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Thermal model for bicycle tire internal temperature evaluation in various contact conditions

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Abstract:

Bicycle mobility has become increasingly popular as a sustainable and healthy means of transportation. Bicycles are not only a cost-effective transportation mode but also help reduce traffic congestion and air pollution. However, the efficiency and safety of bicycling largely depend on the optimization of bicycle components, such as the tires. The importance of bike tire optimization cannot be underestimated as it can affect both bicycle dynamics and bicycle performance.

Due to the lack of multi-physical mathematical models able to analyze and reproduce complex tire/road contact phenomena, useful to predict the wide range of working conditions, this research aims to the development of a bicycle tire thermal model. The main outcome is to provide the full temperature local distribution inside the tire's inner rubber layers and the inflation chamber. Such kind of information plays a fundamental role in the definition of the optimal adherence conditions, for both safety and performance maximization, and as an indicator of the proper tire design for various applications, each requiring specific heat generation and management.

The experimental validation has been carried out thanks to an innovative test-rig developed at Politecnico di Milano. It is known as *VetyT* (acronym of Velo Tyre Testing), and it complies with the standard ISO 9001-2015. It has been specifically instrumented for the activity, acquiring the external tire temperatures to be compared with the respective simulated ones, under various working conditions.

Introduction

The surge in interest and reliance on bicycles for urban mobility has been a noticeable trend, driven by their eco-friendly and health-conscious attributes. Bicycles not only offer a cost-effective and sustainable mode of transportation but also contribute to mitigating traffic congestion and reducing air pollution in urban environments. However, the efficiency and safety of cycling are intricately linked to the optimization of various bicycle components, with a particular emphasis on one of its critical elements – the bicycle tires [18] [5]. The significance of optimizing bicycle tires extends beyond mere comfort, as it has a profound impact on both bicycle dynamics and overall performance [3].

The tire is a pivotal interface between the rider and the road, making its optimization a key focus area [14]. The design and materials used in bicycle tires have been subjects of dedicated research in recent years, with the overarching goal of enhancing their efficiency and safety. Much of this research has encompassed empirical correlation activities [17] and Finite Element Analysis (FEA) models [13]. While these endeavors have yielded valuable insights, there remains a notable gap in the availability of comprehensive multi-physics mathematical models capable of analyzing and replicating the intricate tire-road contact phenomena [16]. Such models are essential for predicting a wide array of working conditions.

This research endeavors to address this gap by developing a comprehensive thermal model tailored specifically for bicycle tires. Building on the expertise and insights gained from similar applications in the realm of motorcycle tires [8], the primary objective is to provide a detailed temperature distribution map within the inner rubber layers of the bicycle tire, extending into the inflation chamber. This information is of paramount importance in configuring optimal adherence conditions, with a dual focus on enhancing both safety and performance. Moreover, it serves as a fundamental indicator for guiding the design and manufacturing of bicycle tires tailored for diverse applications, each with its unique requirements concerning heat generation and management.

The model at the core of this research hinges on the application of the fundamental principles of thermodynamics, utilizing Fourier's Differential Equations in a three-dimensional domain. To enable this model's versatility and applicability, extensive parameterization has been carried out. This process involved the measurement of the footprint extension's variations concerning vertical load, camber, and inflation pressure for a reference bicycle tire. Additionally, measurements of the thermal conductivity and specific heat of the various layers and materials constituting the bicycle tire were conducted via a non-destructive procedure [9].

A pivotal aspect of this research involves the experimental validation of the developed model. To this end, an innovative and meticulously designed test-rig, housed at Politecnico di Milano, has been employed. This test rig is uniquely tailored to measure the lateral characteristics of bicycle tires and complies with ISO 9001-2015 standards [2]. In preparation for this research, the test rig was further instrumented to capture external tire temperatures under various operational conditions (different slip angles, inflation pressures, and vertical loads). These real-world temperature measurements are then meticulously compared with their corresponding simulated counterparts, further bolstering the credibility and reliability of the model.

