

Crack healing under sustained load in concrete: An experimental/numerical study

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ABSTRACT: The need of sustainable resilient structures and infrastructures push towards the use of cementitious materials able to heal micro-cracks and defects. For real structural application under service loading the time-dependent behavior is of the utmost importance, especially in presence of cracks which can lead to a nonlinear creep behavior that might cause the structural failure. Now the new challenge is to study and quantify the effect of crack-healing on the nonlinear creep behavior. This study aims at the following goals: 1) to characterize with experimental investigations the effect of the healing in tests in which the specimens, along the exposure time and under controlled environmental conditions, are under sustained load, the expected service load, determined as a fraction of the pre-cracking load; 2) develop a comprehensive numerical framework for the interpretation and simulation of the experimentally observed results. To this purpose an experimental investigation is currently ongoing at Politecnico di Milano with reference to an Ultra High-Performance Concrete developed in the framework of the H2020 ReSHEALience project for exposure to extremely aggressive environments. The numerical framework is based on the recent developments of the multiphysics lattice particle model.

1 INTRODUCTION

Creep and shrinkage of concrete are time-dependent deformations that influence primarily the serviceability, and in some cases also the safety, of reinforced concrete structures with and without prestressing, and of composite concrete and steel structures. Shrinkage is mainly due to the changes in the moisture content/status dictated by both self desiccation and drying if exposed to lower humidity environments.

Shrinkage alone may cause damage and early age cracking of concrete with a huge impact on the durability of the structure. In addition and in combination to that, the large and widely unrecoverable creep deformations of concrete can cause significant modifications of action effects in structures in terms of internal stress distributions, excessive deflections and loss of prestressing forces, and produce large cracks.

All these effects affect the serviceability and the durability of structures, and may impact on their structural safety as well. In tall structures, absolute and differential shortening and deviations from verticality due to creep and shrinkage of concrete may be a matter of concern requiring serviceability and safety checks and proper remedial actions (Khan et al. 1997; Jirásek & Bažant 2002; Bažant & Jirásek 2018).

For long-term prediction and design the inherent self-healing capability of either ordinary and/or advanced cementitious materials cannot be disregard. The term self-healing refers to the material capacity of repairing the damage in cracked state autonomously.

However, time-dependent behavior of concrete must be contextualized in a more wide comprehensive framework since it is a result of interplay between multiple chemical, physical, and mechanical processes that are functions of the material composition and its curing as well as the surrounding environmental and loading conditions. Chemical processes include the

continued reactions of cementitious materials, aging, healing in the cracks, in addition to a variety of deleterious reactions, such as Alkali-Silica Reaction (ASR), Sulphate Attack, Physical Salt Attack, and others. Physical changes are typically related to thermal and hygral changes. As the hygro-thermal environmental conditions surrounding concrete vary, moisture and heat diffusion processes take place within the material and result in volumetric deformations, such as during thermal shocking, exposure to fire, repeated freeze and thawing, temperature rise in early-age massive structures, or moisture drying that in thick or restrained elements can be induce serious damage.

This paper reports the initial results of an ongoing activity. In the manuscript, first it is presented an experimental investigation on the effect of the crack healing in tests in which the specimens, along the exposure time and under controlled environmental conditions, are under sustained load, close to the expected service load and determined as a fraction of the pre-cracking load. The tests are performed on an Ultra High-Performance Concrete developed in the framework of the H2020 ReSHEALience project for exposure to extremely aggressive environments. Second the mesoscale approach, which is the Multiphysics-Lattice Discrete Particle Model (M-LDPM) (Alnaggar et al. 2017; Yang et al. 2021), resulting from the coupling of the Hygro-Thermo-Chemical (HTC) model (Di Luzio & Cusatis 2009a; Di Luzio & Cusatis 2009b) with the Lattice Discrete Particle Model (LDPM) (Cusatis et al. 2011b), is here for the first time adopted the numerical simulations of the creep and healing phenomena acting simultaneously. In this framework, creep and shrinkage deformations are modeled based on a discrete version (Abdellatef et al. 2015; Alnaggar et al. 2017) of the Microprestress-Solidification theory (Bažant & Prasannan 1989; Bažant et al 1997; Bažant et al. 2004; Di Luzio & Cusatis 2013). This framework has also been recently extended for the crack healing modeling in plain cementitious materials (Cibelli et al. 2022b) and fiber reinforced concrete (Cibelli et al. 2022a), but never considering the shrinkage and creep deformations during crack healing.

