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Offset-free distributed predictive control based on fuzzy logic: Application to a real four-tank plant ⋆

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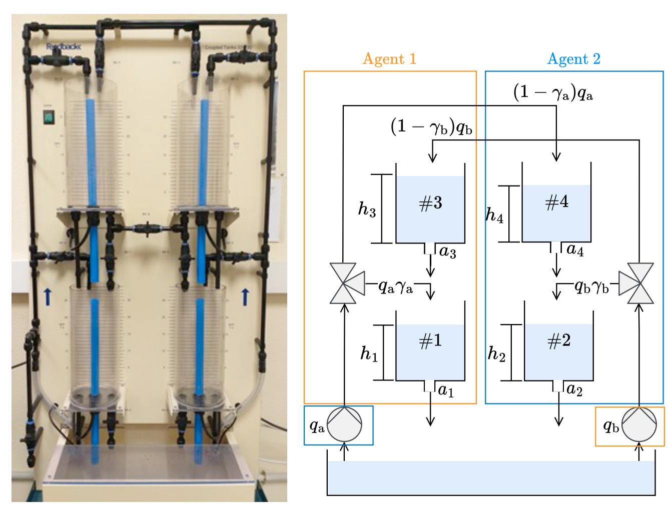
Abstract: This paper proposes an offset-free distributed implementation of a model predictive controller that employs fuzzy negotiation between agents. The scheme is based on model augmentation with additional disturbances to enable zero-offset tracking. Moreover, we code the negotiation criteria as a set of suitable fuzzy rules and consider stability and feasibility guarantees in the controller design for the linearized subsystems. We applied the method to an experimental four-tank plant, showing its effectiveness despite the coupling between subsystems and system-model mismatch.

Keywords: Linear multivariable systems, Predictive control, Fuzzy control systems, Process control, Control of distributed systems.

1. INTRODUCTION

The increasing industrial requirements in efficiency, quality, flexibility, safety, and environmental impact have led to the development of large complex processes involving many interacting units. Non-centralized control techniques seem adequate to handle these complex systems and reduce computational efforts (Negenborn and Maestre, 2014; Maestre et al., 2011). In particular, distributed model predictive control (DMPC) divides the problem into subproblems managed by local agents with local information about the system, and they can communicate with each other to reach an agreement in the overall control action. See (Maestre and Negenborn, 2014) for multiple DMPC schemes based on cooperative, non-cooperative, hierarchical, and others.

In order to validate these and other advanced control techniques, the four-tank plant has been proposed as a benchmark due to its highly coupled states (Johansson, 2000), and its straightforward sectorization. For example, a multi-agent DMPC based on fuzzy negotiation is stated in Francisco et al. (2019) and Morales-Rodelo et al. (2019), and extended by Masero et al. (2021) to eight-coupled tanks with stability guarantees. Grancharova et al. (2018) apply a dual-model DMPC to the four-tank plant, and, in Segovia et al. (2019), agents' coordination is performed limiting the difference between the available solutions inspired by a Lagrangian relaxation problem. However, only few studies (Mercangöz and Doyle III, 2007; Orihuela et al., 2016; Alvarado et al., 2011) are evaluated in an experimental plant.



(a) Plant located in our labs. (b) Schematic diagram.

Fig. 1. Quadruple-tank plant.

In this paper, we propose a fuzzy DMPC scheme based on Masero et al. (2021) and particularized for two agents due to the available experimental setup, but include further improvements to prevent steady-state offset. The offset-free procedure is based on an augmented model that includes disturbances such as noise and modeling errors (Maeder et al., 2009). Therefore, each local MPC works with a local state estimator that provides the state and disturbances to make set-point corrections and enhance steady-state accuracy. The choice of the disturbance model in the augmented model, which is the key to achieving a suitable

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tracking (Pannocchia and Rawlings, 2003), is also studied in the context of the real application. The main benefits of fuzzy negotiation considered in this work include the computation of control inputs that provide smooth responses and the consideration of economic and other process criteria to improve overall performance. Our proposed scheme is tested in the real plant shown in Fig. 1a, which is located in the laboratories of the University of Salamanca, Spain. Therefore, the contribution of the paper is two-fold: further improvement of a distributed MPC method to provide offset-free reference tracking, and its first time assessment in a real plant.