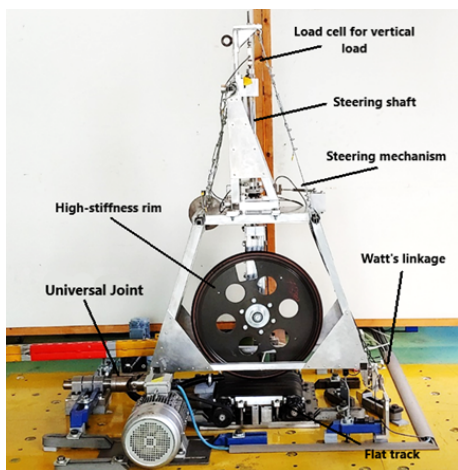
In summary, this research embarks on a crucial endeavor to bridge the existing gap in comprehensive tire modeling for bicycles. The development of a robust thermal model promises to offer a deeper understanding of the intricate interplay between bicycle tire performance, thermal conditions, and safety considerations. As we venture into an era where sustainable and healthy transportation solutions are of paramount importance, the insights garnered from this research stand to significantly contribute to the optimization of bicycle performance and safety, thereby facilitating the broader adoption of this environmentally friendly mode of transportation.

1 Materials and Methods

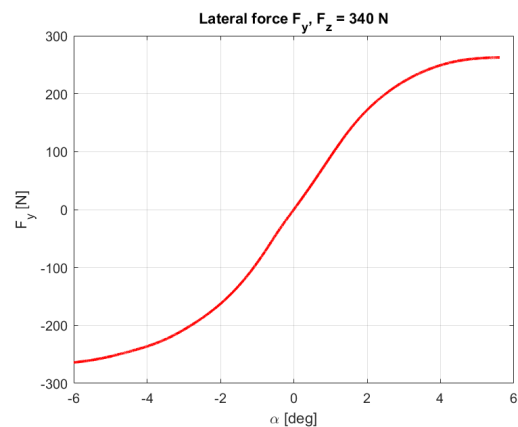
1.1 Experimental measurement of tire characteristics

The lateral characteristics of the bicycle tire under analysis have been measured through *VeTyT*, a test-rig specifically designed for bicycle tires (Figure 1a) [4]. *VeTyT* is an acronym for Velo Tyre Testing. According to our knowledge, it is the only test-rig for bicycle tires complying with the standard ISO 9001-2015. We can measure lateral force and self-aligning torque, as tire parameters vary. It consists of a rigid aluminum frame to hold the bicycle wheel on top of a flat track surface. The chassis can be tilted to simulate the camber angle, while the wheel can be steered to sweep the slip angle range $\pm 7^\circ$. Also, the vertical load can be adjusted, by simply adding masses to the frame. The magnitude of the vertical force applied is measured by a compression load cell on top of the steering shaft.

VeTyT is connected to the ground by means of a universal joint and a Watt's linkage. While the first one acts as a hinge connection, the instrumented Watt's linkage allows a limited vertical motion and it constrains the lateral displacement. The longitudinal axis of the test-rig passes through the center both of the universal joint and the Watt's linkage. In this way, we can derive the lateral force of the tire at the contact point tire-rolling surface through a simple equilibrium equation. Concerning the results presented in this paper, we used a 28 mm wide road racing bicycle tire (ETRTO 700x28c), for different vertical loads (340 N, 400 N, 490 N) and inflation pressures (3, 5 bar, 5, 5 bar, 7, 5 bar). It is configured with inner tube, and mounted on a high-stiffness rim. The latter is six times stiffer in the lateral direction with respect to a commercial aluminum rim, so that we can safely and easily simulate a carbon rim. As an example, in Figure 1b you can see the lateral force F_y as function of the slip angle α , for inflation pressure equal to 5, 5 bar and vertical load 340 N.



(a) Overview of *VeTyT* test-rig. Picture adapted from (Dell'Orto, 2023)



(b) Lateral force F_y as a function of the slip angle. The tire was tested at inflation pressure of 5, 5 bar, and vertical load 340 N

Figure 1. *VeTyT* test-rig and lateral force measurement.

Tests were performed at a rolling speed of 9,3 km/h. The test-rig was updated adding a thermo camera to capture external tire temperatures under different test conditions.

