2022). In this paper, the GeoMed2 project, which is dedicated to the geoid estimation in the Mediterranean area, is introduced. GeoMed2 is the continuation of the GeoMed project that has been carried out in the years 1991–1994. GeoMed2 is an international project that started in 2015. A first estimate has been computed in 2018 (Barzaghi et al. 2018) and new computations are foreseen in 2024. In this paper, a review on geoid computation is given, the GeoMed2 results are revised and the future perspective for the geoid computation in the Mediterranean Sea are described.

2 The geoid estimate based on gravity data and its application

Nowadays, the geoid estimate is usually based on gravity data by solving the geodetic boundary value problem. Analytically, this means that we have to estimate the anomalous potential $T(P)$, harmonic outside the masses, given the gravity anomalies that are observed on the Earth surface. By reducing the gravity data for the topographic masses, one can state the so-called Stokes problem, i.e. to estimate the harmonic function $T(P)$ outside the geoid, regular at infinity, given the boundary condition (here in its spherical approximation)

$$-\frac{\partial T}{\partial r} - 2\frac{T}{r}\bigg|_{\text{mean Earth sphere}} = \Delta g, \qquad (1)$$

where Δg is the gravity anomaly and r is the spherical radial position.

The solution of this boundary value problem is the well-known Stokes' formula

$$T(P) = \frac{R}{4\pi}\int_{\sigma} S(\psi_{PQ})d\sigma_Q \qquad (2)$$

being $S(\psi_{PQ})$ the Stokes' function (see, e.g. Heiskanen and Moritz 1967) and σ the unit sphere.

Once the anomalous potential $T(P)$ is given, the geoid undulation $N(P)$ can be obtained according to the Bruns' formula (Heiskanen and Moritz 1967) as

$$N(P) = \frac{T(P)}{\gamma(Q)}, \qquad (3)$$

where $\gamma(Q)$ is the absolute value of the normal gravity field onto the ellipsoid (that is completely known in analytical form, see, e.g. Moritz 1989).

More rigorously, although with a much larger numerical effort, one can solve the Molodensky problem for the estimation of the anomalous potential $T(P)$ given the gravity data directly on the Earth surface itself (the so-called free-air gravity anomaly). As an example, one can consider the simple Molodensky problem, i.e. to find the potential $T(P)$, harmonic outside the telluroid S (Heiskanen and Moritz 1967), regular at infinity, satisfying the boundary condition

$$-\frac{\partial T}{\partial r} - 2\frac{T}{r}\bigg|_{S} = \Delta g, \qquad (4)$$

where Δg is the free-air anomaly (Heiskanen and Moritz 1967).

Once the potential is known, one can get the height anomaly $\zeta(P)$ as

$$\zeta(P) = \frac{T(P)}{\gamma(Q)}. \qquad (5)$$

Another standard method for solving these geodetic boundary value problems is collocation. This is a method based on the stochastic hypothesis for the anomalous potential $T(P)$ and the Wiener–Kolmogoroff principle (Moritz 1989). Having observed the values of linear functionals of the anomalous potential $L_{P_i}(T)$ at points P_i, the collocation estimate of any other functional $L_P(T)$ of $T(P)$ at a given point P is

$$L_P(T) = \sum \boldsymbol{C}_{L_P L_{P_i}}(\boldsymbol{P}, \boldsymbol{P}_i)\left[\boldsymbol{C}_{L_{P_i} L_{P_k}}(\boldsymbol{P}_i, \boldsymbol{P}_k) + \sigma_n^2 \boldsymbol{I}\right]^{-1}\left(L_{P_k}(T) + n_k\right), \qquad (6)$$

where $\boldsymbol{C}_{L_P L_{P_i}}(\boldsymbol{P}, \boldsymbol{P}_i)$ are the covariances between the functional to be predicted $L_P(T)$ and the observed functionals $L_{P_i}(T)$, the $\boldsymbol{C}_{L_{P_i} L_{P_k}}(\boldsymbol{P}_i, \boldsymbol{P}_k)$ are the covariances between the observed functionals $L_{P_i}(T)$ and $L_{P_k}(T)$ and n_k is the noise at point P_k. As an example, the collocation estimate of the height anomaly based on gravity data is

