

A Bootstrapped Modularised method of Global Sensitivity Analysis applied to Probabilistic Seismic Hazard Assessment

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ABSTRACT

Probabilistic Seismic Hazard Assessment (PSHA) evaluates the probability of exceedance of a given earthquake intensity threshold like the Peak Ground Acceleration (PGA), at a target point for a given exposure time. The stochasticity of the occurrence of seismic events is modelled by stochastic processes and the propagation of the earthquake wave in the soil is typically evaluated by empirical relationships called Ground Motion Prediction Equations (GMPEs). The large uncertainty affecting PSHA is quantified by defining alternative model settings and/or model parametrizations. In this work, we propose a novel Bootstrapped Modularised Global Sensitivity Analysis (BMGSA) method for identifying the model parameters most important for the uncertainty in PSHA, that consists in generating alternative artificial datasets by bootstrapping an available input-output dataset and aggregating the individual rankings obtained with the modularized method from each of those.

The proposed method is tested on a realistic case study that refers to the PSHA of volcanic islands (for example Ischia, Italy). The results are compared with a standard variance-based Global Sensitivity Analysis (GSA) method of literature. The novelty and strength of the proposed BMGSA method are both in the fact that its application only requires input-output data and not the use of a PSHA code for repeated calculations.

Keywords: Probabilistic Seismic Hazard Assessment (PSHA), uncertainty, Modularised Global Sensitivity Analysis (MGSA), Bootstrapped Modularised Global Sensitivity Analysis (BMGSA)

Acronyms

PSHA	Probabilistic Seismic Hazard Assessment
PGA	Peak Ground Acceleration
GMPE	Ground Motion Prediction Equation
SA	Sensitivity Analysis
GSA	Global Sensitivity Analysis
MGSA	Modularised Global Sensitivity Analysis
BMGSA	Bootstrapped Modularised Global Sensitivity Analysis
IM	Intensity Measure
MCS	Monte Carlo Simulation
BU	Bottom-Up
AO	All-Out
BC	Borda Count

Symbols

ΔT	Exposure time window
γ	Threshold value of the given intensity measure
λ_H	Annual rate of exceedance of the γ -th PGA level
λ	Annual rate of mainshock occurrence at a given source location
$f_m(\mathbf{m})$	Probability distribution of different earthquake magnitudes
m_{min}	Minimum magnitude of the probability distribution $f_m(\mathbf{m})$
m_{max}	Maximum magnitude of the probability distribution $f_m(\mathbf{m})$
b	Slope parameter of the probability distribution $f_m(\mathbf{m})$
$f_r(\mathbf{r})$	Probability distribution of the source-to-target distance
r	Source-to-target distance
C	Computational cost of the double-loop Monte Carlo Simulation

ν	Number of input variables
n_1	Sample size for estimating the inner loop of the double-loop MCS
n_2	Sample size for estimating the outer loop of the double-loop MCS
d	Index of the generic bootstrapped dataset
D	Total number of the bootstrapped dataset
g	Model
Y	Output of the model g
\bar{X}	Input parameters
$\bar{\bar{Z}}$	Input-output dataset
s	Index of the input-output pattern of $\bar{\bar{Z}}$
S	Total number of the input-output pattern of $\bar{\bar{Z}}$, i.e., number of rows of $\bar{\bar{Z}}$
N	Number of input parameters, i.e.
n	Index of the input parameter
$Var[\cdot]$	Variance
$\mathbb{E}[\cdot]$	Expectation operator
S_n	Sobol index of the n -th parameter
$\bar{\bar{Z}}_d^*$	Reduced matrix
K	Number of mutually exclusive subset of the reduced matrix
k	Index of the mutually exclusive subsets of the reduced matrix
$\bar{\bar{Z}}_d^{*k}$	Generic mutually exclusive subset of the reduced matrix of the generic bootstrapped dataset
J	Number of rows of the $\bar{\bar{Z}}_d^{*k}$
$S_{n,d}$	Sobol index of the n -th parameter of the generic d -th bootstrapped dataset
$\bar{R}_{BU,d}$	Input ranking (bottom-up strategy) of the generic bootstrapped dataset
\bar{R}_{BU}	Final aggregated input ranking (bottom-up strategy)
BC_n	Borda count for the n -th input variable
$p_{n,d}$	n -th input variable order inside the d -th ranking
\bar{R}_{AO}	Final aggregated input ranking (all-out strategy)
σ_{GMPE}	Standard deviation of the GMPE

1. INTRODUCTION

Probabilistic Seismic Hazard Assessment (PSHA) consists of assessing at a given target location and for a given exposure time window ΔT , the probability that a given intensity measure (IM) of the ground motion, typically the Peak Ground Acceleration (PGA), exceeds a threshold value γ , that is then summarized in hazard curves, where the IM is plotted versus its annual rate of exceedance [1]. To compute the annual rate of exceedance, the most consolidated and widely used method is described in [2]: first, the annual rate of the mainshocks (i.e., the strongest events in a seismic sequence) is estimated; then, the propagation from the source to the site of interest is performed. Usually, the former is based on an Earthquake Rupture Forecast model, that relies on either spatial distributions of the mainshocks, commonly using area source models [3], or smoothed seismicity models [4], coupled with magnitude frequency distributions, ordinarily described by the Gutenberg-Richter law [5].

The latter is typically evaluated by empirical relationships, called Ground Motion Prediction Equations (GMPEs), finally leading to the annual rate of exceedance of a selected IM level, λ_H , that is quantified by means of the total probability theorem as [6]:

$$\lambda_H(PGA > \gamma) = \lambda \int_{m_{min}}^{m_{max}} \int_0^{R_{max}} P(PGA > \gamma | m, r) f_m(m) f_r(r) dm dr \quad (1)$$

where λ is the annual rate of mainshock occurrence at a given source location (i.e., the number of occurrences of earthquakes above a given minimum magnitude threshold m_{min} per year); the distribution $f_m(m)$ describes the probability distribution of different earthquake magnitudes, typically assumed to follow a truncated Gutenberg-Richter distribution within the interval of values $[m_{min}; m_{max}]$ and slope parameter b [6]; $f_r(r)$ describes the probability distribution of the source-to-target distance r , assuming a spatial distribution for earthquakes [1]. These input distributions are typically determined from historical, instrumental, and geological observations [1], [7] but large

uncertainty exists, so that many alternative parametrisations of the model are possible [8], [9]. In this respect, Sensitivity Analysis (SA) can aid the understanding of how the uncertainty in the model is apportioned among the model input parameters uncertainties [10], [11]. In other words, through SA, one can identify the most sensitive parameters and better focus the uncertainty analysis without losing accuracy [12]. Different SA techniques have been proposed in literature, which can be sorted into three main categories: local, regional, and global [13]. Local and regional analyses limit inputs variations to a subset of their overall ranges. Local methods evaluate at low computational costs the effects on the system response of small perturbations in the model input variables around fixed values [13]. Then, local SA provides information on the sensitivity of the model output to the input variability at some fixed points. Regional analyses, on the contrary, focus on calculating the sensitivity of the model output to the variability of the inputs varying in given ranges of the inputs; yet, they do not give complete account to the uncertainty of the model inputs, in terms of their distributions [14], [15]. Global Sensitivity Analysis (GSA) methods, instead, explore the whole distribution range of the model inputs and the effects of their mutual combination on the model output, but they do so at larger computational costs than local and regional methods [14], [15]. GSA methods can be regression-based [16], variance-based [15], [17], [18] distribution-based [19], [20] and expected value of information (EVI)-based [21]. Among the GSA methods, variance decomposition based on Sobol indices is most widely used [22].

