

Modeling of the Vertical Dynamics of a Kick e-Scooter on distributed road irregularity

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Abstract. Vehicle dynamics modelling is a widely used tool to run the design phase without the need of expensive prototypes. Models allow to understand the dynamic properties of the system and run sensitivity analysis faster, cheaper, and easier way with respect to physical testing. Since kick e-scooters are a quite new class of vehicle, generally accepted mathematical models are still not available. In this paper a lumped parameters model for the vertical dynamics simulation of an electric kick scooter is presented. The mechanical impedance of the driver is accounted in the model since it deeply influences the dynamical properties of the whole system. For model validation purposes, acceleration results from computer simulations are compared with those acquired in an experimental campaign on distributed road irregularity. Dealing with a random input, power spectral densities of vehicle acceleration at most interesting points are used to perform the comparison, thus validating the model. The model proposed in this paper can thus be an important tool for the vehicle design stages, allowing to estimate road holding capabilities and the overall driver's comfort while riding kick e-scooter.

Keywords: Electric kick Scooter, e-scooter, Light Vehicles, Vehicle Modelling, Vehicle Dynamics, Comfort, Road Holding.

1 Introduction

Computer simulations are widely used in all engineering fields to fast up the design and to avoid risks and costs of experimental tests. When talking about mechanical systems and vehicles, both full-scale multibody models and lumped parameter ones are used. Recently computer simulations have been introduced in the motorcycle industry [1–3], where the task is much more complex than on cars since the vehicle itself suffers from many sources of instability and it is difficult to simulate open loop maneuvers, meaning the need for a virtual rider [4].

Regarding e-scooters, the simulation approaches that have been developed both for cars and motorcycles are not suitable and require significant modifications. In fact, for e-scooters the driver represents almost 90% of the total mass of the system and its dynamic properties significantly influence the overall dynamic behavior of the vehicle. This has been proven for bicycles, that show a driver/vehicle mass ratio like that of e-scooters. In fact, in bicycles the rider posture influences its comfort [5] and that the modal properties of the system with or without the rider change significantly [6]. Thus a suitable human body model must be adopted.

The human body has been modelled in literature following different approaches through the years. When focusing on people motion, complete multibody models have been developed [7–9]. A completely different approach is used when inspecting the interaction between the human and the structures surrounding him. In this field lumped parameter models are used, with their parameters that generally do not correspond to any physical quantity of the human body [10–15]. Moreover, in many applications, it is also needed to consider that the properties of the human body change as function of posture, vibration magnitude and characteristics of each single subject [16–18].

Being the tire the only element in contact with the ground, tire modelling is also a fundamental part of the dynamic simulation of vehicles. Through the years many models have been built for car tires and then some have recently been extended to motorcycle applications [19,20].

This paper has the scope to study the dynamic behavior of e-scooters to provide hints about their design. The paper proposes a model for the study of the vertical dynamic behavior of e-scooters. It accounts for the mechanical impedance of the rider, providing an important tool to estimate the overall rider's comfort and vehicle road holding capabilities. The model has been tuned and validated through the comparison of simulation and experimental results, having the vehicle running over lumped obstacles [21]. Then the model is tested in a simulation environment giving as input a general road irregularity and the results are directly compared to the experimentally acquired accelerations. The paper thus provides an important tool that can be used to simulate the vehicle vertical response and to estimate rider's comfort while running on several uneven road surfaces.

The paper is organized as follows. Road input modelling and e-scooter modelling are introduced at first. Then the full model validation on general road irregularity is performed by means of experimental tests.

2 Road Modelling

The definition of the input coming from the road surface is of fundamental importance to effectively simulate the behavior of the e-scooter in common operating conditions. As it will be better detailed in the following section, the selected tire model is a single contact point model, requiring road inputs to be introduced in the simulation as imposed displacement to front and rear wheels ground contact points. This kind of model is extensively used since it is simple, and it results in good approximation of the tire forces when running over smooth road profiles characterized by wavelengths greater than the contact patch length. This is the case for this article, in which road irregularity is considered and can be given directly as input to the e-scooter model.