$$\zeta(P) = \sum \boldsymbol{C}_{\zeta_P \Delta g_{P_i}}(\boldsymbol{P}, \boldsymbol{P}_i)\left[\boldsymbol{C}_{\Delta g_{P_i}\Delta g_{P_k}}(\boldsymbol{P}_i, \boldsymbol{P}_k) + \sigma_n^2 \boldsymbol{I}\right]^{-1}(\Delta g_k + n_k), \tag{7}$$

where $C_{\Delta g \Delta g}$ is the auto-covariance of the Δg values and $C_{\zeta \Delta g}$ is the cross-covariance between ζ and Δg.

Finally, another way that is currently applied to get the geoid estimate is the Radial Basis Function method that models the gravity field as linear combination of harmonic basis functions (see, e.g., Schmidt et al. 2007).

The practical numerical computations of the estimation approaches are usually based on the remove–compute–restore method. This technique, as applied to the estimation of the geoid undulation from gravity values, is summarized in the following steps:

(i) Remove step: the observed gravity data $\Delta g_{\text{obs}}(P)$ are reduced for their low-frequency features as well as for their high-frequency components. The low-frequency components $\Delta g_{\text{GGM}}(P)$ are those accounted for by a Global Geopotential Model that is given in terms of a spherical harmonic expansion (see, e.g., Pavlis et al. 2012). The high-frequency part of the gravity field spectrum $\Delta g_{\text{RTC}}(P)$ is related to the topography signal and is estimated via a Digital Terrain Model up to 100 ÷ 200 km from each data point (Forsberg 1984). In this way, the residual gravity values Δg_{r} are obtained

$$\Delta g_{\text{r}}(P) = \Delta g_{\text{obs}}(P) - \Delta g_{\text{GGM}}(P) - \Delta g_{\text{RTC}}(P). \tag{8}$$

(ii) Compute step: the residual values are then used as input values in one of the geoid estimation approaches listed above to get the residual component of the geoid undulation $N_r(P)$ (or the residual component of the height anomaly $\zeta_r(P)$).

$$N_{\text{r}}(P) = \text{CO}\left(\Delta g_{\text{r}}\right), \tag{9}$$

where $\text{CO}(\cdot)$ is the "Compute operator" that is applied to get the geoid undulation estimate. Having reduced the data for the low- and high-frequency components, one can prove that the $\text{CO}(\cdot)$ operator, which should be applied to globally distributed data, can be applied to gravity data in a local area centered on the computation point P (nowadays this area is of 1°÷ 2° degrees) since the contribution of the residual data outside this area to the solution is quite negligible.

(iii) Restore step: in this step, the final geoid estimate is obtained by restoring the $N_{\text{GGM}}(P)$ low- and the $N_{\text{RTC}}(P)$ high-frequency of the geoid, adding them to the residual component $N_{\text{r}}(P)$ estimated in the Compute step.

$$N(P) = N_{\text{GGM}}(P) + N_{\text{RTC}}(P) + N_{\text{r}}(P). \tag{10}$$

The different methods described above, if properly applied, are substantially equivalent. This has been confirmed in the recent Colorado test on geoid computation (Wang et al. 2021) where these geoid estimation methods were applied in an area of the Rocky Mountains with significant height variations (see Fig. 1).

The geoid estimation in an area like this is particularly challenging and thus the test was really a significant comparison among the different approaches.

Gravity data over the area, both ground-based and aerogravimetric, where kindly supplied by United States Geological Survey (USGS) to all the research groups participating into the test. The detailed Shuttle Radar Topography Mission Digital Terrain Model (Farr et al. 2007) was used in the computation of the terrain effect and the precision of the different geoid estimates was tested on geoid undulation values that were available along a levelling line, named GSVS17 line (vanWestrum et al. 2021), that was measured on purpose (see the red line in Fig. 1).

