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# A novel double-hybrid learning method for modal frequency-based damage assessment of bridge structures under different environmental variation patterns

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#### ABSTRACT

Monitoring of modal frequencies under an unsupervised learning framework is a practical strategy for damage assessment of civil structures, especially bridges. However, the key challenge is related to high sensitivity of modal frequencies to environmental and/or operational changes that may lead to economic and safety losses. The other challenge pertains to different environmental and/or operational variation patterns in modal frequencies due to differences in structural types, materials, and applications, measurement periods in terms of short and/or long monitoring programs, geographical locations of structures, weather conditions, and influences of single or multiple environmental and/or operational factors, which may cause barriers to employing stateof-the-art unsupervised learning approaches. To cope with these issues, this paper proposes a novel double-hybrid learning technique in an unsupervised manner. It contains two stages of data partitioning and anomaly detection, both of which comprise two hybrid algorithms. For the first stage, an improved hybrid clustering method based on a coupling of shared nearest neighbor searching and density peaks clustering is proposed to prepare local information for anomaly detection with the focus on mitigating environmental and/or operational effects. For the second stage, this paper proposes an innovative non-parametric hybrid anomaly detector based on local outlier factor. In both stages, the number of nearest neighbors is the key hyperparameter that is automatically determined by leveraging a self-adaptive neighbor searching algorithm. Modal frequencies of two full-scale bridges are utilized to validate the proposed technique with several comparisons. Results indicate that this technique is able to successfully eliminate different environmental and/or operational variations and correctly detect damage.

#### 1. Introduction

Bridges are important civil structures that contribute to the social and economic development by providing important transportation systems between cities or countries in highways and even railways. Bridge structures are subjected to dead and live (traffic) loads along with extreme loads resulting from natural and man-made excitation sources. Some bridges may encounter the hazard of daily emergencies such as overloading, cycle loads, terrible accidents, ship collision, strong wind, flood, etc. Apart from these issues,

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Nomenclature					
DC	Damaged Condition				
DHL	Double-Hybrid Learning				
DPC	Density Peaks Clustering				
FNR	False Negative Rate				
FPR	False Positive Rate				
GMM	Gaussian Mixture Model				
LOF	Local Outlier Factor				
MSD	Mahalanobis-Squared Distance				
NC	Normal Condition				
NPLOF	Non-Parametric LOF				
OMA	Operational Modal Analysis				
PCA	Principal Component Analysis				
SANS	Self-Adaptive Neighbor Searching				
SHM	Structural Health Monitoring				
SNN	Shared Nearest Neighbor				
SVD	Singular Value Decomposition				

those may be exposed to material deterioration, aging, environmental and/or chemical actions that degrade their structural performance and serviceability. Under such circumstances, damage, failure, and even collapse may occur resulting in expensive human and economic losses. In order to preclude such adverse incidents, structural health monitoring (SHM) opens a marvelous opportunity to make sure of the health, safety, serviceability, and sustainability of civil structures [1], particularly bridges [2,3]. In contrast to other kinds of civil structures, bridges are more sensitive to vibration levels due to their slender and light forms. Such structures have different structural components, design, construction, and loading mechanisms. For these reasons, bridge health monitoring has become an important branch of SHM for damage assessment.

Due to rapid development of sensing technologies in recent years, SHM typically contains three main steps of (i) sensing, data acquisition, and transmission systems, (ii) feature extraction, and (iii) feature analysis/classification. After measuring structural responses, one needs to use a feature extractor to discover meaningful engineering features [4]. Operational modal analysis (OMA) is one of the practical and promising feature extraction methods for identifying modal properties (i.e., natural/modal frequency, mode shape, damping ratio) [5], which serve as dynamic features for SHM. Among them, the natural frequencies are highly suitable for assessment and early warning of damage in bridge structures due to their simple identification and the least requirements for sensor deployment. In addition, such vibration features are directly related to inherent structural properties (i.e., mass and stiffness). Although the change in the natural frequencies is an indicator of damage occurrence, this event may also be stemmed from varying environmental and/or operational conditions. In particular, it was demonstrated that temperature, wind speed and direction, and traffic are the key reasons for the alterations in the natural frequencies of various bridge structures [6]. For these reasons, it is important to investigate and interpret the variability in structural modal frequencies [7–9], develop models for representing relationships between such dynamic responses and environmental and/or operational factors, and then predict modal frequencies [10–12]. However, a demanding issue in SHM under varying environmental and/or operational conditions is the problem of outlier making [13]. The nature of this problem emanates from the fact that the presence of outliers leads to irregularities of data, especially sparse data with high-dimensionality [14], and weakness of a model or method [15]. More precisely, the outlier masking problem means that the existence of different groups of outliers (e.g., compounding environmental and/or operational factors) in the structural features/responses can affect their statistical properties, in which case two events may occur. First, such outliers can mask adverse changes (damage) in the structure leading to a false negative error. Second, those can lead to incorrect alarms of damage occurrence when the structure still operates normally implying a false positive error [16]. Because such errors pertain to economic and safety issues, it is imperative to mitigate the deceptive effects of environmental and/or operational variability [6].

Recently, machine learning has become an outstanding and practical method for real-world vibration-based SHM applications [17]. It is a sector of artificial intelligence that set outs to form an intelligent learner (model) developed from training data for solving complex problems [18]. Depending upon the labels of training data, machine learning is typically decomposed into three classes of supervised, semi-supervised, and unsupervised learning [19]. The advantages and disadvantages of these algorithms for SHM applications have been discussed concisely by Entezami et al. [20]. Because an unsupervised learning model does not require fully labeled data (i.e., no information of the current or possibly damaged state of a structure is needed), it is beneficial to most of the SHM applications, particularly early damage assessment. Anomaly detection is the main sector of unsupervised learning for feature analysis/ classification. Generally, this method aims to develop a model called *anomaly detector* for finding anomalies/outliers and discriminating them from normal samples [21]. Development of this idea to SHM brings a practical strategy for damage assessment in civil structures. For this purpose, it suffices to train an anomaly detector by using unlabeled training data (i.e., the information belonging to the only undamaged/normal state), compute anomaly indices (scores) or damage indicators (DIs) of both training and testing data points, and compare these indices with a threshold.

As discussed, the major challenge in applications of anomaly detection to SHM relates to the negative effects of single and multiple

environmental and/or operational factors, which can seriously affect anomaly detectors. Alongside this challenge, the type of anomaly detector is the utmost importance. Generally, anomaly detectors can be classified as parametric and non-parametric models [22]. Parametric anomaly detectors require unknown components called hyperparameters that should be determined before the learning process, while non-parametric anomaly detectors are independent of this prerequisite. Despite simplicity and computational efficiency of non-parametric anomaly detectors, those may fail in obtaining reliable SHM results under severe environmental and/or operational effects [22]. On the other hand, hyperparameter optimization is a big challenge in machine learning [23] that can encounter an additional barrier to SHM projects. The other issue relates to the framework of an anomaly detector that can be developed from a single model or can be combined with different models to make a hybrid anomaly detector. Although the use of single non-parametric and/or parametric anomaly detectors brings some benefits such as computational efficiency, simple forms, and a few requirements, those may not be robust and effective solutions to complex SHM programs under various environmental and/or operational variation patterns and different feature sizes. Thus, this paper intends to propose a novel hybrid method by addressing the aforementioned challenges.

## 1.1. Related works

Removal of environmental and/or operational effects from modal frequencies is a critical issue in SHM. Anomaly detection in conjunction with data normalization is a robust method for SHM under varying environmental and/or operational conditions. In the context of SHM, data normalization is the main solution to eliminating such conditions. Recently, Wang et al. [6] reviewed various data normalization techniques in terms of input–output and output-only algorithms with an emphasis of the impacts of environmental and/or operational changes on structural modal frequencies. The input–output data normalization relies on supervised regression models, for which the measurements of both environmental and/or operational factors (i.e., the independent or predictor data) and structural responses (i.e., the dependent data) are mandatory [24–26]. In contrast, the output-only data normalization uses the structural responses (e.g., modal frequencies) and unsupervised models to eliminate unmeasured variability sources, which are latent in the responses [27–29]. Once the normalized features have been provided, those are applied to anomaly detectors for damage assessment.

In relation to the applications of anomaly detection to SHM under environmental and operational variability, Sarmadi [22] compared various non-parametric and parametric anomaly detectors by using two different structural features extracted from time series modeling and OMA. It was demonstrated that the non-parametric anomaly detectors are seriously influenced by profound environmental and/or operational variability. Deraemaeker and Worden [30] enhanced the performance of the Mahalanobis-squared distance (MSD), which is one of the commonly-used anomaly detectors for SHM but highly sensitive to the variability sources, in the presence of confounding influences caused by environmental and/or operational conditions. The enhancement was concerned with decomposing the covariance matrix of the training data into independent variables. Zhu et al. [31] developed a temperature-driven moving principal component analysis for anomaly detection under a single environmental factor (i.e., temperature), which influenced strain data. Sousa Tomé et al. [32] assessed the occurrence of damage in a cable-stayed bridge under environmental and operational variations by using the cointegration analysis for data normalization and Hotelling's  $T^2$  control chart for anomaly detection. Daneshvar and Sarmadi [33] developed an innovative information-based anomaly detector getting idea from density peaks clustering for health monitoring of civil structures in short- and long-term monitoring programs with different feature sizes. Entezami et al. [34] proposed a novel non-parametric anomaly detector by leveraging the idea of empirical machine learning for damage assessment in bridge structures.