The road irregularity is modelled according to ISO 8608 standard [22] that defines 8 classes of road roughness profiles for simulation purposes basing on the Power Spectral Density (PSD) of the road irregularity. Calling n the spatial frequency

$$n = 1/\lambda \quad (1)$$

where λ is the spatial wavelength in meters, the PSD reads:

$$G_d(n) = G_d(n_0) \left(\frac{n}{n_0} \right)^{-w} \quad (2)$$

where n_0 is the fundamental spatial frequency, and w the exponent of fitted PSD of the road irregularity.

For simulation purposes the road irregularity is quantitatively described through the geometric mean value of the displacement PSD $G_d(n_0)$ at the reference spatial frequency n_0 , that can be selected from the ISO 8608 standard. Then, starting from the road irregularity PSD $G_d(n)$ of a given road class, it is simple to obtain the road irregularity spectrum $S_d(n)$:

$$S_d(n) = \sqrt{2 \Delta n G_d(n)} \quad (3)$$

Since the road irregularity can be considered as a random function, it is possible to obtain the road profile $y(x)$ starting from the spectrum amplitude information and adding a random phase $\varphi(n)$.

$$y(x) = \sum_n |S_d(n)| \cos(2\pi n x + \varphi(n)) \quad (4)$$

Once the road class is selected among the ones provided by the standard ISO 8608, the road profile is automatically generated and given as input to the e-scooter simulation model.

3 E-scooter Modelling

A lumped parameters model is defined to study the e-scooter vertical dynamics, being this directly related to driver comfort and vehicle road holding. This approach is widely used in literature for other vehicles since it allows to get meaningful results with a reduced model complexity. The model used in this work and shown in **Figure 1** has been developed in [21] and it is constituted by rider and tires subsystems assembled on a rigid e-scooter frame.

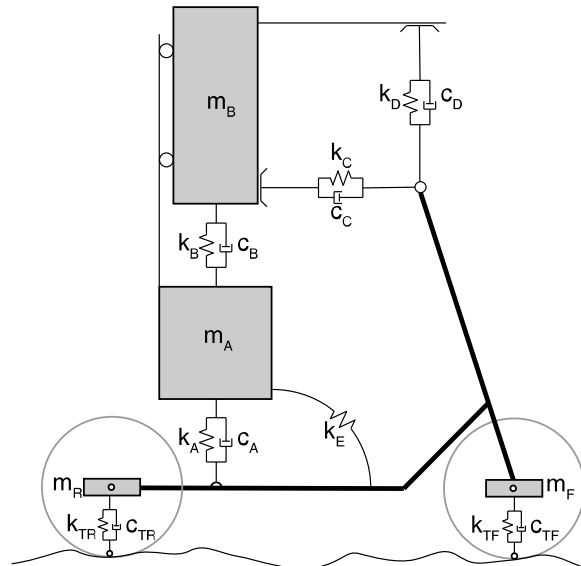


Figure 1. E-scooter lumped parameter model overview

3.1 Tire Model

Tires are represented by means of a single contact point model. Stiffness and damping properties of the tire are reproduced using a spring-damper element that accounts for the tire radial stiffness and damping. The tire static radial stiffness is obtained experimentally by measuring the vertical load at the tire contact patch and the vertical displacement of the wheel hub. Experimental data, as from literature [23], are interpolated by means of a second order polynomial function that relates the hub force F_r to the radial deflection δ_r :

$$F_r = a_1 \delta_r + a_2 \delta_r^2 \quad (1)$$

From previous expression, the tire radial stiffness k_r is obtained by analytically deriving the force expression with respect to the deflection:

$$k_r = a_1 + 2 a_2 \delta_r \quad (2)$$

Figure 2 shows experimental results and fitting curve for the tire inflation pressure of 2.5 bar while **Table 1** reports the fitting coefficients for all the tested inflation pressure values. The radial damping of the tire, instead, is taken from literature for bicycle tires whose shape and dimension are like the ones of those mounted on e-scooters.