The standard deviation of the difference between gravimetric geoid estimates and observed data along the GSVS17 line points ranges between 2 and 4 cm. This can be considered as the nowadays reference value for the precision in geoid estimates.

As described, the different approaches can provide the geoid or the height anomaly estimates using gravity data. The discrepancy between the geoid undulation and the height anomaly is given by the following relationship

$$N - \zeta = \frac{\Delta g_B}{\bar{\gamma}} H \tag{11}$$

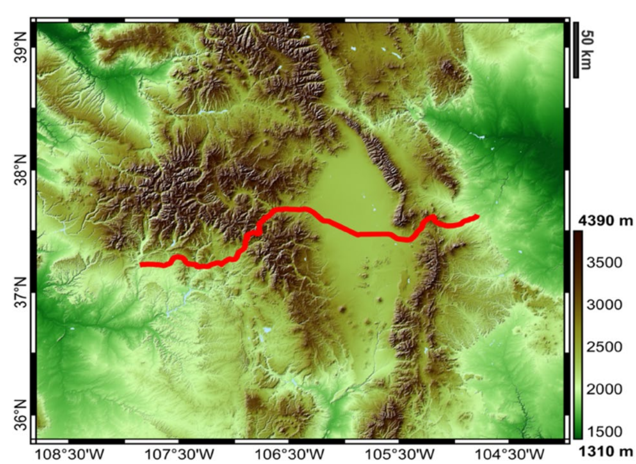


Fig. 1 The Colorado test area and the GSVS17 line (courtesy of NOAA) (Color figure online)

that is function of the Bouguer anomaly Δg_B, of the mean normal gravity $\overline{\gamma}$ along the normal plumb line and of the orthometric height H (Heiskanen and Moritz 1967). This quantity depends on the topography and on the density of the earth crust and it can reach maximum values of the order of 1.5 m in high mountain regions.

Furthermore, in a point P, the ellipsoidal height $h(P)$, the orthometric height $H(P)$ and the normal height $H^*(P)$ are connected to the geoid undulation $N(P)$ and the height anomaly $\zeta(P)$ through the relationship (Heiskanen and Moritz 1967)

$$h(P) = H(P) + N(P) = H^*(P) + \zeta(P). \tag{12}$$

This is a simple but fundamental equation that enlightens the relevance of the geoid (or height anomaly) in defining the height of a point. Furthermore, Eq. (12) is applied in the so-called "GPS/levelling" procedure. If a centimetric geoid precision is available, orthometric heights $H(P)$ can be obtained from GNSS observations, that allow observing $h(P)$, as

$$H(P) = h(P) - N(P). \tag{13}$$

This is currently done instead of using the time-consuming (and thus expensive) spirit-levelling technique. Although spirit levelling can reach millimeter precision, this is not strictly required in most of the current engineering applications that usually require a few centimeters precision in the height determination.

The knowledge of a centimetric precision geoid is also relevant in oceanography. On a point P over the ocean surface it holds that the Sea Surface Height (SSH), which can be obtained by radar altimetry (Wagner 1989), is given as

$$\mathrm{SSH} = h - \rho = N + \mathrm{DOT}, \tag{14}$$

where h is known from the satellite orbit, ρ is the radar observed range, N is the geoid undulation and the DOT is the Dynamic Ocean Topography (see Fig. 2).

Thus, if the geoid undulation N is known over the oceans and the SSH is obtained from altimetry, one can estimate the *DOT* (the instantaneous values which is then averaged in time to get the MDT) that is related to the ocean circulation (Rio and Mulet 2014) as

$$\mathrm{DOT} = \mathrm{SSH} - N. \tag{15}$$

In this kind of application, the geoid is frequently estimated via a Global Geopotential Model (GGM) while the SSH is obtained from global solution like the DTU17 (Andersen and Knudsen 2019).

