A multi-stage machine learning methodology for health monitoring of largely unobserved structures under varying environmental conditions

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Abstract. Structural Health Monitoring (SHM) via data-driven techniques can be based upon vibrations acquired by sensor networks. However, technical and economic reasons may prevent the deployment of pervasive sensor networks over civil structures, thus limiting their reliability in terms of damage detection. Moreover, the effects of environmental (and operational) variability may lead to false alarms. To address these challenges, a multi-stage machine learning (ML) method is here proposed by exploiting autoregressive (AR) spectra as damage-sensitive features. The proposed method is framed as follows: (i) computing the distances between different sets of the AR spectra via the log-spectral distance (LSD), providing also the training and test datasets; (ii) removing the potential environmental variability by an auto-associative artificial neural network (AANN), to set normalized training and test datasets; (iii) running a statistical analysis via the Mahalanobis-squared distance (MSD) for early damage detection. The effectiveness of the proposed approach is assessed in the case of limited vibration data for the laboratory truss structure known as the Wooden Bridge. Comparative studies show that the AR spectrum is a reliable feature, sensitive to damage even in the presence of a limited number of sensors in the network; additionally, the multi-stage ML methodology succeeds in early detecting damage under environmental variability.

Keywords: Structural Health Monitoring; partially observed systems; environmental variability; AutoRegressive time series; neural networks.

1. Introduction

Infrastructures are becoming more and more important in our modern society, so that it is imperative to protect them from any damage due to aging, material deterioration or unexpected large excitations. Structural health monitoring (SHM) is aimed at assessing the health and safety of civil structures via e.g. vibration measurements [1]. A primary step of SHM is represented by early damage detection [2]; although it might look simple to implement such detection strategies, the accuracy of subsequent SHM steps (namely, damage localization and quantification) strongly depends on this first stage.

For this purpose, model-based and data-driven methods can be resorted. A model-based approach needs an accurate model of the real structure [3-8] and, due to possible discrepancies between model prediction and data acquired on the field, algorithms are to be developed to update the model itself [9-11]. On the contrary, a data-based SHM strategy relies upon raw vibration measurements and statistical pattern recognition strategies [12-15]. This latter method, due to the ever-increasing complexity of real structures, seems more suitable for the implementation of SHM strategies.

On their own, data-based methods consist of feature extraction and statistical analysis. Feature extraction digs into the data to provide damage-sensitive features extracted from the vibration measurements; statistical analysis then exploits such features to take a decision regarding the presence of damage via statistical approaches [1]. For this purpose, a comparison between feature datasets relevant to two different structural states is needed, e.g. in terms of a distance measure [16].

Any vibration-based SHM strategy depends on a stream of data coming from the sensors deployed over the structures [17-23]. The effectiveness and reliability of the entire SHM system depend on the sensitivity of the extracted features to damage, as usually obtained thanks to dense sensor networks whose deployment must be optimized [24]. Most of the civil structures in need of an SHM are usually complex and large-scale, so that the installation of a large number of sensors may not be affordable. In such cases, the SHM procedure must be enhanced in order to obtain the maximum possible information from the limited number of sensors [25-27]. This important issue becomes even more difficult to deal with in the presence of environmental and/or operational variability. The said variability may introduce deceptive effects, which look very similar to those induced by a structural damage [12, 28-30]: false alarms and erroneous detections thus become major challenges for the SHM system.

To account for the aforementioned challenges, a parametric spectral-based feature extraction approach is here proposed in conjunction with a multi-stage ML methodology for early damage detection. The feature extraction method first exploits an AR representation of the measured vibration responses, and then estimates their spectra by the Burg's method. The proposed three-stage ML methodology next consists of: computing the distances between different sets of the AR spectra, via the LSD; removing the environmental variability by an AANN; computing the MSD for early damage detection. The proposed method is shown to be able to deal with the limited information regarding the structural behavior, as provided by a small number of sensors, accurately detecting damage of different severity. The effectiveness of the proposed approach is validated against the (limited) vibration data related to the laboratory truss structure known as the Wooden Bridge. Results here collected prove that the AR spectrum is a reliable damage-sensitive feature, even in the case of a largely unobserved structure; furthermore, the multi-stage ML method results successful in detecting early damage in the presence of an environmental variability.

2. Feature extraction by AR modeling

The present analysis is based on the estimation of the PSD of a signal from its representation in the time-domain, see e.g. [31, 32]. The spectral analysis can be carried out by means of nonparametric and parametric methods: the latter ones are model-based and are able to account for a prior knowledge of the signal to get accurate spectral estimates. Starting from the model and relevant tuned parameters, the algorithm provides the corresponding power density spectrum.