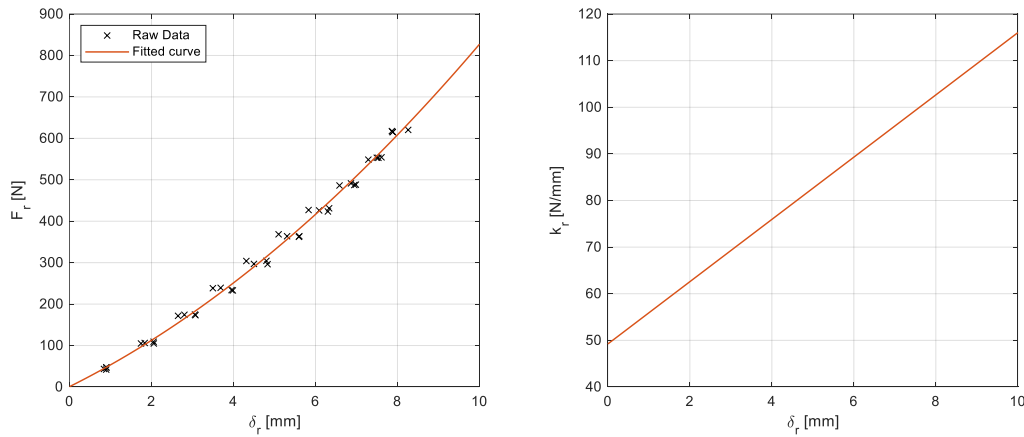


Figure 2. Tire radial characteristics. Left: Tire radial force as function of tire radial deflection. Right: Tire radial stiffness as function of tire radial deflection

Table 1. Tire radial force VS radial deflection fitting parameters

Inflation pressure [bar]	a_1 [N/mm]	a_2 [N/mm ²]
1.5	15.8	4.16
2.0	35.4	3.37
2.5	49.2	3.34

3.2 Rider Model

Rider modelling is carried out considering that its position over the deck is fixed and that there is no feedback from him. This approach is reasonable for vertical dynamics simulations in a straight running condition, where the rider does not move his feet over the deck, and he is not acting on the handle bars to change direction or to correct the trajectory. These assumptions are the ones allowing to use a standard 2 DOFs impedance model for the standing human body exposed to vertical whole-body vibrations [12]. Since the rider is also pitching relatively to the vehicle frame, a rotational elastic element is introduced between the e-scooter deck and the rider lower mass to account for the ankle stiffness, that is retrieved from [24].

The human body parameters for the normal standing posture in [12] have been considered as a starting guess and then have been tuned in [21] by mean of an experimental campaign, with the numerical results that are reported in **Table 2**. These rider body model parameters have been normalized according to the rider own weight to be applicable independently of the mass (M) of the driver itself.

Table 2. Driver body model characteristic parameters

Parameter	Value
m_A [kg]	$0.554 * M$
m_B [kg]	$0.456 * M$
k_A [N/m]	$4.39e3 * M$
k_B [N/m]	$5.53e2 * M$
c_A [Ns/m]	$3.99 * M$
c_B [Ns/m]	$4.37 * M$

Regarding the effects of the human hand-arm system on the handlebar, the model is derived from the 2 DOFs mechanical impedance model (Annex B – Model 1) from [25] and properly modified to fulfill the requirements for this kind of application, as better detailed in [21]. The resulting parameters for the rider hand-arm system model are reported in **Table 3**.

Table 3. Driver hand-arm system model characteristic parameters

Parameter	Value
k_C [N/m]	800
k_D [N/m]	2000
c_C [Ns/m]	40
c_D [Ns/m]	125

4 Model Validation

The model has been tuned in [21] considering its passage over a lumped obstacle, but it is important to validate it by also testing the model performances when running over general road irregularity. Thus, an experimental campaign is run on a quite smooth road. For this purpose an e-scooter without suspensions has been equipped with 6 mono-axial accelerometers according to the scheme of **Figure 3**, where both the location and the measuring direction are reported.



Figure 3. Experimental setup: location of accelerometers for the acquisition of accelerations when passing over a lumped obstacle

Because of the way in which the general road irregularity is introduced in the simulations, it is not possible to directly compare the results in terms of time histories. Moreover, the used road has not been classified according to ISO 8608 standard nor its PSD has been measured. Nevertheless, it is possible to notice that the shape of the PSD is the same for every road class and so it is possible to analyse the results in terms of PSDs, being aware that the shape should be the same, while the amplitude can be different depending on the effective road class given as input.

