

Stochastic inverse modeling of transient laboratory-scale three-dimensional two-phase core flooding scenarios

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Highlights:

- Stochastic model calibration is used for three-dimensional transient two-phase flow
- X-ray scanning allows detection of 3D in-situ fluid saturations
- Interpretation of unique data from oil and brine displacement experiments is given
- Coupling of sensitivity analysis with optimization reduces computational costs
- Roles of uncertain model parameters in two-phase systems are quantified

ABSTRACT

We develop a comprehensive and efficient workflow for a stochastic assessment of key parameters governing two-phase flow conditions associated with core-scale experiments. We rely on original and detailed datasets collected on a Berea sandstone sample. These capture the temporal evolution of pressure

29 drop across the core and three-dimensional maps of phase saturations (determined via X-ray CT) in oil- and
30 brine-displacement flooding scenarios characterized by diverse brine/oil viscosity contrasts. Such
31 experiments are used as a test-bed for the proposed stochastic model calibration strategy. The latter is
32 structured across three main steps: (i) a preliminary calibration, aimed at identifying a behavioral region of
33 the model parameter space; (ii) a Global Sensitivity Analysis (GSA), geared towards identification of the
34 relative importance of model parameters on observed model outputs and assessment of non-influential
35 parameters to reduce dimensionality of the parameter space; and (iii) a stochastic inverse modeling
36 procedure. The latter is based on a differential-evolution genetic algorithm to efficiently explore the reduced
37 parameter space stemming from the GSA. It enables one to obtain a probabilistic description of the relevant
38 model parameters through their frequency distributions conditional on the detailed type of information
39 collected. Coupling GSA with a stochastic parameter estimation approach based on a genetic algorithm of
40 the type we consider enables streamlining the procedure and effectively cope with the considerable
41 computational efforts linked to the two-phase scenario considered. Results show a remarkable agreement
42 with experimental data and imbue us with confidence on the potential of the approach to embed the type of
43 rich datasets considered towards model parameter estimation fully including uncertainty.

44 **1. Introduction**

45 Characterization of two-phase flow processes in porous media is key in environmental and industrial
46 scenarios. In this broad context, multiphase flow processes are critically important in driving, e.g.,
47 sustainable and enhanced use of underground energy resources (e.g., [1]).

48 Real-time three-dimensional (3D) imaging techniques of transient multiphase flow laboratory
49 experiments are becoming increasingly appealing due to their non-invasive nature. A variety of imaging
50 techniques are nowadays applied to study fluid displacement in porous media. These include X-ray
51 Computer Tomography (CT) (e.g., [2–7]) and micro-CT ([8–11]), neutron radiography [12], and magnetic
52 resonance imaging (e.g., [13–17]). A variety of applications of these imaging techniques tackle observation
53 and modeling of multi-phase flow dynamics at the pore-scale (e.g., [9–11]). Here, we focus on continuum-
54 scale patterns of two-phase (oil and brine) flow settings of the kind related to core-flooding experiments.

55 Saturation data based on 3D core imaging have been recently used to characterize CO₂-brine interaction [5]
56 and waterflood dynamics in oil-displacement experiments [6]. Recently, Fannir et al. (2020) [17] rely on
57 three-dimensional (3D) imaging to assess oil saturation in core scale two-phase oil displacement and
58 quantify some relevant aspects of the setting through a set of dimensionless quantities. Wang et al. (2020)
59 [18] employ 3D images of a CO₂/water core flooding scenario to evaluate the effect of errors caused by
60 simplified modeling of heterogeneous systems. **In this context, even as 3D models might be associated with
61 a higher level of operational complexity and computational requirements than their streamlined 1D
62 counterparts, relying on such modeling approaches can provide major insights on the structure of two-phase
63 flow fields that can exhibit complex patterns of displacement fronts due to flow instabilities and/or
64 heterogeneities in the hydraulic properties of the host porous medium (e.g., [2,19,20]).**

65 In this framework, a rigorous and robust approach to quantification of the way uncertainties associated
66 with the key parameters driving the system dynamics are constrained through rich datasets of the kind
67 described above is still lacking. Here, we tackle this objective by merging parallel streams of research.
68 These include Global Sensitivity Analysis (GSA) and stochastic inverse modeling.

69 Considering GSA (e.g., [21]) enables one to diagnose the behavior of a selected interpretive model as
70 well as the relative contribution of uncertain model parameters to model output uncertainty ([22–27]).
71 Furthermore, it has the additional advantage of being conducive to the identification of non-influential (or
72 only minimally influential) model parameters. This enables streamlining the computational efforts linked
73 to stochastic inverse modeling ([28–33]) upon setting some parameter(s) (which are deemed as
74 uninfluential) at prescribed value(s) without significantly affecting modeling results. Our study is patterned
75 after the approach proposed by Morris (1991) [34]. The latter is a versatile and convenient approach in the
76 presence of highly demanding (in terms of computational effort) model simulations. **These types of analyses
77 have been applied to selected basin-scale depositional settings across geologic time scales (e.g., [33] and
78 references therein). Similarly, sensitivity and uncertainty analyses have been recently applied to aid the
79 appraisal of key elements driving the documented complexities associated with multiphase flow through
80 porous media ([35–38]). The solution of an inverse problem is challenging due to a variety of factors,**

81 including e.g., issues related to non-uniqueness (see, e.g., [39], and, [37,38]with reference to two-phase
82 flow scenarios). Here, we consider a stochastic inverse modeling framework. As opposed to a deterministic
83 approach, the latter yields probability distributions of uncertain model parameters (i.e., [40,41]). In this
84 sense, one obtains a collection of possible solutions of the inverse problem, each constrained through (i.e.,
85 conditional on) the available information content. This approach enables one to provide predictions under
86 uncertainty. The latter is quantified in terms of the above mentioned (posterior) distribution of model
87 parameters, i.e., the probability distribution of parameters conditional to available experimental
88 observations (see e.g., [42,43] for recent applications stochastic inverse modeling approaches).

89 The study is structured as follows. Section 2 includes a detailed description of the available
90 experimental investigations and the ensuing dataset. It also introduces the methodological workflow,
91 together with key theoretical elements underpinning the selected Global Sensitivity Analysis and stochastic
92 inverse modeling approach. Implementation of the workflow and the ensuing results are discussed in
93 Section 3. Concluding remarks are offered in Section 4.

94 **2. Materials and Methods**

95 The current study relies on a unique set of core-scale data associated with two-phase laboratory-scale
96 flooding experiments (see Sec 2.1 for further details). Available data encompass: (i) the temporal history
97 of core-scale pressure drop, i.e., $\Delta\hat{P}_t$ (experimentally observed quantities are hereafter denoted with a hat
98 symbol); and (ii) detailed three-dimensional spatial distributions of phase saturations monitored at various
99 times, i.e., $\hat{S}_{\alpha,i,t}$ (subscripts α , i , and t referring to fluid phase, a generic location across the core sample,
100 and time, respectively). While measurements of $\Delta\hat{P}_t$ are commonly available in flooding experiments to
101 support characterization of rock sample attributes, observations about $\hat{S}_{\alpha,i,t}$ are seldom available (e.g.,
102 [44]). These provide a unique source of information on transient two-phase flow dynamics. Such a rich
103 base of information is here used to design a workflow conducive to improved characterization of the
104 hydraulic properties of rock samples within a stochastic inverse modeling context.

105 From a conceptual point of view, we interpret the transient two-phase flow dynamics at the continuum
106 scale through a set of commonly used balance equations and constitutive relationships (see Sec. 2.2) which
107 are then solved numerically. The selected mathematical formulation includes N model parameters θ_j ($j = 1,$
108 \dots, N) collected in vector $\boldsymbol{\theta}$. These embed salient features of processes relevant to the two-phase flood
109 scenario (e.g., relative permeabilities and parameters of the associated formulations) and are here treated as
110 random quantities. Thus, we aim at characterizing the probability density function (*pdf*) of $\boldsymbol{\theta}$ by leveraging
111 on the unique information content associated with the available experimental dataset. To accomplish this
112 objective, we design an operational workflow merging (a) Stochastic Model Calibration (SMC) (see Sec.
113 2.3) and (b) Global Sensitivity Analysis (GSA) (see Sec. 2.4), as detailed in the following. Relying on this
114 strategy enables us to cope with the computational burden related to SMC. The latter might indeed become
115 markedly high in the presence of a large set of parameters because of the computational cost associated
116 with forward numerical simulations of the three-dimensional transient two-phase flow scenario we tackle.

117 Fig. 1 provides a sketch of the workflow adopted in our study. We start by defining the support space
118 of the random model parameters, i.e., $\boldsymbol{\Gamma} = \Gamma_{\theta_1} \times \dots \times \Gamma_{\theta_N}$. At this stage, each parameter θ_j is treated as an
119 independent random variable uniformly distributed within the support Γ_{θ_j} . The latter is assessed on the basis
120 of available literature information and/or expert opinion. We then perform a preliminary model calibration
121 upon estimating model parameters across $\boldsymbol{\Gamma}$. This is aimed at providing: (i) a set of model parameters
122 compatible (in terms of the value of the objective function defined in Sec. 2.3) with the available
123 observations, and (ii) a reference value for the objective function that corresponds to a satisfactory degree
124 of consistency with the experimental data. The results of this step enable us to define a *behavioral* parameter
125 space, i.e., $\boldsymbol{\Gamma}^B$. We then perform a GSA focused on ΔP_t and $S_{\alpha,i,t}$ (i.e., the model-based counterparts of $\Delta \hat{P}_t$
126 and $\hat{S}_{\alpha,i,t}$) across such space. In this second stage, we revise the parameter support ranges. This ensures that
127 the collection of the numerical simulations upon which the GSA is grounded are consistent with the
128 experimental observations, in the sense that the salient qualitative features of $\Delta \hat{P}_t$ and $\hat{S}_{\alpha,i,t}$ are reproduced
129 with a satisfactory quantitative agreement between numerical results and their experimental counterparts.

130 Thus, we tie the GSA results to the available experimental evidence and rely on the ensuing analysis to
 131 (eventually) identify model parameters that can be deemed as non-influential for ΔP_t and $S_{\alpha,i,t}$. Doing so
 132 yields a secondary parameter support space (hereafter denoted as Γ') that includes only model parameters
 133 that are identified as influential. Stochastic model calibration is then performed within Γ' . This allows
 134 identifying sample frequency distributions of influential model parameters conditional on the available data
 135 (non-influential parameters being set at the values obtained from the preliminary model calibration). Note
 136 that, as further detailed in Sec. 2.3, we leverage on the reference value of the objective function obtained in
 137 the preliminary model calibration to define a stopping criterion during the stochastic model calibration
 138 stage.

139 2.1. Experimental set up

140 Fig. 2a depicts a schematic representation of the experimental setup. Key elements include a core
 141 holder, an X-ray apparatus for in-situ detection of fluid saturation, and differential pressure transducers.
 142 The experiments are performed on a cylindrical core of Berea sandstone, whose key properties are listed in
 143 Table 1. **Prior to starting the tests, the core sample is cleaned, washed, and scanned via X-ray CT (NSI X-**
 144 **5000 tomograph; North Star Imaging) at dry and fully brine-saturated conditions (i.e., $S_b = 1$). All of the**
 145 **acquired scans are post-processed (we use a Lanczos filter to this end; see , e.g., [45]) to obtain a voxel**
 146 **resolution of 0.24 mm^3 . The latter is then lowered (through the application of a triangular re-sampling filter)**
 147 **to obtain a spatial resolution of $0.997 \times 0.993 \times 0.996 \text{ mm}^3$. This enables estimating absolute permeability,**
 148 **K , and assessing the distribution of porosity, $\hat{\phi}_i$, (depicted in Fig. 2b), for each voxel i according to which**
 149 the system is discretized as:

$$\hat{\phi}_i = c \cdot (\tau_{b,i} - \tau_{a,i}) \quad (1)$$

150 where $\tau_{a,i}$ and $\tau_{b,i}$ are the i -th voxel linear attenuation coefficient at air- or brine-saturated conditions
 151 respectively. **Fig. 2b depicts the spatial distribution of the porosity at the sub-sample scale. An inspection**
 152 **of the latter highlights the presence of 3D heterogeneous features across the sample, i.e., tilted planes**
 153 **characterized by higher values of porosity.** A pore volume (PV) of 38.2 ml is estimated upon relying on

154 saturation data collected during brine injection, considering the linear relationship between brine saturation
155 and total injected volume (before brine breakthrough):

$$\langle S_b(t) \rangle - \langle S_b(t_0) \rangle = Q_b \frac{(t-t_0)}{PV} \quad (2)$$

156 where $\langle \cdot \rangle$ denotes arithmetic average over all sample voxels, $\langle S_b(t) \rangle$ and $\langle S_b(t_0) \rangle$ represent the average
157 brine saturations at time t and t_0 (i.e., the initial average brine saturation in the sample), respectively; and
158 Q_b is the brine volumetric flow rate. The ratio between the PV and the total core volume provides an
159 estimate of the average core-scale porosity of the sample (here $\langle \hat{\phi} \rangle \approx 0.17$). Note that the value of c in Eq.
160 (1) is assessed as $c = \langle \tau_{b,i} - \tau_{a,i} \rangle / \langle \hat{\phi} \rangle$.