2 thermoRIDE

The thermoRIDE model serves as a reliable tire model that effectively analyzes the complex interactions between a tire, its environment, and the wheel chamber, as depicted in Figure 2.

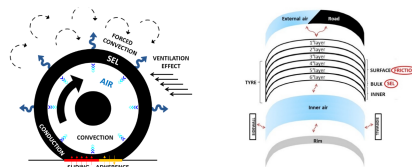


Figure 2. Tire Thermodynamic Model Scheme.

thermoRIDE model is a powerful tool for simulating temperature distributions within tire layers while establishing links to heat exchange mechanisms. It addresses several critical heat-related phenomena, including heat generation and exchange processes in both the tire's structure and its interaction with the external environment [12] [11]:

- **Heat Generation Mechanisms**

- *Friction Power*: this mechanism involves the generation of heat at the tire-road interface due to tangential stresses. It depends on factors like tangential forces, contact area size, and sliding velocities in the contact patch;

- *Strain Energy Loss*: the strain energy loss (SEL) represents the dissipation of energy in the tire caused by cyclic deformations. It results from various phenomena, including intra-ply friction, friction within plies, and the nonlinear viscoelastic behavior of rubbery components [15]. SEL is linked to average interaction forces, wheel rotation frequency, wheel inclination angle, and inflation pressure [6].

• **Heat Exchange Processes**

- *Tire-Road Thermal Conduction*: this process involves heat conduction between the tire tread and the road surface, modeled using Newton’s formula;
- *Internal Thermal Conduction*: it signifies heat conduction within the tire structure, driven by temperature gradients across its different zones;
- *Tire-External Air Thermal Convection*: this complex heat transfer between the tire surface and the surrounding air accounts for both forced convection (occurring during vehicle motion) and natural convection (minimal or no motion). The convection heat transfer can be modeled by Newton’s law of cooling formulation computing the convection coefficient by means of dimensionless analysis;
- *Tire-Inner Air Thermal Convection*: convection phenomena occurring between the inner liner layer and the inner air.

The right modeling of these mechanisms of heat exchange and thermal generation is crucial in achieving the core aim of this research which is to furnish a comprehensive temperature distribution profile meticulously charting the inner rubber layers of the tire, extending to encompass the inflation chamber. This nuanced temperature data plays a pivotal and multifaceted role. It is instrumental in delineating the optimal adherence conditions, critical for enhancing both safety and performance. Additionally, it serves as an indispensable beacon guiding the precise design of tires tailored for a multitude of applications, each necessitating meticulous control over heat generation and management.

As for the inputs and outputs of the model they are summarized in the following Table 1 and Table 2:

Table 1. Model Input

Quantity	Description
F_z	Vertical interaction force
F_x	Longitudinal interaction force
F_y	Lateral interaction force
v_x	Wheel hub longitudinal velocity
v_y	Wheel hub lateral velocity
s_r	Slip ratio
s_a	Slip angle
ω	Wheel angular velocity
γ	Inclination angle
T_{Air}	Ambient air temperature
T_{Track}	Road pavement temperature

Table 2. Model Output

Quantity	Description
$T_{TreadSurf}$	Tread surface temperature
$T_{TreadCore}$	Tread core temperature
$T_{TreadBase}$	Tread base temperature
$T_{InnerLiner}$	Inner liner temperature
$T_{InnerAir}$	Internal air temperature
$T_{PInnerAir}$	Internal air pressure

2.1 Tire Structural Model

The tire is characterized as having a parallelepiped-like shape, and it is divided into discrete nodes using a grid [7]. Within this grid, nodes are designated as points where temperature calculations are made at each moment. The discretization of the tire, which caters

to its unique attributes such as dimensions, diffusivity, and inertia, can substantially vary. This variability is aimed at accurately representing the transient and steady-state thermal dynamics while upholding real-time requirements across diverse tire operational conditions.