The rest of the paper is organized as follows. Section 2 introduces the problem settings. Section 3 details the DMPC scheme and the fuzzy negotiation procedure. The real plant is presented in Section 4, and the results are provided in Section 5. Conclusions are given in Section 6.

2. PROBLEM SETTING

Consider the following discrete-time non-linear dynamics of the real plant:

$$\begin{aligned} x_p(k+1) &= f(x_p(k), u_p(k)), \\ y_p(k) &= g(x_p(k)), \end{aligned} \tag{1}$$

where k is the time instant, and $x_p \in \mathbb{R}^4, u_p, y_p \in \mathbb{R}^2$ are respectively the state, input, and output vectors. The objective is to design an MPC controller to track outputs using the linear internal model:

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k), \\ y(k) &= C\,x(k), \end{aligned} \tag{2}$$

where A, B, C are the state, input and output matrices.

Assumption 1. The pair (A, B) is controllable and the pair (A, C) is observable, with C having a full-row rank.

In light of the possible mismatch between the plant (1) and the system model, the model (2) is augmented with a disturbance model as

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + B_d\,d(k), \\ d(k+1) &= d(k) \\ y(k) &= C\,x(k) + C_d\,d(k), \end{aligned} \tag{3}$$

where d is the disturbance vector that includes the modeling error, and B_d, C_d are the matrices that link disturbances with states and outputs, respectively.

An observer is designed to estimate both the state and disturbance based on the augmented model (3):

$$\begin{aligned} \begin{bmatrix} \hat{x}(k+1) \\ \hat{d}(k+1) \end{bmatrix} &= \begin{bmatrix} A & B_d \\ \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} + \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix} u(k) \\ &+ \begin{bmatrix} L_x \\ L_d \end{bmatrix} \big(-y_p(k) + C\hat{x}(k) + C_d\hat{d}(k) \big), \end{aligned} \tag{4}$$

where $\hat{x}$ and $\hat{d}$ are, respectively, the estimated global state and disturbance vectors, and L_x, L_d are the observer matrices. In this work, the observer is designed as a Kalman filter due to the existence of noisy signals, as detailed in Section 5.

Proposition 2. ((Pannocchia and Rawlings, 2003, Lemma 1)) The observability of the augmented system (3) is guaranteed iff the pair (A, C) is observable and the matrix:

$$\begin{bmatrix} A - I & B_d \\ C & C_d \end{bmatrix} \tag{5}$$

has a full-column rank.

If Assumption 1 and the condition of Proposition 2 hold, the augmented system is observable and, therefore, there exist matrices L_x and L_d in such a way that the stability of the estimator (4) is guaranteed. Let us define $z = H\,y_m$ as the tracked outputs and r as their corresponding references. The overall state and input references (x_r, u_r) should satisfy the following to guarantee offset-free tracking for the MPC controller:

$$\begin{bmatrix} A - I & B \\ H\,C & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_r \\ u_r \end{bmatrix} = \begin{bmatrix} -B_d\,\hat{d}(k) \\ r(k) - H\,C_d\,\hat{d}(k) \end{bmatrix}. \tag{6}$$

For further details of this condition and its proof, the reader is referred to the work of Maeder et al. (2009).

3. DISTRIBUTED MPC CONTROL

We propose a distributed MPC algorithm for two agents based on fuzzy negotiation. The system is partitioned into two subsystems $i \in \mathcal{N} = \{1, 2\}$ coupled by inputs, as shown in Fig. 1b. Note that the partition selection is limited by the approach to only consider input couplings. Each agent i makes use of its corresponding linear subsystem model, which is disaggregated from (2) as:

$$x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + B_{d_i} d_i(k) + w_i(k), \tag{7}$$

with states $x_i \in \mathcal{X}_i$, inputs $u_i \in \mathcal{U}_i$, and $w_i(k) = B_{ij} u_j(k) \in \mathcal{W}_i$ being the input coupling with its neighbor $j \neq i$. Sets $\mathcal{X}_i, \mathcal{U}_i$, and $\mathcal{W}_i$ are compact convex sets that contain the origin in their interiors. The estimated local state and disturbance can also be disaggregated from (3).