Keeping fixed the driver, different conditions in terms of tires inflation pressure and running speed have been simulated, all on an A class road. As an example, **Figure 4** shows the vertical acceleration response at most relevant

points of the e-scooter, i.e. the wheel hubs, the deck center, and the handlebar, when running with tires inflation pressure of 2.5 bar and a forward speed of approximately 23 km/h. Experimental results in solid lines are compared to simulation results in dashed lines. Although the model has not been specifically tuned for that task, it is able to provide a good agreement between the experimentally acquired signals and the simulated ones. For all the considered sensible points the shape is correctly reproduced, with a deviation in modulus at low frequencies which may be due to a different input coming from the real and the simulated road. Moreover, at frequencies above 60 Hz the simulated PSDs strongly deviate from the measured ones and this is due to the fact that the minimum road wavelength which can be provided to the simulation environment without applying any filtering policy has been set to 0.1 m. For all the reported results the wheelbase filtering effect generating a ripple over the main PSD profile can be appreciated and correctly reproduced in the simulations. This phenomenon is due to the fact that front and rear wheels are subject to the same input but phased by a certain quantity, that is in charge of generating the ripple having a characteristic frequency which depends on the vehicle wheelbase and on the running speed as follows

$$f_{wb, filt} = \frac{v}{p} \quad (3)$$

where v is the e-scooter forward running speed and p is the wheelbase. In the case presented in **Figure 4**, this wheelbase filtering happens at a frequency of about 8 Hz. The resonance peak for the front wheel is set at a about 21 Hz, while for the rear wheel it is at a about 50 Hz. Additionally, one can notice the fact that deck centre accelerations are more influenced by the rear wheel, while the handlebar centre accelerations are more influenced by the front wheel. These last two quantities are important in practical applications since they give the acceleration at the vehicle interface points with the driver, that are a direct indication of the comfort level perceived by the driver.

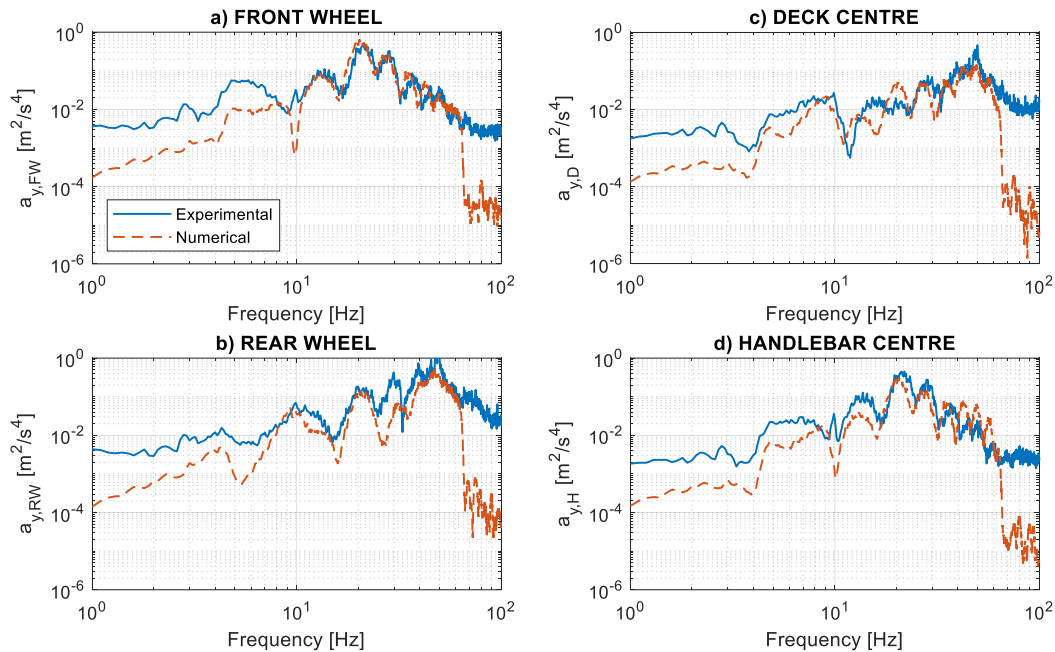


Figure 4. Comparison between experimental and numerical vertical acceleration PSDs at sensible points of the e-scooter running on an A class road profile at a speed of about 23 km/h with tires inflation pressure of 2.5 bar