Hybrid anomaly detection is a new improvement on unsupervised learning. In contrast to most of the state-of-the-art anomaly detectors, which concentrate on single models, a hybrid framework benefits a combination of two or more models that can enhance the overall performance of feature classification and final decision-making. Entezami et al. [35] developed a distance-based hybrid anomaly detector based on a partition-based Kullback-Leibler divergence and the well-known MSD for early damage assessment in a cable-stayed bridge via statistical features extracted from a hybrid time series model. Cadini et al. [36] proposed a hybrid technique to neutralize temperature effects by combining an autoencoder with partial filters. Mousavi and Gandomi [37] merged variational mode decomposition, Johansen cointegration analysis, and a recurrent neural network to assess damage in bridge structures under environmental variability. Entezami et al. [38] proposed an innovative hybrid learning method in the framework of unsupervised meta-learning for early damage assessment in bridges under varying severe environmental conditions. The method consisted of four main stages of an initial data analysis, data clustering, unsupervised subspace learning, and local anomaly detection.

Clustering-based hybrid anomaly detection is among the most successful methods for damage assessment due to preparing local information rather than applying the entire training data. This strategy can yield satisfactory results in SHM when there exists strong variability in engineering features. This is because the outliers caused by environmental and operational changes can impact statistical properties of the whole training data. The use of such data in an anomaly detector keeps the outlier effects and leads to erroneous anomaly values with serious false positive and false negative errors. In the case of partitioning the entire training data into local subsets (clusters), this process can mitigate the outlier effects and improve the performance of the anomaly detector. Most of the clustering-based hybrid anomaly detection methods consist of a data partitioning algorithm for preparing clustered local information and a DI function for determining anomaly values. In this regard, Zhou et al. [39] presented a damage assessment framework by using hierarchical clustering and three dissimilarity functions based on Cosine, Euclidean, and Hausdorff distances. They utilized the transmissibility from structural responses to derive frequency-domain features for damage detection in civil structures. Langone et al. [40] suggested an adaptive kernel spectral clustering along with a DI function through the maximum distance between current data and the centroids of the clusters in the eigenspace for damage assessment. Entezami et al. [20] developed a multi-task unsupervised learning approach in a clustering-based anomaly detection framework for continuous SHM and damage assessment in bridge structures by using spectral clustering and local empirical measures, which the former prepared local information from unlabeled feature data (modal

frequencies) and the latter originated from the theory of empirical machine learning. Sarmadi et al. [41] proposed a novel probabilistic data self-clustering method via the concept of semi-parametric extreme value theory for health monitoring of bridge structures, for which data partitioning and anomaly detection were simultaneously implemented in a single framework. Despite such novel and promising techniques, the effectiveness of clustering-based hybrid anomaly detection strongly depends on its ability to handle the outlier masking problem and correctly detect the current status of a civil structure in terms of being damaged or undamaged. For this issue, the challenges of different environmental/operational variation patterns, the form and type of clustering and anomaly detection algorithms, and the significance of hyperparameters are influential factors for selecting a reliable hybrid method.

#### 1.2. Motivation and novelties

The main motivation behind this paper is to propose a novel double-hybrid learning (DHL) method in terms of clustering-based hybrid anomaly detection for addressing some engineering and technical challenges in relation to real-world SHM applications via the concept of unsupervised learning. The major engineering challenge relates to different environmental and/or operational variation patterns in structural modal frequencies due to differences in structural types, materials, and applications, measurement periods conducted in short and/or long monitoring programs, geographical locations of structures, weather conditions, and influences of single or multiple environmental and/or operational factors. The major technical challenge pertains to the necessity of developing a robust anomaly detector with the minimum hyperparameters. For these issues, the proposed method includes two main stages of data partitioning and anomaly detector modeling, both of which entail two double hybrid algorithms. The first stage sets out to prepare local subsets from training data for anomaly detection with the focus on mitigating the environmental and/or operational effects. Accordingly, this paper proposes an improved hybrid clustering method developed from the ideas of the shared nearest neighbor (SNN) and density peaks clustering (DPC), called here SNN-DPC. The original version of this method proposed by Liu et al. [42] have two main hyperparameters; that is, the number of nearest neighbors for the SNN and the number of clusters needed for the DPC. Hence, the improvement of interest concentrates on determining these hyperparameters. For the number of nearest neighbors, we take advantage of a self-adaptive neighbor searching (SANS) algorithm [43] that enables the SNN-DPC to automatically find adequate nearest neighbors of each data point. Furthermore, a simple but effective approach is developed to determine the number of clusters with an emphasis on mitigating the negative effects of changes in structural modal frequencies resulting from single and/or multiple environmental and operational factors. For the second stage, we propose an innovative non-parametric hybrid anomaly detector based on local outlier factor (LOF). In essence, the LOF is a parametric distance-based anomaly detector, in which the number of nearest neighbors is the main single hyperparameter. On this basis, a non-parametric local outlier factor (NPLOF) method is proposed to address this limitation. For this hybrid anomaly detector, the number of nearest neighbors is obtained from the SANS algorithm. Because the proposed DHL method conforms to an unsupervised learning-based damage assessment framework, some well-accepted assumptions include no damage occurs in the training phase, no environmental and/or operational factors are measured or applied to develop the anomaly detector, and the labels of feature samples (i.e., modal frequencies) are unknown. The key novelties of this paper are summarized as:

- 1) Proposing a novel unsupervised learning method in terms of double-hybrid learning for damage assessment in bridge structures under different environmental variation patterns,
- 2) Leveraging a self-adaptive neighbor searching algorithm with a different searching strategy for automatically determining the number of nearest neighbors needed for both stages of data partitioning and anomaly detection,
- 3) Improving a hybrid clustering technique by concentrating on determining its major hyperparameters and dealing with the problem of different environmental effects
- 4) Proposing a novel hybrid anomaly detector in a non-parametric approach to damage assessment.

Modal frequencies of two full-scale bridges with different environmental variation patterns are used to validate the proposed DHL method along with several comparisons. Results demonstrate that this method succeeds in accurately detecting damage and effectively removing environmental variations. The proposed method is also superior to some state-of-the-art anomaly detection techniques in SHM applications.

# 2. Theoretical backgrounds

# 2.1. Density peaks clustering

The DPC is a new density-based clustering algorithm proposed by Rodriguez and Laio [44], which relies upon density and distance. Compared to many conventional clustering techniques, the greatest advantage of the DPC is the lack of applying additional approaches to choosing the number of clusters. In other words, this hyperparameter is selected based on the intrinsic structure of the DPC and its underlying components; that is, the density and distance. This advantage makes it an easy-to-use and efficient technique for data clustering in engineering and science applications. It can quickly find the dense area of sampling data (i.e., the high-density peaks) without an iteratively computational process or any specific objective function.

The fundamental principal of the DPC lies in this assumption that the cluster centers are characterized by a higher local density than their neighbors and by a relatively large distance from data points with higher density values. Therefore, this method needs two main parameters: (i) a local density of each point and (ii) its distance from the nearest sample with a larger density. The calculation of the local density is usually based on cutoff and Gaussian kernel functions. Given *n* training samples collected into the matrix  $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_n] \in \mathbb{R}^{p \times n}$  and the cutoff kernel function, the local density of the *i*<sup>th</sup> point  $\mathbf{x}_i$ , where i = 1, ..., n, is expressed in the following form:

$$\rho_{i} = \sum_{\mathbf{x}_{i} \neq \mathbf{x}_{j}} \Pi(d_{E}(\mathbf{x}_{i}, \mathbf{x}_{j}) - d_{c}),$$
(1)
$$\Pi(x) = \begin{cases} 1, & x < 0 \\ 0, & x \ge 0 \end{cases}$$
(2)

where  $d_c > 0$  denotes the cutoff distance (i.e., the neighborhood radius of the data point) that should be determined. Moreover,  $d_E(\mathbf{x}_i, \mathbf{x}_j)$  refers to the Euclidean distance between  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , where the latter is the  $j^{\text{th}}$  training sample in **X**. On the other hand, the local density  $\rho_i$  under the Gaussian kernel function is defined as:

$$\rho_i = \sum_{\mathbf{x}_i \neq \mathbf{x}_j} \exp\left(-\left(\frac{d_E(\mathbf{x}_i, \mathbf{x}_j)}{d_c}\right)^2\right)$$
(3)

In Eqs. (1) and (3), it is understood that the local density is dependent upon the number of samples with a distance from  $\mathbf{x}_i$ , which is smaller than  $d_c$ . In addition, the cutoff distance is written as  $d_c = d_u$ , where  $u = [v,\beta]$  and  $d_u$  is the selected distance among all v distance values between every two features; [.] stands for the ceiling function; v is representative of the number of all distance quantities; and  $\beta$  is an arbitrary percentage defined by the user [45]. The other key parameter of the DPC is the local distance  $\delta$ . Having considered the data points  $\mathbf{x}_i$  and  $\mathbf{x}_j$  from the training matrix  $\mathbf{X}$ , the local distance of  $\mathbf{x}_i$  ( $\delta_i$ ) depends on their local density values  $\rho_i$  and  $\rho_j$ . Accordingly,  $\delta_i$  is determined by considering the local density values of  $\mathbf{x}_i$  and  $\mathbf{x}_j$  and the distance between them provided that  $\mathbf{x}_i \neq \mathbf{x}_j$ :

$$\delta_{i} = \begin{cases} \min_{j:\varphi_{j} \ge \rho_{i}} \left( d_{E}(\mathbf{x}_{i}, \mathbf{x}_{j}) \right) \\ \max_{j:\rho_{j} < \rho_{i}} \left( d_{E}(\mathbf{x}_{i}, \mathbf{x}_{j}) \right) \end{cases}$$
(4)

Applying the local density and distance values, the DPC presents a decision value for each data point, which is expressed as  $\gamma_i = \rho_i \times \delta_i$ . Indeed, the calculation of the decision values of all training samples makes a decision graph. Accordingly, it suffices to find the cluster centers that have large decision values. In other words, the DPC chooses the number of clusters and their centers based on large local density and distance quantities [44].

