A NEW ANALYTICAL METHOD FOR ECLIPSE ENTRY/EXIT POSITIONS DETERMINATION CONSIDERING A CONICAL SHADOW AND AN OBLATE EARTH SURFACE

Marco Nugnes,[*] and Camilla Colombo[†]

Satellite eclipse determination is one of the most important tasks to be analyzed in the preliminary design of a planetary space mission. Indeed, the duration of the eclipse is a driver for the sizing of the batteries that must be used when solar energy is not available. Many analytical and numerical algorithms based on several assumptions exist both for the simple determination of the satellite state (i.e., umbra/penumbra/sunlight) and for the definition of the entry/exit anomalies delimiting the umbra and penumbra regions. This paper wants to define a new analytical procedure for the determination of the entry/exit anomalies of a satellite inside a conical shadow generated by the Earth surface modelled as an oblate ellipsoid of rotation. The methodology is tested for different orbit scenarios and is compared with state-of-the art algorithms to check both the effectiveness of the results and the computational performance.

INTRODUCTION

Eclipses represent one of the most studied and spectacular celestial phenomena. The prediction of an eclipse occurrence is not significant when the time spent in the shadow is small as for the Sun eclipses. However, the analysis of the eclipse periods becomes relevant when satellites orbiting the Earth (or another celestial body) are considered because most of their power subsystem are based on solar energy and the eclipse period is important for the sizing of the batteries that should replace the solar energy while the satellite is in the shadow region.

The first detailed eclipse analysis can be found in Escobal who defines an analytical procedure to determine the true anomalies corresponding to the entry and exit points from the cylindrical umbra generated by the Earth assumed as a spherical surface in the framework of Keplerian elements[1]. Escobal suggests also iterative procedures to correct the values of the true anomalies when Earth's flattening and conical shadow are considered. Fixler derives an analytical equation considering the projection of the Sun position vector onto the satellite orbital plane to determine the umbra and penumbra regions assuming a conical shadow and a spherical Earth[2]. However, the equation is nonlinear and transcendental and still requires iterative methods to be solved. In his work Adhya introduces an analytical method for the determination of the state of a satellite (umbra/penumbra/sunlight) when a conical shadow is considered, and the Earth surface is modelled as an oblate ellipsoid of rotation[3]. No information about the exact values of the entry/exit points

[*] Ph.D. Candidate, Department of Aerospace Science and Technology, Politecnico di Milano, marco.nugnes@polimi.it.
[†] Associate Professor of Orbital Mechanics, Department of Aerospace Science and Technology, Politecnico di Milano, camilla.colombo@polimi.it.

of the umbra and penumbra regions are provided. Neta and Vallado describe a method based on Cartesian coordinates which models the conical surface generating the shadow and computes its intersection with the ellipse representing the satellite orbit[4]. The solution is obtained solving a system of two equations numerically and a good initial guess must be provided to obtain the exact solution.

This paper presents a new analytical method for the exact computation of the entry and exit points of the umbra and penumbra regions of a satellite when crossing the conical shadow generated from the Earth's surface modelled as an oblate ellipsoid of rotation. The method is developed in the Cartesian framework and starts from the mathematical expression of a generic conical surface defined in its local reference frame to arrive at a quartic equation in the unknown abscissa of the orbital ellipse in the perifocal frame thanks to a series of rotations and translations. The only inputs required for solving the eclipse problem are the satellite position vector and the epoch to get the Sun position vector. A precise criterion is proposed for the selection of the exact solutions of the quartic equation and no initial guesses or numerically procedures are required to obtain the solution. The method is tested for three different orbital scenarios (a low-Earth orbit, a medium-Earth orbit and a geosynchronous inclined orbit) and compared with other eclipse algorithms to validate the results and test the computational efficiency of the analytical formulation.

MODELLING

The starting point of the method is the mathematical expression of a generic conical surface which is modelling the Earth shadow:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0 \tag{1}$$

with a and b being the semi-major axis and semi-minor axis respectively of the ellipse base of the cone, and c representing the height of the conical surface given by the distance of the cone vertex from the ellipse center. This equation is defined in a general Cartesian reference system centered in the cone vertex with the x-axis along the direction of the ellipse major axis, y-axis along the direction of the semi-minor axis and the z-axis along the cone axis. The generical conical surface in its local reference frame is shown in Figure 1.