3.1 Control objective

The overall objective is to track a predefined set-point:

$$x_r = [x_{r_1}, x_{r_2}]^\top, \quad u_r = [u_{r_1}, u_{r_2}]^\top, \tag{8}$$

while minimizing the sum of local cost functions and satisfying the constraints. At time instant $k \in \mathbb{N}_0$, each local MPC controller $i \in \{1, 2\}$ minimizes its cost function over a predictive horizon H_p:

$$\begin{aligned} J_i(x_i(k), U_i(k), U_j(k)) = \sum_{n=0}^{H_\mathrm{p}-1} \Big(&\|x_i(k+n) - x_{r_i}(k+n)\|^2_{Q_i} \\ &+ \|u_i(k+n) - u_{r_i}(k+n)\|^2_{R_i} \\ &+ \|u_j(k+n) - u_{r_j}(k+n)\|^2_{R_j} \Big) \\ &+ \|x_i(k+H_\mathrm{p}) - x_{r_i}(k+H_\mathrm{p})\|^2_{P_i} \end{aligned} \tag{9}$$

with $j \in \{1, 2\}$ and $j \neq i$, subject to:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_{ii} u_i(k) + B_{ij} u_j(k) + B_{d_i} d_i(k), \\ d_i(k+1) &= d_i(k), \\ y_i(k) &= C_i x_i(k) + C_{d_i} d_i(k), \\ x_i(0) &= \hat{x}_i(k), \\ d_i(0) &= \hat{d}_i(k), \\ x_i(k+n) &\in \mathcal{X}_i, \quad n = 1, \ldots, H_\mathrm{p} - 1, \\ x_i(k+H_\mathrm{p}) &\in \Omega_i \\ u_i(k+n) &\in \mathcal{U}_i, \quad n = 1, \ldots, H_\mathrm{p}, \end{aligned}$$

where x_{r_i} and u_{r_i} are the state and input references that are computed with (6) and (8); $\hat{x}_i$ and $\hat{d}_i$ are, respectively, the estimated state and disturbance; Ω_i is a set of terminal states region used as a constraint for stability (see Remark 4); and $Q_i \geqslant 0$ and $R_i, P_i > 0$ are matrices of appropriate dimensions.

3.2 Control algorithm

We consider the H_p-length control input sequence of agent i at time instant k:

$$U_i(k) \triangleq [u_i(k), u_i(k+1), \ldots, u_i(k+H_\mathrm{p}-1)]^\top, \quad (10)$$

and the optimal input sequence from minimizing (9):

$$U_i^*(k) \triangleq [u_i^*(k), u_i^*(k+1), \ldots, u_i^*(k+H_\mathrm{p}-1)]^\top. \quad (11)$$

A shifted sequence $U_i^s(k)$ can be obtained by adding $K_i\, x_i(k+H_\mathrm{p})$ to the tail of the sequence $U_i(k-1)$ obtained at the previous time instant:

$$U_i^s(k) \triangleq [u_i(k+1), \ldots, u_i(k+H_\mathrm{p}-1), K_i x_i(k+H_\mathrm{p})]^\top, \quad (12)$$

where K_i is a feedback gain for stability (see details in Remark 4), and $x_i(k+H_\mathrm{p})$ is the state predicted at the end of the horizon H_p with the data available at $k-1$.

The proposed hierarchical DMPC algorithm is based on (Maestre et al., 2011) in combination with the fuzzy negotiation process to compute the final control sequences. At each time instant k, the algorithm of Fig. 2 is executed. First, a coordinator agent calculates the set point (6) and sends the local reference to the other agent (Step 0.i), which computes K_i, P_i, and Ω_i if there is a change in reference. Afterwards, agent i solves its MPC problem and exchanges information to calculate its shifted sequence U_i^s (Step 1.i), its optimal sequence U_i^* (Step 2.i) and the sequence that wishes for its neighbor j (Step 3.i). In Step 4.i, each agent i *fuzzifies* the sequences:

$$\{U_i^*, U_i^\mathrm{s}, U_i^{\mathrm{w}_j}\} \quad (13)$$

to calculate its final U_i^f and its cost $J_i(x_i(k), U_i^\mathrm{f}(k))$ (Step 5.i). The overall cost J calculated as the sum of local costs is compared with the cost of the previous instant time. Provided that J decreases, agent i applies U_i^f; otherwise, U_i^s is applied instead (Step 6.i). Finally, agent i measures the current state/output y_{mi} and estimates the state and disturbance in $k+1$ using its local observer (Step 7.i).