The most commonly used (linear) model for this purpose is the AR representation. Given the vibration signal $y(t)$, the representation reads:

$$y(t)+\theta_1 y(t-1)+\cdots+\theta_p y(t-p)=r(t) \tag{1}$$

where: $r(t)$ is the residual error; p is the order of the AR representation; $\theta_1 \ldots \theta_p$ are the model coefficients. To set the order p, the iterative methodology of Entezami and Shariatmadar [33] and based on the Ljung-Box Q-test has been here adopted.

Using the model order and coefficients, the AR spectrum has been obtained with the Burg's method, which is based on the minimization of the forward and backward prediction errors while satisfying the Levinson-Durbin recursion [32]. The AR spectrum $P(\omega)$ is thus computed as:

$$P(\omega)=\frac{\sigma_r^2}{\left|1-\sum_{\mathrm{k}=1}^{p}\theta_{\mathrm{k}}\mathrm{e}^{-\mathrm{j}\omega\mathrm{k}}\right|^2} \tag{2}$$

where σ_r^2 denotes the variance of the model residuals.

3. Multi-stage machine learning methodology

3.1. Stage I: LSD

Multivariate training and test datasets are first obtained through the AR spectra, estimated in relation to the normal and current states. The (dis)similarity between those spectra is then computed via the LSD, which is a symmetric distance measure [34]. Given the two discrete spectra $P(\omega)$ and $\bar{P}(\omega)$, the LSD is given by:

$$LSD=\sqrt{\frac{1}{n_p}\sum_{i=1}^{n_p}\left|\log\bar{P}(i)-\log P(i)\right|^2}=\sqrt{\frac{1}{n_p}\sum_{i=1}^{n_p}\left|\log\frac{\bar{P}(i)}{P(i)}\right|^2} \tag{3}$$

n_p being the number of spectrum samples. Any difference between the two spectra, here assumed to be related with the baseline and to the current state of the structure at each sensor location, leads to an LSD value larger than zero. A deviation of $\bar{P}(\omega)$ from $P(\omega)$ is considered indicative of damage inception.

If the structure of interest is monitored with n_s sensors, and tests are repeated n_m times, we use $S_1 \ldots S_c$ to denote the n_c normal (undamaged) conditions in the baseline phase. The training dataset is thus given by $\mathbf{X}\in\mathbb{R}^{n_x\times n_s}$, where $n_x=n_m\times(n_c\text{-}1)$: each column of this matrix is the LSD value relevant to two different normal conditions at the same sensor location. For the current structural condition S_u in the monitoring phase, the LSD values are computed between the spectra of the normal and the said current states, at the same sensor location: hence, the test matrix is given by $\mathbf{Z}\in\mathbb{R}^{n_z\times n_s}$, where $n_z=n_m\times n_c$.

3.2. Stage II: AANN

A feed-forward AANN is next used to remove the possible effects of the environmental variability from the formerly computed distance values. The adopted AANN consists of three hidden layers, being respectively the mapping, bottleneck, and de-mapping ones. With the adopted AANN architecture, the information is moved in the forward direction only, from the input to the output layers. The AANN provides a filtered representation of the input data, to get rid of variations due to the environmental variability. To set the hyperparameters of the AANN, the procedure has been performed in a least-squares sense via the mean-squared-error (MSE) between input and output, see alsoKramer [35] in relation to possible overfitting issues.

Dealing with the distance values computed in the first stage for all the sensor placements and for all the test measurements, the AANN is initially trained to learn the correlations among the data in **X**. Once trained for the normal conditions, the network output is given as $\bar{\mathbf{X}}$, and the residual $\mathbf{E}_x=\mathbf{X}-\bar{\mathbf{X}}$ is used as the feature for damage detection. In the monitoring stage it is not necessary to re-train the AANN: by removing the environmental variability from **Z**, the output $\bar{\mathbf{Z}}$ is obtained and the residual $\mathbf{E}_z=\mathbf{Z}-\bar{\mathbf{Z}}$ is adopted as the feature for the monitoring stage.