# 2.2. Self-adaptive neighbor searching

Determination of the number of nearest neighbors is the key requirement and hyperparameter for nearest neighbor searching techniques. Despite the development of different approaches to selecting this unknown parameter [46–48], it is still one of the demanding issues in unsupervised learning applications when fully labeled data is not available. Recently, Zhu et. al [43] proposed the idea of SANS that aims to automatically determine the number of nearest neighbors without any additional technique for selection and validation. It should be clarified that a machine learning model can have two types of unknown parameters called model-parameter and hyperparameter [23]. A model-parameter is obtained within the algorithm of the machine learning model or during its learning procedure. In contrast, a hyperparameter needs to be estimated before the learning process by using an additional technique for selection and another one for validation of this selection. With these descriptions, it can be found that one of the great advantages of the SANS method is to modify the role of the number of nearest neighbors from a hyperparameter to a model-parameter. This is because this unknown parameter is obtained and selected within the algorithm of the SANS.

The basis of the SANS method emanates from the friendship in human societies and the idea of mutual friendships [43]. This means that apart from addressing the problem of hyperparameter selection concerning the number of nearest neighbors, this method introduces a new strategy for the neighbor searching. In this regard, the mutual friendships refer to the number of friends of a person in a society/city that he/she takes them as friends and those people also take him/her as a friend. Simply speaking, two people are friends if and only if each of them is mutually friendly towards each other. Hence, it can be made a friendship list of that person by seeking all people in that society/city so that if he/she is more friendly, one can find more friends for him/her and vice versa. Now, suppose that the society/city of interest is a set of features from an SHM program. According to the aforementioned definitions, the SANS method intends to find relevant features, which are equivalent to mutual friends, and disregard irrelevant features, which are similar to strangers in that society/city. This means that if two features are related mutually, those are chosen as the *mutual neighbors*. Such neighbors (features) can be collected into a set in such a way that the number of samples within this set refers to the number of nearest or mutual neighbors. Therefore, one can realize that the SANS presents an automated approach to not only select the mutual neighbors but also determine the number of nearest neighbors.

To find the mutual neighbors of each data point, the SANS requires a search function, a search domain, and a set of mutual neighbors to reach a condition called stable searching state. Given the *n* feature samples in  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n] \in \mathbb{R}^{p \times n}$ , the search function aims at finding the mutual neighbors of the *i*<sup>th</sup> feature ( $\mathbf{x}_i$ ) within the search domain  $1 \le s \le n-1$ . On this basis, the search process should reach the stable searching state for  $\mathbf{x}_i$  when there is a feature such as  $\mathbf{x}_j$  (i.e.,  $\mathbf{x}_i \neq \mathbf{x}_j$ ) so that  $\mathbf{x}_i$  belongs to its mutual neighbor set and  $\mathbf{x}_j$  also belongs to the mutual neighbor set of  $\mathbf{x}_i$ . In this case, both  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are mutual neighbors. Mathematically speaking, this expression can be written as follows:

(5)

$$(\mathbf{x}_i \in \mathbb{M}(\mathbf{x}_j)) \land (\mathbf{x}_j \in \mathbb{M}(\mathbf{x}_i)), \forall \mathbf{x}_i \neq \mathbf{x}_j$$

where

$$\mathbb{M}(\mathbf{x}_i) = \bigcup_{s \in [1, n-1]} \mathbb{F}(\mathbb{M}(\mathbf{x}_i))$$
(6)

where  $\mathbb{M}(\mathbf{x}_i)$  is the set of mutual neighbors of  $\mathbf{x}_i$ . For this, assume that  $\{\widehat{\mathbf{x}}_1^{(i)}, \dots, \widehat{\mathbf{x}}_{i+1}^{(i)}, \widehat{\mathbf{x}}_{i+1}^{(i)}, \dots, \widehat{\mathbf{x}}_n^{(i)}\}$  are *n*-1 nearest neighbors of  $\mathbf{x}_i$ , which are arranged in ascending order based on their distances  $\widehat{d}_E(\mathbf{x}_i, \widehat{\mathbf{x}}_1^{(i)}) \Big\langle \cdots \Big\langle \widehat{d}_E(\mathbf{x}_i, \widehat{\mathbf{x}}_{i+1}^{(i)}) \Big\langle \widehat{d}_E(\mathbf{x}_i, \widehat{\mathbf{x}}_{i+1}^{(i)}) \Big\langle \cdots \Big\langle \widehat{d}_E(\mathbf{x}_i, \widehat{\mathbf{x}}_n^{(i)}) \Big\rangle$  where  $\widehat{d}_E(\mathbf{x}_i, \widehat{\mathbf{x}}_1^{(i)})$  refers to the Euclidean distance between  $\mathbf{x}_i$  and its first nearest neighbor  $\widehat{\mathbf{x}}_1^{(i)}$ . Let  $k_i$  be the number of mutual neighbors of this point, in which case  $\mathbb{M}(\mathbf{x}_i) = [\widehat{\mathbf{x}}_1^{(i)}, \cdots, \widehat{\mathbf{x}}_{k_i}^{(i)}]$ . Moreover, in Eq. (6),  $\mathbb{F}(\mathbb{M}(\mathbf{x}_i))$  denotes the search function for the mentioned set, which can be defined as [43]:

$$\mathbb{F}(\mathbb{M}(\mathbf{x}_i)) = \left\{ \mathbf{x}_h \in \mathbf{X} \land (\mathbf{x}_h \neq \mathbf{x}_i) \middle| \forall \mathbf{x}_j \in \mathbf{X} \land (\mathbf{x}_j \neq \mathbf{x}_h) : d_E(\mathbf{x}_i, \mathbf{x}_h) \le d_E(\mathbf{x}_h, \mathbf{x}_j) \right\}$$
(7)

where i = 1, ..., n and j = h = 1, ..., n-1 so that  $\mathbf{x}_i \neq \mathbf{x}_h$ ,  $\mathbf{x}_i \neq \mathbf{x}_h$ , and  $\mathbf{x}_j \neq \mathbf{x}_h$ ;  $d_E(\mathbf{x}_i, \mathbf{x}_h)$  and  $d_E(\mathbf{x}_h, \mathbf{x}_j)$  are the Euclidean distances between  $\mathbf{x}_i$  and  $\mathbf{x}_h$  as well as  $\mathbf{x}_h$  and  $\mathbf{x}_j$ , respectively. According to Eqs. (6) and (7), it can be constructed the mutual neighbor set of each data point by satisfying the stable searching state expressed in Eq. (5). For all feature samples, their mutual neighbor sets are matrices of p rows and different columns. As a result, the number of columns of each of these matrices is representative of the number of mutual neighbors ( $k_i$ ), which is automatically obtained from the SANS algorithm without any additional techniques for selection and validation.

# 2.3. Local outlier factor

The LOF is a distance-based anomaly detector in the light of a density-based framework [49]. This technique computes an anomaly score (i.e., a LOF value) for each data point by computing the ratio of the local density of the area around that point and local densities of its neighbors (i.e., the average density ratio). On this basis, it is possible to detect anomalies through the relative density of a data point with respect to its surrounding neighborhood. For this purpose, the LOF should initially seek some nearest neighbors of that point and then compute the local reachability densities of the data point of interest and its selected nearest neighbors. Because this technique can assign an anomaly/outlier index to each point, which is the degree of being outlying of that point, one can benefit it as an anomaly detector for damage assessment in civil structures. Despite many distance-based anomaly detectors applicable to SHM, there is less attention to the LOF for vibration-based damage assessment [50–52]. Meanwhile, this paper does not only utilize the classical LOF but also proposes its innovative non-parametric version (i.e., NPLOF) by taking advantage of the SANS method, which merges with the LOF algorithm to develop a hybrid anomaly detector.

Having considered the nearest neighbors of a data point  $\mathbf{x}_i$ , the LOF value of this point is determined in five steps. First, one needs to compute the distances between  $\mathbf{x}_i$  and its q nearest neighbors selected from { $\mathbf{x}_1,...,\mathbf{x}_{i-1},\mathbf{x}_{i+1},...,\mathbf{x}_n$ } by using a user-friendly distance metric such as the Euclidean distance. Second,  $\mathbb{N}(\mathbf{x}_i)$  is constructed as a set of q nearest neighbors of  $\mathbf{x}_i$ . Third, the reachability distance between  $\mathbf{x}_i$  and a new point (vector)  $\mathbf{o} \in \mathbb{N}(\mathbf{x}_i)$  is computed as follows:

$$rd_{q}(\mathbf{x}_{i},\mathbf{o}) = \max(d_{E}(\mathbf{x}_{i},\mathbf{o}), d_{min}(\mathbf{o}))$$
(8)

where  $d_E(\mathbf{x}_i, \mathbf{o})$  stands for the Euclidean distance between  $\mathbf{x}_i$  and  $\mathbf{o}$  and  $d_{min}(\mathbf{o})$  represents the smallest distance among the distances from  $\mathbf{o}$  to its neighbors. Fourth, the local reachability density for the data point  $\mathbf{x}_i$  is derived as follows:

$$lrd_q(\mathbf{x}_i) = \frac{q}{\sum_{\mathbf{o} \in \mathbb{N}(\mathbf{x}_i)} rd_q(\mathbf{x}_i, \mathbf{o})}$$
(9)

Finally, in the fifth step, the LOF value of  $x_i$  is determined by using Eq. (10):

$$LOF(\mathbf{x}_i) = \frac{1}{q} \sum_{\mathbf{o} \in \mathbb{N}(\mathbf{x}_i)} \frac{lrd_q(\mathbf{o})}{lrd_q(\mathbf{x}_i)}$$
(10)

Although Breunig et al. [49] hypothesizes that the LOF value less than or close to one indicates a normal point and a value greater than one refers to an anomaly, this boundary is not necessarily effective and practical for anomaly detection in some engineering problems with many uncertainties and variability conditions. For this reason, in order to develop a rigorous framework for anomaly detection, one should compare any anomaly score (i.e., the LOF value) with a decision threshold. Any deviation from this threshold is indicative of the existence of an anomaly/outlier in sampling data. This procedure is the firm foundation for damage assessment in civil structures.