The default tire structure encompasses six layers along the radial direction:

- *Tread Surface*: the outermost part of the tire that is in contact with the road and external air;
- *Tread Core*: positioned just below the surface, directly tied to grip and also to tire stiffness;
- *Tread Base*: the deepest tread layer, whose temperature affects mostly the tire stiffness rather than the grip level;
- *Belt*: situated beneath the tread base and made of a series of wire cloths arranged with small angles, this layer is significantly linked to the strain energy loss phenomenon;
- *Body Plies*: layer composed by a series of mutually parallel cords of very durable and at the same time flexible material, surrounded by the vulcanized rubber compound; it is another important contributor to the SEL, because of the energy dissipated by the friction among different plies and within the plies;
- *Inner Liner*: last layer of the tire which is in contact with the inner air, not significantly influencing SEL, stiffness, or grip.

When it comes to the lateral discretization of the tire, the standard approach is depicted in Figure 3 on the right. Nonetheless, when dealing with motorcycle or bike tires, the discretization along the y -axis can be tailored to incorporate as many as 16 ribs (Figure 2 displays the default 5-rib arrangement). This customization is dependent on the presence of pre-established boundary conditions maps that are designed for the tire under analysis.

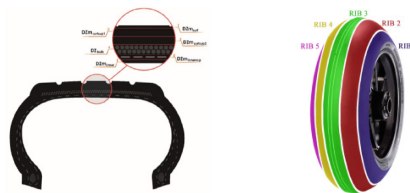


Figure 3. thermoRIDE Tire Mesh Scheme.

2.2 Mathematical Model

The mathematical foundation of the thermoRIDE model relies on Fourier's diffusion equation, applied to a three-dimensional domain. This equation describes the energy within the system, accounting for gains and losses. Fourier's law of heat conduction, a fundamental principle in thermodynamics, states that heat transfer is proportional to the temperature gradient and the thermal conductivity of the material. Mathematically, this relationship is expressed as:

$$\vec{q} = -k\nabla T \quad (1)$$

Where:

- \vec{q} : Local heat flux density (W/m²)
- k : Material's conductivity (W·m/K)
- ∇T : Temperature gradient (K/m)

From Fourier's law, a quadratic partial differential equation suitable for numerical integration in transient thermal conditions can be obtained. To achieve that, in the developed thermal model an infinitesimal volume element $dV = dx dy dz$ has been considered.

Since the change in the internal energy of a closed system is equal to the amount of heat supplied to the system, minus the amount of work done by the system on its surroundings, and the control volume is considered not deformable, the internal energy dU of the infinitesimal volume dV is given by the following expression:

$$dU = \rho \cdot dV \cdot c_v \cdot T \quad (2)$$

Here, ρ represents density, c_v denotes specific heat at constant volume, and T signifies temperature.

Due to the fact that the volume dV is not able to do any work ($dL = 0$), the change in the internal energy dU is linked only to the amount of heat dQ added to the system. The term dQ encompasses two distinct contributions: one related to heat exchanged through the outer surface of the volume dV and the other linked to the heat generated within the volume dV . So it is possible to write the following energy balance equation:

$$\rho dV c_v dT = dQ_{EX} + dQ_G = \dot{q}_G dV dt + dt \nabla \cdot (k \nabla T) dV \quad (3)$$

Here, \dot{q}_G represents the rate of heat generation per unit volume and unit time (in $\text{W/m}^3 \cdot \text{s}$).

Equation 3, divided on both sides by the quantity $\rho \cdot dV \cdot c_v \cdot dt$, defines the Fourier heat equation:

$$\frac{\partial T}{\partial t} = \frac{\dot{q}_G}{\rho c_v} + \frac{\nabla \cdot (k \cdot \nabla T)}{\rho c_v} \quad (4)$$

Equation 4 enables the determination of the three-dimensional temperature distribution $T(x, y, z, t)$, contingent upon the specification of boundary conditions. It governs the temporal variation of temperature concerning a specific thermal gradient and elucidates how temperature evolves over time due to generative effects and heat transport phenomena.