A series of rotation is carried out to move from the cone local reference frame to the perifocal reference frame of the satellite. The equations describing the transformation are summarized in Eq. (2):

$$\begin{cases} x = R_{11}\tilde{x} + R_{12}\tilde{y} + R_{13}\tilde{z} \\ y = R_{21}\tilde{x} + R_{22}\tilde{y} + R_{23}\tilde{z} \\ z = R_{31}\tilde{x} + R_{32}\tilde{y} + R_{33}\tilde{z} \end{cases} \tag{2}$$

A further translation is carried out to move the origin of the reference frame from the vertex of the cone to the Earth center.

$$\begin{cases} \tilde{x} = \overline{x} + x_v \\ \tilde{y} = \overline{y} + y_v \\ \tilde{z} = \overline{z} + z_v \end{cases} \tag{3}$$

with x_v, y_v and z_v representing the cone vertex coordinates with respect to the Earth center. Substituting first Eq. (2) in Eq. (1), and then Eq. (3) it is possible to obtain the mathematical expression of the conical surface in the perifocal frame. This equation must be solved together with the expression of the orbital ellipse defined in the perifocal frame which is reported hereafter.

$$\frac{(\bar{x}-g)^2}{a_{sat}^2}+\frac{\bar{y}^2}{b_{sat}^2}=1 \tag{4}$$

where g is the focal distance of the orbital ellipse, and a_{sat} and b_{sat} are respectively the orbital ellipse semi-major and semi-minor axis. It is possible to combine Eq. (4) with the mathematical expression of the conical surface because they are defined in the same reference frame to obtain a quartic equation that can be solved analytically.

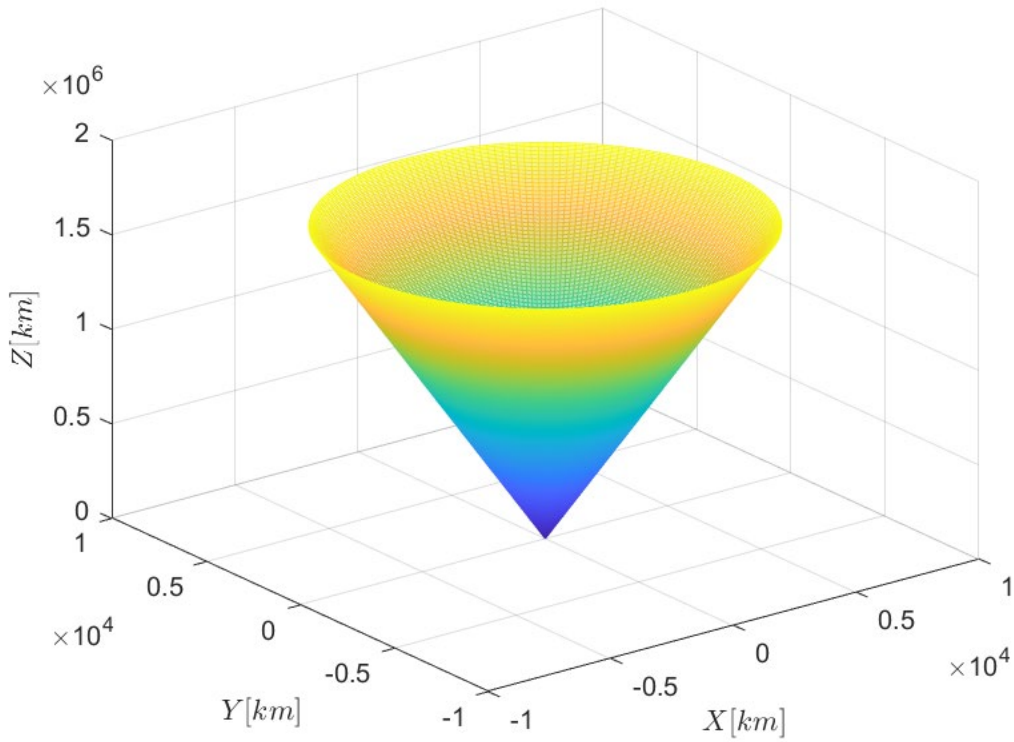

Figure 1. Umbra cone defined in its local reference frame.