3.3. Stage III: MSD

The MSD is a multivariate statistical distance to quantify the (dis)similarity between two datasets, and it is here adopted for novelty detection [36]. To take a decision about the current state, the residual $\mathbf{E}_x$ in the baseline phase is used and the relevant mean vector $\mathbf{m}_x\in\mathbb{R}^{n_s}$ and covariance matrix $\mathbf{S}_x\in\mathbb{R}^{n_s\times n_s}$ are computed. Next, each vector of $\mathbf{E}_z$ is adopted to compute the distances d_{M_i} in the following way:

$$d_{M_i}=\left(\mathbf{e}_{z_i}-\mathbf{m}_x\right)^{\mathrm{T}}\mathbf{S}_x^{-1}\left(\mathbf{e}_{z_i}-\mathbf{m}_x\right), \tag{4}$$

where $i=1,2,\dots,n_z$ and $\mathbf{e}_{\mathbf{z}_i}$ is the i^{th} vector of $\mathbf{E}_z$.

Finally, for damage detection each d_{M_i} value must be compared with a threshold limit: if the current state is undamaged, the corresponding MSD value is smaller than the threshold; if the MSD value is instead larger than the threshold, the current state is targeted as a damaged one. The threshold estimation is based on the MSD values relevant to the training data [37]: each feature vector of $\mathbf{E}_x$ is used as in Eq. (4) by replacing $\mathbf{e}_z$, and by using a standard confidence interval with a significance level the threshold is determined by the mean of the d_{M_i} entries and by the relevant standard deviation, see [36].

4. A case study: The Wooden Bridge

We refer now to the experimental tests relevant to the Wooden Bridge [38]. Such structure was equipped with 15 accelerometers, and randomly excited by an electro-dynamic shaker. The acquired measurements were supposed to represent both undamaged and damaged cases, with uncertainties related to the environment [12].

Details related to the test measurements are briefly reported in Table 1. As damage was represented by an added mass close to Sensor #4, the values reported in the table are to be considered proportional to the damage itself. The number of test measurements n_t=20 was the same for the undamaged and damaged states. As customarily done in ML procedures, undamaged states HC1, HC2 have been adopted to define the baseline, so n_c=2; states HC3 and DC1-5 have been instead used in the monitoring stage.

Table 1. Structural states of the Wooden Bridge

Condition	*Label*	*Damage stage (g)*	*Phase*
Undamaged	HC1	-	Baseline
Undamaged	HC2	-	
Undamaged	HC3	-	Monitoring
Damaged	DC1	23.5	
	DC2	47.0	
	DC3	70.5	
	DC4	123.2	
	DC5	193.7	

The AR model order has been first defined at each sensor location and for each test measurement, on the basis of the iterative approach proposed in [33] and applied to the HC1 and HC2 datasets; the average orders at all sensors have been then used in the monitoring stage. The AR spectrum at each sensor location has been then estimated by means of the Burg method: Fig. 1 shows some exemplary results related to Sensor #4. The main purpose of this comparison is to assess the sensitivity to damage of these features, keeping in mind that states DC1 and DC5 respectively feature the smallest and largest damage severity. Fig. 1(a) shows remarkable differences in the AR spectra for some frequencies; it is instead hard to detect an effect of damage on the AR coefficients. While the AR spectrum proves better than alternate structural features to infer a change in the structural health, it is difficult to detect damage through a direct comparison of the AR spectra. An approach based on this feature only, may not be efficient for cases characterized by a number of sensors and test measurements to handle.

To assess the capability of the proposed procedure to detect damage by means of a limited observability of the structural response, 4 different scenarios have been defined according to Table 2. For each of them, the AR spectra of the sensors there listed have been accounted for by the proposed multi-stage ML methodology. In the first stage of the methodology, the training datasets have been respectively denoted as $\mathbf{X}_1\in\mathbb{R}^{20\times7}$, $\mathbf{X}_2\in\mathbb{R}^{20\times5}$, $\mathbf{X}_3\in\mathbb{R}^{20\times3}$, and $\mathbf{X}_4\in\mathbb{R}^{20\times1}$; the relevant distance calculation has been implemented to compute the LSD values of the AR spectra of HC2 with respect to the corresponding spectra of HC1. Next, the distance calculation has been run to measure the LSD values of the AR spectra regarding each of the current states and the corresponding spectra associated with the baseline: the resulting test datasets in the four scenarios have been denoted as $\mathbf{Z}_1\in\mathbb{R}^{240\times7}$, $\mathbf{Z}_2\in\mathbb{R}^{240\times5}$, $\mathbf{Z}_3\in\mathbb{R}^{240\times3}$, and $\mathbf{Z}_4\in\mathbb{R}^{240\times1}$.