Sobol indices measure the contribution the input variables provide, individually or in groups, to the variability of the model output [14], [23]. They are usually computed via a double-loop Monte Carlo Simulation (MCS), with computational cost equal to $C = v \cdot n_1 \cdot n_2$, where v is the number of input variables, n_1 the sample size for estimating the inner loop and n_2 the sample size for the outer loop [24], [25]. When the model runs are time consuming, the computational cost is high and strategies have been proposed to reduce it, including: reduced order models calibrated on input-output data

obtained by few runs of the original model, e.g., Bayesian approaches [26], kriging [12], [27], and polynomial chaos expansion [28], [29]; sampling schemes tailored to efficiently characterize the sensitivity of the model inputs, e.g., Fourier Amplitude Sensitivity Test (FAST) [30], [31] and Effective Algorithm for computing global Sensitivity Indices (EASI) [32]; finally, data-driven approaches allow exploiting available datasets to calculate sensitivity measures [22], [33], [34], [35], which is quite of interest for many practical applications in which an input-output dataset is available and models to perform a GSA using MCS-based methods cannot be run [22], [36].

In this work, we propose a novel data-driven method based on bootstrapping, Modularised GSA (MGSA) and ensemble strategies, to identify the input variables of a seismic model (for example, $\bar{X} = (\sigma_{GMPE}, \lambda, m_{max}, m_{min}, b, r)$, where σ_{GMPE} is the standard deviation of the GMPE, λ is the annual rate of seismic activity at the source location, m_{min} and m_{max} are the minimum and the maximum magnitude parameters of the truncated Gutenberg-Richter distribution, whose slope is b [6], [37], and r is the source-to-target distance [6]) whose output $Y = \lambda_H(PGA > \gamma)$ is most sensitive to, assuming that only an input-output dataset is given and with no need of repeating hazard computations. The method can be used a posteriori, i.e., when the PSHA has been performed, as well as a priori, when an update of the PSHA is required, by informing the seismologists about the parameters that drive most the uncertainty, and allowing for scenarios exploration, saving modelling efforts and computational time. The procedure of the novel method is sketched in the flowchart of Figure 1. It consists of, first, applying a Bootstrap technique to the available input-output dataset $[\bar{X}; Y]$ to artificially increase the amount of data available [38], [39]. Then, for each d -th dataset, a sensitivity index is calculated for each input variable. Without loss of generality, in this work we propose to calculate the first-order Sobol index, which measures the input variables individual contributions to the variability of the model output [14], [23]: in practice, for each input variable, the d -th dataset is modularised (i.e., partitioned) into sub-sets that are used

to calculate the variance of the model output Y and the first-order Sobol index [25]. Finally, the D independent rankings of the input variables, obtained based on their first-order Sobol indices values, are ensembled to provide an aggregated ranking of the input variables which the output is sensitive to. Typical ensemble strategies are Bottom-Up (BU) and All-Out (AO) strategies: the former computes a ranking order of the input variables out of each d -th dataset and combines the D alternative rankings a posteriori to generate a final aggregated ranking order [10]; on the contrary, the latter merges a priori the information from the D datasets by averaging the D Sobol indices for each input parameter and, then, provides the final ranking [10].

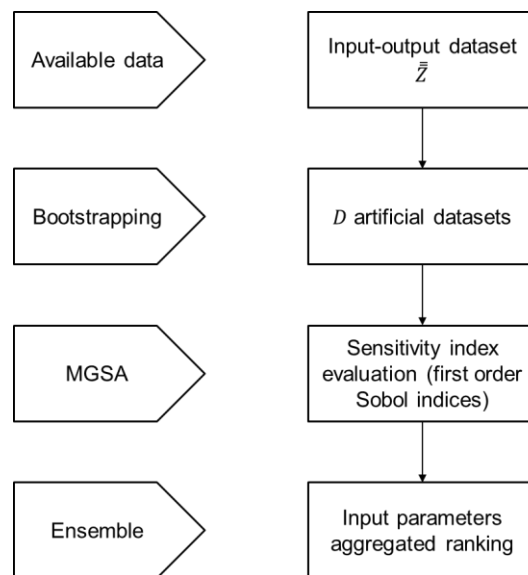


Figure 1 – Flowchart of the proposed method.

The proposed method is tested on with respect to the calculation of the hazard intensity corresponding to 10% probability of being exceeded in 50y for a realistic case study with a point seismic source nearby the target point, as it occurs for volcanic islands with short source-to-site distance and high b -values (for example, Ischia (Italy) [40]). The results are compared to those obtained by a standard variance-based GSA method [11], which is the state-of-practice approach when the simulation model is available.

The remainder of the paper is organised as follows. Section 2 explains the novel BMGSA method. Section 3 presents the case study. Section 4 shows the results of the application of the proposed method to the data of the case study and the comparison with the results obtained with the standard variance-based GSA of literature. Conclusions are drawn in Section 5.

2. THE NOVEL BOOTSTRAPPED MODULARISED GSA

Let us consider a seismic model g whose output value $Y \in \mathbb{R}$ depends on the values of uncertain input parameters $\bar{X} = (X_1, X_2, \dots, X_N)$, with N being the number of input parameters:

$$Y = g(\bar{X}) \quad (2)$$

where, without loss of generality, $\bar{X} = (\sigma_{GMPE}, \lambda, m_{max}, m_{min}, b, r)$ and $Y = \lambda_H(PGA > \gamma)$.

Let us also assume that an input-output data set \bar{Z} is given (i.e., the analyst may not have the simulation code available):

$$\bar{Z} = \begin{pmatrix} \bar{Z}^1 \\ \vdots \\ \bar{Z}^S \end{pmatrix} = \begin{pmatrix} x_1^1 & \dots & x_N^1 & y^1 \\ \vdots & \ddots & \vdots & \vdots \\ x_1^S & \dots & x_N^S & y^S \end{pmatrix} \quad (3)$$

where $\bar{Z}^s = [\bar{x}^s, y^s]$, $\forall s = 1, \dots, S$ is the s -th input-output pattern of $\bar{Z} = (X_1, X_2, \dots, X_N, Y)$.

The proposed methodology consists in:

1. Generating D alternative bootstrapped artificial datasets from the available input-output dataset \bar{Z} [41];
2. From each d -th alternative dataset and for each input variable, calculating a sensitivity index (here the first-order Sobol index) with the modularised method [25];
3. Aggregating the D individual rankings (one for each alternative dataset) with Bottom-Up/All-Out strategies [10].

GENERATION OF THE BOOTSTRAPPED DATASETS

Bootstrap is a computer-based method usually employed to assess the accuracy of statistical estimates with minimum assumptions [39]. The main benefit is avoiding additional computational burden (for example, when simulation codes are computationally demanding or not available, as in the current case) by relying only on the available data [39], [42], which makes it particularly fit for the purpose of this work.

The basic idea is to generate a number D of artificial datasets \bar{Z}_d by random sampling, with replacement from \bar{Z} , the input-output patterns. The generic bootstrapped \bar{Z}_d , thus, consists in a $S \times (N + 1)$ matrix.