The selection of the exact points representing the entry and exits positions of the umbra and penumbra regions is carried out considering the property that the correct points are in the half-plane opposite to the one where the Sun is present.

An example of the results obtained using the new analytical methodology is presented in Figure 2 where the geometrical conical surface is cutting the orbital ellipse in the exact points corresponding to the umbra region.

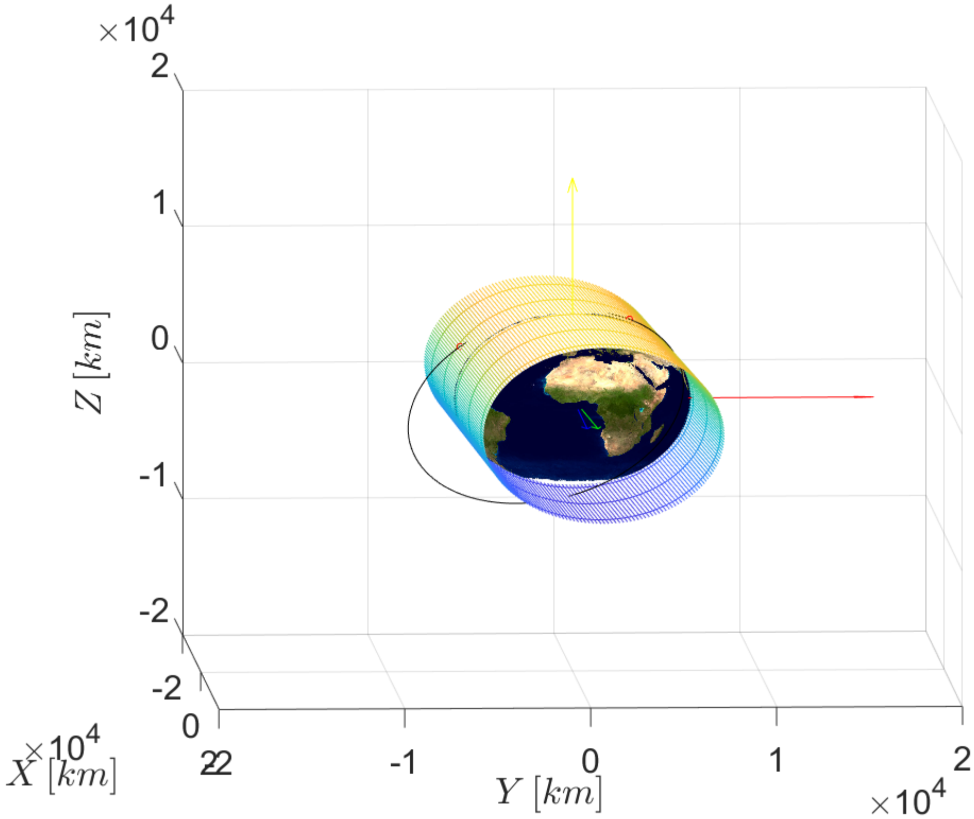

Figure 2. Umbra Cone generated by Earth surface cutting the orbital ellipse.

REFERENCES

[1] Escobal, P. R., *Methods of Orbit Determination,* Ch. 4, pp. 162-174, Krieger, 1965.

[2] Fixler, S. Z., "Umbra and penumbra eclipse factors for satellite orbits", *AIAA Journal*, Vol. 2, pp. 1455-1457, 1964.

[3] Adhya S., Sibthorpe A., and Ziebart, M. ´Oblate Earth eclipse state algorithm for low-Earth orbiting satellites", *Journal of Spacecraft*, Vol. 41, pp. 157-159, 2003.

[4] Neta B., and Vallado D., "On Satellite Umbra/Penumbra Entry and Exit Positions", *The Journal of the Astronautical Sciences*, Vol. 46, No. 1, pp. 91-103, 1998.