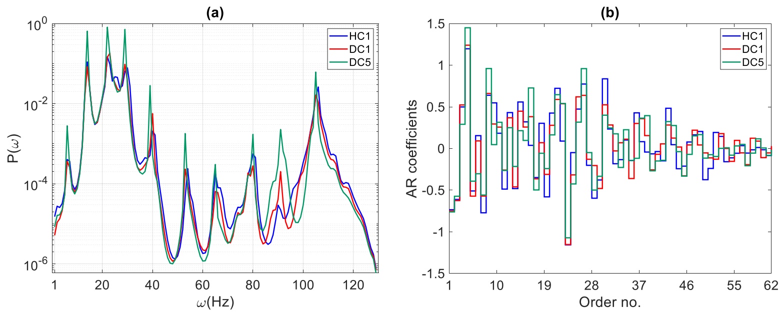

Fig. 1. Comparison among the (a) AR spectra and (b) AR coefficients relevant to states HC1, DC1 and DC5, at Sensor #4 location.

Table 2. Considered sensor deployment scenarios

Sensor deployment scenario	*Sensor labels*
1	2, 4, 6, 7, 9, 11, 15
2	1, 3, 5, 12,14
3	4, 8, 13
4	14

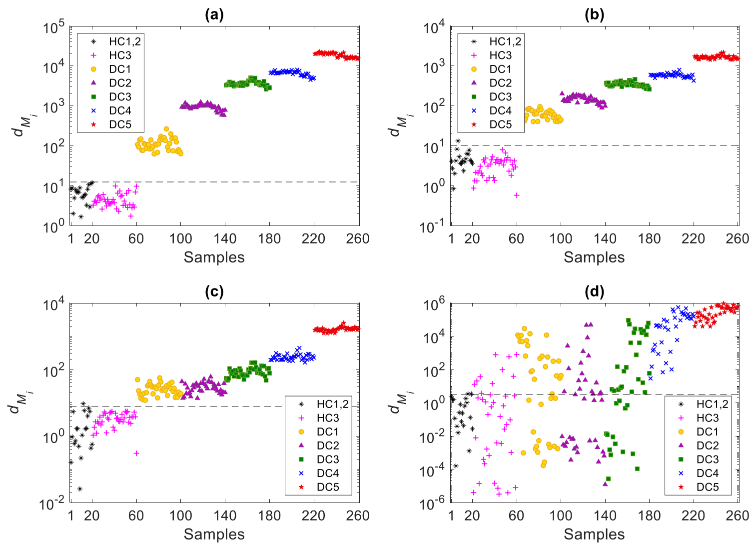

Fig. 2. Damage detection by the multi-stage ML methodology and the AR spectrum, using the statistical distance measure d_{M_i}: (a) scenario 1, (b) scenario 2, (c) scenario 3, (d) scenario 4

In the second stage, the AANN has been used to remove the environmental effects. The number of neurons in the hidden layers has been determined with a cross-validation technique. The optimal number of neurons of the mapping and de-mapping layers and of the bottleneck has been set for all the scenarios. It is to be remarked that the bottleneck layer has turned out to feature only 2-3 neurons for all the considered scenarios, which looks remarkable to make the AANN representation parsimonious enough.

Using the trained AANN, the normalized training and test matrices $\mathbf{E_x}$ and $\mathbf{E_z}$ have ben next adopted for early damage detection. In Fig. 2, results are reported in terms of the d_{M_i} values for the different sensor deployment scenarios; in the graphs, the dashed horizontal lines represent the threshold limit related to the 95% confidence interval of the MSD values from training. Fig. 2(a)-(b), relevant to the two denser sensor networks, clearly show that states DC1-DC5 are correctly detected as damaged, as the d_{M_i} values all exceed the threshold. Additionally, the values are split appropriately so that the different states are characterized by MSD values proportional to the damage severity. The other way around, the d_{M_i} values relevant to the undamaged state HC3 look similar to those corresponding to states HC1 and HC2. Similar conclusions are drawn by Fig. 2(c), even if the classification relevant to small damage severity can result hard. The other way around, Fig. 2(d) shows an inaccurate damage detection in the fourth scenario, for all the states but DC4 and DC5. Hence, one sensor only may not be effective for damage detection in the entire structure.

To assess the beneficial effects of the AANN to remove the environmental effects, results have been also obtained without plugging in the corresponding stage of the proposed procedure. Fig. 3 shows the obtained results, as before for all the scenarios: graphs in Fig. 3(a)-(c) report dissimilarities in the d_{M_i} values, which are specifically highlighted in Fig. 3(b). Even if a still good detection capability is reported for states DC1-DC5, the values relevant to state HC3 partially exceed the threshold limit and lead to false alarms. Results in Fig. 3(d) are instead considered not acceptable, due to the number of fluctuations around the threshold limit.