2 EXPERIMENTAL INVESTIGATION

Three different UHPC mixes – two with nano-additives, alumina nano-fibres (ANF) and cellulose nano-crystals (CNC), and one as reference – were characterised with both non-destructive and destructive tests; the tests were performed on $500 \times 100 \times 30 \text{ mm}^3$ thin beams obtained cutting pre-cast slabs. The main goal of the experiments was to analyse the material behaviour under real service conditions and its evolution over time; for this purpose, both mechanical loading and environmental exposure were simultaneously applied to the specimens. The experimental campaign was carried out in three different stages, defined as follows: (i) uncracked specimens, (ii) pre-cracked specimens, and (iii) cured/healed specimens (i.e., after exposure). Material properties were first evaluated on beams in virgin conditions, then the specimens were pre-cracked by means of 4-point bending, targeting a crack opening displacement (COD) of $200 \mu\text{m}$ under load, resulting in a residual COD of $75\text{-}125 \mu\text{m}$. The properties were then investigated again through non-destructive measurements, setting the reference for the effectiveness of the self-healing process. A new setup was specifically designed to apply a constant sustained load on the beams during the environmental exposure. After the pre-cracking process, beams were coupled referring to the peak load achieved during the process itself. The couple was braced and loaded with a steel frame, as shown in Figure 1; two cylinders (pink colour) act as hinges in the mid span of the beams, while four bars (black colour) are used to hold the specimens on the sides. With the system described, it was possible to emulate the 4-point bending condition during the curing and healing process, guaranteeing the presence of a constant moment in the central region of the beams. The loading process can be described in three different stages, starting from unloaded and pre-cracked specimens; (a) two load cells (blue colour) are positioned above the second layer of steel bars and braced with the top bars, (b) the targeted load, derived from the pre-cracking process, is first applied to the cells tightening the top nuts (red colour), ensuring that the exact value will be transferred to the beams, then (c) the load is finally applied to the specimens tightening the nuts in the middle (brown colour) and simultaneously releasing the load cells, allowing to remove them. The sustained loading setup was previously tested on a couple of samples to validate the effective maintenance of a constant load over time on the braced pair, keeping the load cells in position for the whole

duration of the test; the system was able to guarantee an almost constant load, with less than 10% reduction. An extensive summary of the results has been provided in Al-Obaidi et al. (2023).

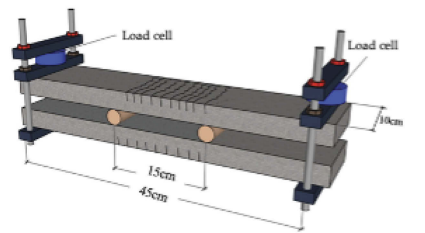


Figure 1. Sustained loading setup.

The specimens paired and braced with the sustained loading system were exposed into chloride solution (3.3% NaCl), geothermal water, and tap water as reference, to simulate the chloride and sulphate environmental attack respectively. The curing period varied between 1 and 12 months, and the specimens were tested at 1, 2, 3, 6, 9, and 12 months. The effect of healing and the crack sealing – promoted by crystalline admixture and nano-additives – were evaluated at the end of each exposure period, allowing to define the evolution of the performance that can be expected in a real structure. The material proved to be substantially resilient even under aggressive conditions such as sustained loads and different chemical attacks, maintaining constant mechanical performance over time.

3 GENERAL MULTI-PHYSICS FRAMEWORK

This section presents the numerical framework that merges the Hygro-Thermo-Chemical (HTC) model with a discrete meso-scale (LDPM) model for concrete. For time-dependent mechanical analysis, it is essential to use a multi-physics model that simulates all the relevant chemical/physical phenomena in a cement based material, such as the chemical reactions of the binder coupled with moisture transport and heat transfer in interaction with the environmental changes. Especially in the cracks chemical/physical phenomena are fundamental to characterize the healing evolution.