THE MODULARISED METHOD TO CALCULATE THE SOBOL INDEX

The Sobol index is the result of the application of a variance-based method that apportions the output variance into the single (or groups of) variables variances [43], [44]. No hypotheses are made on the structure of the model g from which the data have been generated. The variance $Var[Y]$ of the output Y can be, indeed, decomposed as follows [14], [15]:

$$Var[y] = Var_{X_n}[\mathbb{E}_{X_{\sim n}}(y|x_n)] + \mathbb{E}_{X_n}[Var_{X_{\sim n}}(y|x_n)] \quad (4)$$

where:

- $Var_{X_n}[\mathbb{E}_{X_{\sim n}}(y|x_n)]$ is the variance of Y caused by X_n without considering its interactions with other input variables (i.e., $X_{\sim n}$)
- $\mathbb{E}_{X_n}[Var_{X_{\sim n}}(y|x_n)]$ is the variability of Y depending on all variables but on n (i.e., $X_{\sim n}$);
- $\mathbb{E}(\cdot)$ is the expectation operator;

The first-order Sobol index for the n -th generic input variable is defined as [14], [15] :

$$S_n = \frac{Var_{X_n}[\mathbb{E}_{X_{\sim n}}(y|x_n)]}{Var[y]}, \forall n = 1, \dots, N \quad (5)$$

The larger S_n , the more X_n contributes to the variance of Y [43]. As mentioned in the Introduction, the computation of the Sobol indices usually requires a double-loop MCS, which can be computationally burdensome. Since, in practice, the numerator is solved by a double-loop MCS [15]:

- The inner loop computes $\mathbb{E}_{X_{\sim n}}(y|x_n)$ using n_1 random samples of $\bar{X}_{\sim n}$ with fixed \bar{X}_n ;
- The outer loop computes $Var_{X_n}[\mathbb{E}_{X_{\sim n}}(y|x_n)]$ by iterating the inner loop n_2 times, with different values of X_n ;

In total, for each S_n , the number of model evaluations is $C_n = n_1 \cdot n_2$, that is unaffordable if each evaluation is time-consuming (notice that, in many practical applications, each loop must be of order greater than 1000 [13], [25]). To address this issue and avoid calling the simulation code, we propose a modularised approach, that partitions the $\bar{\bar{Z}}_d$ into subsets and proceeds as follows [24], [25].

Step 1: construct the reduced matrix

For each n -th input variable, append the n -th input column of $\bar{\bar{Z}}_d$ to the output column; then, shuffle the rows in ascending order to obtain $\bar{\bar{Z}}_d^*$, where $x_n^{1*} \leq x_n^{2*} \leq \dots \leq x_n^{S*}$:

$$\bar{\bar{Z}}_d^* = \begin{pmatrix} x_n^{1*} & y^{1*} \\ \vdots & \vdots \\ x_n^{S*} & y^{S*} \end{pmatrix} \quad (6)$$

Step 2: partition the reduced matrix in subsets

Partition the support of X_n in $k = 1, \dots, K$ mutually exclusive subsets $\bar{\bar{Z}}_d^{*k}$, such that $\bigcup_{k=1}^K X_n^k = X_n \wedge X_n^k \cap X_n^{l \neq k} = \emptyset$. Operatively, divide the resulting matrix $\bar{\bar{Z}}_d^*$ into $k = 1, \dots, K$ submatrices $\bar{\bar{Z}}_d^{*k}$ of J rows, each retaining the order of the Step 1:

$$\bar{\bar{Z}}_d^{*k} = \begin{pmatrix} x_n^{1*k} & y^{1*k} \\ \vdots & \vdots \\ x_n^{J*k} & y^{J*k} \end{pmatrix} \quad (7)$$

Note that $J \cdot K = S$, where S is the total input-output pattern size and $K = \text{int}(\sqrt{S})$ [24], [33].

Notice that the large K improves the accuracy of the estimation of $\text{Var}_{X_n}[\cdot]$, while worsening the accuracy of the estimation of $\mathbb{E}_{X_n}(y|x_n)$, and vice versa [24][45].

Step 3: estimation of the Sobol index

The Sobol index $S_{n,d}$ is, finally, calculated as in Eq. (8):

$$S_{n,d} = \frac{\text{Var}_{X_n}[\mathbb{E}_{X_{\sim n}}(y|x_n)]}{\text{Var}[y]} \approx \frac{\frac{1}{K} \sum_{k=1}^K (\bar{y}_k - \bar{y})^2}{\frac{1}{S} \sum_{s=1}^S (y_s - \bar{y})^2} \quad (8)$$

where: $\bar{y} = \frac{1}{S} \sum_{s=1}^S y^s$ and $\bar{y}_k = \frac{1}{J} \sum_{j=1}^J y^{j*k}$.

The calculation of $S_{n,d}$, as a result of the modularisation, depends solely on X_n and Y , and can be performed even if the input values of $\bar{X}_{\sim n}$ are not available [24], [25].

ENSEMBLE OF THE ALTERNATIVE RANKINGS

Each input variable X_n has been, thus, assigned a $S_{n,d}$. Input variables can, accordingly, be ranked from the most important (largest $S_{n,d}$) to the least contributor to the variance (smallest $S_{n,d}$). Two strategies are explored for ensembling the D available alternative rankings: the BU and AO strategies [10].

2.3.1 BOTTOM-UP STRATEGY

Each d -th bootstrapped dataset \bar{Z}_d is treated separately from the others to generate its input ranking $\bar{R}_{BU,d}$. Then, the set of input rankings obtained from all the D datasets is processed a posteriori to give the final aggregated ranking order \bar{R}_{BU} (Figure 2) [10]. For each \bar{Z}_d the $S_{n,d}$ are computed and, by sorting in ascending order, the corresponding ranking $\bar{R}_{BU,d}$ is obtained. The final ranking order \bar{R}_{BU} is obtained applying the Borda method, that consists in computing the Borda

Count (BC) for each input variable [10]. Denoting by $p_{n,d}$ the n -th variable order inside the d -th ranking, the BC for the input variable X_n is given by [10] :

$$BC_n = \sum_{d=1}^D p_{n,d}. \quad (9)$$

A small value of BC_n means that the n -th input variable is among the most important (top ranked) input variables [10] .

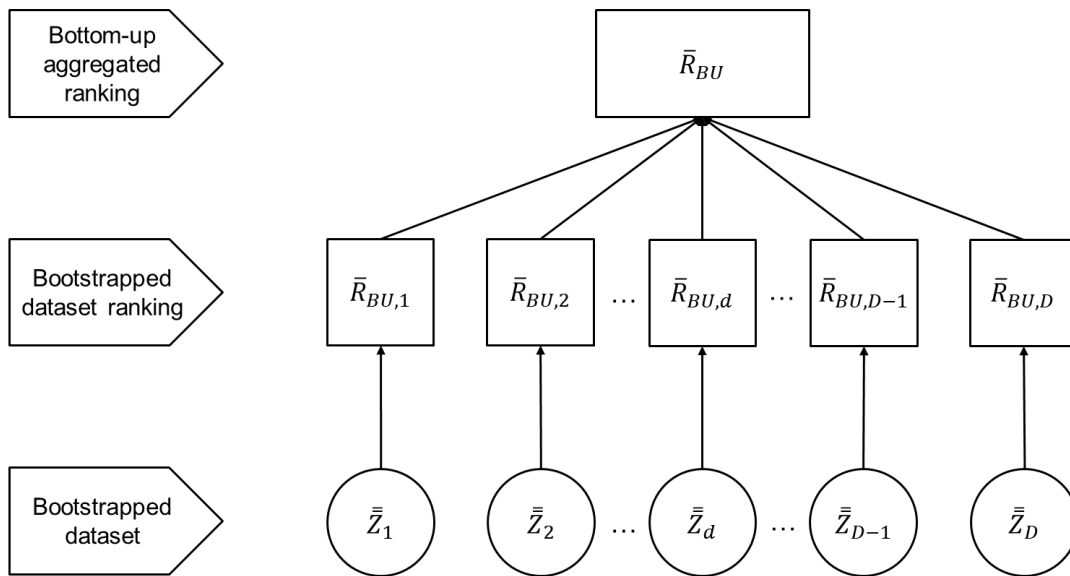


Figure 2 – The proposed bottom-up aggregation strategy.