Over the Mediterranean Sea and the surrounding land areas, there are several gravity data collected since the 50s and a dense set of altimetry data from different satellite

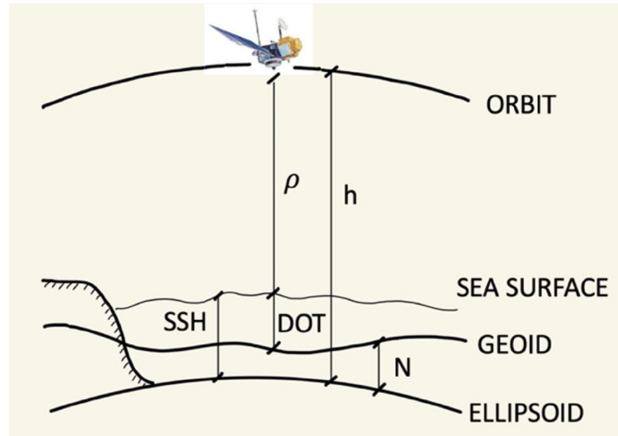


Fig. 2 The satellite altimetry scheme. *SSH* is the sea surface height, *N* is the geoid undulation, *DOT* is the dynamic ocean topography. ρ is the measured range satellite-sea surface and h is the height of the satellite above the ellipsoid known from orbit

missions. It is thus possible to estimate a geoid which, in principle, should be more detailed than a high-frequency global model and compare this geoid undulation with a detailed *SSH* based on the available Mediterranean Sea satellite data. This is the main goal of the GeoMed2 project that will be described in the next section.

3 The GeoMed2 project for the geoid estimation on the Mediterranean Sea

GeoMed2 is the continuation of the GeoMed project that was carried out in the nineties (Barzaghi et al. 1992). At that time, due to restrictions in the computation facilities, the geoid was estimated in three areas covering the Western, the Central and the Eastern Mediterranean. The three solutions were then merged to get a unique solution over the whole Mediterranean Sea. This solution suffered for a poor data distribution and for the adopted estimation methodology that implied discontinuities due to edge effects that were only mitigated by the patching procedure. Also, the GGMs and the DTM/bathymetry data that were available at that time were not so reliable in the Mediterranean area.

This situation has sharply changed in the last decades. New GGMs based on satellite only data have been estimated and the STRM DTM/bathymetry improved a lot the knowledge of the topography, thus allowing a more detailed computation of the high frequency component of gravity and geoid. Furthermore, new gravity data either on the Mediterranean Sea and the surrounding countries are nowadays available. Finally, recent satellite altimetry missions provided very valuable data (Egido and Smith 2017). Therefore, a new project for estimating the geoid and the DOT in the Mediterranean Sea was promoted. This international project led to a first solution that is documented in Barzaghi et al. (2018).

The collected gravity data (either ground- and marine-based) were validated for outliers and a pre-processing of some old marine gravity data was also performed (Lequentrc-Lalancette et. al. 2016). Gravity data were then reduced for the low-frequency components by means of the EIGEN-6C4 model (Förste et al. 2014) to d/o 1000. The high-frequency part of the ground-based gravity data spectrum was accounted for by the SRTM3 DTM while marine gravity data were not reduced for the terrain effect. The obtained residual values were then gridded on a regular $2' \times 2'$ geographical grid over the area 10° S $< \varphi <$ 40° N and 29° E $<$ λ $<$ 48° E and are presented in Fig. 3.

The residual geoid component was computed using the 1D-FFT spherical Stokes convolution method (Haagmans et al. 1993) and the final geoid undulation was obtained by restoring the low- and high-frequency components as previously described.

This geoid estimate was compared with the CNES-CLS MSS (Pujol et al. 2018) to get the MDT over the Mediterranean Sea. This MDT was then compared with the one inferred from the same MSS and the EIGEN-6C4 GGM to full resolution (i.e. to d/o 2190) which was assumed to be the benchmark solution. As a whole, the locally estimated geoid was less accurate than the EIGEN-6C4 model when considering the computation of the inferred MDT. However, in some parts of the Mediterranean Sea the local gravimetric geoid was more accurate in the comparison between drifter-observed and geoid derived geostrophic currents.

Thus, improvements are required to get a better geoid solution allowing a more reliable estimate of the MDT. In view of that, some experiments have been performed in the Central Mediterranean area (see Fig. 4).