3.1 *Hygro-Thermo-Chemical (HTC) model*

The HTC model (Di Luzio & Cusatis 2009a; Di Luzio & Cusatis 2009b) simulates the moisture and temperature evolution in a cementitious material considering the simultaneous chemical reactions – hydration of cement and pozzolanic reactions. The reaction kinetics are formulated in terms of reaction degrees that represent the progress of a chemical reaction as the ratio between the amount of reacted material and the total initial amount of it. Similarly, if a pozzolanic material is utilized in the mixture, such as silica-fume, the rate of its reaction degree can be describe using a similar approach (Di Luzio & Cusatis 2009a).

The coupling between chemical reactions and transport processes (water and heat) is achieved by combining the water mass and enthalpy balance equations with the reaction degree kinetic equations, in which the cement hydration degree and the pozzolanic material reaction degree are assumed as an internal variables, leading to a system of partial differential equations with only two state variables, i.e. the temperature T and the pore relative humidity h . All the details of the formulation can be found in Di Luzio & Cusatis (2009a); Pathirage et al. (2019); Bousikhane et al. (2018).

3.2 *Discrete mesoscale mechanical model*

The mesoscale discrete model employed in the framework is the so-called Lattice Discrete Particle Model (LDPM) (Cusatis et al. 2011a; Cusatis et al. 2011b). It simulates the heterogeneity of concrete considering the mechanical interaction of coarse aggregates in the cementitious matrix. The

discretized concrete mesostructure creates a system of polyhedral cells interacting through triangular facets and a lattice system composed by the line segments connecting the particle centers. A rigid body kinematics is assumed to describe the deformation of the LDPM lattice/particle system. By summing up the stress contributions of all the facets and equating the total internal work with the total external work, one can obtain the discrete equilibrium equations of the LDPM formulation.

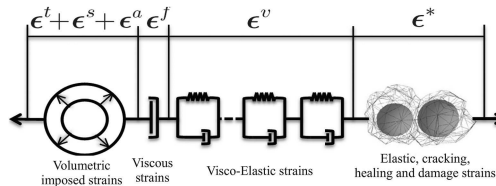


Figure 2. Equivalent rheological model based on strain additivity at the meso-scale.

The LDPM has been used successfully to simulate concrete behavior under a large variety of loading conditions (Cusatis et al. 2011a). It has been also formulated for fiber reinforced concrete (Schauffert & Cusatis 2012) and for ultra-high performance concrete (UHPC) (Smith et al. 2014). Assuming the additivity of strains at mesoscale level of each facet, one can write

$$\dot{\epsilon} = \dot{\epsilon}^* + \dot{\epsilon}^a + \dot{\epsilon}^s + \dot{\epsilon}^t + \dot{\epsilon}^v + \dot{\epsilon}^f \quad (1)$$

where $\dot{\epsilon}$ is the total strain, $\dot{\epsilon}^*$ represents the effect of instantaneous elasticity and damage, $\dot{\epsilon}^a$ represents the strain rate due to material degradation such as ASR; $\dot{\epsilon}^s$ and $\dot{\epsilon}^t$ are shrinkage and thermal strain rates, respectively; $\dot{\epsilon}^v$ is the viscoelastic strain rate and $\dot{\epsilon}^f$ is the purely viscous strain rate. The strain additivity of Eq. 1 can be represented as a rheological model of elements in series that is depicted in Figure 2.

3.2.1 Elastic, cracking, and damage behavior

In the elastic regime, the normal and shear stresses are proportional to the corresponding strains. In vectorial form, one has $\dot{\epsilon}^* = \frac{1}{E_0} \mathbf{G} \boldsymbol{\sigma}$, where $\mathbf{G} = [1, 0, 0; 0, 1/\alpha, 0; 0, 0, 1/\alpha]$, E_0 = effective normal modulus, and α = shear-normal coupling parameter. It must be observed that, E_0 represents the instantaneous deformation since all creep strains, which always occurs during quasi-static loading, are included in the Kelvin chain of the rheological model. However, the Kelvin chain has a limited number of elements and, in this case, E_0 will also include the effect of very short term creep whose characteristic time is smaller than the smallest of the discrete chain. All the detail on the LDPM constitutive law can be found in Cusatis et al. (2011b).