2.3.2 ALL OUT STRATEGY

The AO strategy a priori merges the information coming from each dataset \bar{Z}_d . For each input variable X_n , the expected value $\mathbb{E}(S_n)$ of each Sobol index S_n is computed over the D datasets [10]:

$$\mathbb{E}(S_n) = \frac{1}{D} \sum_{d=1}^D S_{n,d} \quad (10)$$

Sorting $\mathbb{E}(S_n)$ in ascending order provides the AO aggregated ranking \bar{R}_{AO} (the larger the value of $\mathbb{E}(S_n)$, the more important the n -th input variable) [10].

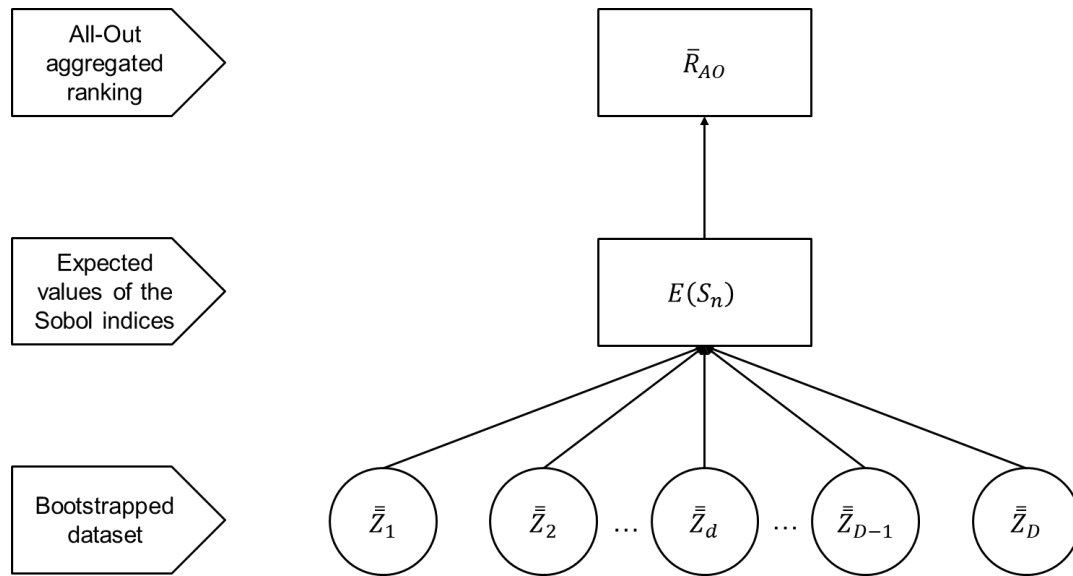


Figure 3 –The proposed all-out aggregation strategy.

3. CASE STUDY

The proposed GSA methodology has been tested on a benchmark case study regarding the PSHA of one hypothetical seismic point source and one target point located in its proximity. Such configuration is of particular interest when volcanic zones, e.g., volcanic islands, are of concern, since these are usually characterized by short source-to-site distance and high b -values [40]. We can, for example, realistically assume Ischia (Italy) to be the target point.

The source model consists of a point source with a given annual rate and generating seismicity with magnitudes following a truncated Gutenberg-Richter distribution [37]. The source model is, then, coupled with a standard Ground Motion Prediction Equation (GMPE) for the propagation of the earthquake waves [46]. A reference target point is selected in the near field, at distance of approximately 10 km.

The uncertainty of the PSHA is evaluated by means of Eq. (1) with respect to six input parameters, accounting for a reasonable large number of 16384 alternative computational settings (i.e. 2^{14} , possibly larger than in [11], where, as done in this work, a classical crude Monte Carlo sampling

strategy is used to sample 2^{12} alternative settings), capable of guaranteeing a thorough enough scenario exploration within a tractable computational time. This results in $\bar{\bar{Z}} = [16384 \times 7]$, as in Eq. (11) below. The purpose is to quantify the impact of these parameters on the uncertainty of the IM value corresponding to an exceedance probability of 10% in 50 years, that is with a return period of 475 years. The input parameters are $\bar{X} = (\sigma_{GMPE}, \lambda, m_{max}, m_{min}, b, r)$ where: σ_{GMPE} is the standard deviation of the GMPE, λ is the annual rate of seismic activity at the source location (i.e., the number of earthquakes per year of intensity magnitude m a minimum magnitude m_{min}), m_{min} and m_{max} are the minimum and the maximum magnitude parameters of the truncated Gutenberg-Richter distribution, whose slope is b [6], [37], and r is the source-to-target distance [6]. The latter parameter is here added to the other five input to emulate the dependence of GMPEs on source characteristics like earthquake depth, size or geometry. The output variable is the PGA , i.e., the reference peak ground acceleration at the target location that has annual rate of exceedance λ_H assumed to be equal to $1/475$ (corresponding in this case to a PGA of 0.07g).

$$\bar{\bar{Z}} = \begin{pmatrix} \sigma_{GMPE_1} & \lambda_1 & m_{max_1} & m_{min_1} & b_1 & r_1 & PGA_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{GMPE_S} & \lambda_S & m_{max_S} & m_{min_S} & b_S & r_S & PGA_S \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{GMPE_S} & \lambda_S & m_{max_S} & m_{min_S} & b_S & r_S & PGA_S \end{pmatrix} \quad (11)$$

The distributions of all the six input variables \bar{X} are reported in Table 1, all assumed to be normal as customary [11]. Notably, the source-target distance is set around 10 km, thus in the very near field, in which the dependence of distance on the source characteristics (geometry, depth, and dimension) is more pronounced. Consequently, a quite large variance is set for r . The other parameters are inspired from the ones adopted in the areal sources of the PSHA study that is enforced by law in Italy, MPS04 [47], [48]. In particular, the parameters of the source model ($\lambda, m_{min}, m_{max}$ and b) are inspired by source zone 920 (Val di Chiana-Ciocciaria) of MPS04, with a reduced value m_{min} (from 4.76 to 4.5) and λ (as we are considering a point source). The central

value of σ_{GMPE} is instead taken from [46]. Variance value representing the uncertainty on the parameters, are set based on expert judgement.

Table 1 - Model input variables and output, with their associated distributions.

Input variable	Units	Type of distribution	Mean value	Standard deviation
σ_{GMPE}	$g_0 (m/s^2)$	Normal	0.3446	0.0490
λ	yr^{-1}	Normal	0.0600	0.0021
m_{max}	-	Normal	5.6791	0.2430
m_{min}	-	Normal	4.5005	0.1000
b	-	Normal	1.9597	0.0580
r	km	Normal	10.0142	2.9639

The results of the proposed method are compared to those obtained by a standard variance-based GSA method [11], which is the state-of-practice approach when the simulation model is available.