Refinements in the pre-processing of the Morelli marine gravity data (Allan and Morelli 1971) were performed. A track-by-track bias and tilt removal in the residuals between these data and the EIGEN-6C4 model has been performed thus improving the consistency of the data of those marine gravity campaigns (Lequentrc-Lalancette et. al. 2016). Contrary to what was done in the previous computation, the contribution of the bathymetric effect was considered in the reduction of the marine gravity data. This has been done by using the global bathymetry and topography grid SRTM15 (15 arc sec resolution) (Tozer et al. 2019) that has been recently released. Furthermore, a new global geopotential model, namely the XGM2019 GGM (Zingerle et al. 2020), to d/o 1000 was used for reducing the low-frequency component of the gravity data. The empirical covariance values and the best-fit covariance model of the residual marine gravity data Δg_r are plotted in Fig. 5.

Finally, the collocation method (Moritz 1989; Sansò and Sideris 2013) was applied in estimating the residual geoid

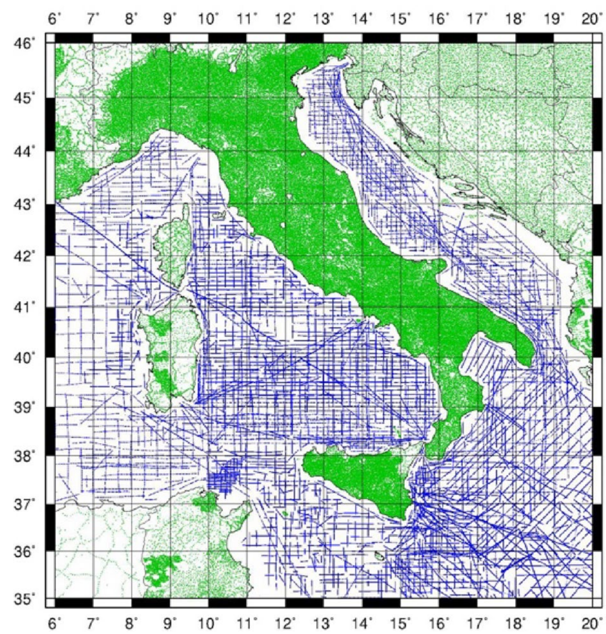
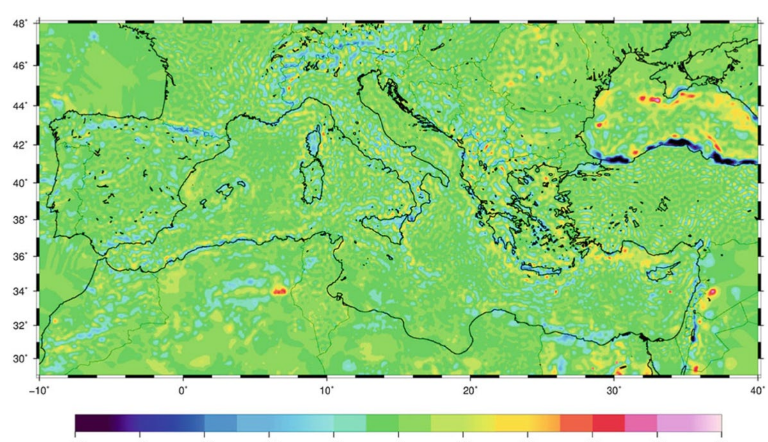


Fig. 4 The gravity data in the test area (marine gravity in blue, ground-based data in green) (Color figure online)

Fig. 3 The residual gridded gravity field in the Mediterranean area (values in mGal)



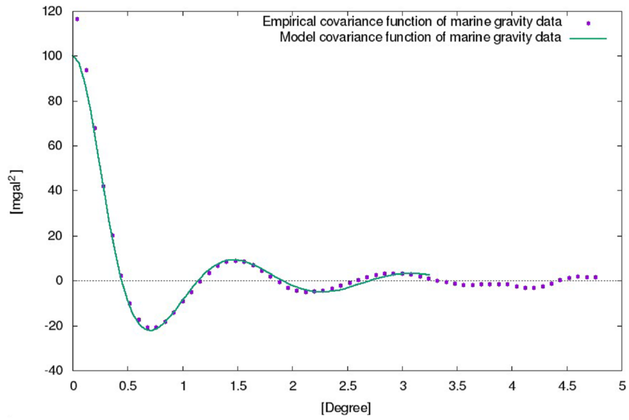


Fig. 5 The empirical covariance values and the best-fit covariance model of the residual marine gravity data Δg_r