#### 3. Proposed double-hybrid learning method

The main aim of the proposed DHL method is to benefit the existing algorithms described in Section 2 and develop novel algorithms for data partitioning and anomaly detection. As explained earlier, these algorithms intend to address some engineering and technical challenges concerning health monitoring of bridge structures under different environmental variation patterns. For simplicity, Fig. 1

shows the workflow of the proposed DHL method. In this figure, the collection of the SNN, DPC, and SANS refers to the improved data clustering, which is altered from a non-parametric hybrid clustering algorithm (i.e., the original SNN-DPC) to a parametric hybrid clustering approach. Furthermore, the coupling of the LOF and SANS is concerned with the proposed NPLOF method for anomaly detection, which enables the classical LOF to automatically select adequate nearest neighbors and become non-parametric.

# 3.1. Improved hybrid clustering

## 3.1.1. SNN-DPC

Although the DPC has become one of the widely-used clustering methods in many disciplines in a short period from 2014, it contains some important limitations that directly affects its overall performance. This technique does not consider the local structure of data during calculating the local density. Despite the possibility of selecting the cluster centers without any additional technique, the DPC may yield an inaccurate number of clusters, especially when an outlier inserts into a cluster. Moreover, the cluster center selection depends on a decision graph and the user expertise. Apart from such important limitations, the major drawback of the DPC pertains to its poor performance in the case of severe variability in data. As Liu et al. [42] described, this technique directly computes the distance and density between points without any attention to the environment in which the points are located and their distribution. Regarding the problem of SHM, Daneshvar and Sarmadi [33] demonstrated that the direct use of the DPC is not a robust way for damage assessment under strong environmental changes.

To address these limitations, Liu et al. [42] suggested merging the DPC with the SNN. In this case, the SNN-DPC makes a hybrid clustering approach, for which one can consider the only relevant points (features) and ignore irrelevant ones with high variability. The basic idea behind the SNN is that two data samples resemble together if they have common neighbors. Given the feature samples  $\mathbf{x}_i$  and  $\mathbf{x}_j$ ,  $\mathbb{H}(\mathbf{x}_i)$  and  $\mathbb{H}(\mathbf{x}_j)$  denote the sets of their nearest neighbors. Accordingly, the shared neighbors of  $\mathbf{x}_i$  and  $\mathbf{x}_j$  denoted  $\mathbb{S}(\mathbf{x}_i, \mathbf{x}_j)$  are their common neighbor sets, which can be expressed as follows:

$$\mathbb{S}(\mathbf{x}_i, \mathbf{x}_j) = \mathbb{H}(\mathbf{x}_i) \cap \mathbb{H}(\mathbf{x}_j), \forall \mathbf{x}_i \neq \mathbf{x}_j$$
(11)

An important note is that this strategy is somehow equivalent to the concept of SANS method. Based on this definition, one can derive the shared neighbor similarity measure  $d_s(\mathbf{x}_i, \mathbf{x}_i)$  in the following form:

$$d_{S}(\mathbf{x}_{i}, \mathbf{x}_{j}) = \begin{cases} \frac{N_{s}^{2}}{\sum_{\mathbf{x}_{i} \in \mathbb{S}(\mathbf{x}_{i}, \mathbf{x}_{i})} \left(d_{E}(\mathbf{x}_{i}, \mathbf{x}_{i}) + d_{E}(\mathbf{x}_{j}, \mathbf{x}_{i})\right), & \text{if } \mathbf{x}_{i}, \mathbf{x}_{j} \in \mathbb{S}(\mathbf{x}_{i}, \mathbf{x}_{j}) \\ 0 & \text{if } \mathbf{x}_{i}, \mathbf{x}_{j} \notin \mathbb{S}(\mathbf{x}_{i}, \mathbf{x}_{j}) \end{cases}$$
(12)

where  $N_s$  is the number of shared neighbors of the features  $\mathbf{x}_i$  and  $\mathbf{x}_j$ ;  $d_E(\mathbf{x}_i, \mathbf{x}_l)$  denotes the dissimilarity between  $\mathbf{x}_i$  and  $\mathbf{x}_l$  gained by the Euclidean distance. From Eq. (12), it is understood that the shared neighbor similarity measure holds if two samples appear in their nearest neighbor sets; otherwise,  $d_S(\mathbf{x}_i, \mathbf{x}_i) = 0$ . For non-zero samples, Eq. (12) can be rewritten as follows:

$$d_{S}(\mathbf{x}_{i},\mathbf{x}_{j}) = N_{s}\left(\frac{1}{\frac{1}{N_{s}}\sum_{\mathbf{x}_{i}\in\mathbb{S}(\mathbf{x}_{i},\mathbf{x}_{j})}\left(d_{E}(\mathbf{x}_{i},\mathbf{x}_{i})+d_{E}(\mathbf{x}_{j},\mathbf{x}_{i})\right)}\right)$$
(13)

where the right-hand-side of Eq. (13) represents the reciprocal of the average distance quantity from the samples  $x_i$  and  $x_j$  with



Fig. 1. The workflow of the proposed DHL method consisting of an improved parametric hybrid clustering algorithm for data partitioning and a novel non-parametric hybrid algorithm for anomaly detection developed from some existing algorithms.

respect to all shared neighbors of these samples. This concept refers to the density around them for a specific level. Using the shared neighbors and the density amounts of two samples  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , the shared neighbor similarity measure can be adapted properly to any variability in data. On the other hand, one can apply this measure to determine a new local density for  $\mathbf{x}_i$ . Let  $\mathbb{L}(\mathbf{x}_i)$  be the set of all shared neighbors of  $\mathbf{x}_i$ ; that is,  $\mathbb{L}(\mathbf{x}_i) = \{\tilde{\mathbf{x}}_1, \dots, \tilde{\mathbf{x}}_{N_i}\}$ , containing  $N_s$  samples with the maximum similarity to  $\mathbf{x}_i$ . Accordingly, the local density  $\tilde{\rho}_i$  can be expressed as the sum of all similarity values of  $N_s$  points in  $\mathbb{L}(\mathbf{x}_i)$  as follows:

$$\widetilde{\rho}_i = \sum_{h=1}^{N_s} d_S(\mathbf{x}_i, \widetilde{\mathbf{x}}_h) \tag{14}$$

Unlike the local density of the DPC algorithm, in the one hand, the SNN-related local density  $\tilde{\rho}$  uses the distance information, on the other hand, it obtains content about the cluster framework using the number of shared neighbors ( $N_s$ ). The other important component in the SNN-DPS is the nearest larger density point  $\tilde{\delta}$ . For the data point  $\mathbf{x}_i$ , the calculation of  $\tilde{\delta}_i$  is based on finding a sample such as  $\mathbf{x}_j$  whose local density  $\tilde{\rho}_i$  is greater than  $\tilde{\rho}_i$ . On this basis,  $\tilde{\delta}_i$  is expressed as follows:

$$\widetilde{\delta}_{i} = \min_{j:\widetilde{\rho}_{j}>\widetilde{\rho}_{i}} \left( d_{E}(\mathbf{x}_{i}, \mathbf{x}_{j}) \cdot \left( \sum_{\mathbf{x}_{v} \in \mathbb{H}(\mathbf{x}_{i})} d_{E}(\mathbf{x}_{i}, \mathbf{x}_{v}) + \sum_{\mathbf{x}_{w} \in \mathbb{H}(\mathbf{x}_{j})} d_{E}(\mathbf{x}_{j}, \mathbf{x}_{w}) \right) \right)$$
(15)

This equation makes sense that the local distance  $\tilde{\delta}_i$  is based on the minimization of the distance between  $\mathbf{x}_i$  and  $\mathbf{x}_j$  multiplied by the distances from  $\mathbf{x}_i$  and  $\mathbf{x}_j$  to their  $N_s$  shared neighbors. When  $\tilde{\rho}_j$  is smaller than  $\tilde{\rho}_i$ , the distance  $\tilde{\delta}_i$  is the maximum value among other samples, which can be written as:

$$\widetilde{\delta}_{i} = \max_{j:\rho_{j} < \rho_{i}} \left( d_{E}\left(\mathbf{x}_{j}, \mathbf{x}_{o}\right) \right), \forall \mathbf{x}_{o} \neq \mathbf{x}_{i}$$
(16)

After obtaining the values regarding  $\tilde{\rho}_i$  and  $\tilde{\delta}_i$ , the center cluster can be determined by computing the decision value  $\tilde{\gamma}_i = \tilde{\rho}_i \times \tilde{\delta}_i$ . Similar to the DPC, the feature samples with the largest decision values should be selected as the cluster centers. From the aforementioned descriptions, it can be found that the shared neighbor and cluster numbers are two underlying hyperparameters of the original SNN-DPC. As Liu et al. [42] did not present any specific approach how the number of shared neighbors should be determined, the SANS method is suggested here to deal with this shortcoming.