4. RESULTS

The BMGSA methodology described in Section 2 has been applied to the case study presented in Section 3. The original dataset \bar{Z} has been replicated by bootstrap to generate $D = 1000$ datasets. Each replicate matrix \bar{Z}_d is comprised of $S = 16384$ rows (the input-output patterns) and 7 columns (6 input variables, 1 output). The partition size chosen to test the proposed methodology is $K = int(\sqrt{S}) = 128$.

The results of the assessment carried out with the standard GSA on the case study of Section 3 are taken as a benchmark, i.e., as correct ranking. This ranking is reported in Table 2, along with the values of the Sobol indices.

Notably, the main drivers of the uncertainty on the reference PGA are m_{min} and σ_{GMPE} . While the dependence on σ_{GMPE} is a well-established result [49], the dependence on m_{min} is not that straightforward, and it is probably due to the selection of a target point in the very near field (about 10 km) and the use of a very large value for the slope b -value (about 2). The combined effect of these two parameters is to produce a very large number of events with a magnitude very close to m_{min} , resulting in a critical dependence of the reference PGA on the selected minimum magnitude value. Also, notice that the first order Sobol indices sum to greater than one, meaning that the variables considered are correlated [50]. Finally, it is worth mentioning that this result cannot be taken as general, because obtained under specific modelling assumptions (e.g., large return period and large PGA values): the analysis of the results to other settings/assumptions/constraints and the generality of such results is the object of future work.

Table 2 – Input variables ranking obtained with the standard GSA.

Rank	1	2	3	4	5	6
Input variable	m_{min}	σ_{GMPE}	λ	r	m_{max}	b
S_n	0.5896	0.3811	0.0470	0.0320	0.0264	0.0221

4.1 RESULTS OF THE BOTTOM-UP STRATEGY

The application of the ensemble BU strategy produces the final ranking obtained by the Borda method, shown in Table 3. The major limitation of the BU strategy is that the result is lumped in a ranking table that is not transparent with respect to the actual Sobol indices that generate that ranking and, finally, the analyst is not provided with any confidence measure on the resulting rank: in other words, it cannot be quantitatively assessed how much the generic n -th input X_n contributes to the variance of Y .

Table 3 – Input variables ranking obtained through MGSA, $D=1000$ bootstrapped datasets, BU strategy.

Rank	1	2	3	4	5	6
Input variable	m_{min}	σ_{GMPE}	λ	r	m_{max}	b

4.2 RESULTS OF THE ALL-OUT STRATEGY

The ranking and the expected values of the Sobol indices obtained with the ensemble strategy AO are reported in Table 4. As stated for the BU strategy, one can observe that:

1. The variables σ_{GMPE} and m_{min} are the first two (by far) more relevant inputs (Figure 4);
2. The other input variables bring a negligible contribution to the variability of the output.

Table 4 – Input variables ranking obtained through MGSA, $D=1000$ bootstrapped datasets, AO strategy.

Rank	1	2	3	4	5	6
Input variable	m_{min}	σ_{GMPE}	λ	m_{max}	r	b
$\mathbb{E}(S_n)$	0.5760	0.3693	0.0508	0.0359	0.0336	0.0249

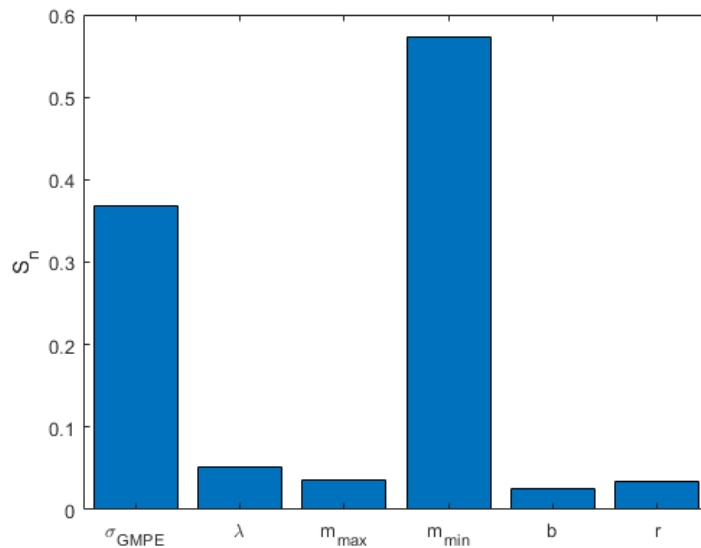


Figure 4 – Sobol indices obtained with the AO strategy.

4.3 COMPARISON WITH THE BENCHMARK RESULTS

The results of the proposed methodology have been then compared with the ranking results of the standard GSA, reported in Table 5.

Table 5 – Input variables rankings (sample size $S=16384$).

Rank	1	2	3	4	5	6
Standard GSA	m_{min}	σ_{GMPE}	λ	r	b	m_{max}
BU (BMGSA)	m_{min}	σ_{GMPE}	λ	m_{max}	r	b
AO (BMGSA)	m_{min}	σ_{GMPE}	λ	m_{max}	r	b
No bootstrap (MGSA)	m_{min}	σ_{GMPE}	r	m_{max}	λ	b

Both ensemble strategies and the standard GSA identify the σ_{GMPE} and m_{min} as the most important variables, whereas the sensitivity indices of the other input variables are negligible (Table 5, Figure 5). The disagreement regarding the ranking for the positions 4-6 may be due to i) hidden dependences and/or correlations between the input variables that are neglected when the alternative datasets are generated, and ii) the paucity of data upon which the rankings are drawn.

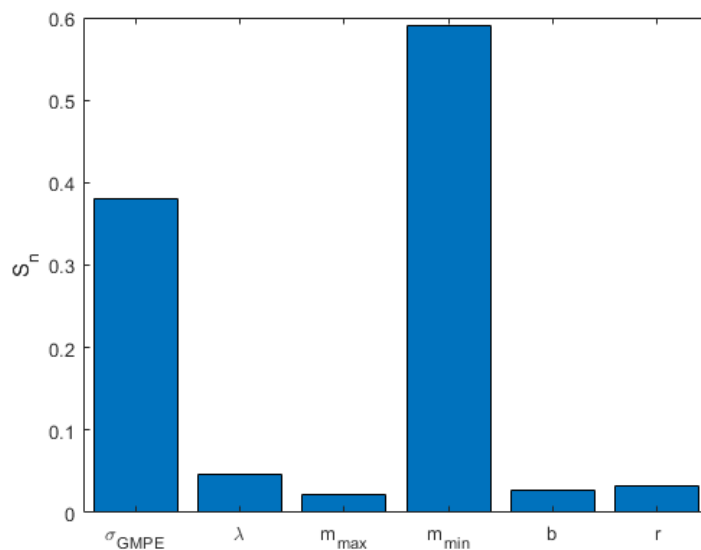


Figure 5 – Sobol indices obtained with the standard GSA.