Remark 3. A coordinator agent (in this case, $i = 1$) computes x_r and u_r with (6) in a centralized manner due to the coupling between the subsystems. There is no significant increase in the computational load for this agent because it is a straightforward algebraic calculation.

Remark 4. In this work, due to the mild nonlinearity of the plant, we consider the stability approach of Maestre et al. (2011) by using feedback gains K_i and terminal regions Ω_i for each local MPC problem. However, a rigorous stability and feasibility analysis would require the inclusion of terms arising from linearization and estimation errors, as well as characteristics of the input-to-state stability framework (Limon et al., 2009; Huang et al., 2013).

3.3 Fuzzy negotiations

Once agent i has its tuple of control sequences (13), the idea is to *fuzzy* them to calculate a final input U_i^f that reduces the cost-to-go, and guarantees the stability of the linearized DMPC scheme. One of the advantages of fuzzy negotiation is that the computation of the final control inputs does not require numerous communication steps to reach a consensus, only merging the input proposals of each agent. In this work, the specific parameters for membership functions and fuzzy rules have been selected heuristically by trial and error considering experimental results for this case study. The main steps of the fuzzy negotiation process are the following:

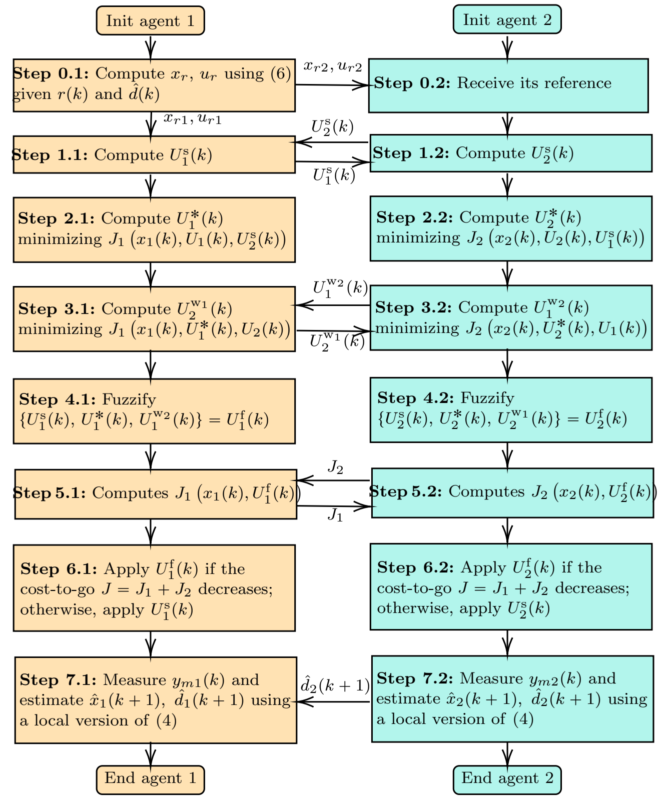
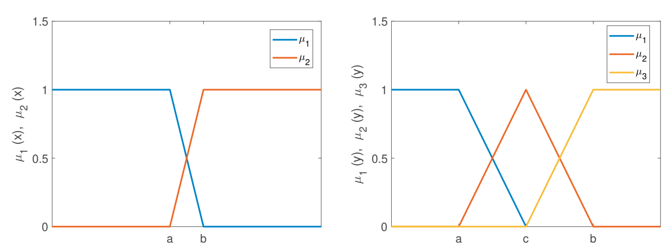


Fig. 2. Fuzzy-based distributed control algorithm.



(a) Two-alternative criterion. (b) Three-alternative criterion.

Fig. 3. Fuzzy sets employed in the fuzzy inference process with two and three alternatives ('low', 'medium', and 'high'), assuming typical triangular and trapezoid shapes for the sets.

(1) *Fuzzification:* Crisp numerical values are converted into fuzzy numbers by considering the degree of membership in the fuzzy sets based on specific criteria (refer to Fig. 3).
(2) *Rule evaluation:* The fitness of control actions is obtained evaluating all fuzzy rules N_r using the fuzzy numbers obtained in the previous step. The number of fuzzy rules is determined by all possible combinations of linguistic variables and negotiation criteria.
(3) *Defuzzification:* This step transforms linguistic variables into crisp numbers that represent the fitness of a control action considering all rules. The defuzzification method employed is a Sugeno-type fuzzy inference with constant singleton output membership functions.