The complexity of the phenomena being modeled, along with the level of precision required for its intended applications, necessitates the incorporation of temperature-dependent thermodynamic properties. Additionally, the non-uniform composition of the tire highlights the importance of addressing variations in these properties throughout its thickness. For these reasons, the Fourier law takes the following state-space formulation:

$$\frac{\partial T}{\partial t} = \frac{\dot{q}_G}{\rho c_v} + \frac{1}{\rho c_v} \left(\frac{\partial^2 k(z, T) T}{\partial x^2} + \frac{\partial^2 k(z, T) T}{\partial y^2} + \frac{\partial^2 k(z, T) T}{\partial z^2} \right) \quad (5)$$

2.3 Tire Thermal Characterization

It is clear from Equation 5 that in order to use the thermal model and for the model to give robust results, it is essential to find the laws of variation of thermodynamic properties with temperature for the layers that make up the tire. A non-destructive procedure was introduced by C. Allouis et al. for this purpose [1]. This procedure involves heating a small area on the tire tread surface using a laser beam, at different powers, focused by a lens as can be seen in Figure 4.

Two thermal cameras are used to measure the temperatures on both the heated area (tread) and the corresponding inner liner surface. These temperature measurements are crucial for the evaluation of the radial and circumferential temperature gradients across the tire.

For the purpose of evaluating the tire thermodynamic properties, a unique thermal model known as the Thermo Racing Tire Laboratory (TRTLab) model is utilized. It relies on the measured temperature gradients to determine the thermal diffusivity of various layers of the tire. To achieve this, Fourier's law of diffusion is applied to a three-dimensional domain. The heat produced by the laser at specific points on the tread surface serves as the heat generation term in the Fourier equation. This term is only applied to the nodes within the laser spot area. The total heating flux power is then divided among these nodes based on their share of the laser spot area. Along with the laser-induced heat, the heat exchange mechanisms linked to the conduction between the different tire layers and the convection with the air are also taken into account. However, since the tire is stationary in this instance, the other mechanisms mentioned earlier are not present.

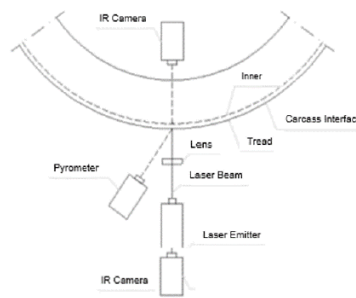


Figure 4. TRTLab Scheme.

With the TRTLab model, it is possible to simulate both surface and internal temperature patterns. By comparing the temperature curves generated by the simulation with the experimental data gathered through the outlined process, a reverse engineering process can be executed. This process entails tweaking the values of density, specific heat, and thermal conductivity until the simulated curves accurately align with the collected data.

When the simulated curves align with the experimental data, the values of these physical properties for each tire layer are considered as the actual parameters of the tire. In the case of the bike tires under consideration, the TRTLab model provided the following results regarding thermal conductivity and the specific heat coefficient:



Figure 5. Tire Thermal Properties.

where:

- the solid blue curve represents the thermodynamic properties of the tread compound;
- the dashed blue curve corresponds to the properties of the inner liner compound;
- the orange curve relates to the belt, which contains metal wires and so has different values.

From these curves, it can be observed that for this particular tire, the material composition of the tread conducts heat more effectively compared to the inner liner. Regarding specific heat, the inner liner compound shows a similar trend as the tread compound but has higher values.

2.4 Contact Patch Evaluation

In order to accurately analyze the heat transfer that occurs in a tire, it is necessary to gather information on the tire's contact area with the test-rig surface, including its dimensions and configuration. This data can be obtained through specialized testing procedures that use a scanner and a custom tool designed with Matlab. These tests involve scanning the tires under different operating conditions, including various vertical loads (F_z) and internal air pressures (p_{in}^{air}) during static testing.