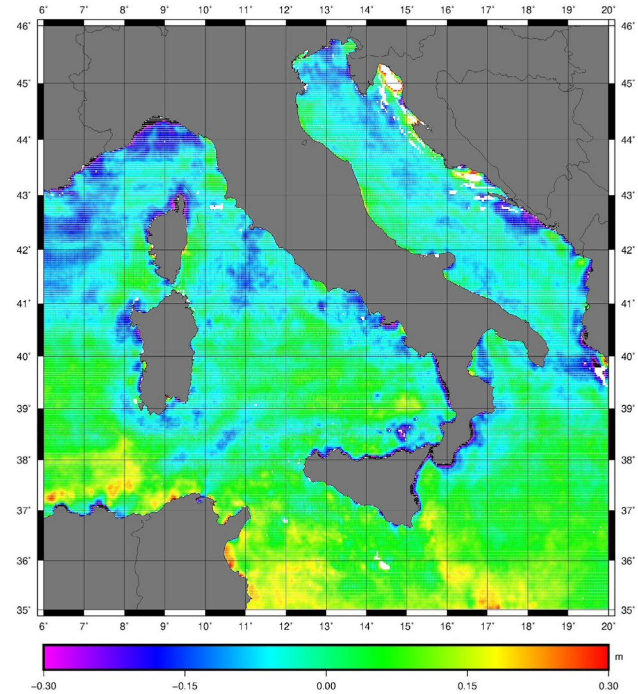


Fig. 6 The MDT in the central Mediterranean area computed as MSS(DTU15)-N(GeoMed$_{test}$) (Values in m)

component N_r. The estimated geoid N(GEOMED$_{test}$) was then compared with the DTU15 MSS (Andersen et al. 2016) to compute the MDT in the central Mediterranean area (see Fig. 6).

As one can see in Fig. 6, the computed MDT displays a quite high-frequency pattern that requires a proper smoothing before any use of this estimate in computing a reliable ocean circulation pattern.

4 Conclusions

The estimate of a precise gravimetric geoid in the Mediterranean area is a relevant topic in scientific applications of geodesy and oceanography. The comparison between the geoid undulation and the altimetry derived MSS gives the estimate of the MDT that is the base for estimating the ocean circulation. This is of relevance in the Mediterranean Sea as this can be the base for environmental research in this highly anthropized area. To this aim, gravity data were collected, either on ground and on the Mediterranean Sea, during two projects, namely the GeoMed project and its recent continuation, the GeoMed2 project. The availability of new reliable GGM and of global bathymetry/DTM data allowed a remarkable improvement in the geoid estimate over this area (the solution obtained in the GeoMed project has a precision which is around 50 cm, see Benciolini et al. 1991 and Barzaghi et al. 1993, while the one computed in the GeoMed2 project has a precision of 9 cm, see Barzaghi et al. 2018). Nevertheless, as a whole, the solution that was obtained in the framework of the GeoMed2 project was still quite poor if compared with the undulation coming from the EIGEN-6c4 GGM to full resolution (i.e. to d/o 2160). As a matter of fact, the MDT based on the CNES-CLS SSH and the EIGEN-6c4 undulation was globally more effective in representing the circulation in Mediterranean Sea than the one obtained using the same SSH and the GeoMed2 geoid undulation. The refinement test presented in this paper was still not completely satisfactory as, in the comparison with the DTU15 MSS, a noisy pattern of the MDT was obtained. Partially, this noisy pattern reflects some geophysical signals that seem to be not adequately contained in the smooth altimetric MSS. On the other side, some noisy components are related to gravity data inconsistencies (mainly along coastal areas) that must be further investigated and checked.

