Virtualization of Guitar Pickups through Circuit Inversion

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***Abstract*—A method for circuit system inversion has been recently employed to develop digital algorithms for loudspeaker virtualization. In this brief, building on such promising results, we propose an algorithm that flips the paradigm by virtualizing sensors rather than actuators. In particular, we derive direct and inverse nonlinear circuital models of guitar pickup systems by assuming string vertical excitations, and we then present a virtualization algorithm based on circuit inversion. The proposed circuital models are then implemented in the discrete-time domain in a fully explicit fashion (i.e., with no use of iterative solvers) by employing Wave Digital Filter principles. Finally, we validate and test the designed algorithm on the physical output voltage of a guitar pickup system, making it sound as if acquired by two other magnetic pickups characterized by a different nonlinear behavior.**

***Index Terms*—sensor virtualization, guitar pickups, nonlinear model, Wave Digital Filters**

I. Introduction

Since their first step into the musical scene, magnetic guitar pickups completely disrupted the way music is conceived. Their story dates back to the mid-1920s when G. Beauchamp and A. Rickenbacker designed and produced the very first single-coil pickup, known as “horseshoe pickup” due to its U shape [1]. In particular, a magnetic guitar pickup is a nonlinear sensor that captures the vibrations of the strings and converts them into an electrical signal [2], [3]. The vibration of a ferromagnetic string in close proximity of a magnetic pickup yields a variation in the magnetic flux generated by the coil itself, inducing an electrical voltage at the coil terminals according to the Faraday-Lenz law [2], [4]. The attractive sound of electric guitars is indeed due to the nonlinearities affecting the pickups mounted on their body, which contribute to shaping the timbre by imparting different tonal characteristics (e.g., warmth, clarity, etc.). With the advent of digital audio effects, it became thus interesting to model such nonlinear systems in order to derive digital signal processing techniques able to synthesize such distinct sounds [5].

Different models of magnetic pickups are available in the literature. Some of them are linear [3], others nonlinear [6]–[10], and still others combine both linear and nonlinear blocks together [4]. In particular, in [4] it is shown that the characteristic sound of a magnetic pickup is mainly due to: (i) the distance d_0 between strings and pickup (the smaller, the more pronounced the nonlinear behavior); (ii) the filtering action of the processing chain; (iii) the nonlinear relation between string displacement and magnetic flux. As experimentally verified in [9], besides being static (i.e., independent of frequency) [4], the nonlinearity curve of magnetic pickups is also independent of the pickup/string distance d_0. As a consequence, a magnetic pickup can be completely described by a single nonlinear static curve which is invariant to the string (or pickup) starting position and the maximum string displacement.

In this brief, we propose a novel algorithm for the virtualization of guitar pickups by means of circuit inversion. Starting from the work presented in [9], we first derive a nonlinear circuital model of guitar magnetic pickup systems which includes tone and volume controls, as well as cable and interface (e.g., amplifier, direct injection box, etc.) input impedances. Then, given the physical direct circuital system, we design its inverse by exploiting Leuciuc’s theorem [11], recently reworded in [12]–[14], after augmenting the circuit representation with a two-port element known in circuit theory as *nullor*. Building on the promising results obtained in [13], [14] for the virtualization of loudspeakers, we later propose an algorithm based on a flipped *Direct-Inverse-Direct Chain* (DIDC) able to alter the guitar signal making it sound as if acquired by another magnetic pickup. The chain is composed of: the physical pickup, a digital model of the inverse pickup, and a digital model of the target pickup. The algorithm is implemented exploiting Wave Digital Filter (WDF) principles [15]. Finally, we test the proposed DIDC-based algorithm on three different magnetic pickups, considered also in [9], successfully achieving the virtualization in real-time.

II. Circuital Model of Guitar Pickups

In this section, we first introduce the nonlinearity affecting magnetic pickups and provide the empirical model chosen to describe such a curve. Then, we present a possible circuit model of a nonlinear magnetic pickup system and its inverse, which will be employed further ahead for deriving the virtualization algorithm.

A. Magnetic Pickup Nonlinearity

As experimentally verified in [4], [9], there exists a nonlinear relation between the string displacement x_{in} and the time-integral of the voltage ϕ, which can be directly measured across the pickup. The quantity ϕ has the dimensions of the magnetic flux, i.e., V$\cdot$ s, and differs of a multiplicative constant from the real magnetic flux generated by the coil. In addition, the nonlinear curve is also proved to be independent of the

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TABLE I: Model parameters for the three pickup nonlinear curves [9].