161 Two unsteady-state displacement tests have been performed on the core. The first experiment
162 corresponds to an oil-displacement setting, in which a low-viscosity (LV) oil (soltrol 130) is displaced by
163 injecting brine in the sample. Note that, to mimic a typical reservoir scenario, the oil initially in place in
164 this experiment is the result of an injection in a preliminary brine-saturated pore space. The second
165 experiment corresponds to a brine-displacement scenario. Here, a high viscosity (HV) oil (OBI 10) is
166 injected into the initially fully brine-saturated sample. Note that the core was washed before each
167 experiment. Density and viscosity of the fluids employed in the experiments are listed in Table 2. Brine
168 composition is characterized by NaCl (84.36 g/L), CaCl₂ (23.12 g/L), KCl (32.14 g/L), and NaI (54.09 g/L).
169 The latter is used to enhance X-ray contrast between water and oil phase, to improve measurement accuracy.
170 A constant temperature of 30 °C is maintained in both experiments. Fluid injection takes place from the
171 bottom of the sample. Pressure difference between core inlet and outlet is continuously monitored. Ambient
172 pressure is maintained at the outlet section. The temporally-varying spatial distribution of oil saturation is
173 monitored periodically via X-Ray CT scans. As an example, Fig. 2 collects three-dimensional spatial
174 distributions of oil saturation corresponding to three observation times during the oil- (Fig. 2c) and brine-
175 displacement (Fig. 2d) experiments. Visual inspection of the 3D maps of oil saturation reveals the presence
176 of sub-sample heterogeneities. It can be seen that the displacement front appears to be quite dispersed during
177 the oil-displacement (low viscosity contrast). Otherwise, the front transition is sharper during the brine-

178 displacement (high stabilizing viscosity contrast) while being clearly non-uniform along the transverse
 179 cross-section as a consequence of the spatially heterogeneous nature of the hydraulic properties of the
 180 sample. The X-Ray beam employed to infer oil saturation is generated by applying an electric potential of
 181 140 kV. A scan time of 15 min is used to collect high-quality images and is employed at the equilibrium or
 182 for slow changes in saturation distribution. Otherwise, in the transient regime we take scans every 1 min
 183 and 13 s, to enhance characterization of the rapidly-evolving fluid dynamics. As a consequence, we expect
 184 the acquisitions in the transient time frame to be more affected by experimental errors than their steady
 185 state counterparts. This issue has been considered in the definition of the objective function to be minimized
 186 in the model calibration process (see Sec. 2.3).

187 2.2 Two-phase flow model

188 Mass conservation for each (incompressible) fluid phase α reads

$$\phi \frac{\partial}{\partial t} (S_\alpha) + \nabla \cdot (\mathbf{q}_\alpha) = 0 \quad (3)$$

189 where, ϕ [-] is the (spatially variable, see Sec. 2.1) porosity; S_α [-] is saturation of fluid phase α ; \mathbf{q}_α [LT⁻¹]
 190 is the extended Darcy flux vector for fluid phase α , which can be expressed as

$$\mathbf{q}_\alpha = - \frac{\mathbf{k} k_{r\alpha}}{\mu_\alpha} (\nabla P_\alpha - \rho_\alpha \mathbf{g}) \quad (4)$$

191 where \mathbf{k} [L²] is the absolute permeability tensor; $k_{r\alpha}$ [-] is relative permeability for fluid phase α ; P_α [ML⁻¹T⁻²],
 192 μ_α [ML⁻¹T⁻¹], and ρ_α [ML⁻³] are pressure, dynamic viscosity, and density of phase α , respectively;
 193 and \mathbf{g} [LT⁻²] is gravity. Note that we consider a two-phase system composed by brine ($\alpha = b$) and oil ($\alpha =$
 194 o). Saturation of the two phases must satisfy

$$S_b + S_o = 1 \quad (5)$$

195 Solutions of Eqs. (3)-(5) require an additional constraint. The latter concerns the capillary pressure, $P_c(S_b)$
 196 [ML⁻¹T⁻²], (i.e., the pressure difference across the interface between the two-phases in the system), which
 197 can be expressed as a function of brine saturation. Preliminary tests of the calibration framework aimed at

198 evaluating the potential ability of various $P_c(S_b)$ formulations to grasp the main patterns associated with
 199 the experimental observations revealed that a numerical solution consistent with the available datasets could
 200 be obtained only by considering negligible capillary effects (details not shown).

201 Absolute permeability is treated as isotropic and spatially heterogeneous, i.e., $\mathbf{k}(\mathbf{x}) = \mathbf{I}k(\mathbf{x})$, where \mathbf{I}
 202 is the identity matrix. We leverage on the knowledge about the spatial distribution of porosity across the
 203 core to (at least partially) capture the heterogeneous distribution of k which can then be considered in the
 204 inverse modeling context. To this end, we employ the following widely used relationship [43]:

$$\log_{10}(k) = m\phi + w \quad (6)$$

205 Note that k appearing in Eq. (6) is expressed in m^2 . We include m [-] in our probabilistic analysis workflow.
 206 Otherwise, we set $w = -16$ (which is equivalent to imposing a permeability of 0.1 mD for porosity values
 207 that tend to 0) based on a series of previous applications of Eq. (6) to interpret an extensive set of core-scale
 208 two-phase flow experiments performed on several Berea samples at the internal experimental facilities in
 209 ENI, Italy (not shown). These imbue us with prior knowledge to guide the modeling choice about w .

210 With reference to relative permeability, we recall that a variety of empirical formulations are available
 211 to render the dependence of relative permeability on the degree of fluid saturation (e.g., [7,44,46–50]).
 212 Here, we rely on the Corey formulation [46] due to its simplicity and parsimony (in terms of the number of
 213 parameters that are to be estimated), i.e.,

$$k_{rb} = k_{rb}^* (S_b^*)^{N_b} \quad (7)$$

$$k_{ro} = k_{ro}^* (1 - S_b^*)^{N_o} \quad (8)$$

214 Here, k_{rb}^* [-] and k_{ro}^* [-] are the end-point relative permeabilities for brine and oil, respectively; N_b [-]
 215 and N_o [-] are exponents; and S_b^* is the normalized brine saturation, i.e.,

$$S_b^* = \frac{S_b - S_b^{irr}}{1 - S_b^{irr} - S_{or}} \quad (9)$$

216 where S_b^{irr} [-] and S_{or} [-] are the irreducible brine and residual oil saturations, respectively. Note that we
 217 treat S_b^{irr} as a spatially heterogeneous quantity, by viewing it as a fraction (i.e., through a proportionality
 218 factor, F_r) of (i) the initial brine saturation in the oil-displacement scenario; or (ii) the steady-state brine
 219 saturation, for the brine-displacement experiment. The heterogeneous distribution of S_{or} is assessed
 220 according to the formulation of Spiteri (2008) [51], i.e.,

$$S_{or} = \xi(\gamma)S_{oin} - \beta(\gamma)S_{oin}^2 \quad (10)$$

221 where the coefficients $\xi(\gamma)$ [-] and $\beta(\gamma)$ [-] are functions of the contact angle, i.e., γ [-] ([52]; see Fig. 10),
 222 and S_{oin} is the initial oil saturation.

223 In summary, the two-phase flow model detailed in Eqs. (3)-(10) embeds a total of $N = 7$ parameters,
 224 i.e., $\theta = (m, k_{rb}^*, k_{ro}^*, N_b, N_0, F_r, \gamma)$. These are then subject to estimation through stochastic model
 225 calibration (see Sec. 2.3).

226 Various open-source numerical codes are available to cope with transient two-phase flow settings (e.g.
 227 [20,53]). Similar to Manasipov et al. (2020) [54], we solve the transient two-phase flow scenario associated
 228 with Eqs. (3)-(10) upon relying on the well known and widely tested open-source Matlab Reservoir
 229 Simulation Toolbox (MRST, see [55]) environment in light of its straightforward adaptability to our
 230 context. We employ the finite volume discretization method with a two-point flux-approximation scheme,
 231 as embedded in MRST. Consistent with the previously noted spatial heterogeneity of the sub-sample
 232 hydraulic properties that, in turns, impacts the dynamics of the displacing front (see Sec. 2.1 and Fig. 2),
 233 we employ a three-dimensional (structured Cartesian) grid. The latter comprises $N_v = 720$ elements of
 234 size 6.88 mm^3 . We test the robustness of the employed discretization upon considering several synthetic
 235 scenarios (characterized by various combinations of the model parameters) designed considering the same
 236 geometry of the Berea sample and the same initial conditions in terms of phase saturations employed in the
 237 experiments. We obtain satisfactory stochastic calibrations at an affordable computational burden for all of
 238 the scenarios analyzed, thus imbuing us with confidence about the selected discretization (see [56]). Note
 239 that we transfer the (spatially variable) experimental values of porosity and phase saturations obtained at

240 the original spatial resolution (see Sec. 2.1) to the one associated with the adopted computational grid to
 241 ensure consistency between observations and numerical results. For simplicity, we do so upon relying on a
 242 straightforward arithmetic and weighted (by the porosity) averaging approach for the porosity and the phase
 243 saturations, respectively.

244 2.3. Stochastic Model Calibration

245 Model calibration is grounded on the minimization of the following objective function

$$\begin{aligned}
 J(\boldsymbol{\theta}) = & W_S \frac{\sum_t^{N_{ts}} \sum_i^{N_v} (\hat{S}_{o,i,t} - S_{o,i,t}(\boldsymbol{\theta}))^2}{N_{ts} N_v} + W_P \frac{\sum_t^{N_{tp}} \left(\frac{\Delta \hat{P}_t - \Delta P_t(\boldsymbol{\theta})}{\Delta \hat{P}_{max}} \right)^2}{N_{tp}} \\
 & + W_{S_{ss}} \frac{\sum_i^{N_v} (\hat{S}_{o,i,ss} - S_{o,i,ss}(\boldsymbol{\theta}))^2}{N_v}
 \end{aligned}
 \tag{11}$$

246 $\Delta P_t(\boldsymbol{\theta})$ and $S_{o,i,t}(\boldsymbol{\theta})$ corresponding to model outputs at the N_{ts} and N_{tp} experimental observation times
 247 for the phase saturations and core-scale pressure drop, respectively; $S_{o,i,ss}(\boldsymbol{\theta})$ is the oil saturation under
 248 steady-state condition; $\Delta \hat{P}_{max}$ is the maximum value of the measured pressure drop; and W_S, W_P , and $W_{S_{ss}}$
 249 are the weights of the terms appearing in the objective function. These typically depend on the experimental
 250 error affecting $\hat{S}_{o,i,t}$, $\Delta \hat{P}_t$, and $\hat{S}_{o,i,ss}$, respectively. We recall here that a common working assumption relies
 251 on considering a Gaussian probability distribution to characterize measurement errors (e.g., [39]). At the
 252 same time, we also note that the exact values of the standard deviation of the latter are rarely known. Thus,
 253 various combinations of the values of the weights in Eq. (11) are typically tested to determine their optimal
 254 values (see e.g., [38]). Distinct triplets of the weights - i.e., $(W_S = W_{S_{ss}} = W_P = 1)$, $(W_S = W_{S_{ss}} =$
 255 $1; W_P = 10)$ and $(W_S = 1; W_{S_{ss}} = W_P = 10)$ - have been analyzed in a preliminary study. The analysis
 256 revealed that (a) the magnitude of the errors associated with transient and steady-state saturations (i.e., $S_{o,i,t}$
 257 and $S_{o,i,ss}$) does not vary significantly across the set of triplets $(W_S, W_P, W_{S_{ss}})$; (b) the residual associated
 258 with ΔP_t decreases as $W_{S_{ss}}$ and W_P increase. The latter finding is consistent with the higher degree of
 259 reliability associated with the experimental values of $\Delta \hat{P}_t$ and $\hat{S}_{o,i,ss}$ relative to those of $\hat{S}_{o,i,t}$.

260 In this context, minimization of Eq. (11) with respect to $\boldsymbol{\theta}$ is performed through a Differential Evolution
 261 (DE) algorithm (e.g., [57]). The latter is a direct-search method in which one starts upon introducing a
 262 population, \mathcal{S} , of candidate solutions composed by N_S members (where each member, s_i , is a vector of
 263 dimension N_m). In the present context, a member of the population represents a possible model parameter
 264 combination, $\boldsymbol{\theta}$. Hence, $N_m = N = 7$. We implement the algorithm according to [57] and set $N_S = 10 \times N =$
 265 70 . The initial population of candidate solutions, i.e., $\mathcal{S}^0 = [\mathbf{s}_1^0, \dots, \mathbf{s}_{N_S}^0]$, is defined by randomly selecting
 266 the j -th element of the i -th member, $s_{i,j}^0$ (for $j = 1, \dots, N$ and $i = 1, \dots, N_S$), from the support Γ_{θ_j} of the model
 267 parameter θ_j . This enables the DE algorithm to be initialized through a uniform coverage of the parameter
 268 space. At a subsequent k -th step, the members of the population are updated to obtain $\mathcal{S}^k = [\mathbf{s}_1^k, \dots, \mathbf{s}_{N_S}^k]$ by
 269 selecting the i -th member \mathbf{s}_i^k between (i) a trial member $\tilde{\mathbf{s}}_i^k$ and (ii) the i -th population member at the
 270 previous step, \mathbf{s}_i^{k-1} , according to

$$\mathbf{s}_i^k = \begin{cases} \tilde{\mathbf{s}}_i^k, & J(\tilde{\mathbf{s}}_i^k) < J(\mathbf{s}_i^{k-1}) \\ \mathbf{s}_i^{k-1}, & J(\tilde{\mathbf{s}}_i^k) \geq J(\mathbf{s}_i^{k-1}) \end{cases} \quad (12)$$

271 We recall that the trial member $\tilde{\mathbf{s}}_i^k$ is determined as a mutation of \mathbf{s}_i^{k-1} . The algorithm randomly
 272 determines which of the N elements of \mathbf{s}_i^{k-1} undergoes a mutation according to the following: (i) a random
 273 sample $\mathbf{r} \sim U(0,1)^N$ is drawn; (ii) mutations take place only for the elements of \mathbf{s}_i^{k-1} for which the
 274 corresponding elements of \mathbf{r} are smaller than a given crossover value, CR , the mutating elements of \mathbf{s}_i^{k-1}
 275 being collected in the indexing vector \bar{j} ; (iii) the values of the mutating elements of \mathbf{s}_i^{k-1} are determined
 276 according to

$$\tilde{\mathbf{s}}(\bar{j})_i^k = \mathbf{s}(\bar{j})_i^{k-1} + F \cdot (\mathbf{s}(\bar{j})_a^{k-1} - \mathbf{s}(\bar{j})_b^{k-1}) \quad (13)$$

277 where F is a DE algorithm parameter called *differential weight* and $\mathbf{s}(\bar{j})_a^{k-1}$ and $\mathbf{s}(\bar{j})_b^{k-1}$ (with $a, b \neq i$)
 278 correspond to two random members of the population. Considering the results obtained on the synthetic
 279 scenarios (discussed above), we set $CR = 0.3$ and $F = 0.4$.