To highlight the important role played by the bootstrapping (Section 2.2) in obtaining such results, we show (from Figure 6 to Figure 10) the results that would have been obtained with a given input-output dataset \bar{Z} of decreasing size ($S = 16384, 8192, 4096, 2048, 1024$) (that correspond to successive halves of the dataset size), employing the more transparent AO ensemble strategy (green squares in Figure 6 to Figure 10). These results are compared with i) the benchmark values (Standard GSA, blue diamonds in the Figure 6 - Figure 10) and ii) the results obtained with the MGSA without bootstrap (magenta circles in the Figure 6 - Figure 10). The relative rankings are reported in Tables 6-9.

When $\bar{Z} = [16384 \times 7]$ and $\bar{Z} = [8192 \times 7]$, the Standard GSA (\diamond), the BMGSA (green squares in the Figure 6) and the MGSA agree on the identification of m_{min} and σ_{GMPE} as the most important variables, whereas for the third most important variable only Standard GSA and BMGSA agree on λ . Then, the approaches provide different rankings for lower ranking positions.

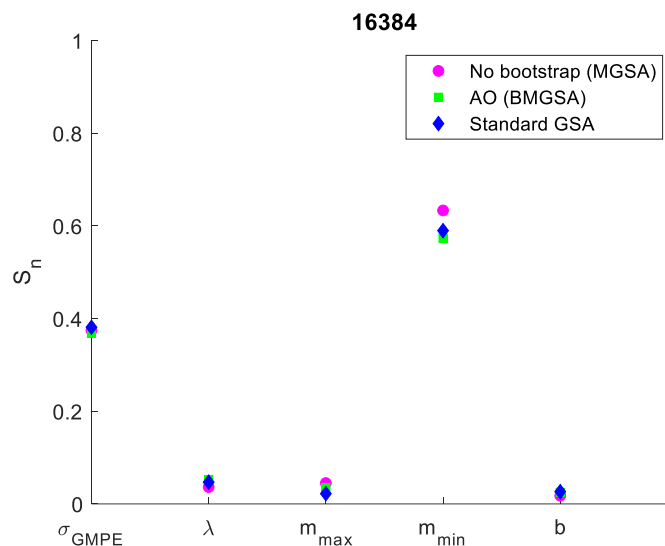


Figure 6 – Sobol indices estimates at sample size $S=16384$.

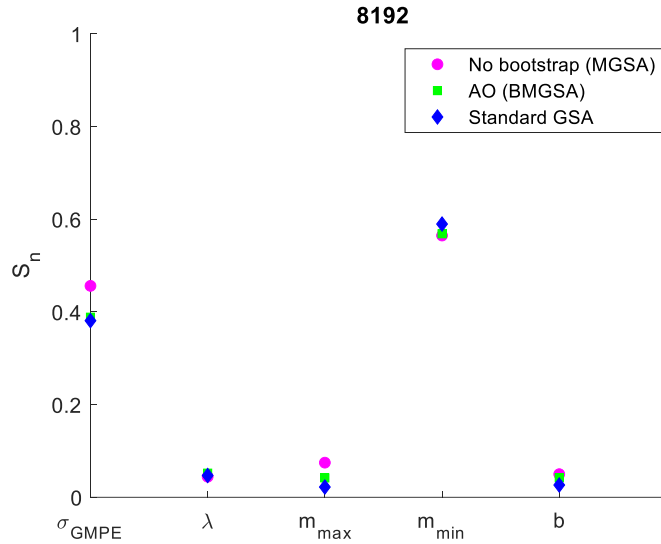


Figure 7 – Sobol indices estimates at sample size $S=8192$.

Table 6 – Input variables rankings (sample size $S=8192$).

Rank	1	2	3	4	5	6
Standard GSA	m_{min}	σ_{GMPE}	λ	r	b	m_{max}
AO (BMGSA)	m_{min}	σ_{GMPE}	λ	b	m_{max}	r
No bootstrap (MGSA)	m_{min}	σ_{GMPE}	m_{max}	b	λ	r

When $\bar{Z} = [4096 \times 7]$ and $\bar{Z} = [2048 \times 7]$, the Standard GSA and the BMGSA agree on the identification of m_{min} and σ_{GMPE} as the most important variables, as well as on the third (λ) and fourth (r) most important variables. Then, the approaches provide different rankings for lower ranking positions. The MGSA instead yields a completely different ranking (except for positions 4 and 5, when $\bar{Z} = [4096 \times 7]$ and $\bar{Z} = [2048 \times 7]$, respectively). Notice that, when the dimension of \bar{Z} decreases, even if the most important variables are correctly identified, a less accurate estimation of the Sobol indices is provided and the differences between the GSA, the BMGSA and MGSA increase.

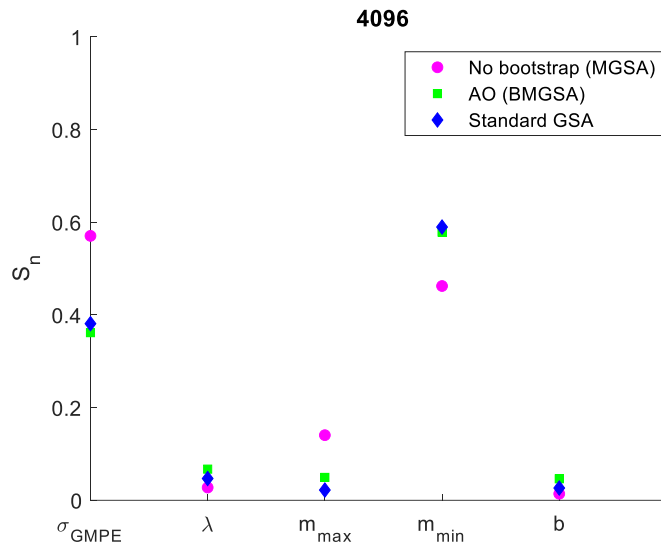


Figure 8 – Sobol indices estimates at sample size $S=4096$.

Table 7 – Input variables rankings (sample size $S=4096$).

Rank	1	2	3	4	5	6
Standard GSA	m_{min}	σ_{GMPE}	λ	r	b	m_{max}
AO (BMGSA)	m_{min}	σ_{GMPE}	λ	r	m_{max}	b
No bootstrap (MGSA)	σ_{GMPE}	m_{min}	m_{max}	r	λ	b

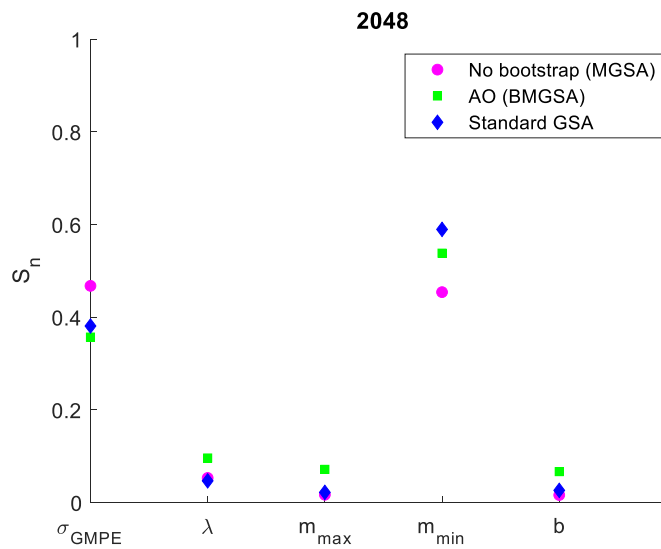


Figure 9 – Sobol indices estimates at sample size $S=2048$.