#### 3.1.2. Cluster number selection

The number of clusters is the other important hyperparameter of the SNN-DPC. Although the manual graphical approach related to the DPC is applicable to this problem, this strategy is not necessarily effective and practical for some engineering projects such as SHM because it depends on the user expertise, analysis, and decision. To cope with this limitation, a simple but effective method is presented here to find the optimal number of clusters with the purpose of mitigating the unfavorable effects of different environmental variation patterns. In contrast to the original version of the SNN-DPC, as the limitation of the number of shared neighbors is addressed by the SANS method, one can state that the improved hybrid clustering is able to decrease the number of hyperparameters to one.

Algorithm 1. Cluster number selection of the improved hybrid clustering

**Inputs**: Training data, the number of the initial cluster ( $c_0$ ), the number of the final cluster ( $c_f$ ), and the number of initial mutual neighbors ( $k_0$ ) **Note**: Steps 4 and 5 are implemented in the second stage of the proposed DHL method related to the hybrid anomaly detector (see Section 3.2).

For  $i = c_0$ :  $c_f$  Do

- 2) Run the SANS algorithm to select the final number of mutual neighbors from  $k_0$  samples.
- 3) Re-cluster the training data into *i* clusters by considering their selected mutual neighbors.
- 4) Compute the anomaly values of the training samples by the proposed NPLOF method.
- 5) Estimate a threshold limit through the anomaly values of the training samples.

- 7) Calculate the false positive rate (FPR) via Eq. (17).
- 8) Store the FPR of the *i*<sup>th</sup> trial in a vector.
- End For

**Output**: The optimum cluster number (*c*)

To choose the optimum cluster number, the proposed approach relies on the performance of the proposed DHL method in yielding the minimum rate of false positive error. Simply speaking, this procedure considers a cluster domain and compute the false positive rate (FPR) of the DHL method during the training period. The sample cluster (trial) with the minimum FPR is chosen as the optimum cluster number. The great advantage of this approach is its engineering problem-solving attribute. As mentioned earlier, the main objective of the first stage of the proposed DHL method is to prepare local information with the focus on mitigating the environmental and/or operational effects. Hence, the proposed cluster number selection enforces the improved hybrid clustering to provide clusters whose features allow the proposed hybrid anomaly detector to yield the minimum FPR. On the other hand, the other advantage of the proposed approach is to avoid applying some complex algorithms of the original SNN-DPC related to choosing inevitable and possible subordinate points [42]. Because the first stage of the DHL method is carried out during the training period, it suffices to apply the

<sup>1)</sup> Run the SNN-DPC to divide the training data into i clusters by selecting  $k_0$  mutual neighbors.

<sup>6)</sup> Compare the anomaly values with the threshold limit.

<sup>9)</sup> Find the minimum FPR value in the vector obtained from Step 8.

training data related to the normal condition of the structure. In this case, the proposed approach divides the training data points into different clusters in an effort to compute their anomaly indices by the proposed hybrid anomaly detector (see Section 3.2), estimate a threshold limit for each sample cluster, and finally compare the anomaly indices with the threshold. On this basis, the FPR is computed as follows:

$$FPR(\%) = 100 \left(\frac{N_p}{n}\right) \tag{17}$$

where  $N_p$  denotes the number of false positives, which is equivalent to the number of all anomaly indices of the training points exceeded the threshold limit, and *n* is the number of all training samples. For simplicity, Algorithm 1 gives the step-by-step strategy for selecting the cluster number as regards the improved hybrid clustering in conjunction with the SANS. Eventually, once the main requirements of the improved hybrid clustering have been determined, this technique divides the training data  $\mathbf{X} \in \mathbb{R}^{p \times n}$  into *c* clusters { $\mathbf{C}_1,...,\mathbf{C}_c$ }, each of which is a matrix of *p* rows and different columns. Indeed, these clusters are the main outputs of the first stage of the proposed DHL method, which are subsequently applied to the hybrid anomaly detector.

### 3.2. Hybrid anomaly detector

The proposed hybrid anomaly detector is based on the coupling of the LOF and SANS as shown in Fig. 1. The main motivation for proposing this method is to cope with the major limitation of the classical LOF concerning its parametric nature and dependency on the number of nearest neighbors, which needs an additional technique for selection and another one for validation. Accordingly, the SANS method is merged with the LOF to make it a non-parametric anomaly detector by automatically determining the number of nearest or mutual neighbors. The other significant matter is related to the implementation of anomaly detection for SHM based on the outputs of the first stage of the proposed DHL method. Generally, this process is performed in the training and monitoring phases, which are equivalent to offline learning and online damage assessment. During the training phase in a fixed period, local subsets or clusters of the training samples are provided by the improved hybrid clustering. These subsets are related to the known state of the structure (i.e., the undamaged/normal condition). In the monitoring phase, the current status of the structure is unknown, which can be undamaged or damaged. Therefore, the local subsets from the training stage are utilized to compute a new anomaly score for any test point.

Having considered the *c* local clusters { $C_1,...,C_c$ }, the NPLOF-based anomaly detection begins by finding the mutual neighbors of the *i*<sup>th</sup> feature  $\mathbf{x}_i$ , where i = 1,...,n, from the clustered features of each subset by using the SANS algorithm. Subsequently, one can construct the mutual neighbor set of  $\mathbf{x}_i$  regarding the cluster/subset of interest, which is defined here as  $M_j(\mathbf{x}_i)$ , where j = 1,...,c. Using these outputs and Eqs. (8)-(10), it can be computed an anomaly value for  $\mathbf{x}_i$  with respect to the *j*<sup>th</sup> cluster in the following form:

$$NPLOF_{j}(\mathbf{x}_{i}) = \frac{1}{\widehat{k}_{i}^{(j)}} \sum_{\mathbf{o} \in \mathcal{M}_{j}(\mathbf{x}_{i})} \frac{lrd_{k}(\mathbf{o})}{lrd_{k}(\mathbf{x}_{i})}$$
(18)

where

$$drd_{\hat{k}}(\mathbf{x}_{i}) = \frac{\hat{k}_{i}^{(j)}}{\sum_{\mathbf{o}\in\mathcal{M}_{j}(\mathbf{x}_{i})} rd_{\hat{k}}(\mathbf{x}_{i},\mathbf{o})}$$
(19)

$$rd_{i}(\mathbf{x}_{i}, \mathbf{0}) = \max(d_{E}(\mathbf{x}_{i}, \mathbf{0}), d_{\min}(\mathbf{0})), \forall \mathbf{0} \in \mathbb{M}_{i}(\mathbf{x}_{i})$$

$$\tag{20}$$

where  $\hat{k}_i^{(j)}$  represents the number of mutual neighbors of the *i*<sup>th</sup> training point regarding the *j*<sup>th</sup> cluster. Once the anomaly scores of  $\mathbf{x}_i$  for all clusters have been determined, the final score is the minimum quantity of all computed values:

$$NPLOF(\mathbf{x}_i) = \min\{NPLOF_j(\mathbf{x}_i)\}, \forall j = 1, \cdots, c$$
(21)

At the end of the training stage, one can derive *n* anomaly indices of all training points  $\{DI(\mathbf{x}_1),...,DI(\mathbf{x}_n)\}$ , which are corresponding to  $\{NPLOF(\mathbf{x}_1),...,NPLOF(\mathbf{x}_n)\}$ .

Regarding the current structural state as regards the monitoring phase, it is assumed that only one test point ( $z_l$ ) is available at each time, where l = 1,...,m and m denotes the total number of test points collected sequentially. The process of anomaly score calculation of  $z_l$  exactly resembles the training feature  $x_i$ . Applying the local clusters { $C_1,...,C_c$ }, it suffices to find the mutual neighbors of  $z_l$  within each cluster and make its mutual neighbor set defined as  $M_j(z_l)$ . In the following, one needs to compute the NPLOF value of  $z_l$  with respect to each cluster as follows:

$$NPLOF_{j}(\mathbf{z}_{l}) = \frac{1}{\tilde{t}_{l}^{(j)}} \sum_{\mathbf{v} \in \mathcal{M}_{j}(\mathbf{z}_{l})} \frac{lrd_{i}(\mathbf{v})}{lrd_{i}(\mathbf{z}_{l})}$$
(22)

where

$$lrd_{i}(\mathbf{z}_{l}) = \frac{\hat{t}_{l}^{(j)}}{\sum_{\mathbf{v} \in \mathbb{M}_{j}(\mathbf{z}_{l})} rd_{i}(\mathbf{z}_{l}, \mathbf{v})}$$
(23)

(24)

$$rd_{\hat{t}}(\mathbf{z}_l, \mathbf{v}) = \max(d_E(\mathbf{z}_l, \mathbf{v}), d_{min}(\mathbf{v})), \forall \mathbf{v} \in \mathbb{M}_i(\mathbf{z}_l)$$

where  $\hat{t}_l^{(j)}$  stands for the number of mutual neighbors of the  $l^{\text{th}}$  test feature concerning the  $j^{\text{th}}$  cluster. Similar to Eq. (21), the minimum value among all *c* NPLOF quantities is selected as the final anomaly value of  $\mathbf{z}_l$ . By collecting *m* test data points during the monitoring period, one can determine *m* anomaly indices { $DI(\mathbf{z}_1),...,DI(\mathbf{z}_m)$ }, which are corresponding to { $NPLOF(\mathbf{z}_1),...,NPLOF(\mathbf{z}_m)$ }.