3.2.2 Rate effect

The loading rate has a direct effect on the mechanical properties of concrete as it has been shown by many authors, among others Watstein (1953), Hughes & Gregory (1972), Reinhardt (1985). This phenomenon is explained by three main contributions: the creep of the material, the rate effect on the bond rupture in the mesoscale fracture, and the effect of inertia force on the crack propagation. For quasi-static loads, the first two mechanisms mentioned above are dominant. While increasing the strain rate loading the third contribution becomes more and more dominant. Since the fracture can be seen as the consequence of the rupture of atomic or molecular bonds dominated by a thermal activation of the bond breakages, the rate effect can be modeled by the concept of a rate dependence on the fracture process (Di Luzio & Cedolin 2005).

As proposed in Wu & Bažant (1993), the cohesive crack model can be made a time-dependent process characterized by its aforementioned activation energy. This concept has been implemented in LDPM (Smith & Cusatis 2016; Boumakis et al. 2018) by scaling the tensile stress-strain boundary by the function with the normal strain rate.

3.2.3 Solidification Theory for visco-elastic deformations

According to the Solidification Theory (Bažant & Prasanna 1989; Bažant et al 1997; Bažant et al. 2004; Di Luzio & Cusatis 2013), the visco-elastic behavior of concrete comprises the

visco-elastic strain of the aging hardened cement gel. A non-aging micro-compliance function of cement gel produces strains that reduces overtime with a function that represents the volume fraction of cement gel produced by early-age chemical reactions. This formulation is expressed in an incremental stress/strain relation using the approximation of the creep function with a Dirichlet series with the introduction of a continuous retardation spectrum based on the Post-Widder formula to calculate the parameters of the Dirichlet series (Bažant & Xi 1995; Bažant & Jirásek 2018; Di Luzio et al. 2020).

3.2.4 *Microprestress Theory for viscous deformations*

The purely viscous strain rate represents the totally unrecoverable part of the creep strain and it is associated to long-term creep, drying creep effect (also called Pickett effect), and transitional thermal creep. From the microprestress theory the purely viscous strain rate can be expressed with the dashpot in Figure 2 whose viscosity is function of the microprestress (Bažant et al 1997; Bažant et al. 2004; Di Luzio & Cusatis 2013).

Recently it has been shown that the microprestress theory arises in nanoscale numerical simulations of C-S-H in which the logarithmic creep and power-law microprestress relaxation emerge from generic deformation kinetics in such disordered systems (Masoero & Di Luzio 2020).

3.2.5 *Hygral and thermal deformation*

The variation of the relative humidity in the material pores causes free hygroscopic strain ϵ^h (swelling or shrinkage, for positive or negative relative humidity change, respectively). Similarly, temperature changes cause thermal strain rates, $\dot{\epsilon}^t$ (Abdellatef et al. 2019).

3.2.6 *Self-healing modeling*

The modelling approach consider the matrix cracks at the mesoscale, and the fibre-matrix interface cracks (*tunnel* cracks). The modelling approach relies on the idea for which the self-healing of matrix and tunnel cracks affect the material mechanical behaviour differently. For this reason, the autogenous repairing of the matrix cracks is implemented within the constitutive fracture law at the mesoscale, whereas the effect of healing on the fibres response is taken into account within the calculation of the bridging force carried by the steel reinforcement. In the proposed model, the moisture permeability is assumed to be affected only by the matrix cracks, then only their closure contributes to the recovery in water-tightness that healed material might experience.

The healing kinetic is described by an internal variable that characterized the crack close. It is formulated for plain cementitious materials can be also employed for fibre-reinforced composites as well. For all the details on the formulation an interested reader has to refer to Di Luzio et al. (2018); Cibelli et al. (2022a); Cibelli et al. (2022b).

3.2.7 *Aging*

As widely known concrete properties typically improve in course of time, especially in the early weeks after casting. This complex process is called aging and its principal source is the ongoing hydration, which depends on the mix design and the environmental boundary conditions (Cernuschka et al. 2022).

The evolution of mechanical properties are expressed as a function of an aging degree internal variable making some meso-scale parameters age-dependent that are the normal modulus, the tensile strength, the tensile characteristic length, and the shear strength ratio. For all the details on the formulation an interested reader has to refer to Di Luzio & Cusatis (2013); Wan et al. (2016).