280 The DE algorithm is employed in the preliminary model calibration step as well as in the final stochastic
 281 model calibration (see Fig. 1). In our implementation of the preliminary calibration, we terminate the

282 algorithm when the value of the objective function in Eq. (11) does not vary over 50 consecutive iterations.
283 We do so for simplicity and considering that only a single parameter combination is required in this phase.
284 Otherwise, when tackling the final stochastic model calibration phase, we end the progress of the DE
285 algorithm when the objective function attains a value which is 5% larger than the reference value obtained
286 from the preliminary calibration. We recall that our ultimate purpose is to obtain a distribution of
287 (influential) model parameter values conditional on available information through the imposed convergence
288 criterion. As such the strategy we consider enables one to avoid many model iterations within a region of
289 the parameter space where the objective function does not change significantly, while still maintaining a
290 satisfactory consistency between model results and experimental data. We remark that in this context our
291 results can be interpreted in terms of a frequency distribution of a collection of model parameter estimates.
292 These can then be employed to propagate residual (i.e., after calibration on available data) parameter
293 uncertainty onto target model outputs.

294 **2.4 Global Sensitivity Analysis**

295 The computational burden required to estimate empirical frequency distributions of uncertain model
296 parameters can be alleviated by reducing the dimensionality of the parameter space. This can be
297 accomplished through a rigorous sensitivity analysis. The latter is aimed at diagnosing the model behavior
298 in the presence of uncertain parameters and enables us to discriminate between (i) non-influential and (ii)
299 influential sets of parameters (with respect to the simulated state variables of interest, i.e., $\Delta P_t, S_{o,i,t}$).

300 We note that during the identification of non-influential parameters one cannot disregard the
301 experimental evidence about the two-phase flow dynamics taking place in the particular core-sample under
302 investigation. Thus, one needs to guarantee that the set of model simulations upon which the **sensitivity**
303 **analysis is** grounded are *behavioral* (see e.g., [58–60]), in the sense that the set of ensuing model outputs
304 must be consistent with their experimental counterparts. We ensure this aspect by demarcating a *behavioral*
305 parameters space (also referred as *active subspace*), $\mathbf{\Gamma}^B = \Gamma_{\theta_1}^B \times \dots \times \Gamma_{\theta_N}^B$, where $\Gamma_{\theta_i}^B$ is the *behavioral*
306 **support (or range of values) of the i -th parameter. This *behavioral* parameter space is only employed to**

307 **perform the sensitivity analysis.** We design it through a simple trial-and-error procedure. We select
 308 candidate upper and lower bounds of Γ^B by acknowledging (i) the results of the preliminary model
 309 calibration and (ii) the corresponding model outputs. We then ensure that the whole set of model simulations
 310 **upon which the sensitivity analysis is grounded** are indeed *behavioral*. Due to the limited number of model
 311 simulations required by the selected sensitivity analysis (see below), we ensure the latter requirement by
 312 the simple inspection of the juxtaposition of the set of model results and their experimental counterparts. **In**
 313 **the event that some simulations lead to unacceptable results (e.g., very distinct trends associated with the**
 314 **time evolution of the core scale pressure drop), we proceed to manually adjusting the extent of Γ^B to ensure**
 315 **that all of the sampled parameter combinations lead to *behavioral* responses. Moreover, the ranges of the**
 316 ***behavioral* space are also controlled after the stochastic calibration step, i.e., we check that for each i -th**
 317 **parameter the probability associated with values outside the support range $\Gamma_{\theta_i}^B$ tend to be lowest (ideally**
 318 **null). While more complex and automated approaches are available in the literature for the definition of Γ^B**
 319 **(e.g., [61] and references therein), we note that the associated computational burden is typically higher and**
 320 **can possibly reduced through the introduction of a surrogate model. Thus, here we prefer to rely on a simple**
 321 **trial-and-error approach with an affordable computational cost.**

322 Considering a model output of interest, $y(\boldsymbol{\theta})$, it is possible to evaluate the so-called elementary effect
 323 associated with the j -th uncertain model parameter θ_j as

$$EE_{\theta_j} = \frac{y(\theta_1, \dots, \theta_j + \Delta\theta_j, \dots, \theta_N) - y(\theta_1, \dots, \theta_j, \dots, \theta_N)}{\Delta\theta_j} \quad (14)$$

324 The value of EE_{θ_j} is a *local* (in the parameter space) measure of the sensitivity of y with respect to θ_j , that
 325 is quantified in terms of the variation in the value of the former due to a variation of the latter. A *global*
 326 measure of sensitivity is obtained upon evaluating the elementary effect (EE_{θ_j}) for a variety of model
 327 parameter combinations. To this end we rely on the radial-sampling strategy detailed in [62]. Key summary
 328 statistics of the ensuing distribution of EE_{θ_j} are then evaluated, such as [34] and [62]:

$$\mu_{\theta_j}^* = \frac{1}{M} \sum_{k=1}^M |EE_{\theta_j}^k| \quad (15)$$

$$\sigma_{\theta_j} = \sqrt{\frac{1}{M} \sum_{k=1}^M (EE_{\theta_j}^k - \mu_{\theta_j}^*)^2} \quad (16)$$

329 where M is the number of values of EE_{θ_j} associated with θ_j . The quantities $\mu_{\theta_j}^*$ (15) and σ_{θ_j} (16) are
330 estimates of the mean and standard deviation of the distribution of EE_{θ_j} . Note that the absolute value is
331 introduced for the evaluation of $\mu_{\theta_j}^*$ to avoid compensation between positive and negative valued EE_{θ_j} .
332 Non-influential parameters are then characterized by low values of $\mu_{\theta_j}^*$ (i.e., variations of θ_j do not
333 correspond to significant variations in y). A high value of σ_{θ_j} indicates that θ_j influences y in a non-linear
334 fashion or through interactions with other model parameters. Thus, inspection of the Morris' indices (15)-
335 (16) enables us to identify non-influential model parameters at an affordable computational cost (see also
336 [63] and references therein). **We follow the sampling strategy proposed by [64] for each experimental**
337 **scenario. We then find that 80 parameters combinations within Γ^B are sufficient to yield robust evaluations**
338 **of the Morris' indices.** We then set the identified non-influential parameters to the value associated with
339 their counterparts obtained during the preliminary calibration. We perform the final stochastic model
340 calibration upon considering the parameter space of reduced dimensionality, i.e., Γ' . The extent of the
341 **support of the influential parameters within Γ' coincides with its counterpart in Γ , while the non-influential**
342 **parameters are set at the values obtained from the preliminary model calibration** (we recall here that the
343 numerosity of the population in the DE algorithm scales with N , i.e., the dimensionality of the space within
344 which solutions are searched, see Sec. 2.3).

345 **3. Results**

346 In this Section we detail our results for the (i) oil-displacement and (ii) brine-displacement experiments.

347 **3.1. Oil-displacement scenario**

348 As detailed in Section 2.1, available observations comprise three-dimensional distributions of: (i)
349 porosity across the core sample; and oil saturation (ii) at the beginning of the experiment, and during the
350 (iii) transient (for a total of six observation windows) and (iv) the stationary regime. We complement this
351 information with temporal histories of the core-scale pressure drop. During the experiment brine is injected
352 at a constant flow rate of 15 ml/h.

353 As a first step in the workflow (see Fig. 1), we perform a preliminary model calibration upon relying
354 on the parameter space Γ (see Table 3) and leveraging on the DE algorithm (see Sec. 2.3). Considering the
355 stopping criterion stated in Sec. 2.3 yields a total of 183 model iterations (attaining a final value of the
356 objective function $J = 0.0096$). The resulting combination of parameter values is listed in Table 3. The
357 corresponding model results are depicted in Fig. 3. The latter includes (a) core-scale pressure drop versus
358 time (blue and red curves corresponding to experimental data, $\Delta\hat{P}_t$, and numerical results ΔP_t , respectively);
359 (b) scatter plots of simulated ($S_{o,i,t}$) versus measured ($\hat{S}_{o,i,t}$) oil saturations at each voxel of the simulation
360 grid and for the available acquisition times. Inspection of Figs. 3a-3b denotes an overall satisfactory
361 agreement between experimental and numerical results that imbues us with confidence on the
362 appropriateness of this preliminary model calibration step. **At the same time, we note that our model tends**
363 **to underestimate the steady state oil distribution. This might be due to some restriction in the former to rend**
364 **the full dynamics of the oil-displacement experiment.**

365 We then evaluate the Morris' indexes (see Sec. 2.4) by considering the behavioral parameter space Γ^B
366 (see Table 3). Fig. 4 depicts the values of $\mu_{\theta_j}^*$ and σ_{θ_j} associated with temporal dynamics of (a) section-
367 averaged oil saturations (which are here considered for ease of representation, as opposed to their three-
368 dimensional counterparts) and (b) pressure drop. Inspection of Fig. 4 reveals that (i) $[k_{ro}^*, m, \gamma]$ and (ii)
369 $[k_{ro}^*, \gamma]$ are characterized by small values of both $\mu_{\theta_j}^*$ and σ_{θ_j} . Hence, we regard these sets of parameters as
370 non-influential with respect to the spatial distribution of saturation (Fig. 4a) and pressure drop (Fig. 4b),
371 respectively. We interpret these results by recalling that: (i) parameter m has a marked influence on the
372 distribution of the absolute permeability and thus on the corresponding pressure drop required to sustain

373 water flow through the sample; (ii) variations of γ across Γ_γ^B do not lead to significant variations of ξ and
 374 β (see also [39]) and therefore do not impact markedly on the investigated system state variables; (iii)
 375 according with Corey's formulation (Eq. (8)) k_{ro} depends linearly on k_{ro}^* (consistent with the small values
 376 of $\sigma_{k_{ro}^*}$ depicted in Fig. 4a). Moreover, the low oil saturations recorded (see Figs. 2c and 3b) during the oil-
 377 displacement experiment is associated with a low relative permeability of this phase, i.e., $k_{ro} \approx 0$. Thus,
 378 variations in k_{ro}^* within $\Gamma_{k_{ro}^*}^B$ have a limited effect on k_{ro} and, consequently, on the simulated state variables
 379 (as indicated by the small values of $\mu_{k_{ro}^*}^*$ in Fig. 4a). Eqs. (8)-(9) highlight that the exponent N_o controls the
 380 (nonlinear) variation of k_{ro} with the normalized oil saturation, $S_o^* = 1 - S_b^*$. This observation is consistent
 381 with the high values of σ_{N_o} for $S_{o,i,t}$ in Fig. 4a.

382 This set of results leads us to consider k_{ro}^* and γ as non-influential. In the stochastic model calibration
 383 stage, we set $k_{ro}^* = 0.01$ and $\gamma = 146.76$ (as obtained from the preliminary model calibration) and focus on
 384 $(m, k_{rb}^*, N_b, N_o, F_r)$ to minimize the objective function in Eq. (11) within the support Γ' (see Table 3).
 385 Following the DE algorithm convergence criterion (Sec. 2.3), Fig. 5 depicts the empirical frequency
 386 distributions for $(m, k_{rb}^*, N_b, N_o, F_r)$ obtained on the basis of 100 inverse modeling solutions (vertical red
 387 lines demarcate the bounds of Γ^B). Inspection of Fig. 5 suggests that: (i) most of the instances of the
 388 frequency distribution of each parameter are confined within the bounds of Γ^B , supporting the selected
 389 ranges for the GSA; (ii) the frequency distributions for m and k_{rb}^* display well defined peaks, in agreement
 390 with the high sensitivity of ΔP_t to these parameters (see Fig. 4b) and with the weight associated with
 391 pressure data in the objective function in Eq. (11); and (iii) the wide range of values spanned by the
 392 frequency distribution of N_o seems to be consistent with the low impact of this parameter on ΔP_t (see Fig.
 393 4b) as well as with the high value of σ_{N_o} (Fig. 4a). The latter element suggests the presence of a non-linear
 394 contribution to the impact of N_o on $S_{o,i,t}$, or the effect of interactions between N_o and other model
 395 parameters. As such, it is possible that the combination of these two factors could lead to a higher
 396 uncertainty for N_o than for the other parameters. We further note that F_r is characterized by a similar
 397 behavior to that observed for N_o . Finally, the presence of a sharp peak in the empirical frequency

398 distribution of N_b does not seem to correlate with a high sensitivity of ΔP_t and/or $S_{o,i,t}$ (see Fig. 4) with
399 respect to this parameter (note that ΔP_t is still sensitive to N_b , sensitivity being a necessary while otherwise
400 not a sufficient condition for parameter identifiability, see e.g., [65], [66]). **For completeness, in Appendix**
401 **A we investigate the cross correlation for the estimated parameters that are relevant for the oil-displacement**
402 **experiment.**

403 **3.2. Brine-displacement experiment**

404 The brine-displacement experiment is designed upon setting a lower injection rate (i.e., 2 ml/h) of the
405 invading phase than in the oil-displacement scenario. This enables us to collect (i) 14 snapshots of three-
406 dimensional spatial distributions of oil saturation during the transient regime and (ii) one image under
407 stationary conditions, in addition to continuously recorded core-scale pressure drop.