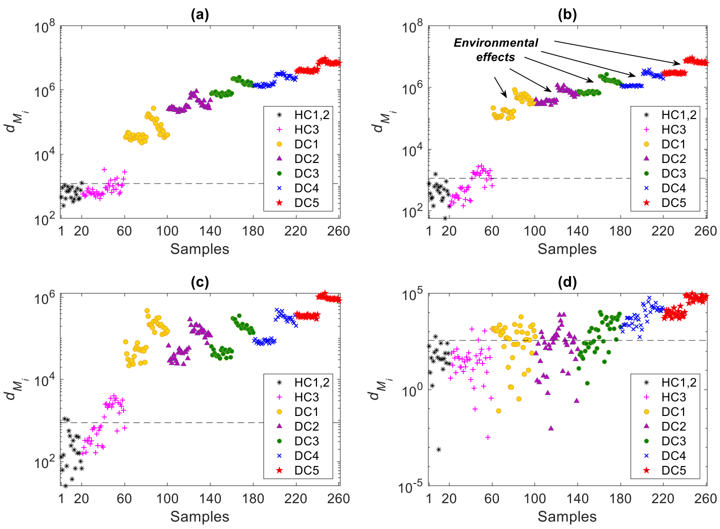

Fig. 3. Damage detection by the multi-stage ML methodology and the AR spectrum, avoiding to exploit the capability of the AANN to remove the environmental variability: (a) scenario 1, (b) scenario 2, (c) scenario 3, (d) scenario 4

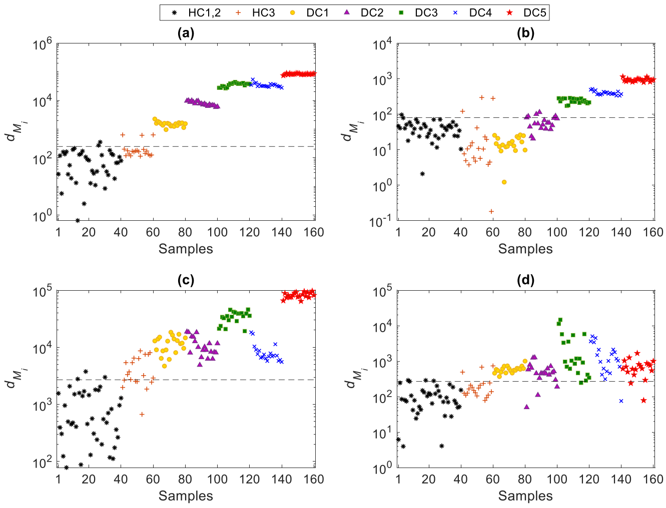

Fig. 4. Damage detection by the conventional MSD technique to handle the AR coefficients: (a) scenario 1, (b) scenario 2, (c) scenario 3, (d) scenario 4

A comparative analysis has been finally carried out by adopting the conventional MSD technique to handle the AR coefficients, all collected to build the training and test matrices. The Burg's method has then been used to estimate the model coefficients. Fig. 4 shows the relevant results, in a case characterized by 20 MSD values. The plot relevant to scenario 1 shows that there exist some false alarms, especially for state HC3. The results concerning scenarios 2, 3 and 4 prove instead that the traditional MSD technique fails in accurately detecting damage, due to number of false alarms. The proposed method thus outperforms the traditional MSD one in handling the AR coefficients.

5. Conclusions

In this paper, a methodology has been proposed to deal with damage detection in largely unobserved structures, accounting also for possible environmental effects. A parametric spectral-based feature extraction approach based on AR modeling has been adopted to estimate the AR spectrum, that has been shown to be a reliable damage-sensitive feature of the monitored structure. A multi-stage ML methodology has been also proposed to assemble the dataset by the LSD (stage 1), remove the environmental variability by an AANN (stage 2), and detect early damage by the MSD (stage 3). The experimental data related to the so-called Wooden Bridge have been finally exploited to assess the performance of the offered method.

The AR spectrum has shown a noteworthy potential to detect damage in case of limited sensor deployments. As expected, robust and reliable results are not always guaranteed in such cases. Furthermore, the AANN enhances the performance of the proposed methodology by removing the effects of environmental variability. The method has been shown to outperform a traditional MSD technique handling the AR coefficients.

In future activities, the proposed approach will be further developed to deal with structures subjected to ambient vibrations. Furthermore, still within the frame of limited sensor deployments, the optimal configuration of the network and the corresponding damage detection capability will be evaluated.

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