An example of the obtained contact patch is shown in the following Figure 6:

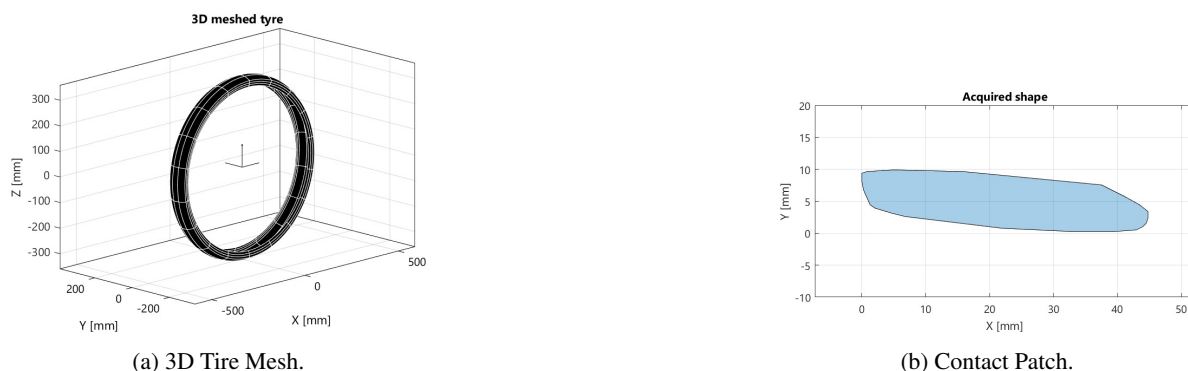


Figure 6. Example of Measured Tire Contact Patch.

At this point, for each contact patch, a set of four values is measured to effectively characterize each footprint. After ensuring their physical consistency within the limits established through experimental tests, these computed values serve as input for a tool that generates a map of the contact footprints, taking into account the specific working conditions selected for each instance. Nevertheless, it is important to acknowledge that the instantaneous dynamic extension and shape of the contact patch can vary significantly due to transient conditions related to wheel loading, the centrifugal effect on the rolling tire, and the viscoelastic properties inherent to the tire. To implement the contact patch dynamic characteristics an MBD/FEA tire model able to fit both static and dynamic experimental data can constitute a valid instrument.

3 Results

This section presents the results of simulations done with the thermal model and the presented test-rig *VeTyT*. Understanding tire layer temperatures could be of paramount importance when utilizing tires on the test-rig. This knowledge serves as a critical factor in optimizing performance, safety, and the overall lifespan of the tire [10] and in better understanding in general how the tire behaves internally when subjected to lateral interaction.

Figure 7 shows the result of the thermal model tuning. The thermoRIDE model tuning is a process that aims to find a fitting between the temperatures obtained experimentally and those simulated by the physical-analytical model just introduced. During this phase, to tune correctly the model, the heat exchange mechanisms described previously are modeled in a proper way keeping in mind the physical mechanisms inherent in the situation under consideration.

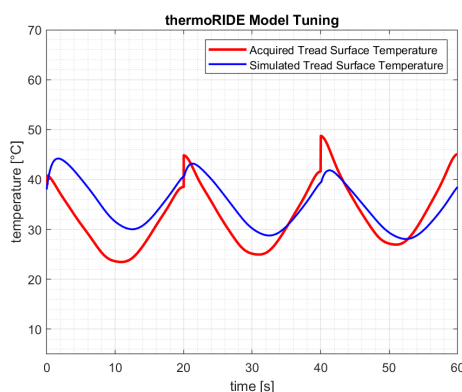


Figure 7. thermoRIDE Model Tuning.

It is observed how, downstream of the model tuning process, the blue curve simulated by the model manages to follow the thermal trend and dynamics acquired through the thermal camera. At this point, the estimated internal temperatures can be obtained.



Figure 8. thermoRIDE Model Results.

Figure 8a shows the temperature readings of three layers of the tire. The blue line represents the outermost layer in contact with the test rig surface. The red line shows the temperature of the middle layer of the tread, while the black line shows the temperature of the tread layer in contact with the tire carcass. The graph clearly indicates that the outermost layer has the lowest temperature due to the lack of longitudinal friction (there is more or less only lateral friction in this test), as well as the cooling effect of the convection with air. On the other hand, the two innermost layers of the tread have the highest temperature as they are affected by heat-related to SEL generated by tire rolling and are not affected by convection phenomena.

From Figure 8b, it is also possible to see how, since this test is designed to stress the tire from a lateral interaction point of view, the link between temperatures and lateral force is clear. Indeed, this specific test, even for the low vertical loads and low rotational speed imposed in this first instance, developed mostly lateral friction power.