4 MODEL APPLICATIONS

The initial results of the application of the computational framework presented in the previous Section are reported here. The proposed computational framework has been first calibrated for the aging mechanical properties of the considered material, by simulating an extensive laboratory campaign conducted at the Politecnico di Milano and looking for the best match between numerical and experimental results. As an example, in Figure 3 the comparison between numerical and experimental curves is shown for the fracture behaviour of notched specimens after three different ages from casting: 6, 28 and 180 days.

The same approach has been adopted also for the healing model, which has been calibrated and validated by using the evidences presented in Cuenca et al. (2021). In Figure 4 the model capability of capturing (i) the effect of cracks on the moisture transport phenomena and (ii) the time evolution of crack sealing due to the self-healing (Index of Crack Sealing, ICS) is shown.

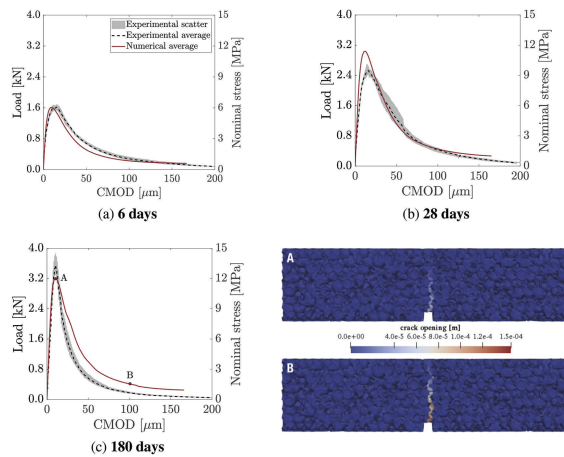


Figure 3. Ageing model – Comparison between experimental and numerical results relevant to three different ages from casting: 6, 28 and 180 days.

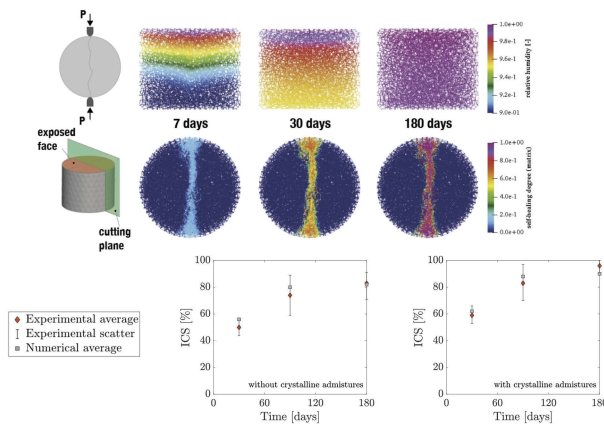


Figure 4. Healing model – Moisture gradient and healing evolution in cracked conditions over time.

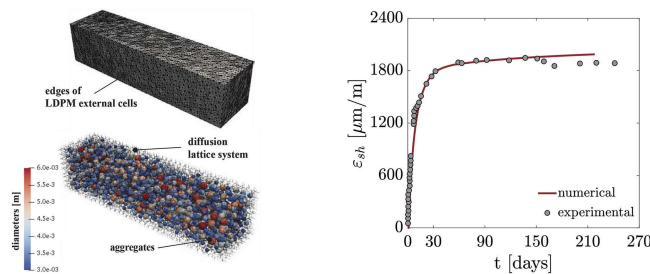


Figure 5. Shrinkage model – Comparison between experimental and numerical autogenous shrinkage in UHPC.

Finally, the proposed computational framework is being calibrated for the shrinkage and creep properties of the considered mixes. Some preliminary results are reported in Figure 5.

5 CONCLUSIONS

This study presents the initial results of an experimental/numerical research on the cross effects of the crack healing and creep in typical expected service load condition. The numerical model is based on the mesoscale Multiphysics-Lattice Discrete Particle Model (M-LDPM) for the characterization of the long- and short-term mechanical behavior with coupled thermal, shrinkage, creep deformations, and self-healing. Preliminary results are presented in this manuscript. This research allows to learn something new about the behavior of crack healing under loading condition, which is of utmost importance for practical applications.

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