Thus, a more detailed investigation has to be performed. Particularly, a more thorough analysis of the available gravity data must be operated as well as a more effective gridding procedure has to be used. Furthermore, different estimation procedures should be used in estimating the geoid. This would allow having comparisons among the different methods as it was done in the Colorado test, thus having a deeper insight on the impact that possible data inconsistencies can have in the geoid estimate.

Through these refinements, improvements in the gravity geoid estimate and a detailed MDT pattern are expected, which in turn will allow defining a reliable ocean circulation pattern in the Mediterranean Sea.

Author contributions RB and DC contributed equally to the paper.

Funding Open access funding provided by Politecnico di Milano within the CRUI-CARE Agreement.

Declarations

Conflict of interest The authors declare no competing interests.

References

Allan TD, Morelli C (1971) A geophysical study of the Mediterranean Sea. NATO Subcommittee on oceanographic Research, Technical reports n 55, p 141

Andersen OB, Knudsen P (2019). The DTU17 Global Marine Gravity Field: First Validation Results. In: Mertikas S, and Pail R (eds) Fiducial Reference Measurements for Altimetry. International Association of Geodesy Symposia, vol 150. Springer, Cham. https://doi.org/10.1007/1345_2019_65

Andersen OB, Stenseng L, Piccioni G, Knudsen P (2016) The DTU15 MSS (mean sea surface) and DTU15LAT (lowest astronomical tide) reference surface. ESA Living planet symposium 2016, orbit.dtu.dk

Barzaghi R (2016) The remove-restore method. In: Grafarend E (ed) Encyclopedia of geodesy. Springer, Cham. https://doi.org/10.1007/978-3-319-02370-0_19-1

Barzaghi R, Brovelli M, Sanso' F, Tscherning CC (1992) Geoid computation in the Mediterranean area. Mare Nostrum, GeoMed Report n. 1, DIIAR - Politecnico di Milano, Milano

Barzaghi R, Brovelli M, Sanso' F (1993) The gravimetric geoid and the sea surface topography in the central Mediterranean area. Mare Nostrum, GeoMed Report n. 3, DIIAR - Politecnico di Milano, Milano.

Barzaghi R, Carrion D, Vergos GS, Tziavos IN, Gigoriadis VN, Natsiopoulos DA, Bruinsma S, Reinquin F, Seoane L, Bonvalot S, Lequentrec-Lalancette MF, Salaun C, Andersen O, Knudsen P (2018) GEOMED2: high-resolution geoid of the Mediterranean. International Association of Geodesy Symposia. Springer, Berlin, Heidelberg, https://doi.org/10.1007/1345_2018_33

Benciolini B, Manzino A, Sguerso D (1991) ITALGEO '90: risultati globali, validazioni e confronti. Atti del 10° Convegno GNGTS, CNR, Roma

Drinkwater MR, Floberghagen R, Haagmans R, Muzi D, Popescu A (2003) GOCE: ESA's first earth explorer core mission. In: Beutler G, Drinkwater MR, Rummel R, Von Steiger R (eds) Earth gravity field from space—from sensors to earth sciences. Space Sciences Series of ISSI, vol 17. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-1333-7_36

Egido A, Smith WHF (2017) Fully focused SAR altimetry: theory and applications. IEEE Trans Geosci Remote Sens 55(1):392

Farr TG, Rosen PA, Caro E, Crippen R, Duren R, Hensley S, Kobrick M, Paller M, Rodriguez E, Roth L, Seal D, Shaffer S, Shimada J, Umland J, Werner M, Oskin M, Burbank D, Alsdorf D (2007) The SHUTTLE radar topography mission. Rev Geophys. https://doi.org/10.1029/2005RG000183

Forsberg R (1984) A study of terrain reduction, density anomalies and geophysical inversion methods in gravity field modelling. OSU Report no. 5, Columbus, Ohio.