To assess the current state of the structure, it is necessary to compare any anomaly score with a decision threshold, which can be determined under the significance level  $\alpha$  in the following form:

$$\tau_a = \mu_x + \xi_a \sigma_x \tag{25}$$

where  $\mu_x$  and  $\sigma_x$  denote the scalar values relating to the mean and standard deviation of  $\{DI(\mathbf{x}_1), ..., DI(\mathbf{x}_n)\}$ . Moreover,  $\xi_\alpha$  is defined as the  $(1-\alpha)$  critical value of the distribution concerning the mentioned anomaly indices, which is expressed as follows:

$$\xi_{\alpha} = \sqrt{2} \mathscr{F}^* (1 - \alpha) \tag{26}$$

where  $\mathscr{F}^*(1-\alpha)$  is the inverse error function for the  $(1-\alpha)$  critical value.<sup>‡</sup> The error function is mainly defined as [53]:

$$\mathscr{F}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt \tag{27}$$

Since the anomaly quantities { $DI(\mathbf{x}_1),...,DI(\mathbf{x}_n)$ } relate to the undamaged structural state, it is anticipated that these insert in the area below the threshold limit. For any anomaly value of the test point, if  $DI(\mathbf{z}_l) > \tau_a$ , this means that the structure suffered from damage and the current state is damaged. In contrast, if  $DI(\mathbf{z}_l) \le \tau_a$ , one can deduce that the structure still operates normally and the current state is undamaged.

### 4. Results and discussions

# 4.1. The Z24 Bridge

This structure was a three-span concrete box-girder bridge built in Switzerland [54]. Fig. 2 displays an actual image of the Z24 Bridge along with the elevation view and the span lengths. The superstructure of the Z24 Bridge contained a two-cell box girder along with tendons in the three webs. The major piers were constructed from concrete diaphragms entirely attached to the superstructure. A monitoring test in a long-term manner within one year between 1997 and 1998 was carried out to measure structural (acceleration) responses and different environmental factors (i.e., air temperature, humidity, rain, wind speed, and wind direction). Furthermore, vibrations caused by traffic underneath the bridge were considered as the main unmeasured excitation source [55]. In 1998, it was decided to demolish this structure to construct a new bridge with wider spans. Before this process, realistic damage cases were considered in a controlled manner to simulate a damaged condition.

An automated OMA was performed by Peeters and De Roeck [54] to extract modal properties from acceleration responses under different measured environmental factors. In this paper, the final set of the identified natural frequencies concerning four modes are incorporated to validate the proposed method for damage assessment. This set consists of 3932 frequency samples associated with the undamaged (normal) and damaged structural states as shown in Fig. 3. As can be seen, there are many important variations (i.e., sudden jumps) in the natural frequencies of the undamaged state, particularly in the samples 1–1500. As Peeters and De Roeck [54] described, despite the measurement of various environmental factors such as the wind characteristics, rainfall, and humidity, the air temperature was the underlying reason for the variability in the bridge modal frequencies. Notably, the asphalt layer in the continuous deck system of the Z24 Bridge was stiffened in freezing days (i.e., the measurements approximately around the samples 500–1500), when the air temperature was below 0 °C. This phenomenon increased the bridge stiffness leading to considerable increases (i.e., sudden jumps) in the modal frequencies. This is a critical effect of the air temperature on long-term monitoring of civil structures during freezing weather so that the structural natural frequencies often emerge as sharp increases [38]. The other important conclusion is that the air temperature is dominant the other measured environmental parameters such as humidity, wind speed and direction, and rainfall. Therefore, the SHM problem of the Z24 Bridge encounters the single environmental factor.

Using the natural frequencies of the undamaged and damaged conditions, the training data is comprised of 3127 samples (n = 3127), i.e.,  $\mathbf{X} \in \mathbb{R}^{4 \times 3127}$ , which is equivalent to 90% of all features of the normal/undamaged state. Moreover, the test data is a collection of 805 samples ( $m = \tilde{n} + \tilde{m} = 805$ ), i.e.,  $\mathbf{Z} \in \mathbb{R}^{4 \times 805}$ , including 348 natural frequencies of the undamaged condition, which are utilized as the validation data ( $\tilde{n}$ ), and 457 natural frequencies of the damaged condition ( $\tilde{m}$ ). Notice that it is assumed that the 805 test points are collected sequentially in order to meet the prerequisite for online damage assessment. Considering the training points, the proposed DHL method begins by selecting its single hyperparameter; that is, the cluster number *c*. For this purpose, the inputs of the cluster number selection include the training data  $\mathbf{X}$ , the initial and final cluster numbers  $c_0 = 2$  and  $c_f = 20$ , and the initial mutual neighbor number  $k_0 = 50$ . Based on Algorithm 1, Fig. 4(a) displays the variations in the FPR amounts under 19 trials (i.e., the sample clusters 2–20). As this figure appears, the optimum cluster number coincides with the seventh sample; that is, *c* = 7. Moreover, Fig. 4(b) illustrates the numbers of the final mutual neighbors of the seven clusters related to Step 2 of Algorithm 1. Based on these outputs, the

<sup>&</sup>lt;sup>‡</sup> Under a significance level *a*, the inverse error function can be determined by the MATLAB default function "erfinv".



Fig. 2. The Z24 Bridge: (a) an actual image, (b) the elevation view and the span lengths.



Fig. 3. The natural frequencies of the Z24 Bridge.



Fig. 4. Cluster number selection regarding the Z24 Bridge: (a) selection of the optimum cluster, (b) the number of the final mutual neighbors of each optimum cluster.

improved hybrid clustering splits the training data into the seven clusters  $\{C_1, ..., C_7\}$ . For more details, Fig. 5(a) displays the distribution of the training samples in these clusters and their cluster labels, which vary from 1 to 7. Additionally, Fig. 5(b) shows the number of samples in each cluster.

After finishing the data partitioning, the local clusters are moved to the second stage of the proposed DHL method to compute the anomaly indices of the training and test data based on the proposed NPLOF approach. In this regard, the numbers of mutual neighbors related to the minimum NPLOF values (i.e., the final anomaly indices) in the training and monitoring phases (i.e., the values of  $\hat{k}_i$  and  $\hat{t}_i$ , where i = 1,...,3127 and l = 1,...,805) are shown in Fig. 6. From this figure, one can see that the minimum and maximum mutual neighbor numbers are  $\hat{k}_{min} = 8$  and  $\hat{k}_{max} = 17$  for the training points and  $\hat{t}_{min} = 7$  and  $\hat{t}_{max} = 12$  for the test points. Using 3127 training samples related to the training phase, it can be determined their anomaly quantities  $\{DI(\mathbf{x}_1), \dots, DI(\mathbf{x}_{2127})\}$  and then applied them to estimate a threshold through a small significance level. In order to ensure the estimate of a reliable threshold, a simple but effective grid search process is considered here to determine the optimal value of the significance level ( $\alpha$ ) under some samples. Because the process of threshold estimation should be carried out by using the anomaly indices of the training features, the main criterion for choosing the optimal value of  $\alpha$  is the FPR as expressed in Eq. (17). On this basis, the optimal significance level is one that yields the minimum number or percentage of the FPR. Table 1 lists the values of  $\xi_{\alpha}$  as well as the numbers and percentages of the FPR using six samples of the significance level. As the FPR quantities appear, the best performance occurs in the significance level  $\alpha = 0.00001$  (i.e.,  $\xi_{\alpha}$  = 4.41). It should be clarified that  $\alpha$  = 0.000001 yields the same outputs, while we select the fifth sample in order to prevent a redundant increase in the threshold value leading to increasing the false negative rate (FNR). Furthermore, it needs to mention that the outputs related to the FPR are based on the number of training samples (n = 3127) and they differ from the corresponding outputs regarding the overall performance of the proposed method based on the total number of undamaged features (i.e.,  $n + \tilde{n} = 3475$ ).

In relation to the monitoring phase, each test point is incorporated to compute its anomaly score and compare with the estimated threshold. By gathering the DIs of all training and test data, Fig. 7 displays the result of damage assessment in the Z24 Bridge via the proposed DHL method, where the horizontal line is indicative of the decision threshold. It is seen that nearly all of the anomaly indices of the training and validation data labeled as "Training Phase (NC)" and "Monitoring Phase (NC)" are below the threshold line. Although few false positive errors are noticeable (i.e.,  $FPR(\%) = 100(N_p/(n + \tilde{n})) = 0.49\%$ , where  $N_p = 17$ , n = 3127, and  $\tilde{n} = 348$  denote the number of anomaly indices of both training and validation samples incorrectly exceeded the threshold line, the total number of training samples, and the total number of validation samples, respectively), one can discern that the sudden jumps in the natural frequencies, especially between the samples 500–1500, are no longer visible in Fig. 7. This conclusion clearly demonstrates the robustness and effectiveness of the proposed DHL method in eliminating the serious environmental effect stemming from the temperature variability. Alternatively, most of the anomaly indices of the test points belonging to the damaged state labeled as "Monitoring Phase (DC)" are over the threshold with the exception of three samples, which denote the number of false negatives ( $N_n = 3$ ) leading to the FNR equal to 0.65% (i.e.,  $FNR(\%) = 100(N_n/\widetilde{m})$ , where  $\widetilde{m} = 457$  denotes the total number of unknown test samples regarding the damaged state). Without the threshold, one can perceive that the anomaly indices of the damaged state are sufficiently larger than the corresponding indices concerning the undamaged state. Therefore, this conclusion verifies that the proposed DHL method could provide reasonable detectability of damage so that the occurrence of damage is clearly distinguishable.

In spite of the effective performance of the proposed method, it is compared with some state-of-the-art techniques. For this purpose, the first comparison is concerned with the process of data partitioning. The improved hybrid clustering depends on the SNN algorithm and some mutual neighbors. Although the well-known DPC can also provide local information for anomaly detection, it is investigated



Fig. 5. Data partitioning via the improved hybrid clustering regarding the Z24 Bridge: (a) the distribution of training data points in seven clusters and their labels, (b) the number of clustered training points in each cluster.