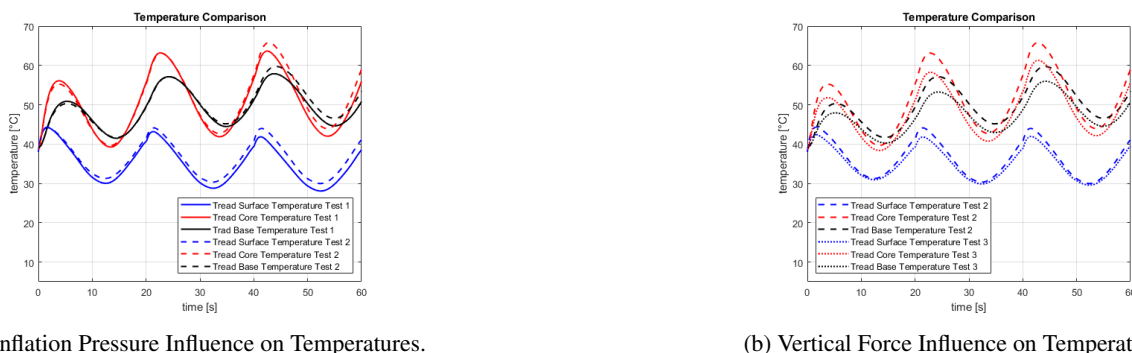


Figure 9. thermoRIDE Model Sensitivity.

The comparison of two tests, Test 1 and Test 2, can be observed in Figure 9a. Test 1 was conducted at a pressure of $3.5bar$ and a load range of $[340N - 490N]$, while Test 2 was conducted at a pressure of $7.5bar$ and the same load range. All other parameters were kept the same between the two tests. The thermal model outputs are presented as continuous curves for Test 1 and dashed curves for Test 2. These results demonstrate the high sensitivity of thermoRIDE to changes in pressure, with all three tread temperatures increasing as these parameters increase.

Furthermore, Figure 9b illustrates the model’s sensitivity when only varying the load on the tire while keeping pressure and other parameters constant. The results of Test 2, with a load range of $[340N - 490N]$, and Test 3, with a load range of $[440N - 590N]$, are compared. As the vertical load increases, there is a noticeable variation in all three temperatures, highlighting the excellent sensitivity of the thermoRIDE model.

These preliminary results highlight how the thermal model is also able to respond correctly to inputs from the test rig under examination, and thus how it can also be used for this type of application so that important information about the tires under examination can also be obtained from a thermal point of view without the need to have data from track tests.

Conclusion

The temperature distribution within a tire's layers directly influences its grip and traction on the road surface. So, where precise control and maneuverability are essential, having the right tire temperature can mean the difference between maintaining control in tight corners and losing traction. Furthermore, temperatures that are too cold or too hot can accelerate tire wear. By monitoring and managing tire layer temperatures, it is possible to extend the life of tires, reducing the frequency of replacements and associated costs. Incorrect temperatures and pressures across tire layers can lead to unpredictable and potentially dangerous phenomena. Knowing and maintaining the appropriate temperature range ensures consistent and predictable performance, enhancing rider safety. Finally, such kind of information can help the tire manufacturer in the direction of developing that type of product to improve its characteristics. In summary, having a comprehensive understanding of tire layer temperatures is very important for several reasons related not only to performance and durability.

This paper showcases the feasibility of precisely calibrating the thermal model by utilizing the test rig's thermal outcomes. This fine-tuning permits the estimation of internal temperatures, which hold significant importance in comprehending the tire's conduct when subjected to particular stress. As a result, the testing instrument's effectiveness increases in identifying the tire's thermal strengths and vulnerabilities, thus eliminating the requirement for expensive and time-consuming outdoor testing.

To improve these aspects even more, as future developments it might be important to carry out tests at higher rotational speeds so as to improve the formulation adopted for strain energy loss and also it might be useful to monitor the temperature not only of the tread but also of the inner liner so as to allow a more robust thermal model calibration.

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