Table 8 – Input variables rankings (sample size $S=2048$).

Rank	1	2	3	4	5	6
Standard GSA	m_{min}	σ_{GMPE}	λ	r	b	m_{max}
AO (BMGSA)	m_{min}	σ_{GMPE}	λ	r	m_{max}	b
No bootstrap (MGSA)	σ_{GMPE}	m_{min}	r	λ	m_{max}	b

When $\bar{Z} = [1024 \times 7]$, the Standard GSA, the BMGSA and the MGSA agree on the identification of m_{min} as the most important variable, whereas for the second (σ_{GMPE}) and third (λ) most important variables only Standard GSA and BMGSA agree. Then, the approaches provide different rankings for lower ranking positions. Nevertheless, as Figure 10 clearly shows, the numerical values of the Sobol indices obtained with the proposed BMGSA may not be considered satisfactory. Furthermore, Figure 11 to Figure 15 shows that, when the dimension of \bar{Z} decreases, the distributions of $S_{n,d}$ become wider (i.e., bootstrap replicates are subject to noise and, as a result, the Sobol indices estimate are not precise).

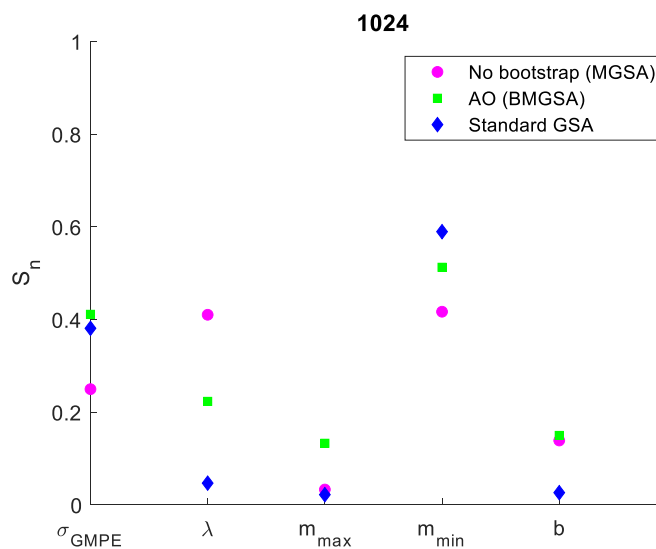


Figure 10 – Sobol indices estimates at sample size $S=1024$.

Table 9 – Input variables rankings (sample size $S=1024$).

Rank	1	2	3	4	5	6
Standard GSA	m_{min}	σ_{GMPE}	λ	r	b	m_{max}
AO (BMGSA)	m_{min}	σ_{GMPE}	λ	b	r	m_{max}
No bootstrap (MGSA)	m_{min}	λ	σ_{GMPE}	b	m_{max}	r