The fitness of control action U for rule R_r is computed as

$$\alpha_{R_r}(U) = w_r^x \cdot \mu_r(x) \cdot w_r^y \cdot \mu_r(y) \quad (14)$$

where r denotes the r-th rule, $\mu_r(x)$ and $\mu_r(y)$ are fuzzy sets with w_r^x, w_r^y being their corresponding weights regarding the considered linguistic variables ('low', 'medium', and 'high') and x, y representing two algebraic variables representing criteria for negotiation (in our case, control efforts and hydraulic residence time). Thus, the total fitness of the control action for all rules N_r is:

$$\alpha(U) = \sum_{r=1}^{N_r} \alpha_{R_r}(U). \quad (15)$$

Regarding Step 4.i of the proposed algorithm, the final control action U_i^{f} of agent i is calculated as a linear combination of the triplet of control sequences (13):

$$U_i^{\mathrm{f}} = \frac{U_i^s \cdot \alpha(U_i^{\mathrm{s}}) + U_i^* \cdot \alpha(U_i^*) + U_i^{\mathrm{w}_j} \cdot \alpha(U_i^{\mathrm{w}_j})}{\alpha(U_i^{\mathrm{s}}) + \alpha(U_i^*) + \alpha(U_i^{\mathrm{w}_j})} \quad (16)$$

Remark 5. Since neighbor j calculates the control sequence $U_i^{\mathrm{w}_j}$ without considering the state constraints of agent i, it is necessary to check if the state constraints of agent i are satisfied. Otherwise, $U_i^{\mathrm{w}_j}$ is excluded from (16).

4. A REAL CASE STUDY

The plant is composed of four coupled tanks (Johansson, 2000), which are interconnected as shown in Fig. 1b. The aim is to track reference water levels considering pumping energy and other operational requirements. The plant has two centrifugal pumps (q_{a} and q_{b}) and two manual three-way valves (γ_1, γ_2) that distribute the flow rate according to their opening. The selected partition leads to agent 1 (tanks #1 and #3, and pump b) and agent 2 (tanks #2 and #4 and pump a). This choice is based on the coupling because the effect of q_{a} is more significant on the agent 2, and the same holds for q_{b} with agent 1.

Taking into account the parameters from Table 1, each subsystem is defined by matrices:

$$A_1 = \begin{bmatrix} \frac{-1}{\tau_1} & \frac{S_3}{S_1 \tau_3} \\ 0 & \frac{-1}{\tau_3} \end{bmatrix}, \quad A_2 = \begin{bmatrix} \frac{-1}{\tau_2} & \frac{S_4}{S_2 \tau_4} \\ 0 & \frac{-1}{\tau_4} \end{bmatrix},$$

$$B_{11} = \frac{1}{3600} \begin{bmatrix} \frac{\gamma_1}{S_1} \\ 0 \end{bmatrix}, \quad B_{21} = \frac{1}{3600} \begin{bmatrix} 0 \\ \frac{1-\gamma_1}{S_4} \end{bmatrix},$$

$$B_{12} = \frac{1}{3600} \begin{bmatrix} 0 \\ \frac{1-\gamma_2}{S_3} \end{bmatrix}, \quad B_{22} = \frac{1}{3600} \begin{bmatrix} \frac{\gamma_2}{S_2} \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where $\tau_n = \frac{S_n}{a_n} \cdot \sqrt{\frac{2h_n^0}{g}}$ with $n \in \{1, \ldots, 4\}$. The inputs and states are constrained as follows:

$$\begin{aligned} 0 < h_n(k) \leqslant 0.3, \ \forall n \in \{1, \ldots, 4\}, \\ 0 < q_m(k) \leqslant 0.5, \quad \forall m \in \{\mathrm{a}, \mathrm{b}\}. \end{aligned} \quad (17)$$

Table 2 shows the tuning parameters, the local feedback gains K_i, and terminal costs P_i. In Fig. 4, we display the maximal RPI set of each subsystem i, which has been computed considering constraints (17) and disturbance set $\mathcal{W}_i = B_{ij}\mathcal{U}_j$. The prediction horizon for both agents is $H_{\mathrm{p}} = 30$, and the sample time is $T_{\mathrm{s}} = 2$ s.

Table 1. Plant parameters.