Pickup	A	l_{m}	r_{m}
SSL-5	$21.51 \cdot 10^{-3}$	12.98 mm	2.77 mm
SH-2N	$40.67 \cdot 10^{-3}$	9.86 mm	1.34 mm
STHR-1B	$47.46 \cdot 10^{-3}$	13.11 mm	1.88 mm

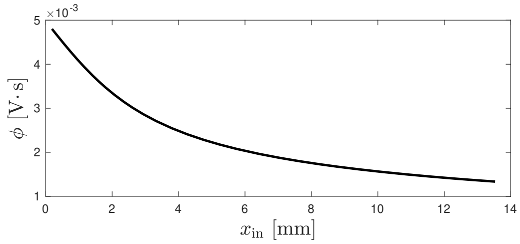


Fig. 1: Nonlinear curve (time integral of voltage vs. displacement) of Seymour Duncan SSL-5 pickup.

rest pickup-string distance d_0, further suggesting that it is possible to derive an analytical model suitable to describe such a relation. Figure 1 shows the nonlinear curve of a Seymour Duncan SSL-5 pickup; the curve is obtained from direct measurements provided in [9]. Drawing on the findings shown in [4], in [9] such a nonlinear characteristic is described by the following model based on the physics of the device itself

$$\phi = A\left(\frac{x_{\mathrm{in}} + l_{\mathrm{m}}}{\sqrt[3]{r_{\mathrm{m}}^2 + (x_{\mathrm{in}} + l_{\mathrm{m}})^2}} - \frac{x_{\mathrm{in}}}{\sqrt[3]{r_{\mathrm{m}}^2 + x_{\mathrm{in}}^2}}\right), \qquad (1)$$

where r_{m} and l_{m} are the radius and length of an equivalent cylindrical magnet (or solenoid), respectively, whereas A is a constant that takes into account the residual flux and the string geometrical characteristics as well as its material properties. The electrical voltage of the pickup is then computed by simply taking the time derivative of ϕ. The model is inspired by the equation describing the magnetic field of a cylindrical magnet (or solenoid), and turns out to be really effective in fitting the measured nonlinear curves of magnetic pickups. It is worth pointing out that, however, such a model assumes only vertical excitations. Besides Seymour Duncan SSL-5 (single-coil) pickup, we take into account Seymour Duncan SH-2N (humbucker) and Seymour Duncan STHR-1B (humbucker rail) pickups (also considered in [9]). Table I reports the parameters estimated in [9] for modeling the nonlinearities of the three pickups. Finally, more details on the physics of guitar string vibrations can be found in [16].

B. Circuital Model

Figure 2(a) shows the proposed (nonlinear) circuital model of a magnetic guitar pickup system. The first subsystem on the left is a circuital realization of the nonlinear block considered in [4], [9] and represents a fictitious domain where the "through" and "across" variables are the time integral of voltage ϕ and the string displacement x_{in}, respectively. In such a domain, the pickup nonlinearity is thus represented as a nonlinear resistor (rectangular one-port in Fig. 2(a)). The output pickup voltage is then obtained by means of the second subsystem that provides the time derivative ψ of ϕ at the inductor terminals. In fact, by considering $L = 1$ and the inductor constitutive relation, we obtain $\psi = \frac{d\phi}{dt}$. Finally, the third subsystem models the analog processing chain that characterizes the path from the pickup to the input of the interface to which the guitar is connected. In particular, capacitor C_{t} and resistor R_{t} model the tone control, resistor R_{v} the volume control, whereas capacitor C_{t} and resistor R_{i} model the capacitance of the cable and the interface input impedance, respectively.

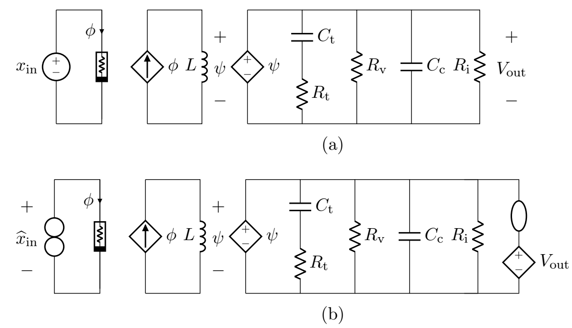


Fig. 2: (a) Proposed circuit model of a guitar pickup system; (b) inverse circuit model of the guitar pickup system.