Fig. 6. Number of mutual neighbors of the minimum NPLOF (the final anomaly value) regarding the Z24 Bridge: (a) the training phase, (b) the monitoring phase.



Parameters	Sample significance levels					
	0.05	0.01	0.001	0.0001	0.00001	0.000001
$\xi_{\alpha}$ FPR (No.)	1.96 36	2.57 30	3.29 24	3.89 20	4.41 16	4.89 16
FPR (%)	1.15	0.95	0.76	0.64	0.51	0.51



Fig. 7. Damage assessment in the Z24 Bridge via the proposed DHL method.

to demonstrate the effects of the SNN, SANS, and the proposed algorithm of the cluster number selection on damage assessment of the Z24 Bridge under severe temperature variability. In other words, these techniques are disregarded to implement anomaly detection via the proposed NPLOF by using the outputs (i.e., new local clusters) of the DPC. Employing  $\beta = 0.05$  and the Gaussian kernel function, the DPC begins by calculating the cutoff distance  $d_c$  and local density quantity for each training sample. Finally, the local distance  $\delta$  is computed. Having considered the distance and local density values of all training points, it can be obtained their decision values (i.e.,  $\gamma_i = \rho_i \times \delta_i$ ) and drawn a decision graph as shown in Fig. 8(a). As can be seen, the DPC needs two clusters to partition the training data. Furthermore, Fig. 8(b) indicates the result of damage assessment through the anomaly indices gained by the NPLOF, where the horizontal line is a threshold gained by the same approach presented in Section 3.2. As this figure appears, the combination of the DPC and NPLOF, which also makes a hybrid learning method called DPC-NPLOF, could not succeed in detecting damage as good as the proposed DHL method. Alongside the limitation of the graphical way for choosing the cluster number needed for the DPC and the elimination of the sudden jumps in the anomaly indices of the normal condition, it is clear in Fig. 8(b) that the DPC-NPLOF yields much



Fig. 8. Damage assessment in the Z24 Bridge by the DPC-NPLOF: (a) the manual cluster selection related to the DPC, (b) the evolution of anomaly values.

more false positive errors compared to the proposed DHL method (see Table 2). It should be noted that the preparation of local subsets and also utilization of NPLOF are the primary reasons for removing the environmental effects from the anomaly values of the DPC-NPLOF.

The second comparison aims to investigate the performance of the proposed DHL method against some state-of-the-art anomaly detection techniques developed from the SVD [22], MSD, Gaussian mixture model (GMM) coupled with the MSD [22], and principal component analysis (PCA) coupled with the MSD [56]. It is worth remarking that the first two techniques present single anomaly detectors, in which case the entire training data is employed to develop the SVD and MSD models. In contrast, the GMM-MSD makes a hybrid anomaly detector similar to the proposed DHL method, for which the GMM undertakes preparing local information (i.e., clusters) and then transfers them to the MSD for anomaly detection. In addition, the PCA-MSD is another hybrid anomaly detector based on unsupervised (output-only) data normalization conducted by the PCA and anomaly detection via the MSD. As such, the second comparison contains various types of techniques for anomaly detection. The other note is that both the GMM and PCA are parametric techniques and their hyperparameters (i.e., the number of components for the GMM and the number of principal components related to the PCA) should be determined priorly. In this paper, these hyperparameters are selected by using the Bayesian criterion information (BIC) [29] and the eigenvalue ratio indicator [30]. Under 20 sample components, the optimal number of components based on the BIC is equal to 8. Moreover, the optimal number of principal components corresponds to 2 under a threshold identical to 99%. The results of damage assessment via the SVD, MSD, GMM-MSD, and PCA-MSD are shown in Fig. 9(a)-(d), respectively, where the horizontal lines are the threshold limits gained by the same approach, i.e., Eq. (25). It needs to clarify that the same significance level as the proposed method is considered to have fair comparisons. Furthermore, Table 2 lists the decision-making errors including false positive, false negative, and misclassification associated with the aforementioned techniques as well as the DHL and DPC-NPLOF.

Apart from the GMM-MSD, the SVD, MSD, and PCA-MSD are seriously influenced by the environmental (temperature) variability, particularly in the samples 500–1500. The worst performance is concerned with the SVD-based anomaly detector so that some anomaly indices of the mentioned samples are greater than the corresponding indices of the damaged condition. In addition, as the quantities in Table 2 reveal, the SVD yields much more false positive errors compared to the other techniques. In relation to the anomaly indices of the damage state, one can perceive in Fig. 9 that nearly all of them are either similar or smaller than the corresponding indices of the undamaged state. Having considered Table 2, all the state-of-the-art techniques provide considerable false negative errors, which seriously degrade their accuracy rates (i.e., seriously increase their misclassification rates). In comparison with the DHL and DPC-NPLOF, it can be realized that the proposed non-parametric hybrid anomaly detector greatly outperforms the SVD and MSD-based anomaly detectors as regards the FNR. On the other hand, regardless of the threshold lines, it is discerned that the state-of-the-art techniques could not provide discriminative DIs for differentiating the damaged condition from the undamaged one. This is because the majority of the anomaly indices of the damaged state are similar to the corresponding indices concerning the normal condition. The other important conclusion is that the clustering-based hybrid anomaly detectors such as the DHL, DPC-NPLOF, and

Table 2

Numerical performance evaluations of the proposed and state-of-the-art methods in the damage assessment problem of the Z24 Bridge.

Methods	Performance evaluation metrics				
	False positive	False negative	Misclassification		
DHL	17 (0.49%)	3 (0.65%)	20 (0.50%)		
DPC-NPLOF	147 (4.23%)	2 (0.43%)	149 (3.78%)		
SVD	104 (2.99%)	305 (66.88%)	409 (10.40%)		
MSD	103 (2.96%)	245 (53.73%)	348 (8.85%)		
GMM-MSD	0 (0.00%)	185 (40.57%)	185 (4.70%)		
PCA-MSD	89 (2.56%)	222 (48.68%)	311 (7.91)		



Fig. 9. Damage assessment in the Z24 Bridge by some single and hybrid anomaly detection techniques: (a) SVD, (b) MSD, (c) GMM-MSD, (d) PCA-MSD.

GMM-MSD surpass the single anomaly detectors for removing profound environmental effect. This validates the great advantage of methods developed from the concept of local learning and data clustering for providing more reliable results compared to techniques based on global learning.

In the accomplished comparison, there is an ambiguity in the good performance of the DHL in contrast with the GMM-MSD related to the FPR. To better demonstrate the superiority of the proposed method and directly compare it with the GMM-MSD, the results of damage assessment without the threshold lines are re-drawn and shown in Fig. 10. In Fig. 10(a), despite the elimination of the temperature variability available in the normal features between the samples 500–1500 (see Fig. 3), it is clear that the majority of the anomaly indices of the damaged state have similar values with the corresponding indices related to the normal condition. In contrast, as Fig. 10(b) appears, the proposed DHL method not only removes the temperature variability (i.e., the sudden jumps in the samples 500–1500 are no longer visible) but also improves the detectability of damage so that only a few anomaly indices of the damaged state (i.e., three points) have similar quantities with the normal condition. Thus, it can be concluded that the proposed method is superior to the state-of-the-art techniques.

# 4.2. The Yonghe Bridge



This structure is a five-span cable-stayed concrete bridge in China, as shown in Fig. 11(a) [57], which was constructed from continuous pre-stressed box-girder and precast segments [58]. The total length of this bridge corresponds to 510 m containing the

Fig. 10. Graphical comparisons regarding the damage assessment problem of the Z24 Bridge with the threshold limits: (a) GMM-MSD, (b) DHL.

major span of 260 m and two side spans of 25.15 and 99.85 m as shown in Fig. 11(b). In 2005, some serious cracks were found at the bottom of one girder segment over the mid-span. Moreover, some cables near the anchors were corroded severely. On this basis, a major rehabilitation program was carried out to replace the damaged girder segment and all cables. In 2007, it was decided to monitor the Yonghe Bridge using an SHM system. Nevertheless, in August 2008, some new damage cases including a considerable crack at the left side of the bridge (i.e., the side towards Tianjin) and separation between the girder and the piers at both sides were found over a regular inspection.

Due to the installation of the SHM system, it is feasible to employ vibration and environmental data for SHM. The bridge vibration responses are acceleration time histories recorded from fourteen uniaxial accelerometers permanently mounted on the bridge deck. In addition, an anemoscope and a temperature sensor were attached on the top of the south tower and the mid-span of the girder to measure the wind speed and air temperature, respectively. Because the environmental data is not available, this paper only considers the acceleration responses of thirteen accelerometers (i.e., the acceleration time histories of the tenth accelerometer were not recorded properly) measured on January 1, January 17, February 3, March 19, March 30, April 19, May 5, May 18, and July 31, 2008 [35] to investigate the performance of the proposed DHL method. According to Li et al. [59], the first eight days and last day are equivalent to the undamaged and damaged states of the Yonghe Bridge, respectively.

Using an automated OMA based on the fast Fourier decomposition conducted by Sarmadi et al. [41], 24 natural frequencies per day were identified as the dynamic features. In total, 216 natural frequencies were obtained so that the first 192 features relate to the undamaged state and the other 24 features are associated with the bridge damaged state. Fig. 12 shows the identified natural frequencies. As Li et al. [60] discussed, the air temperature and wind speed had the significant influences on the variability in the bridge natural frequencies. In particular, since the low wind speed was dominant, the positive aerodynamic stiffness effect on the bridge under such a wind condition led to increases in the natural frequencies. This is most likely the main reason for the increases in the identified natural frequencies of the normal condition in Fig. 12. Alternatively, there exist decreases in the structural frequencies, which might be attributable to relative increases of daily temperature or traffic volume. This means that in contrast to the Z24 Bridge with the single dominant environmental factors based on the measured temperature and wind speed and other unmeasured environmental and/or operational conditions. Thus, this case study allows us to further assess the performance of the proposed DHL method under compounding environmental effects.