Parameter	Value	Description [units]
S_i	0.0123	Cross-sectional area of tanks [m^2] ($i = 1, ..., 4$)
a_1	7.21e-5	Tank 1 discharge constant [m^2]
a_2	7.28e-5	Tank 2 discharge constant [m^2]
a_3	7.72e-5	Tank 3 discharge constant [m^2]
a_4	7.83e-5	Tank 4 discharge constant [m^2]
h_1^0	0.181	Steady state level of tank 1 [m]
h_2^0	0.206	Steady state level of tank 2 [m]
h_3^0	0.031	Steady state level of tank 3 [m]
h_4^0	0.032	Steady state level of tank 4 [m]
q_1^0	0.34	Steady state of q_1 [m^3/h]
q_2^0	0.33	Steady state of q_2 [m^3/h]
γ_1	0.33	Opening of manual valve 1
γ_2	0.35	Opening of manual valve 2
g	9.81	Acceleration of gravity [m/s^2]

Table 2. Summary of MPC matrices.

	Agent 1	Agent 2
Q_i:	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
R_i:	$R_1 = 1, \quad R_2 = 1$	$R_1 = 1, \quad R_2 = 1$
K_i:	$\begin{bmatrix} -0.0728 & -0.0769 \end{bmatrix}$	$\begin{bmatrix} -0.0817 & -0.0856 \end{bmatrix}$
P_i:	$\begin{bmatrix} 8.9809 & 6.0872 \\ 6.0872 & 6.1129 \end{bmatrix}$	$\begin{bmatrix} 9.4428 & 6.4978 \\ 6.4978 & 6.5073 \end{bmatrix}$

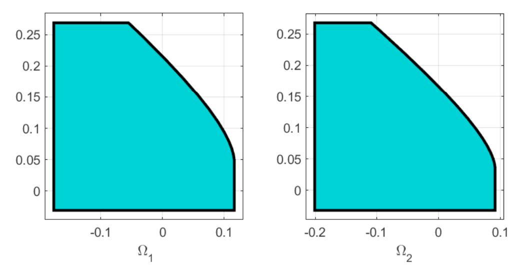


Fig. 4. Invariant sets (terminal regions) for both agents.

Regarding the fuzzy criteria, we just include the control efforts (directly linked to pumping energy) in the fuzzy negotiation because their effects are the most significant for this case study. In particular, we consider the three fuzzy sets with trapezoid shapes shown in Fig. 3b. The control effort of agent i is calculated as:

$$\Delta u_i(k) = |u_i(k) - u_i^{\mathrm{f}}(k-1)|, \quad \forall i \in \{1, 2\}, \quad (18)$$

where $u_i(k)$ is the first element of the corresponding control sequence available for negotiation in each agent, and $u_i^{\mathrm{f}}(k-1)$ is the control action applied to the plant at the previous time instant. The knowledge base for the fuzzy inference system consists of $N_r = 3$ fuzzy rules, where the fitness of each control sequence U_i according to each rule is:

$$\alpha_r(U_i) = w_r \cdot \mu_r(\Delta u_i), \quad \forall r \in \{1, 2, 3\}, \ \forall i \in \{1, 2\},$$

and the total fitness is calculated as (15).

5. EXPERIMENTAL RESULTS

5.1 Estimator

The offset-free procedure requires an observer (3) to estimate the disturbance d that captures the plant-model mismatch, and update the states of each agent, making use

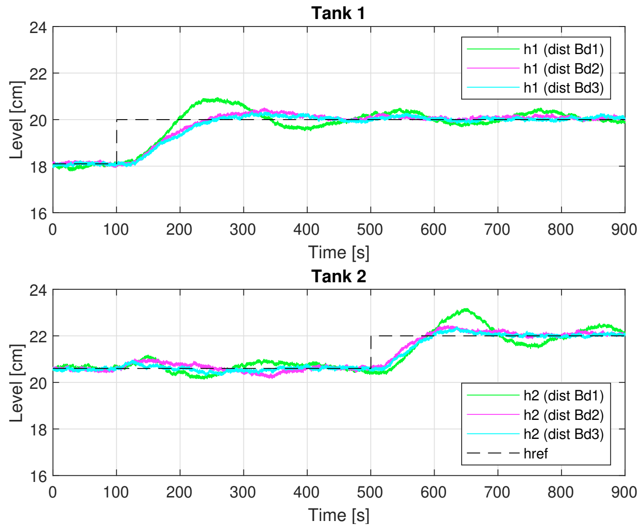


Fig. 5. Tanks levels #1 and #2 for the disturbance matrices considered.