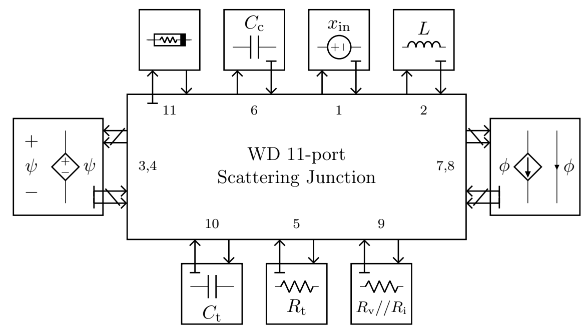
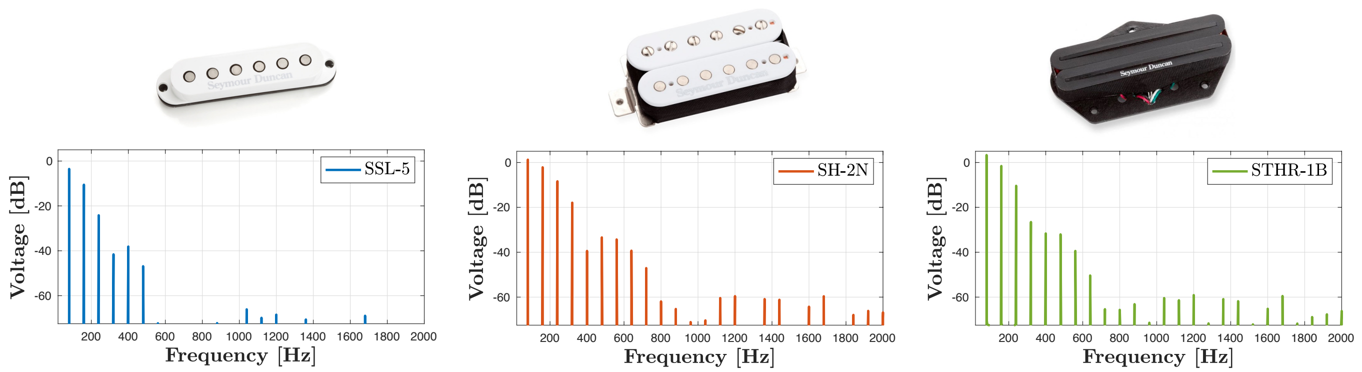


Fig. 3: WDF implementation of the circuit shown in Fig. 2(a). The T-shaped stubs indicate port adaptation.

Fig. 3 shows a WDF realization of the proposed model characterized by a 11-port scattering junction to which all the elements, implemented as input-output blocks characterized by scattering relations, are connected. In the Wave Digital (WD) domain, port currents and port voltages are turned into linear combinations of incident and reflected waves with the addition of a free parameter per port called *port resistance* [15]. In this work, voltage waves are considered; moreover, all the linear elements are *adapted* by properly setting the corresponding free parameters to remove the delay-free loops as done in [15], [17]. The input generator x_{in} is implemented as a resistive source, by considering a series resistance small enough to not modify the circuit behavior (e.g., 1 μΩ), such that it can be adapted. Continuous-time derivatives of dynamic elements are discretized using the Backward Euler method [17], whereas both Current-Controlled Current Source (CCCS) and Voltage-Controlled Voltage Source (VCVS) are implemented by means of vector waves [18]. The scattering matrix $\mathbf{S}$ of the WD 11-



(a) (b) (c)

Fig. 4: Discrete Fourier Transforms (DFTs) of the outputs V_{out} for Seymour Duncan SSL-5 (a), Seymour Duncan SH-2N (b), and Seymour Duncan STHR-1B (c).

TABLE II: Parameters of the magnetic pickup system shown in Fig. 2.

Param.	Value	Param.	Value	Param.	Value
L	1	C_{t}	1 nF	R_{t}	500 kΩ
R_{v}	800 kΩ	C_{c}	750 pF	R_{i}	1 MΩ

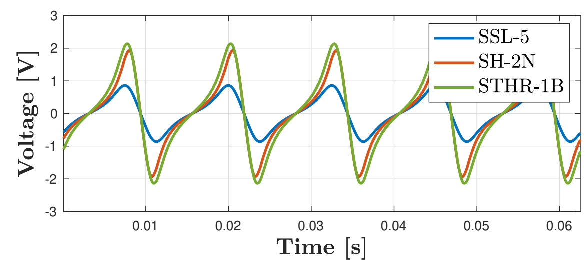


Fig. 5: Time-domain comparison of the output signals V_{out} of Seymour Duncan SSL-5 (blue), Seymour Duncan SH-2N (red), and Seymour Duncan STHR-1B (green) pickup models.