408 Following our workflow, we obtain a first set of model parameters compatible with the experimental
409 observations through a preliminary model calibration relying on the parameter space \mathbf{I} (see Table 4).
410 Achievement of convergence of the DE algorithm (according to the criterion outlined in Sec. 2.3) require a
411 total of 118 iterations (attaining $J = 0.020$). Note that the high density and viscosity of the oil injected in
412 the brine-displacement experiment leads to a reduced degree of control on the experimental value of the
413 flow rate, some mild fluctuations being observed during the course of the experiment. This might hinder
414 the quality of the match between ΔP_t and $\Delta \hat{P}_t$ (see Fig. 6a), as also suggested by the increased value of J
415 documented here as compared against its counterpart for the oil-displacement scenario.

416 The resulting parameter combination is listed in Table 4. The corresponding inverse modeling results
417 are collected in Figs. 6a, 6b. The former depicts core-scale pressure drop versus time (blue and red curves
418 corresponding to experimental data and numerical results, respectively). The latter shows a scatter plot of
419 simulated versus observed oil saturations at each voxel and for all acquisition times (i.e., $S_{o,i,t}$ versus $\hat{S}_{o,i,t}$).
420 Inspection of Fig. 6 suggests an overall satisfactory agreement between experimental and numerical results
421 stemming from the preliminary model calibration. Moreover, we note that the strong viscosity contrast
422 leads to a sharp interface between the invading oil and the displaced brine phase. The latter behavior is

423 visible from the experimental results collected in Fig. 2d. It is also clearly documented in Fig. 6b, where it
424 is possible to recognize a set of voxels characterized by (i) high or (ii) low oil saturation (i.e., upstream and
425 downstream region of the advancing front), jointly with (iii) a decreased amount of voxels characterized by
426 intermediate values of saturation (i.e., at locations corresponding to the front region) as compared to the
427 oil-displacement experiment. It is observed that the numerical results tend to overestimate the degree of oil
428 saturation, especially during the initial period of the experiment (i.e., until approximately 7080 s) when
429 high values of oil saturation are observed. This observation suggests that during the early stages the
430 numerical model tends to render the advancement of the oil front at a rate that is faster than its
431 experimentally observed counterpart. Nevertheless, there is a markedly high consistency between model
432 results and experimental oil phase saturation values at steady state.

433 We then evaluate the Morris' indices by considering the behavioral parameter space Γ^B (see Table 4).
434 Fig. 7 depicts the values of $\mu_{\theta_j}^*$ and σ_{θ_j} for (a) (section-averaged) oil saturation (as a function of time and
435 depth along the core) and (b) pressure drop across the core. Inspection of Figs. 7a-7b reveals that N_o and γ
436 exhibit a very limited influence on both state variables. In contrast with what observed for the oil-
437 displacement experiment, we obtain low values also for σ_{N_o} . We interpret this result by considering that,
438 due to the higher viscosity contrast, the advancing front is considerably sharper in the brine-displacement
439 than in the oil-displacement scenario (see Figs. 2c and 2d). As a consequence, there is a lower portion of
440 voxels that is characterized by intermediate values of $S_{o,i,t}$. Thus, variations in N_o (i.e., in the relative
441 permeability curve far from the endpoints) are not associated with marked variations of the system
442 dynamics. Another difference with respect to the oil-displacement experiment is that $S_{o,i,t}$ increases
443 progressively in time and assumes values in the whole range [0, 1]. For these reasons (i) the quantity S_{or} is
444 not relevant in the description of this experiment, consistent with the low impact of γ (see Eq. (10)) and (ii)
445 the oil phase is characterized by a non-negligible relative permeability ($k_{ro} \gg 0$). This makes k_{ro}^* an
446 influential parameter, in particular for the pressure drop (see Fig. 7b).

447 We then complete the stochastic model calibration stage by focusing on the reduced parameter space
448 $(N_b, k_{rb}^*, k_{ro}^*, m, F_r)$. We minimize the objective function Eq. (11) within the support Γ' (see Table 4) while
449 setting N_o and γ to the values obtained in the preliminary model calibration step. Fig. 8 depicts the empirical
450 frequency distributions (grounded on 100 inverse modeling solutions) for $(N_b, k_{rb}^*, k_{ro}^*, m, F_r)$ (vertical red
451 lines correspond to the bounds of Γ^B). Most of the frequency distributions obtained are contained within
452 the limits of Γ^B , some exceptions being noted for k_{ro}^* and m . This observation suggests that, for these
453 parameters, the actual behavioral ranges could be wider than those considered in the GSA. Furthermore,
454 the frequency distributions for the influential parameters tend to be more broadly distributed than their
455 counterparts obtained in the oil-displacement experiment. This result can be attributed to the generally
456 lower degree of sensitivity of ΔP_t and $S_{o,i,t}$ with respect to the influential parameters, as quantified by the
457 values of $\mu_{\theta_j}^*$ and σ_{θ_j} (Fig. 7) when compared to those resulting for the oil-displacement experiment (Fig.
458 4). This, in turn, prevents the identification of a clear peak in the resulting frequency distributions of the
459 parameters. Considering the constraint associated with Eqs. (8) and (9), the parameter sets obtained lead to
460 a sustained increase in oil relative permeability with oil saturation, i.e., relative oil permeability is
461 significantly higher than the water relative permeability for intermediate and high oil saturations. **For**
462 **completeness, in Appendix A we investigate the cross correlation for the estimated parameters that are**
463 **relevant for the brine-displacement experiment.**

464 **4. Conclusions**

465 We propose a novel stochastic inverse modeling framework to assist interpretation laboratory-scale
466 two-phase fluid displacement experiments. Our methodology is conducive to frequency distributions of
467 model parameters and combines the Differential Evolution optimization algorithm and the use of Morris'
468 indices for global sensitivity analysis (GSA). Including GSA in the workflow enables us to mitigate the
469 challenges related to large computational costs typically associated with the application of population-based
470 optimization algorithms in stochastic inverse settings.

471 The methodology is applied to interpret uniquely rich datasets. These include detailed temporal series
472 of (i) three-dimensional spatial distributions of phase saturations and (ii) core-scale pressure drops. To cover
473 a wide variety of scenarios, two sets of experiments are analyzed. These encompass an oil-displacement
474 and a brine-displacement scenario, characterized by different density and viscosity contrasts between brine
475 and oil. Our work leads to the following major conclusions.

- 476 1. The results of our analyses clearly show that assisting stochastic inverse modeling through Global
477 Sensitivity Analysis can considerably contribute to (a) clarification and quantification of the role
478 of uncertain model parameters in driving the two-phase system dynamics and (b) reduction of the
479 dimensionality of the uncertain model parameter space. The latter is a critical element to be
480 considered in the context of a computationally intensive forward modeling scenario of the kind we
481 consider.
- 482 2. Amongst the uncertain model parameters embedded in the formulation considered, only 5 have
483 been recognized as key sources of uncertainties significantly affecting the prediction of model
484 outputs (i.e., temporal histories of detailed three-dimensional distribution of fluid saturations and
485 pressure drop across the sample). These relevant parameters include, for both the experiments, the
486 coefficient m in the porosity/permeability relationship (Eq. (6)); k_{rb}^* and N_b , controlling brine
487 relative permeability curve (Eq. (7)); and F_r , associated with the irreducible brine saturation (Eq.
488 (9)). Moreover, N_o appeared to be relevant only in the oil-displacement scenario, while k_{ro}^* affected
489 appreciably the model outputs only in the brine-displacement experiment.
- 490 3. The amount of information employed in the stochastic model calibration is very detailed and
491 consistent with modern experimental capabilities. Our study fully takes advantage of these
492 experimental evidences and provides a comprehensive analysis of the dynamics of two-phase flow
493 in fractured/porous media. In all cases investigated, the stochastic approach is documented to be
494 effective to identify frequency distributions of model parameters rendering a satisfactory agreement
495 between experimental and numerical results (in a probabilistic sense). It forms a robust basis to

496 effectively assist (a) interpretation of data associated with coreflooding practices, (b) further design
497 of laboratory coreflooding experiments.

498 From a practical perspective, the proposed workflow is structured across various steps (spanning from
499 a preliminary model calibration to a stochastic calibration) to readily assist the efficient assessment of
500 probability distributions of model parameters upon leveraging on the information content of the available
501 experimental observations. We recall here that the workflow is fully compatible with the use of alternative
502 specific techniques to complement those we selected in our application. For example, one can embed (i)
503 other models for relative permeability curves (e.g., the LET model of [67]); (ii) different GSA approaches
504 (e.g., the moment-based sensitivity indices of [27]) for parameter screening; or (iii) other stochastic
505 calibration strategies (e.g., Monte Carlo Markov Chain as in [35,36]). The layout selected for the current
506 applications is inspired by our expertise and knowledge and aims at attaining a balance among a variety of
507 factors including, e.g., the representativeness of the numerical modeling approach, its accuracy and the
508 associated computational burden. The latter might be reduced through the introduction of a surrogate model
509 (e.g., [27,35,36,68]). Otherwise, it is remarked that the efficient assessment of a surrogate model for time-
510 dependent quantities, such as three-dimensional fluid phase distributions, is a non-trivial task. Thus, we
511 opted to rely on the numerical model detailed in Sec. 2.2 across the various steps of the proposed workflow
512 for the current application while deferring to future studies the detailed analysis of the effects of introducing
513 a surrogate model.

514 **CRedit author statement:**

515 **A. Dell’Oca:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing -
516 Original Draft, Writing - Review & Editing; **A. Manzoni:** Conceptualization, Methodology, Software,
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522 **Guadagnini:** Conceptualization, Methodology, Validation, Formal analysis, Writing - Original Draft,
523 Writing - Review & Editing, Supervision.

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528 **Appendix A. Estimated parameters cross correlations**

529 Figure A1 depicts the scatter plots between the sets of calibrated parameters for (a) the oil- and (b) the
530 brine-displacement scenario, respectively. Inspection of Fig. A1a does not reveal any clear degree of
531 correlation between the pairwise sets of estimated parameters, with the sole exception of N_o and F_r that
532 appear to display a mild positive correlation. The emergence of such pattern is imprinted/driven by the
533 information content provided by the available data. It suggests that oil displacement in case of an increased
534 brine saturation (corresponding to an increase of N_o) tends to be harder when the fraction of irreducible
535 brine increases (i.e., F_r increases). Otherwise, inspection of Fig. A1b suggests a negative correlation
536 between N_b and F_r . This reflects a tendency towards a facilitated displacement of brine (i.e., a decrease of
537 N_b) as the irreducible water content increases (i.e., F_r increases). Stochastic model calibration on the
538 available data also suggests that the coefficient m is (i) negatively correlated with k_{rb}^* , k_{ro}^* and (ii) positively
539 correlated with N_b . These results denote a tendency to balance between those parameters that control the
540 overall resistance to flow of the fluid phases through the sample, i.e., the absolute values of the
541 permeabilities (an increase in m leads to a reduced flow resistance) and the values attained by k_{rb}^* (higher
542 values lead to less flow resistance) and N_b (lower values correspond to less flow resistance) for the brine
543 and the values of k_{ro}^* (higher values lead to less flow resistance) for the oil phase. The same type of
544 conceptual picture could be associated with the positive correlation that is noted between k_{rb}^* and k_{ro}^* .
545 Overall, these sets of results highlight that parameters associated with an equifinality (in particular with
546 respect to the core-scale pressure drop) are harder to be identified for our two scenarios, especially for the

547 brine-displacement scenario (note that parameters displaying correlation with some others tend to be
548 characterized by broad frequency distributions; see Fig. A1).