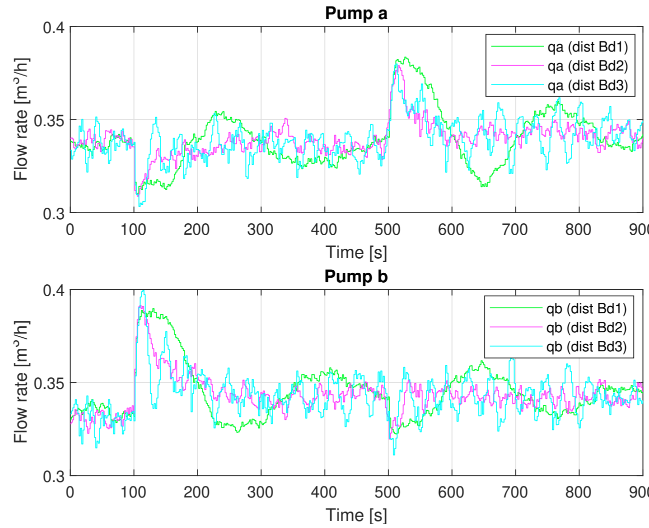


Fig. 6. Flow rate of pumps for the disturbance matrices.

Table 3. Performance indices for multiple disturbance matrices

Indices	B_d^1	B_d^2	B_d^3
M_{p1} [%]	4.74	2.53	1.71
M_{p2} [%]	5.36	2.02	1.76
ISE_1	259.56	217.73	227.10
ISE_2	207.48	86.91	93.19
Δu_1	13.25	21.03	36.03
Δu_2	16.24	27.97	50.73

of the current local measurements and the previous control action. At each sampling instant, the supervisor agent provides the set-point (8) to each agent. The estimator gain L_d is the identity matrix, and L_x has been selected based on a Kalman filter:

$$L_x = \begin{bmatrix} 0.0093 & 0.0019 & 0.0083 & 0.0020 \\ 0.0019 & 0.0080 & 0.0016 & 0.0082 \\ 0.0068 & 0.0015 & 0.0136 & -0.0001 \\ 0.0019 & 0.0069 & -0.0001 & 0.0149 \end{bmatrix}. \quad (19)$$

Due to the significant plant-model mismatch affecting the states and outputs, constant non-zero disturbances affecting input and output must be considered. Specifically, matrices $C_d = I$ and B_d are designed to achieve suitable performance (Pannocchia and Rawlings, 2003) while fulfilling the controllability requirements of (5). A performance comparison is presented in Fig. 5 and Table 3 given three different disturbance-model matrices:

$$B_d^1 = \text{diag}([0.03, 0.04, 0.03, 0.04]), \quad (20)$$

$$B_d^2 = \text{diag}([0.12, 0.12, 0.12, 0.12]), \quad (21)$$

$$B_d^3 = \text{diag}([0.24, 0.24, 0.24, 0.24]). \quad (22)$$

As shown in Fig. 5, overshoot decreases when B_d increases, but at the expense of increasing control efforts (see Fig. 6). In this case, comprising all performance objectives, B_d^2 provides the best results. For the performance analysis given in Table 3, ISE is the integral square error, Δu_i is defined as in (18) but integrated for the entire simulation time, and $M_{pi} = ((x_{i,\max} - x_{r_i})/x_{r_i})$ is the overshoot with $x_{i,\max}$ being the peak value of x_i.

Table 4. Fuzzy rules weights

	Alternative	**Case 1**	**Case 2**	**Case 3**
Δu_i	Low (w_1)	1	1	1
	Medium (w_2)	1	0.2	0.2
	High (w_3)	1	0.1	0.01

Table 5. Performance indices.