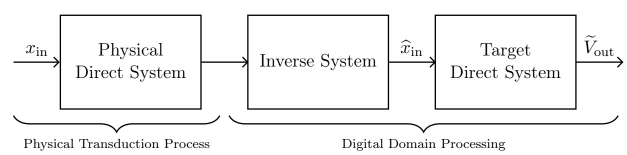


Fig. 6: Direct-Inverse-Direct Chain (DIDC) for the virtualization of sensors.

port junction is computed following the methodology shown in [19], [20]. The nonlinear element is, instead, realized in the WD domain by means of Canonical Piecewise-Linear (CPWL) functions which enable a global explicit and data-driven implementation of the pickup nonlinearity [21]. In particular, we consider a smoothed representation (based on natural exponential and logarithmic functions) of CPWL functions [22] such that the curve is still characterized by a continuous first-order derivative, avoiding thus problems with the differentiation introduced by the second subsystem. In order to accomplish such a representation, we compute ϕ via (1) and extract 31 points by considering x_{in} in the typical range of string displacement (i.e., $0.5 - 12.5$ mm). Then, such points are expressed in the WD domain by following the same procedure described in [23] considering as port resistance of the nonlinear element the one which makes port 11 of the topological junction reflection free. This enables a fully explicit solution of the WD structure [24]. The accuracy of the WDF implementation is then verified through a comparison with a MathWorks Simscape implementation of the proposed circuit, taking into account the parameters shown in Table II.

Finally, we perform three different simulations considering the nonlinearities of Seymour Duncan SSL-5, Seymour Duncan SH-2N, and Seymour Duncan STHR-1B pickups. To this aim, we set the input displacement equal to $x_{\text{in}} = d_0 + d_{\text{max}} \sin(2\pi k f_0 / f_{\text{s}})$, with k as the sample index, $d_0 =$ 3 mm (typical value of string rest position), $d_{\text{max}} = 2.8$ mm, fundamental frequency $f_0 = 80$ Hz, and sampling frequency $f_{\text{s}} = 44.1$ kHz. The output signal V_{out} of the simulations of the three pickups is shown in Fig. 5, pointing out their different behaviors. Such differences are even more evident in Fig. 4, where the Discrete Fourier Transforms (DFTs) of the three signals are depicted. This further justifies the development of virtualization algorithms for recreating the sound of different pickups.

C. Inverse Circuital Model

Fig. 2(b) shows the inverse of the system in Fig. 2(a). This is obtained by applying the theorem presented in [11], [14] after augmenting the direct system (i.e., circuit in Fig. 2(a)) with a theoretical two-port called *nullor* [25]. A nullor is composed of a *nullator* (represented as an ellipse), having both port voltage and current equal to zero, and a *norator* (represented by two circles), having unconstrained port variables. According to the theorem, the voltage $\widehat{x}_{\text{in}}$ across the norator in the inverse system is equal to the input signal x_{in} of the direct system, and it is obtained by driving the inverse system with V_{out}, i.e., the output of the direct system. The WD implementation of the inverse is similar to the one shown in Fig. 3, but port 1 is connected to voltage generator V_{out}, implemented as a resistive source with small series resistance. The output, instead, is taken across the nonlinear element given that it is connected in parallel to the norator. Finally, following the methodology explained in [26], we embed the nullor inside the WD 11-port

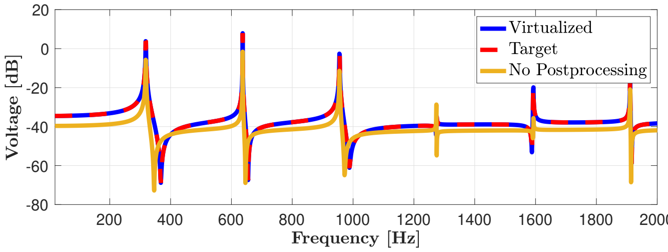


Fig. 7: Validation of the DIDC-based virtualization algorithm.

scattering junction and we then compute the corresponding scattering matrix. Since such a scattering matrix is different with respect to the one of the WD direct system, we have to recompute the adaptation condition for port 11. Once obtained the new port resistance, we employ it for expressing the 31 points in the WD domain such that smoothed CPWL functions can be considered once again for the implementation of the pickup nonlinearities.