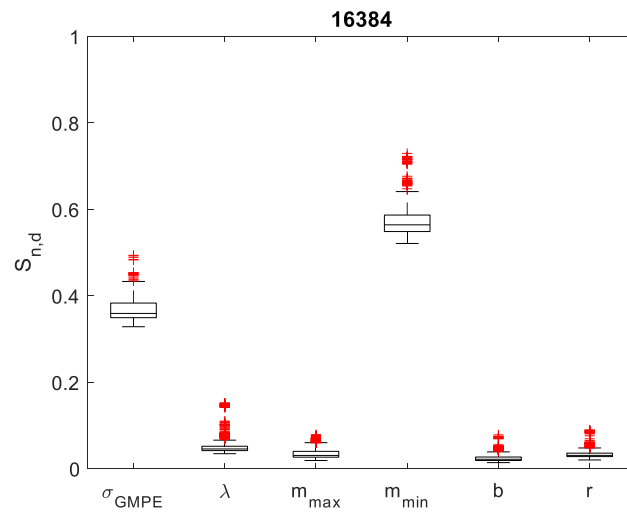


Figure 11 - $S_{n,d}$ distributions at sample size $S=16384$

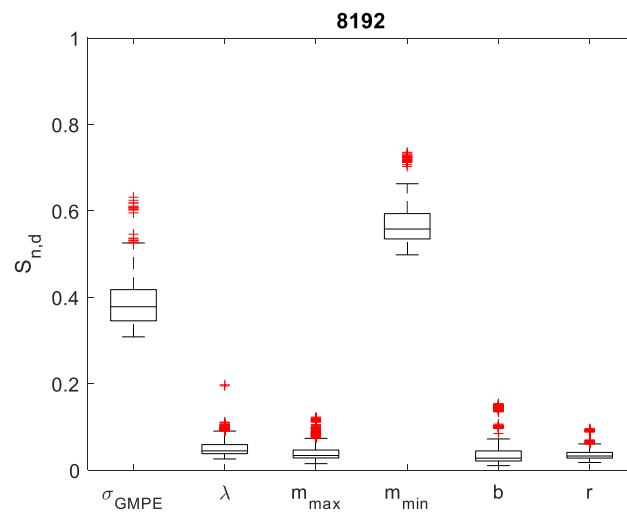


Figure 12 - $S_{n,d}$ distributions at sample size $S=8192$

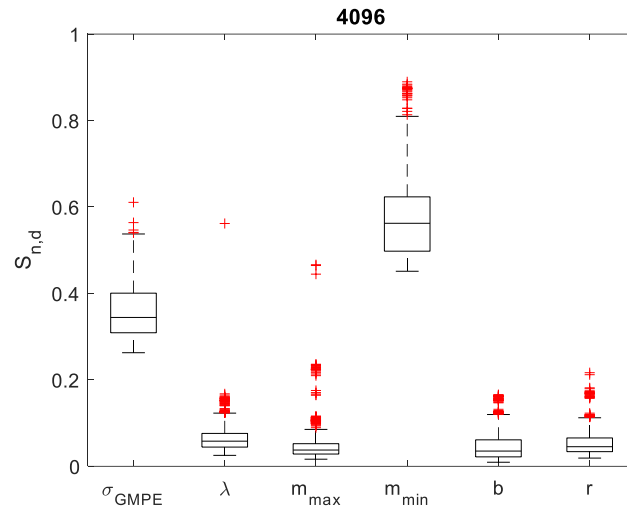


Figure 13 - $S_{n,d}$ distributions at sample size $S=4096$

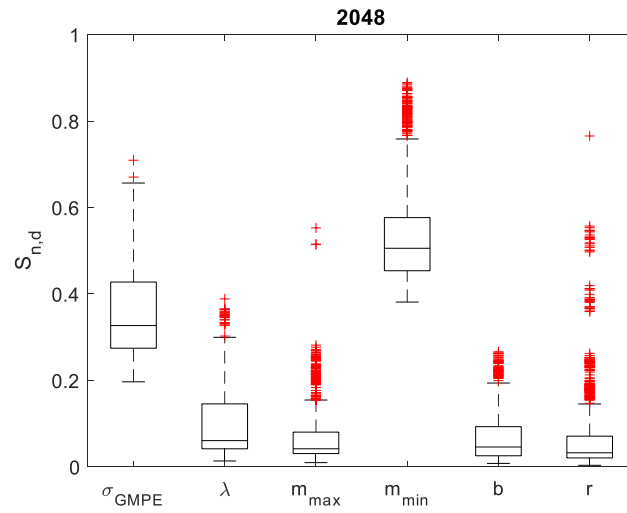


Figure 14 - $S_{n,d}$ distributions at sample size $S=2048$

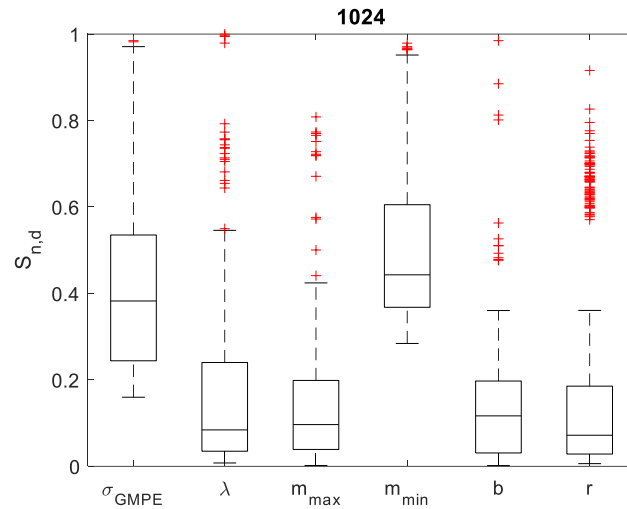


Figure 15 - $S_{n,d}$ distributions at sample size $S=1024$

As general conclusion, we can state that bootstrapping allows relying on a very small dataset. Indeed, a sample size of $S = 2048$ (Figure 9) allows correctly identifying the most important input variables, whereas $S = 4096$ (Figure 8) yields already a very satisfactory estimate of the Sobol indices values (compared with the GSA estimates). Thus, as a general recommendation, we may conclude that a ratio of 4:1 of $S:D$ (dataset size vs number of bootstrap replicates) is enough to guarantee satisfactory results, without resorting further to demanding computations.

For the case study at hand, we can conclude that, m_{min} , σ_{GMPE} , and λ have been identified as the input variables which most influence the reference PGA, whereas r , b , and m_{max} influence is negligible (for whatever dataset size $S = 16384, 8192, 4096, 2048, 1024$). The analyst, once identified the input parameters which most influence the uncertainty on reference PGA, may decide to further investigate the choice made regarding such inputs and proceed with the uncertainty analysis.

We underline that the numerical results obtained are relative to the specific case study and cannot be generalized to other PSHA cases. In particular, while the strong impact of σ_{GMPE} , and λ on hazard quantifications is well known (e.g. [11]), the reasons behind the importance of m_{min} must be further

investigated. In Figure 16, we show the impact of small events (i.e., with a magnitude around Mw 4.5). The relative short source-to-site distance (10 km) and the low PGA level (0.07g) make the contribution of small events of utmost relevance: the conditional probability of exceedance of such PGA level for a magnitude Mw 4.5 is larger than 0.2. In the case of larger distances (e.g., 40km) and larger PGA levels (e.g., 0.20g), the impact of small events decreases noticeably.

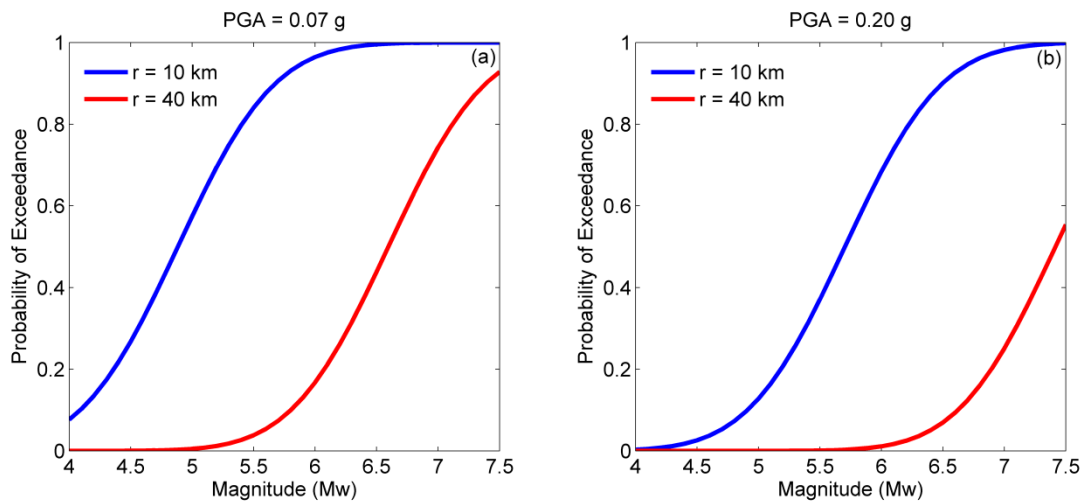


Figure 16 – Probability of exceedance of a specific PGA level, 0.07g for panel (a), 0.20g for panel (b), as a function of the magnitude, using the GMPE adopted in this study, for two different distances (10 km for the blue curve, 40 km for the red curve)

The large probabilities shown by the blue curve ($r=10$ km) in Figure 16 (a) explain the unexpected large importance of m_{min} as input variable in the PSHA computation (see [49] or [51]) and calls for a deeper investigation of the effect of small magnitude events in PSHA, as it is in the case of volcanic islands [40]. This highlights that the proposed method is useful to be used for sanity check of hazard assessments, as PSHA results are assumed independent from the selection of m_{min} [11]. In this case, indeed, we show that, for small magnitudes, the tails of the GMPE are sufficiently populated to strongly impact the hazard quantification also at a relatively high return period (475 years in this case), at least in the near field of the source areas.

Notably, this effect may be due to the extension of the validity of the GMPEs to small magnitude events that, in some cases, can even lead to a bias in the hazard estimation, as pointed out also in other studies (see [49], [51]),.

5. CONCLUSIONS

In this work, we have proposed a novel Bootstrapped Modularised Sensitivity Analysis (BMGSA) method based on bootstrapping, MGSA and ensemble strategies to identify the input parameters which the output of a PSHA model is most sensitive to, assuming that only an input-output dataset is given whereas the model is not available. The novelty and strength of the proposed BMGSA method is that to be applied it only needs data and not the source simulation code. The capability of the proposed method is tested on a benchmark case study. The results have been compared with a standard variance-based GSA method of literature, showing that the proposed method and the standard GSA agree on the identification of the three by-far most important input variables. Furthermore, the BMGSA has proved to be reliable even when applied to very small datasets.

The application of the developed technique to a realistic PSHA demonstrates its capability of scoring correctly the importance of existing uncertainty factor, needing only the input and the output data.

This allows applying the technique to any hazard model in which uncertainty is to be evaluated. Its systematic application to hazard studies to detect the most influential parameters would allow hazard practitioners to both improve the sanity checks during the assessment and to focus future research toward the reduction of uncertainty, by further characterisation of the important factors.

The results of our applications, for example, highlight the importance of small magnitudes near to the seismic source areas, showing the importance of the definition of the minimum magnitude and the potential impact of the tail of the uncertainty distributions on GMPE on seismic hazard evaluation. Also, the application of the proposed technique can result practically useful when the PSHA is to be updated on a time basis (usually 5/10 years): in this case, a standard GSA would not allow the seismologist to obtain the results in reasonable time; on the contrary, the BMGSA here proposed enables a lean update of the PSHA, by informing the seismologists about the parameters

that rule most the uncertainty; focusing on these only, scenarios exploration and conclusions would be more swiftly performed and drawn, respectively.

Acknowledgments

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