549 References

- 550 [1] M. Taheriotaghsara, Enhanced oil recovery methods targeting danish north sea chalk
551 reservoirs PhD Thesis, Denmark Technical University, 2020.
- 552 [2] S. Berg, S. Oedai, H. Ott, Displacement and mass transfer between saturated and
553 unsaturated CO₂-brine systems in sandstone, *International Journal of Greenhouse Gas*
554 *Control*. 12 (2013) 478–492. <https://doi.org/10.1016/j.ijggc.2011.04.005>.
- 555 [3] S.C.M. Krevor, R. Pini, L. Zuo, S.M. Benson, Relative permeability and trapping of CO₂
556 and water in sandstone rocks at reservoir conditions, *Water Resour Res*. 48 (2012).
557 <https://doi.org/10.1029/2011WR010859>.
- 558 [4] T. Kurotori, R. Pini, A general capillary equilibrium model to describe drainage
559 experiments in heterogeneous laboratory rock cores, *Adv Water Resour*. 152 (2021)
560 103938. <https://doi.org/10.1016/j.advwatres.2021.103938>.
- 561 [5] K.-Y. Kim, J. Oh, W.S. Han, K.G. Park, Y.J. Shinn, E. Park, Two-phase flow visualization
562 under reservoir conditions for highly heterogeneous conglomerate rock: A core-scale study
563 for geologic carbon storage, *Sci Rep*. 8 (2018) 4869. [https://doi.org/10.1038/s41598-018-](https://doi.org/10.1038/s41598-018-23224-6)
564 [23224-6](https://doi.org/10.1038/s41598-018-23224-6).
- 565 [6] M. Mascle, E. Rosenberg, B. Roboele, E. Kowalewski, S. Youssef, Investigation of
566 waterflood front digitations during immiscible displacements in porous media, *Oil & Gas*
567 *Science and Technology – Revue d'IFP Energies Nouvelles*. 76 (2021) 71.
568 <https://doi.org/10.2516/ogst/2021053>.
- 569 [7] L. Moghadasi, A. Guadagnini, F. Inzoli, M. Bartosek, D. Renna, Characterization of two-
570 and three-phase relative permeability of water-wet porous media through X-Ray saturation
571 measurements, *J Pet Sci Eng*. 145 (2016) 453–463.
572 <https://doi.org/10.1016/j.petrol.2016.05.031>.
- 573 [8] M.J. Blunt, B. Bijeljic, H. Dong, O. Gharbi, S. Iglauer, P. Mostaghimi, A. Paluszny, C.
574 Pentland, Pore-scale imaging and modelling, *Adv Water Resour*. 51 (2013) 197–216.
575 <https://doi.org/10.1016/j.advwatres.2012.03.003>.
- 576 [9] L. Leu, S. Berg, F. Enzmann, R.T. Armstrong, M. Kersten, Fast X-ray Micro-Tomography
577 of Multiphase Flow in Berea Sandstone: A Sensitivity Study on Image Processing, *Transp*
578 *Porous Media*. 105 (2014) 451–469. <https://doi.org/10.1007/s11242-014-0378-4>.
- 579 [10] M. Andrew, H. Menke, M.J. Blunt, B. Bijeljic, The Imaging of Dynamic Multiphase Fluid
580 Flow Using Synchrotron-Based X-ray Microtomography at Reservoir Conditions, *Transp*
581 *Porous Media*. 110 (2015) 1–24. <https://doi.org/10.1007/s11242-015-0553-2>.
- 582 [11] S. Hasan, V. Niasar, N.K. Karadimitriou, J.R.A. Godinho, N.T. Vo, S. An, A. Rabbani, H.
583 Steeb, Direct characterization of solute transport in unsaturated porous media using fast X-
584 ray synchrotron microtomography, *Proceedings of the National Academy of Sciences*. 117
585 (2020) 23443–23449. <https://doi.org/10.1073/pnas.2011716117>.

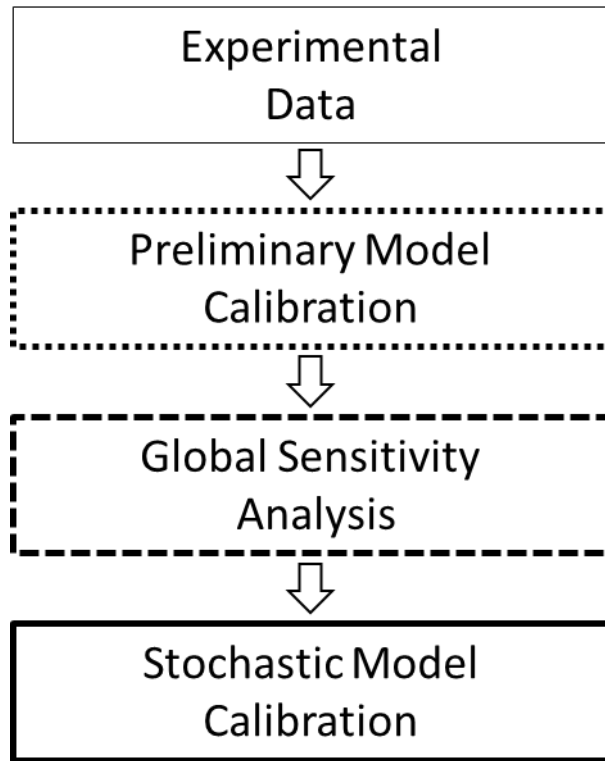
- 586 [12] Y. Zhao, S. Xue, S. Han, L. He, Z. Chen, Characterization of unsaturated diffusivity of
587 tight sandstones using neutron radiography, *Int J Heat Mass Transf.* 124 (2018) 693–705.
588 <https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.090>.
- 589 [13] S. Chen, K.-H. Kim, F. Qin, A.T. Watson, Quantitative NMR imaging of multiphase flow
590 in porous media, *Magn Reson Imaging.* 10 (1992) 815–826. [https://doi.org/10.1016/0730-](https://doi.org/10.1016/0730-725X(92)90417-X)
591 [725X\(92\)90417-X](https://doi.org/10.1016/0730-725X(92)90417-X).
- 592 [14] Y. Zhao, Y. Song, Y. Liu, H. Liang, B. Dou, Visualization and Measurement of CO₂
593 Flooding in Porous Media Using MRI, *Ind Eng Chem Res.* 50 (2011) 4707–4715.
594 <https://doi.org/10.1021/ie1013019>.
- 595 [15] Y. Song, L. Jiang, Y. Liu, M. Yang, Y. Zhao, N. Zhu, B. Dou, A. Abudula, An
596 experimental study on CO₂/water displacement in porous media using high-resolution
597 Magnetic Resonance Imaging, *International Journal of Greenhouse Gas Control.* 10 (2012)
598 501–509. <https://doi.org/10.1016/j.ijggc.2012.07.017>.
- 599 [16] M. Li, L. Romero-Zerón, F. Marica, B.J. Balcom, Polymer Flooding Enhanced Oil
600 Recovery Evaluated with Magnetic Resonance Imaging and Relaxation Time
601 Measurements, *Energy & Fuels.* 31 (2017) 4904–4914.
602 <https://doi.org/10.1021/acs.energyfuels.7b00030>.
- 603 [17] J. Fannir, I. Panfilova, S. Leclerc, D. Stemmelen, Studying of parameters of two-phase
604 displacement in porous media with MRI technique, *Mechanics & Industry.* 21 (2020) 524.
605 <https://doi.org/10.1051/meca/2020058>.
- 606 [18] D. Wang, Z. Zheng, Q. Ma, P. Hu, X. Huang, Y. Song, Effects of the errors between core
607 heterogeneity and the simplified models on numerical modeling of CO₂/water core
608 flooding, *Int J Heat Mass Transf.* 149 (2020) 119223.
609 <https://doi.org/10.1016/j.ijheatmasstransfer.2019.119223>.
- 610 [19] M.S. Krause, S. Krevor, J.C. Perrin, R. Pini, S. Benson, A Procedure for Accurate
611 Determination of Sub-Core Scale Permeability Distributions with Error Quantification, in:
612 *7 Th Annual GCEP Research Symposium, 2011.*
- 613 [20] J.G. Maas, N. Springer, A. Hebing, Defining a sample heterogeneity cut-off value to obtain
614 representative Special Core Analysis (SCAL) measurements, in: *International Symposium*
615 *of the Society of Core Analysts Held in Pau, France, 2019: pp. 26 Aug.-30 Aug.*
- 616 [21] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, S.
617 Tarantola, *Global sensitivity analysis: The primer*, John Wiley & Sons, Ltd, Chichester,
618 UK, 2008. <https://doi.org/10.1002/9780470725184>.
- 619 [22] M.K. Muleta, J.W. Nicklow, Sensitivity and uncertainty analysis coupled with automatic
620 calibration for a distributed watershed model, *J Hydrol (Amst).* 306 (2005) 127–145.
621 <https://doi.org/10.1016/j.jhydrol.2004.09.005>.
- 622 [23] F. Pappenberger, K.J. Beven, M. Ratto, P. Matgen, Multi-method global sensitivity
623 analysis of flood inundation models, *Adv Water Resour.* 31 (2008) 1–14.
624 <https://doi.org/10.1016/j.advwatres.2007.04.009>.

- 625 [24] T. Wagener, K. van Werkhoven, P. Reed, Y. Tang, Multiobjective sensitivity analysis to
626 understand the information content in streamflow observations for distributed watershed
627 modeling, *Water Resour Res.* 45 (2009). <https://doi.org/10.1029/2008WR007347>.
- 628 [25] M.V. Ruano, J. Ribes, A. Seco, J. Ferrer, An improved sampling strategy based on
629 trajectory design for application of the Morris method to systems with many input factors,
630 *Environmental Modelling & Software.* 37 (2012) 103–109.
631 <https://doi.org/10.1016/j.envsoft.2012.03.008>.
- 632 [26] E. Hill, M.C. Foglia, S. Christensen, O. Rakovec, Model Validation, in: *The Handbook of*
633 *Groundwater Engineering*, CRC Press, Taylor & Francis Group, 6000 Broken Sound
634 Parkway NW, Suite 300, Boca Raton, FL 33487-2742, 2016: pp. 597–620.
635 <https://doi.org/10.1201/9781315371801-22>.
- 636 [27] A. Dell’Oca, M. Riva, A. Guadagnini, Moment-based metrics for global sensitivity
637 analysis of hydrological systems, *Hydrol Earth Syst Sci.* 21 (2017) 6219–6234.
638 <https://doi.org/10.5194/hess-21-6219-2017>.
- 639 [28] D. Degenring, C. Froemel, G. Dikta, R. Takors, Sensitivity analysis for the reduction of
640 complex metabolism models, *J Process Control.* 14 (2004) 729–745.
641 <https://doi.org/10.1016/j.jprocont.2003.12.008>.
- 642 [29] A. van Griensven, T. Meixner, S. Grunwald, T. Bishop, M. Diluzio, R. Srinivasan, A
643 global sensitivity analysis tool for the parameters of multi-variable catchment models, *J*
644 *Hydrol (Amst).* 324 (2006) 10–23. <https://doi.org/10.1016/j.jhydrol.2005.09.008>.
- 645 [30] J. Nossent, P. Elsen, W. Bauwens, Sobol’ sensitivity analysis of a complex environmental
646 model, *Environmental Modelling & Software.* 26 (2011) 1515–1525.
647 <https://doi.org/10.1016/j.envsoft.2011.08.010>.
- 648 [31] J. Chu, C. Zhang, G. Fu, Y. Li, H. Zhou, Improving multi-objective reservoir operation
649 optimization with sensitivity-informed dimension reduction, *Hydrol Earth Syst Sci.* 19
650 (2015) 3557–3570. <https://doi.org/10.5194/hess-19-3557-2015>.
- 651 [32] V. Punzo, M. Montanino, B. Ciuffo, Do We Really Need to Calibrate All the Parameters?
652 Variance-Based Sensitivity Analysis to Simplify Microscopic Traffic Flow Models, *IEEE*
653 *Transactions on Intelligent Transportation Systems.* 16 (2015) 184–193.
654 <https://doi.org/10.1109/TITS.2014.2331453>.
- 655 [33] S.E. Patani, G.M. Porta, V. Caronni, P. Ruffo, A. Guadagnini, Stochastic Inverse Modeling
656 and Parametric Uncertainty of Sediment Deposition Processes Across Geologic Time
657 Scales, *Math Geosci.* 53 (2021) 1101–1124. <https://doi.org/10.1007/s11004-020-09911-z>.
- 658 [34] M.D. Morris, Factorial Sampling Plans for Preliminary Computational Experiments,
659 *Technometrics.* 33 (1991) 161. <https://doi.org/10.2307/1269043>.
- 660 [35] A.R. Valdez, B.M. Rocha, G. Chapiro, R. Weber dos Santos, Uncertainty quantification
661 and sensitivity analysis for relative permeability models of two-phase flow in porous
662 media, *J Pet Sci Eng.* 192 (2020) 107297. <https://doi.org/10.1016/j.petrol.2020.107297>.
- 663 [36] A.R. Valdez, B.M. Rocha, J.M. da Fonseca Façanha, A.V.O. de Souza, A. Pérez-
664 Gramatges, G. Chapiro, R.W. dos Santos, Foam-Assisted Water–Gas Flow Parameters:

- 665 From Core-Flood Experiment to Uncertainty Quantification and Sensitivity Analysis,
666 *Transp Porous Media.* 144 (2022) 189–209. <https://doi.org/10.1007/s11242-021-01550-0>.
- 667 [37] S. Berg, E. Unsal, H. Dijk, Non-uniqueness and uncertainty quantification of relative
668 permeability measurements by inverse modelling, *Comput Geotech.* 132 (2021) 103964.
669 <https://doi.org/10.1016/j.compgeo.2020.103964>.
- 670 [38] S. Berg, E. Unsal, H. Dijk, Sensitivity and Uncertainty Analysis for Parameterization of
671 Multiphase Flow Models, *Transp Porous Media.* 140 (2021) 27–57.
672 <https://doi.org/10.1007/s11242-021-01576-4>.
- 673 [39] J. Carrera, S.P. Neuman, Estimation of Aquifer Parameters Under Transient and Steady
674 State Conditions: 1. Maximum Likelihood Method Incorporating Prior Information, *Water*
675 *Resour Res.* 22 (1986) 199–210. <https://doi.org/10.1029/WR022i002p00199>.
- 676 [40] A. Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*,
677 Society for Industrial and Applied Mathematics, 2005.
678 <https://doi.org/10.1137/1.9780898717921>.
- 679 [41] H. Zhou, J.J. Gómez-Hernández, L. Li, Inverse methods in hydrogeology: Evolution and
680 recent trends, *Adv Water Resour.* 63 (2014) 22–37.
681 <https://doi.org/10.1016/j.advwatres.2013.10.014>.
- 682 [42] L. Ceresa, A. Guadagnini, G.M. Porta, M. Riva, Formulation and probabilistic assessment
683 of reversible biodegradation pathway of Diclofenac in groundwater, *Water Res.* 204 (2021)
684 117466. <https://doi.org/10.1016/j.watres.2021.117466>.
- 685 [43] A. Timur, An investigation of permeability, porosity, and residual water saturation
686 relationships, in: *SPWLA 9th Annu. Logging Symp.*, 1968.
- 687 [44] G.L. Chierici, Novel Relations for Drainage and Imbibition Relative Permeabilities,
688 *Society of Petroleum Engineers Journal.* 24 (1984) 275–276.
689 <https://doi.org/10.2118/10165-PA>.
- 690 [45] C.E. Duchon, Lanczos filtering in one and two dimensions, *Journal of Applied*
691 *Meteorology.* 19 (1979).
- 692 [46] A.T. Corey, The interrelation between gas and oil relative permeabilities, *Prod. Mon.*
693 (1954) 38–41.
- 694 [47] M.R.J. Wyllie, A Note on the Interrelationship Between Wetting and Nonwetting Phase
695 Relative Permeability, *Journal of Petroleum Technology.* 3 (1951) 17–17.
696 <https://doi.org/10.2118/951381-G>.
- 697 [48] M. Honarpour, L.F. Koederitz, A.H. Harvey, Empirical Equations for Estimating Two-
698 Phase Relative Permeability in Consolidated Rock, *Journal of Petroleum Technology.* 34
699 (1982) 2905–2908. <https://doi.org/10.2118/9966-PA>.
- 700 [49] P. Xu, S. Qiu, B. Yu, Z. Jiang, Prediction of relative permeability in unsaturated porous
701 media with a fractal approach, *Int J Heat Mass Transf.* 64 (2013) 829–837.
702 <https://doi.org/10.1016/j.ijheatmasstransfer.2013.05.003>.