In order to validate the inverse circuital model, we cascade the direct system (in Fig. 2(a)) and the inverse system (in Fig. 2(b)), and we verify that the input signal of the direct system x_{in} is equal to the output signal of the inverse system $\widehat{x}_{\text{in}}$, feeding the output of the direct system in input to the inverse system. The input signal considered for this test is the displacement of an ideal plucked string of length 0.65 m with damping implemented as in [27]. In particular, x_{in} is chosen to be the displacement of a point close to the plucking point, which is located at $1/5$ of the total string length. We perform such a test with the direct system modeling Seymour Duncan SSL-5 pickup, obtaining a perfect correspondence between x_{in} and $\widehat{x}_{\text{in}}$ with a Normalized Root Mean Square Error (NRMSE) of 1.27×10^{-8}.

III. Virtualization Algorithm

Let us now introduce the proposed pickup virtualization algorithm, which is characterized by the signal flow shown in Fig. 6. The algorithm is based on a flipped version of the *Direct-Inverse-Direct Chain* (DIDC) proposed in [14] for the virtualization of loudspeakers. In fact, in our application we aim at virtualizing sensors rather than actuators. The chain is composed of three main blocks: the *Physical Direct System*, the *Inverse System*, and the *Target Direct System*; the first representing the actual magnetic pickup, i.e., the physical transduction process, while the second and the third representing the digital signal processing structure, which can be implemented in the WD domain as shown in Section II. The output signal $\widetilde{V}_{\text{out}}$ of the digital (virtual) *Target Direct System* is equal to the output V_{out} of its physical counterpart, provided that the cascade of the actual *Physical Direct System* and the *Inverse System* is equivalent to the identity operator. The *Target Direct System* is designed according to the specific application. In fact, different tasks can be accomplished with a DIDC-based algorithm, including but not limited to linearization, equalization, or virtualization of sensors/actuators, having thus linear or nonlinear blocks. In the following, we will address the case of virtualization considering a nonlinear *Target Direct System*.

IV. Examples of Application

In this section, we provide some examples of application of the DIDC-based algorithm presented in Section III. For instance, in order to validate the processing chain, we perform a test by simulating first the output of the Seymour Duncan SH-2N pickup system using the nonlinear WDF presented in Section II-B. We then compare it to the output of the DIDC-based virtualization algorithm having the Seymour Duncan SSL-5 and the Seymour Duncan SH-2N models as *Physical Direct System* and *Target Direct System*, respectively. As input signal, instead, we consider the same displacement employed for validating the inverse system in Section II-C. The results of the frequency-domain analysis are shown in Fig. 7, where “No Postprocessing” (yellow) refers to the simulated output of the Seymour Duncan SSL-5 model, “Target” (blue) refers to the output of the Seymour Duncan SH-2N model acting as *Target Direct System*, while “Virtualized” (red) is the output of the DIDC-based virtualization algorithm. The “Target” curve is superimposed to the “Virtualized” pickup curve yielding a NRMSE $= 9.6 \times 10^{-8}$. We then perform other tests taking into account all the possible permutations of the three pickups in the DIDC-based algorithm (e.g., Seymour Duncan SH-2N as *Physical Direct System* and Seymour Duncan STHR1-B as *Target Direct System*, etc.), obtaining an accuracy comparable to the one reported above. It follows that the proposed algorithm is suitable to accomplish the task of virtualization, more generally, paving the way toward the virtualization of nonlinear sensors.

As a last test, we apply the same virtualization algorithm considering a guitar audio signal of 16 s acquired by means of Seymour Duncan SSL-5, and we virtualize the behaviors of the other two pickups. The algorithm is implemented in a MATLAB script with an average execution time of approximately 4.73 µs per sample, which is lower than $T_{\text{s}} = 1/f_{\text{s}} = 22.7$ µs. Therefore, the MATLAB implementations of the proposed algorithm can be executed in real-time, being thus suitable to be integrated into digital audio effects for the virtualization of guitar sounds.

V. Conclusions

In this letter, we proposed a novel and efficient algorithm for the virtualization of magnetic guitar pickups. We derived, for the first time, a nonlinear circuit able to describe the whole magnetic pickup system, and we designed its inverse circuit model according to Leuciuc’s theorem of inversion [11]. We employed WDFs for obtaining a fully explicit and efficient implementation of both direct and inverse systems. Then, we presented a virtualization algorithm based on a flipped version of the *Direct-Inverse-Direct Chain* [14], which we employed to invert the behavior of a (nonlinear) *Physical Direct System*, i.e., the actual magnetic pickup, and apply the nonlinear characteristics of a *Target Direct System*, i.e., another magnetic pickup. The proposed algorithm was able to run in real-time, and is promising for applying new kinds of effects to electric guitar sounds, but also for re-amping, re-miking, etc. Future work might concern models of both vertical and horizontal excitations [10], the virtualization of piezoelectric pickups as well as other kinds of sensors (such as microphones).

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