For damage assessment in the Yonghe Bridge, the training matrix consists of 172 natural frequencies of the structural normal condition, i.e.,  $\mathbf{X} \in \mathbf{R}^{4 \times 172}$  (n = 172), which is equivalent to the training rate of 90%. Furthermore, the test matrix comprises the remaining 20 natural frequencies of the undamaged state, which serve as the validation set ( $\tilde{m} = 20$ ), and all 24 natural frequencies of the damaged state ( $\tilde{n} = 24$ ), i.e.,  $\mathbf{Z} \in \mathbf{R}^{4 \times 44}$ , where  $m = \tilde{m} + \tilde{n} = 44$ . Using the training data, Fig. 13(a) illustrates the variations in the



Fig. 11. The Yonghe Bridge: (a) an actual image [57], (b) the elevation view and span lengths [58].



Fig. 12. The natural frequencies of the Yonghe Bridge.

FPR values for nine sample clusters (i.e., the inputs of Algorithm 1 are  $c_0 = 2$ ,  $c_f = 10$ ,  $k_0 = 20$ ). As can be seen, the optimum cluster coincides with the third trial (i.e., c = 3), which yields FPR = 0%. Furthermore, the mutual neighbor numbers of the three clusters are presented in Fig. 13(b). Using these outputs, the improved hybrid clustering divides the training data into three clusters {C<sub>1</sub>,...,C<sub>3</sub>}. On this basis, Fig. 14 shows the distribution of the training samples in these clusters along with their labels and the number of clustered training features in each cluster.

Once the step of data partitioning has been terminated, the NPLOF-based anomaly detector is utilized to compute the anomaly values of the training and testing points. Fig. 15 shows the number of mutual neighbors of these points, where  $\hat{k}_{min} = 5$ ,  $\hat{k}_{max} = 9$ ,  $\hat{t}_{min} = 3$  and  $\hat{t}_{max} = 6$ . Applying 172 training samples, their anomaly quantities { $DI(\mathbf{x}_1), ..., DI(\mathbf{x}_{172})$ } are determined by the proposed NPLOF method and then used to estimate a threshold. Similar to the previous example, the grid search process with six samples of the significance level is implemented to select the optimal value. On this basis, Table 3 lists the numbers and percentages of the FPR of the six samples. As can be observed, the optimal significance level is identical to the fifth sample; that is,  $\alpha = 0.00001$ . Although the sixth sample reaches the same result as the fifth sample, it is ignored to be applied in order to prevent redundant increases in the threshold limit and the FNR.

For all test points, their anomaly values { $DI(\mathbf{z}_1),...,DI(\mathbf{z}_{44})$ } are also computed by the NPLOF and then compared with the estimated threshold for damage assessment. Fig. 16 presents the result of damage assessment concerning the Yonghe Bridge. As can be seen, all anomaly indices associated with the undamaged condition fall below the threshold line without any false positive error (i.e., FPR = 0%). This conclusion clearly verifies how the proposed DHL method could effectively cope with the compounding environmental variability and correctly predict the normal condition of the bridge. The other result is that all anomaly indices of the damaged state accurately exceed the threshold line implying no false negative error (i.e., FNR = 0%). Without considering the threshold, it is obvious that the DI values of the damaged state are clearly distinguishable from the values related to the undamaged state indicating reasonable detectability of damage provided by the proposed method.

For further investigations, the same comparative analyses as the preceding case study are implemented here. For the first comparison via the hybrid technique DPC-NPLOF, Fig. 17(a) shows the decision graph of the DPC. Accordingly, one can realize that this clustering approach needs two clusters. In addition, the result of damage assessment is shown in Fig. 17(b). Even though the DPC-NPLOF technique correctly detected the damaged state of the Yonghe Bridge with no false negative error, one can observe in Fig. 17(b) that this technique could not be properly successful in addressing the problem of compounding environmental effects due to many false positive errors (i.e., FPR = 9.37% for 18 false positives out of 192 samples). In particular, it is discerned that some anomaly indices related to the normal condition are larger than the corresponding indices associated with the damaged state. Therefore, this comparison confirms the positive influence and great performance of the proposed DHL method for damage assessment under the compounding environmental variability.

In the other comparison, Fig. 18 illustrates the results of damage assessment in the Yonghe Bridge using the single and hybrid anomaly detection techniques; that is, SVD, MSD, GMM-MSD, and PCA-MSD. Moreover, Table 4 numerically evaluates and compares the proposed and state-of-the-art methods in terms of the numbers and percentages of the false positive, false negative, and misclassification errors. From Fig. 18(a),(b), and (d) as well as the error values in Table 4, one can perceive that the compounding environmental variations in the bridge natural frequencies cause false positives in the anomaly indices of the SVD, MSD, and PCA-MSD. Among the state-of-the-art techniques, the PCA-MSD gave the worst performance in the false positive, false negative, and misclassification errors. In contrast, as the data in Table 4 reveals, the best performance belongs to the proposed DHL method without any error. An interesting note in Fig. 18 is that most of the false positive errors occurred on the initial and last days of measurements, which coincide with the samples 1–48 and 144–192. This is also compatible with the variability in the bridge natural frequencies within these domains (see Fig. 12). Therefore, these observations prove that the SVD, MSD, and PCA-MSD techniques could not properly remove the



Fig. 13. Cluster number selection regarding the Yonghe Bridge: (a) selection of the optimum cluster, (b) the number of final mutual neighbors of each optimum cluster.



**Fig. 14.** Data partitioning via the improved hybrid clustering regarding the Yonghe Bridge: (a) the distribution of training data points in three clusters and their labels, (b) the number of clustered training points in each cluster.



Fig. 15. Number of mutual neighbors of the minimum NPLOF (the final anomaly value) regarding the Yonghe Bridge: (a) the training phase, (b) the monitoring phase.

#### Table 3

Selection of the o	ptimal significance	level for the threshold	estimation regarding	the Yonghe Bridge.
	F · · · · · · · · · · · ·			

Parameters	Sample significance levels						
	0.05	0.01	0.01	0.0001	0.00001	0.000001	
FPR (No.)	11	7	4	2	0	0	
FPR (%)	6.3953	4.0697	2.3255	1.1627	0	0	



Fig. 16. Damage assessment in the Yonghe Bridge by the proposed DHL method.



Fig. 17. Damage assessment regarding the Yonghe Bridge by the DPC-NPLOF: (a) the manual cluster selection related to the DPC, (b) the evolution of anomaly values.

compounding environmental factors. In Fig. 18(c) and Table 4, although GMM-MSD did not indicate any false positive error similar to the proposed DHL method, it performs poorly by considering the false negative and misclassification errors. Thus, one can conclude that the proposed DHL method is still superior to the aforementioned techniques for damage assessment when the compounding environmental and/or operational variability conditions are dominant (see Fig. 18).

# 5. Conclusions

This paper proposed a novel double-hybrid learning method entailing data partitioning via an improved hybrid clustering algorithm and a hybrid anomaly detector based on a non-parametric LOF model for damage assessment in bridge structures under different environmental and/or operational variation patterns. The improved hybrid clustering relied upon the SNN-DPC in conjunction with the SANS and an automated approach to selecting the number of clusters. The proposed hybrid anomaly detector (NPLOF) combined



Fig. 18. Damage assessment in the Yonghe Bridge by some single and hybrid anomaly detection techniques: (a) SVD, (b) MSD, (c) GMM-MSD, (d) PCA-MSD.

Table 4

Numerical performance evaluations of the proposed and state-of-the-art methods in the damage assessment problem of the Yonghe Bridge.

Methods	Performance evaluation metrics				
	False positive	False negative	Misclassification		
DHL	0 (0.00%)	0 (0.00%)	0 (0.00%)		
DPC-NPLOF	18 (9.37%)	0 (0.00%)	18 (8.33%)		
SVD	12 (6.25%)	0 (0.00%)	12 (5.55%)		
MSD	7 (3.64%)	0 (0.00%)	7 (3.24%)		
GMM-MSD	0 (0.00%)	9 (37.50%)	9 (4.16%)		
PCA-MSD	12 (6.25%)	21 (87.5%)	33 (15.27%)		

the SANS method with the classical LOF to alter it from a parametric model to a non-parametric one. The natural frequencies of two full-scale bridge structures with different variability patterns caused by the single and multiple environmental factors were used to prove the proposed DHL method as well as several comparisons.

The results of this paper demonstrated that the proposed DHL method is successful and reliable for early warning of damage occurrence under the single and multiple (compounding) environmental factors. In particular, this method could effectively remove the profound environmental variability observed in the natural frequencies related to the Z24 Bridge. It was observed that the SNN, SANS, and the proposed approach to selecting the number of clusters of the DPC have profoundly positive impacts on mitigating the environmental variations and obtaining reliable SHM results. The proposed DHL method outperformed the state-of-the-art anomaly detection techniques by obtaining smaller decision-making errors and better damage detectability. Regardless of the type of environmental and/or operational variability, clustering-based hybrid anomaly detectors developed from the concept of local learning are more reliable for removing environmental and/or effects compared to the anomaly detectors based on global learning, which consider the entire training data.

# CRediT authorship contribution statement

**Alireza Entezami:** Conceptualization, Methodology, Software, Writing – original draft, Funding acquisition. **Hassan Sarmadi:** Conceptualization, Methodology, Software, Writing – review & editing, Resources. **Bahareh Behkamal:** Methodology, Writing – review & editing, Validation.

#### **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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