Indices	**Case 1**	**Case 2**	**Case 3**	**Centralized MPC**
M_{p1} [%]	3.75	3.32	–	3.62
M_{p2} [%]	3.72	2.79	–	3.48
ISE_1	230.37	308.35	927.92	213.06
ISE_2	141.67	195.01	626.65	119.64
Δu_1	13.93	7.54	3.43	22.59
Δu_2	17.54	9.60	1.92	27.45

5.2 Reference tracking

The experiment consists of applying two successive step references. First, from $h_{r_1} = 18$ cm to $h_{r_1} = 20$ cm for h_1, and then, from $h_{r_2} = 20$ cm to $h_{r_2} = 22$ cm for h_2, as illustrated in Fig. 7. In this experiment, $B_d = \text{diag}([0.04, 0.06, 0.04, 0.06])$ have been slightly detuned to produce a noticeable overshoot for better validation of the fuzzy negotiation impact. We study the performance indices of the DMPC for three cases with different weights for the fuzzy rules (see Figs. 7, 8 and Table 5) together with a centralized MPC, showing no relevant performance loss with the proposed distributed framework. Case 1 represents an equal activation of rules for different Δu_i to give the total fitness of a control sequence, and Cases 2 and 3 have a higher fitness for 'low' Δu_i (see Table 4). The control performance is suitable because the output trajectories follow the reference signals with small overshoots, except Case 3, because an excessive penalization of control moves prevents reaching set-point.

6. CONCLUSIONS

We present a fuzzy-based distributed predictive controller for reference tracking, which has been applied to a real four-coupled tank plant. A procedure to cancel the offset

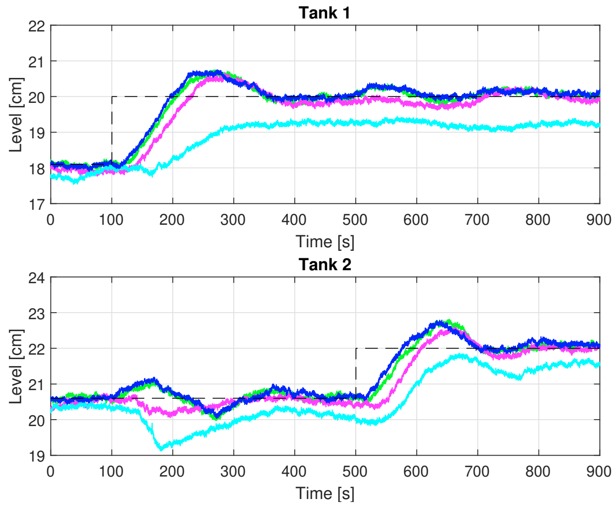


Fig. 7. Tank levels #1 and #2 for multiple weights in fuzzy rules: Case 1 (green), Case 2 (magenta), Case 3 (cyan), Centralized MPC (blue)

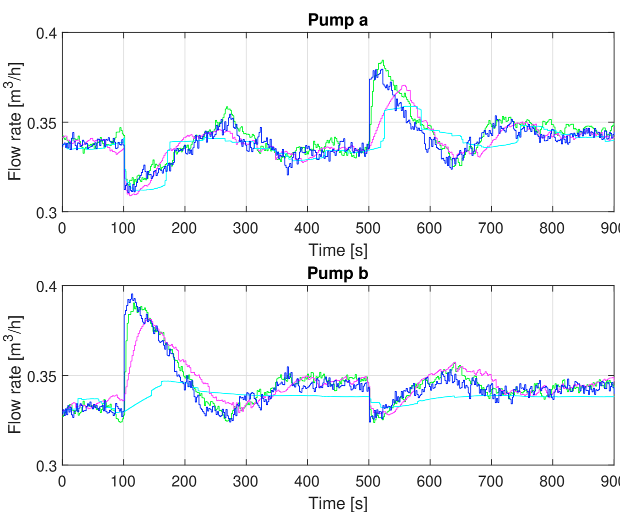


Fig. 8. Pump flows for multiple weights in fuzzy rules: Case 1 (green), Case 2 (magenta), Case 3 (cyan), Centralized MPC (blue).

has also been included to account for plant-model mismatch. In particular, the mismatch is caused by phenomena such as the actual opening parameters of the three-way valves, which are critical for the plant coupling, and the sensors' noise. Suitable results for typical reference changes have been achieved through proper selection and tuning of the estimator. The use of fuzzy negotiation in our approach avoids the need to evaluate all combinations of available local control actions (as in the cooperative game of Maestre et al. (2011)), providing smooth responses unlike the work of (Alvarado et al., 2011), where some abrupt changes occurred. This method can be easily extended to larger processes with more than one non-coordinator agent.

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