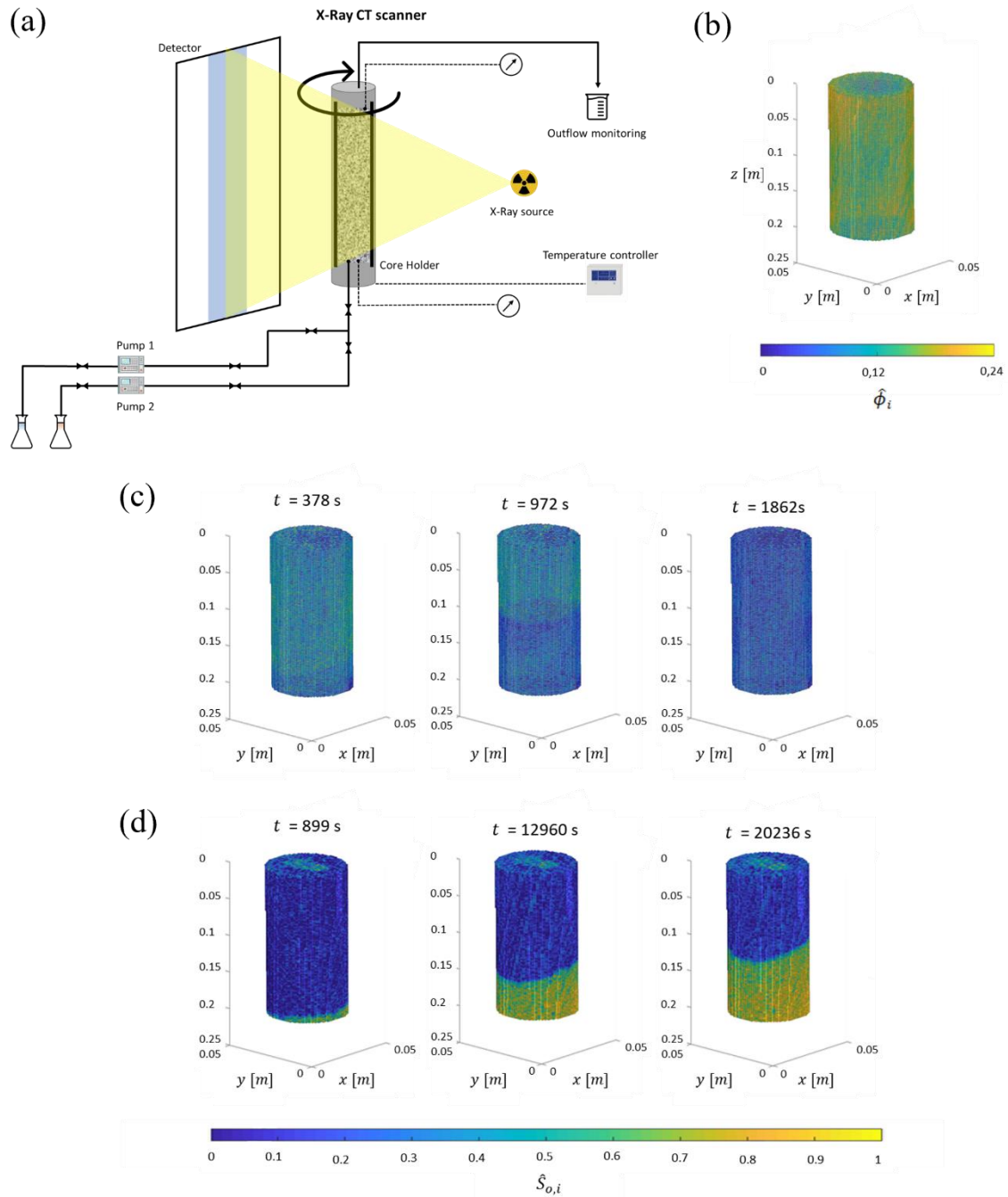
- 703 [50] E. Bianchi Janetti, M. Riva, A. Guadagnini, Effects of Pore-Scale Geometry and
704 Wettability on Two-Phase Relative Permeabilities within Elementary Cells, *Water (Basel)*.
705 9 (2017) 252. <https://doi.org/10.3390/w9040252>.
- 706 [51] E.J. Spiteri, R. Juanes, M.J. Blunt, F.M. Orr, A New Model of Trapping and Relative
707 Permeability Hysteresis for All Wettability Characteristics, *SPE Journal*. 13 (2008) 277–
708 288. <https://doi.org/10.2118/96448-PA>.
- 709 [52] E.J. Spiteri, R. Juanes, M.J. Blunt, F.M. Orr, A new model of trapping and relative
710 permeability hysteresis for all wettability characteristics, *SPE Journal*. 13 (2008) 277–288.
711 <https://doi.org/10.2118/96448-PA>.
- 712 [53] J.M.M. Regtien, G.J.A. Por, M.T. van Stiphout, F.F. van der Vlugt, *Interactive Reservoir*
713 *Simulation*, in: *All Days, SPE, 1995*. <https://doi.org/10.2118/29146-MS>.
- 714 [54] R. Manasipov, B. Jenei, *Automated Interpretation Tool for Synchronous History Matching*
715 *of Multiple Scal Experiments with Advance Nurbs Representations of Relevant Functions*,
716 in: *Day 3 Thu, December 03, 2020, SPE, 2020*. <https://doi.org/10.2118/200559-MS>.
- 717 [55] K.-A. Lie, *An Introduction to Reservoir Simulation Using MATLAB/GNU Octave*,
718 Cambridge University Press, 2019. <https://doi.org/10.1017/9781108591416>.
- 719 [56] A. Manzoni, *Three-dimensional inverse modelling of two-phase flows at the core scale*,
720 *Politecnico di Milano, 2020*.
- 721 [57] R. Storn, K. Price, A Simple and Efficient Heuristic for Global Optimization over
722 Continuous Spaces, *Journal of Global Optimization*. 11 (1997) 341–359.
723 <https://doi.org/10.1023/A:1008202821328>.
- 724 [58] M. Teixeira Parente, S. Mattis, S. Gupta, C. Deusner, B. Wohlmuth, Efficient parameter
725 estimation for a methane hydrate model with active subspaces, *Comput Geosci*. 23 (2019)
726 355–372. <https://doi.org/10.1007/s10596-018-9769-x>.
- 727 [59] D. Erdal, O.A. Cirpka, Technical Note: Improved sampling of behavioral subsurface flow
728 model parameters using active subspaces, *Hydrol Earth Syst Sci*. 24 (2020) 4567–4574.
729 <https://doi.org/10.5194/hess-24-4567-2020>.
- 730 [60] D. Erdal, O.A. Cirpka, Global sensitivity analysis and adaptive stochastic sampling of a
731 subsurface-flow model using active subspaces, *Hydrol Earth Syst Sci*. 23 (2019) 3787–
732 3805. <https://doi.org/10.5194/hess-23-3787-2019>.
- 733 [61] D. Erdal, O.A. Cirpka, Technical Note: Improved sampling of behavioral subsurface flow
734 model parameters using active subspaces, *Hydrol Earth Syst Sci*. 24 (2020) 4567–4574.
735 <https://doi.org/10.5194/hess-24-4567-2020>.
- 736 [62] F. Campolongo, J. Cariboni, A. Saltelli, An effective screening design for sensitivity
737 analysis of large models, *Environmental Modelling & Software*. 22 (2007) 1509–1518.
738 <https://doi.org/10.1016/j.envsoft.2006.10.004>.
- 739 [63] G. Porta, D. la Cecilia, A. Guadagnini, F. Maggi, Implications of uncertain bioreactive
740 parameters on a complex reaction network of atrazine biodegradation in soil, *Adv Water*
741 *Resour*. 121 (2018) 263–276. <https://doi.org/10.1016/j.advwatres.2018.08.002>.

- 742 [64] F. Campolongo, A. Saltelli, J. Cariboni, From screening to quantitative sensitivity analysis.
743 A unified approach, *Comput Phys Commun.* 182 (2011) 978–988.
744 <https://doi.org/10.1016/j.cpc.2010.12.039>.
- 745 [65] H. v. Gupta, S. Razavi, Revisiting the Basis of Sensitivity Analysis for Dynamical Earth
746 System Models, *Water Resour Res.* 54 (2018) 8692–8717.
747 <https://doi.org/10.1029/2018WR022668>.
- 748 [66] R. Brun, P. Reichert, H.R. Künsch, Practical identifiability analysis of large environmental
749 simulation models, *Water Resour Res.* 37 (2001) 1015–1030.
750 <https://doi.org/10.1029/2000WR900350>.
- 751 [67] F. Lomeland, E. Ebeltoft, W.H. Thomas, A New Versatile Relative Permeability
752 Correlation History matching View project Curve fitting flow functions View project Frode
753 Lomeland OREC AS-Consultants Einar Ebeltoft A NEW VERSATILE RELATIVE
754 PERMEABILITY CORRELATION, 2005.
755 <https://www.researchgate.net/publication/242724867>.
- 756 [68] M. Riva, A. Guadagnini, A. Dell’Oca, Probabilistic assessment of seawater intrusion under
757 multiple sources of uncertainty, *Adv Water Resour.* 75 (2015) 93–104.
758 <https://doi.org/10.1016/j.advwatres.2014.11.002>.
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763 **Fig. 1.** Sketch of the proposed workflow: (i) collection of experimental datasets; (ii) preliminary model
764 calibration to assess a plausible combination of parameters (see Sec. 2.3); (iii) Global Sensitivity Analysis
765 (see Sec. 2.4) across a behavioral parameter space to identify influential parameters; (iv) Stochastic model
766 calibration to assess frequency distributions of influential model parameters (see Sec. 2.4).

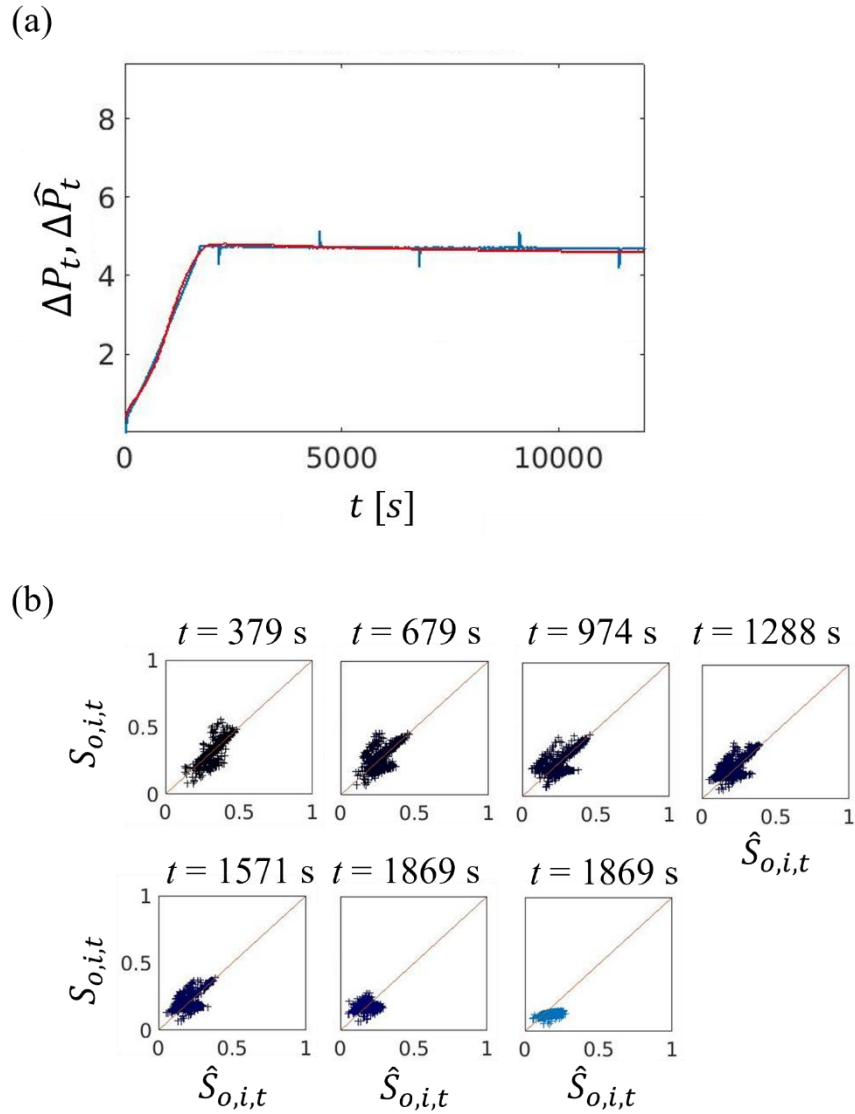


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768 **Fig. 2.** Sketch of the experimental set-up (a); three-dimensional distributions of sample porosity, $\hat{\phi}_i$ (b);
 769 three-dimensional distributions of oil saturation, $\hat{S}_{o,i,t}$, at various acquisition times for the oil- (c) and brine-
 770 displacement experiments. Spatial resolution associated with experimental data is $0.997 \times 0.993 \times 0.996$
 771 mm^3 .