Förste C, Bruinsma SL, Abrikosov O, Lemoine JM, Marty JC, Flechtner F, Balmino G, Barthelmes F, Biancale R (2014) EIGEN-6C4 The latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse. GFZ Data Serv. https://doi.org/10.5880/icgem.2015.1

Haagmans R, de Min E, van Gelderen M (1993) Fast evaluation of convolution integrals on the sphere using 1D-FFT, and a comparison with existing methods for Stokes' integral. Manuscr Geodaet 18:82–90

Heiskanen W, Moritz H (1967) Physical Geodesy. W.H. Freeman and Company, San Francisco.

Jahanmard V, Delpeche-Ellmann N, Ellmann A (2022) Towards realistic dynamic topography from coast to offshore by incorporating hydrodynamic and geoid models. Ocean Model 180:102124

Lequentrec-Lalancette MF, Salaün C, Bonvalot S, Rouxel D, Bruinsma S (2016) Exploitation of marine gravity measurements of the Mediterranean in the validation of global gravity field models. International association of geodesy symposia. Springer, Berlin

Moritz H (1989) Advanced physical geodesy. Herbert Wichmann Verlag GmbH, Karlsruhe

Pavlis NK, Holmes SA, Kenyon SC, Factor JF (2012) The development and evaluation of Earth Gravitational Model (EGM2008). J Geophys Res Atmos. https://doi.org/10.1029/2011JB008916

Pujol M-I, Schaeffer P, Faugere Y, Raynal M, Dibarboure G, Picot N (2018) Gauging the improvement of recent mean sea surface models: A new approach for identifying and quantifying their errors. J Geophys Res: Oceans. https://doi.org/10.1029/2017JC013503

Rio M-H, Mulet S (2014) Beyond GOCE for the ocean circulation estimate: synergetic use of altimetry, gravimetry, and in situ data provides new insight into geostrophic and Ekman currents. Geophys Res Lett. https://doi.org/10.1002/2014GL061773

Sansò F, Sideris MG (2013) Geoid determination: theory and methods. Springer, Berlin

Schmidt M, Fengler M, Mayer-Gürr T et al (2007) Regional gravity modeling in terms of spherical base functions. J Geodesy 81:17–38. https://doi.org/10.1007/s00190-006-0101-5

Smith AD, Véronneau M, Roman DR, Huang J (2013) Towards the unification of the vertical datum over the North American Continent. IAG Symposia 138, https://doi.org/10.1007/978-3-642-32998-2_36

Tozer B, Sandwell DT, Smith WHF, Olson C, Beale JR, Wessel P (2019) Global bathymetry and topography at 15 Arc Sec: SRTM15+. Earth Space Sci 6(10):1874–1864

Tscherning CC (2015) Least-squares collocation. In: Grafarend E (ed) Encyclopedia of geodesy. Springer, Cham. https://doi.org/10.1007/978-3-319-02370-0_51-1

vanWestrum D, Ahlgren K, Hirt C, Guillaume S (2021) A geoid slope validation survey (2017) in the rugged terrain of Colorado, USA. J Geod 95:9. https://doi.org/10.1007/s00190-020-01463-8

Wagner CA (1989) Summer school lecture notes on satellite altimetry. In: Sansò F, Rummel R (eds) Lecture noted in earth sciences 25. Springer-Verlag, Cham

Wang YN, Sánchez L, Ågren J, Huang J, Forsberg R, Abd-Elmotaal HA, Ahlgren K, Barzaghi R, Bašić T, Carrion D, Claessens S, Erol B, Erol S, Filmer M, Grigoriadis VN, Isik MS, Jiang T, Koç Ö, Krcmaric J, Li X, Liu Q, Matsuo K, Natsiopoulos DA, Novák P, Pail R, Pitoňák M, Schmidt M, Varga M, Vergos GS, Véronneau M, Willberg M, Zingerle P (2021) Colorado geoid computation experiment: overview and summary. J Geodesy 95:127. https://doi.org/10.1007/s00190-021-01567-9

Zingerle P, Pail R, Gruber T et al (2020) The combined global gravity field model XGM2019e. J Geod 94:66. https://doi.org/10.1007/s00190-020-01398-0

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.