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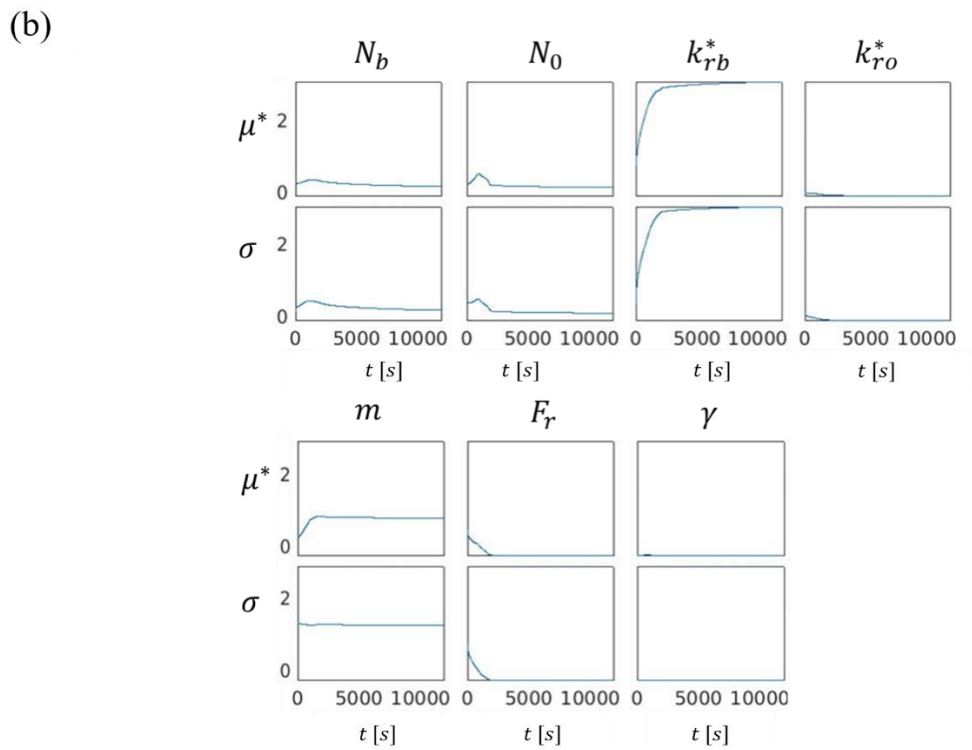
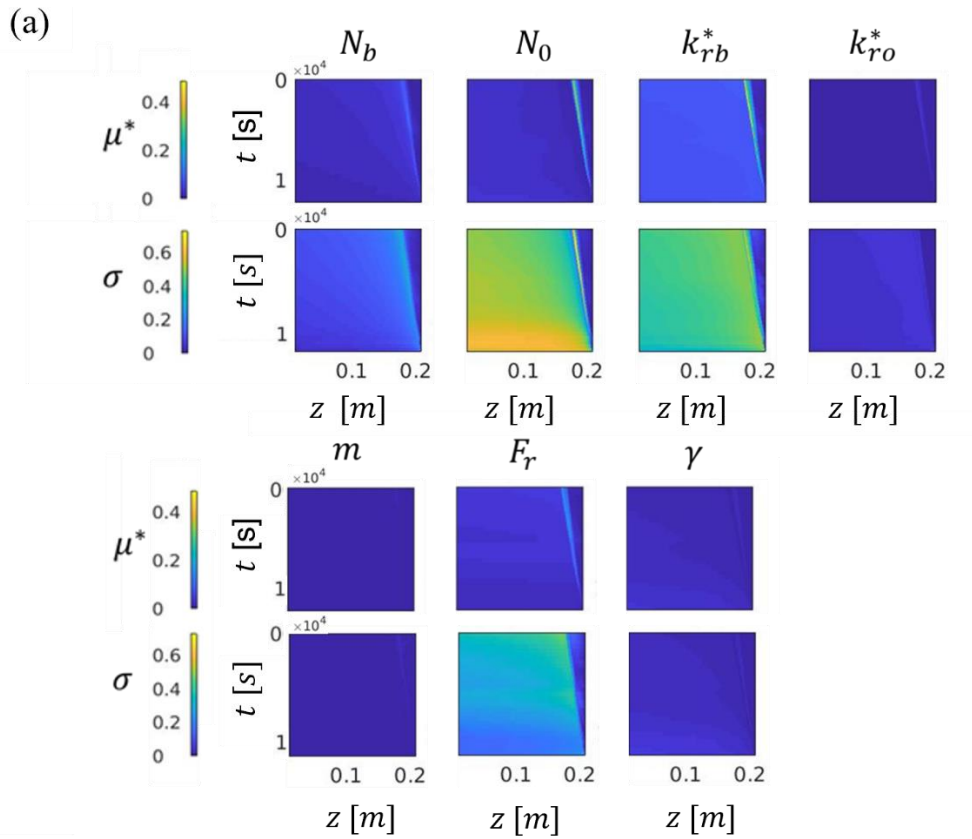
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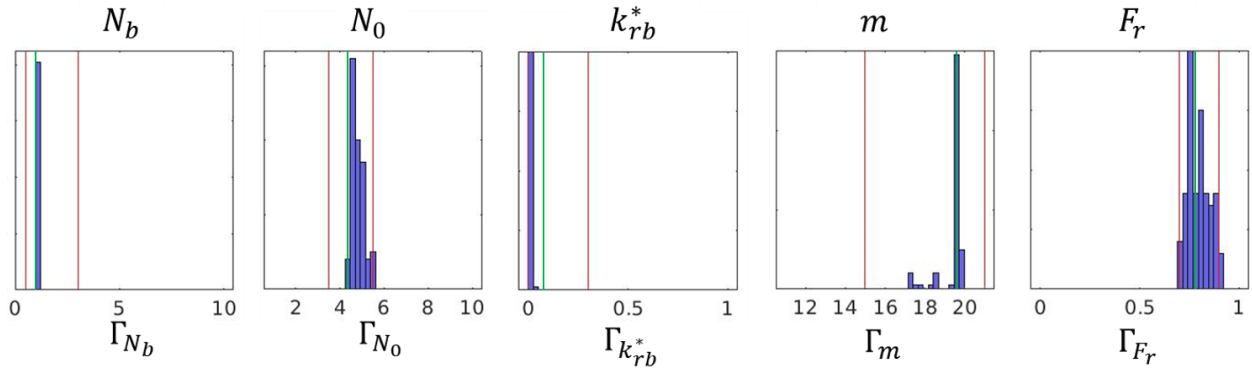
775 **Fig. 3.** Application of the proposed workflow to the oil-displacement scenario. Preliminary model
 776 calibration results: (a) experimental ($\Delta\hat{P}_t$, blue curve) and numerical (ΔP_t , red curve) temporal pattern of
 777 core-scale pressure drop; (b) scatter plots of experimental oil saturations ($\hat{S}_{o,i,t}$) versus their numerical
 778 counterparts ($S_{o,i,t}$, symbols) at available observation times.

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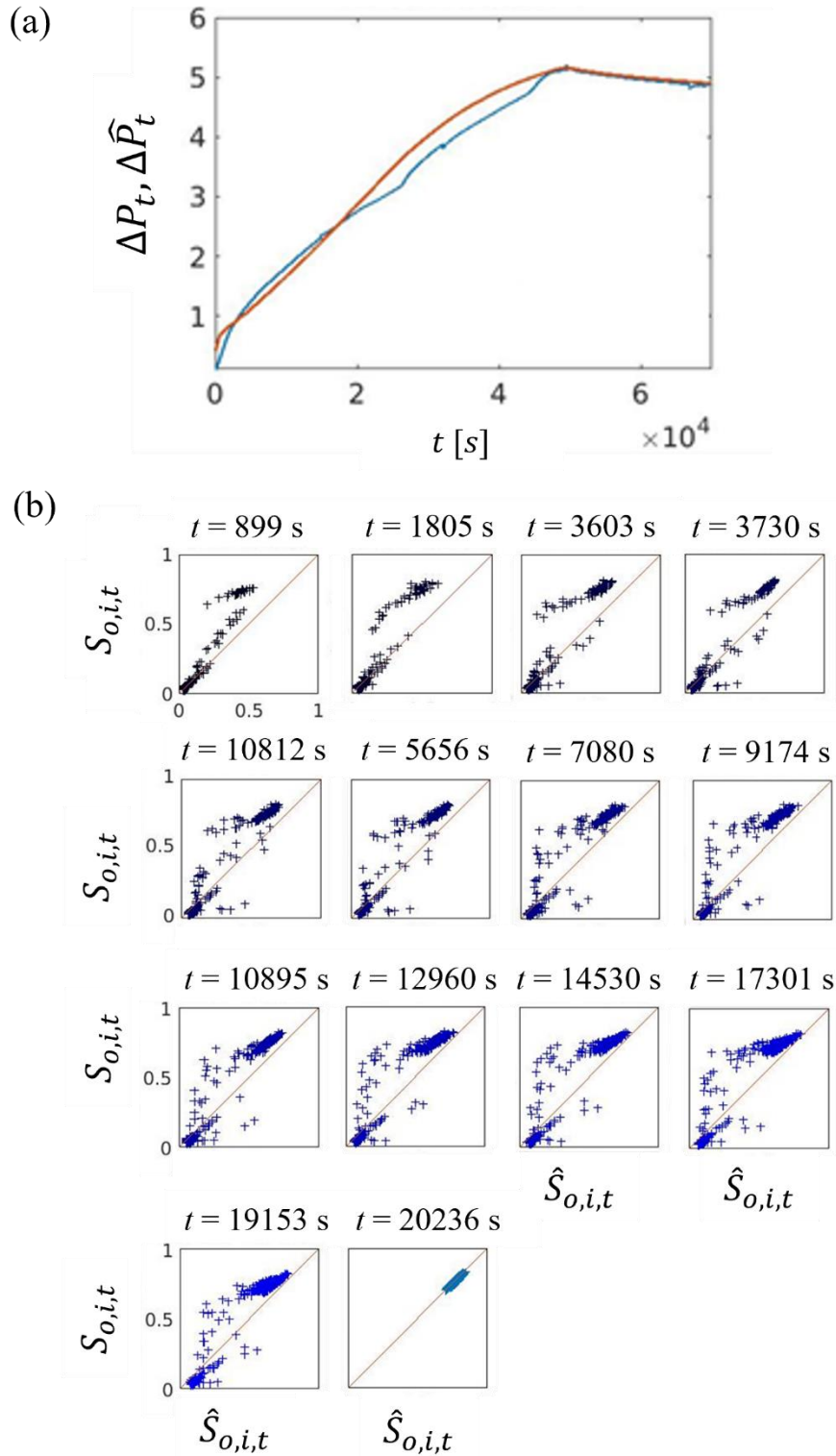
781 **Fig. 4.** Application of the proposed workflow for the oil-displacement scenario. Global Sensitivity Analysis
 782 results: Morris' sensitivity indices (i.e., $\mu_{\theta_j}^*$ (15) and σ_{θ_j} (16)) for (a) (section-averaged) oil saturation and
 783 (b) core-scale pressure drop.



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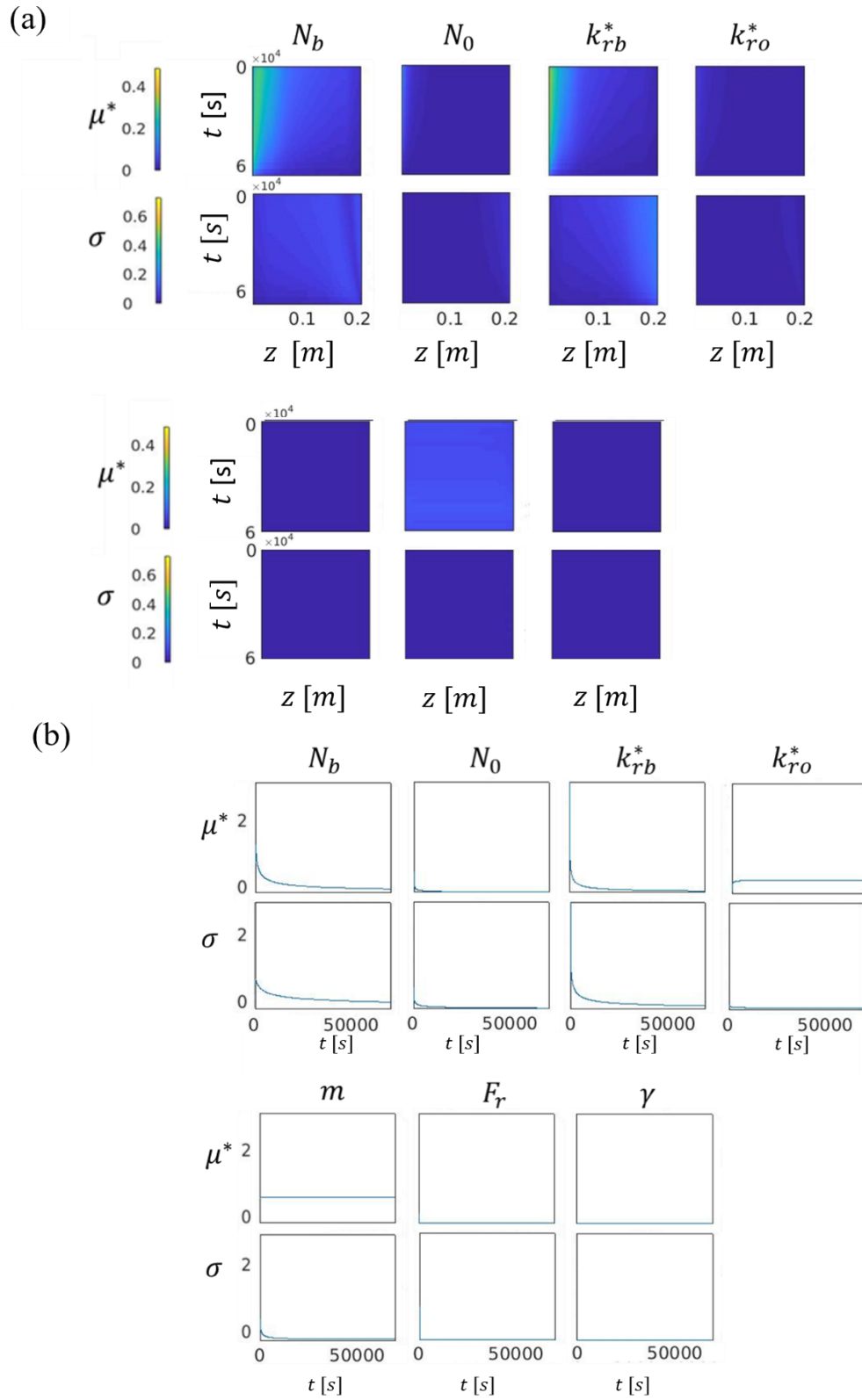
785 **Fig. 5.** Application of the proposed workflow for the oil-displacement scenario. Stochastic model
 786 calibration results: empirical frequency distributions for the set influential parameters (see Table 3). Red
 787 lines indicate the boundaries of the corresponding behavioral range; the green line correspond to the
 788 parameter values obtained through the preliminary model calibration stage.

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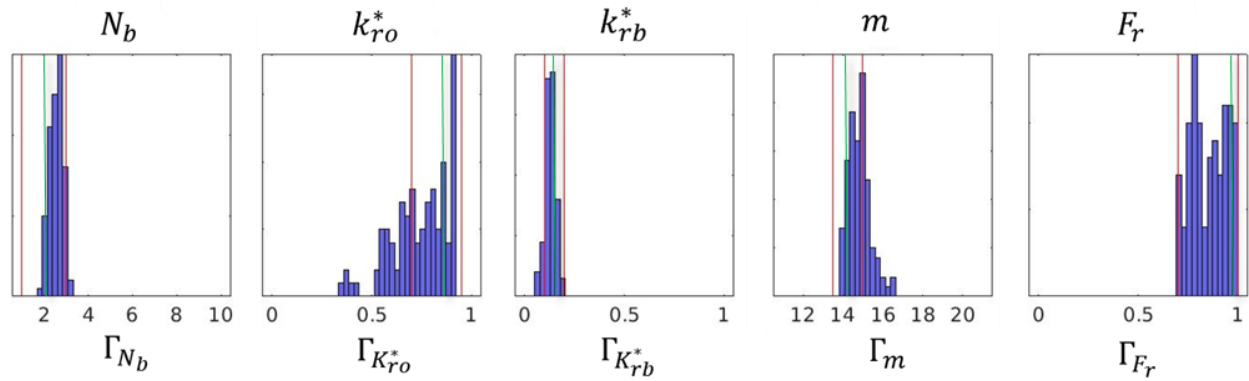
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791 **Fig. 6.** Application of the proposed workflow for the brine-displacement scenario. Preliminary model
 792 calibration results: (a) experimental ($\Delta\hat{P}_t$, blue curve) and numerical (ΔP_t , red curve) temporal pattern of
 793 core-scale pressure drop; (b) scatter plots of experimental oil saturations ($\hat{S}_{o,i,t}$) versus their numerical
 794 counterparts ($S_{o,i,t}$, symbols) at available observation times.



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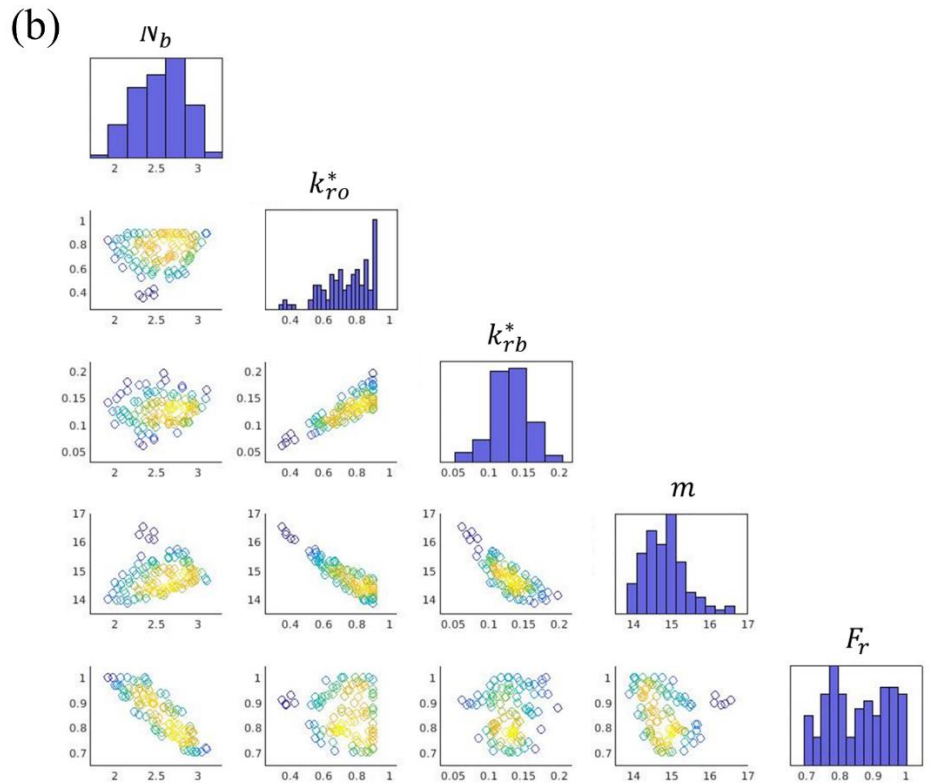
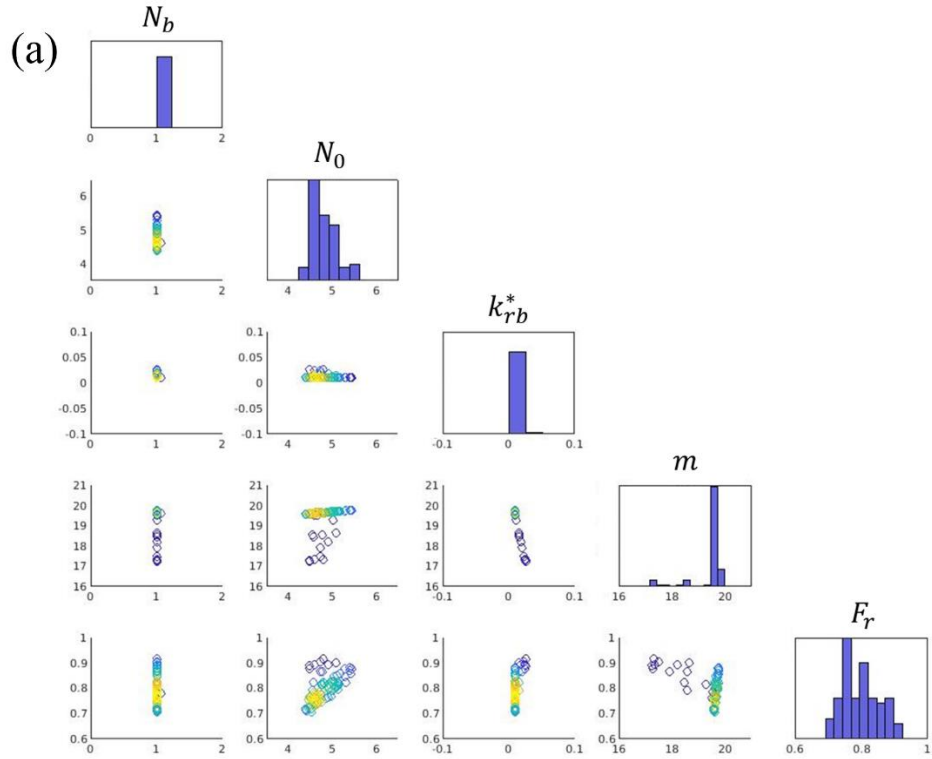
796 **Fig. 7.** Application of the proposed workflow for the brine-displacement scenario. Global Sensitivity
 797 Analysis results: Morris' sensitivity indices (i.e., $\mu_{\theta_j}^*$ (15) and σ_{θ_j} (16)) for (a) (section-averaged) oil
 798 saturation and (b) core-scale pressure drop.



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800 **Fig. 8.** Application of the proposed workflow for the brine-displacement scenario. Stochastic model
 801 calibration results: empirical frequency distributions for the set influential parameters (see Table 4). Red
 802 lines indicate the boundaries of the corresponding behavioral range; the green line correspond to the
 803 parameter values obtained through the preliminary model calibration stage.

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Figure A1. Scatter plot matrix for pairwise sets of parameter values stemming from stochastic model calibration considering the (a) oil and (b) brine displacement scenarios. Color gradation is indicative of the underlying bivariate probability function.

PROPERTY	SYMBOL	VALUE
LENGTH (cm)	l	20.1
DIAMETER (cm)	D	3.8
PORE VOLUME (ml)	PV	38.2
POROSITY (%)	$\langle \hat{\phi} \rangle$	16.7
ABSOLUTE PERMEABILITY (mD) AT BRINE SATURATION $S_B = 1$	K	31.0

810 **Table 1.** Key properties of the Berea sandstone core sample.

811

PROPERTY	SYMBOL	OIL-DISPLACEMENT	BRINE-DISPLACEMENT
		EXP	EXP
OIL DENSITY (g/ml)	ρ_o	0.76	0.86
OIL VISCOSITY (cp)	μ_o	1.41	93.2
BRINE DENSITY (g/ml)	ρ_b	1.13	1.13
BRINE VISCOSITY (cp)	μ_b	1.11	1.11

812 **Table 2.** Key properties of the fluids employed in the oil-displacement and brine-displacement experiments.

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	PRELIMINARY MODEL CALIBRATION RESULTS	PARAMETER RANGE FOR SUPPORTS Γ AND Γ'	PARAMETER RANGE FOR Γ^B
N_b	1.00	[0-10]	[0-2.5]
N_o	4.73	[0-10]	[3.5-5.5]
k_{rb}^*	0.09	[0-1]	[0-0.3]
k_{ro}^*	0.01	[0-1]	[0-0.2]
m	19.61	[10-22]	[15-22]
F_r	0.78	[0-1]	[0.7-0.9]
γ	146.76	[0-180]	[120-160]

815 **Table 3.** Results of the preliminary model calibration stage and ranges of model parameter values
816 associated with the preliminary model calibration (Γ), reduced (Γ' , only bold ranges) and *behavioral* (Γ^B)
817 parameter spaces. Oil-displacement experiment.

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	PRELIMINARY MODEL CALIBRATION RESULTS	PARAMETER RANGE FOR SUPPORTS Γ AND Γ'	PARAMETER RANGE FOR Γ^B
N_b	2.02	[0-10]	[1-3]
N_o	4.15	[0-10]	[3-5]
k_{rb}^*	0.15	[0-1]	[0.1-0.2]
k_{ro}^*	0.85	[0-1]	[0.7-0.95]
m	14.01	[10-22]	[13.5-15]
F_r	0.97	[0-1]	[0.8-1]
γ	95.89	[0-180]	[80-100]

820 **Table 4.** Results of the preliminary model calibration stage and ranges of model parameter values
821 associated with the preliminary model calibration (Γ), reduced (Γ' , only bold ranges) and *behavioral* (Γ^B)
822 parameter spaces. Brine-displacement experiment.

823

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Alberto Guadagnini reports financial support was provided by Eni SpA. Aronne Dell'Oca, Martina Siena, Andrea Manzoni, Alberto Guadagnini has patent #4D-CoreINV software licensed to SIAE.

CRedit author statement:

A. Dell’Oca: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing; **A. Manzoni:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing; **M. Siena:** Conceptualization, Methodology, Software, Writing - Original Draft, Writing - Review & Editing; **N. G. Bona:** Investigation, Data Curation, Writing - Review & Editing, Supervision; **L. Moghadasi:** Investigation, Data Curation, Writing - Review & Editing; **M. Miarelli:** Investigation, Data Curation, Writing - Review & Editing; **D. Renna:** Investigation, Data Curation, Writing - Review & Editing; **A. Guadagnini:** Conceptualization, Methodology, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing, Supervision.