To further prove the reliability of the model, **Figure 5** shows the vertical acceleration response at the same sensible points previously analyzed but with the e-scooter running at a forward speed of a about 17 km/h with tires inflation pressure of 1.5 bar. The same considerations drawn for the previous case are still valid, with the numerical road input cut-off frequency that switches to about 50 Hz because of a lower speed associated to the same minimum wavelength limitation of 0.1 m. Still because of a different forward e-scooter speed the frequency associated with the wheelbase filtering effect is reduced to about 6 Hz. In this second case it is also possible to notice a resonance peak at a about 4.5 Hz for all the inspected points and this frequency is associated with the rigid pitch mode of the vehicle. Moreover, the front wheel resonance frequency remains around 21 Hz meaning the tire radial stiffness to be almost unchanged with a change in tires inflation pressure and assuming a fixed weight distribution between the two cases. In general one can expect the tire radial stiffness to reduce when decreasing the inflation pressure, but thanks to the non-linearity a greater radial compression generates a greater radial stiffness and in this case the two

opposite contributions are equal, generating a null net contribution. Instead, for what concerns the rear wheel resonance frequency, it switches from 50 Hz to about 45 Hz, meaning the tire radial stiffness has decreased with the reduction in tires inflation pressure.

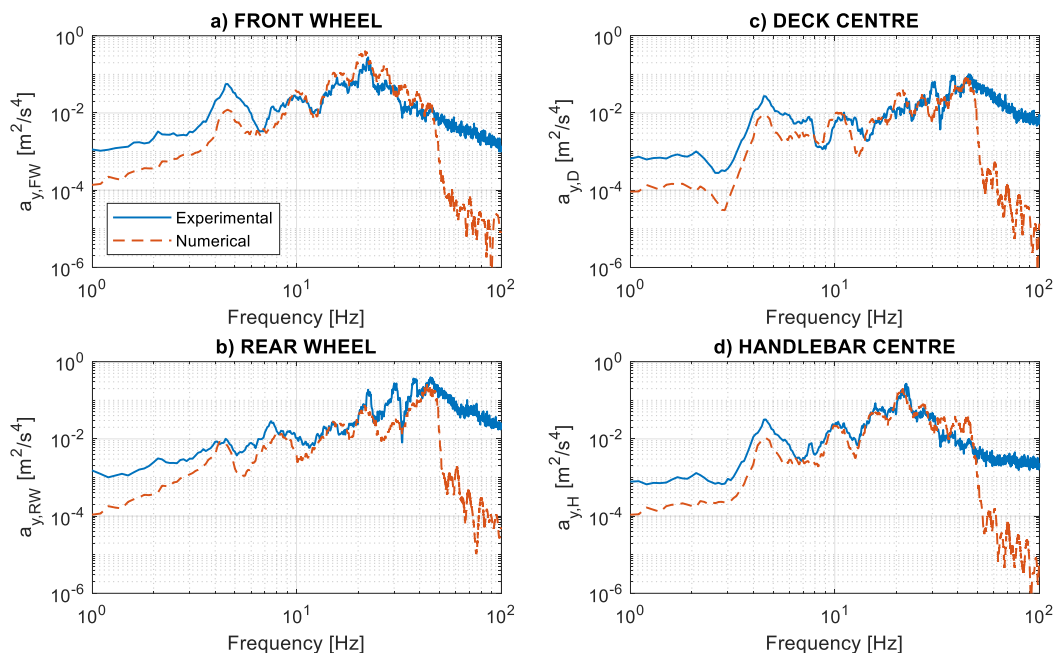


Figure 5. Comparison between experimental and numerical vertical acceleration PSDs at sensible points of the e-scooter running on an A class road profile at a speed of about 17 km/h with tires inflation pressure of 1.5 bar

5 Conclusions

The paper presents a lumped parameter model for the simulation of kick e-scooter vertical dynamics. The most challenging task for the model is to consider the mechanical impedance of the rider that is deeply influencing the whole system dynamics. Starting from common approaches in the modelling of the mechanical impedance of the human body for the excitation of structures, the necessary simplifications have been made to obtain a model that is suitable for the new scope. The parameters characterizing the driver lumped parameter model are still close to those reported in literature for different applications. Moreover, an experimental campaign has been run to acquire accelerations at relevant points on the e-scooter, that have shown to properly match with those reproduced by the simulation model.

This contribution provides an important tool for the design of e-scooters since it allows to have a deeper understanding of the dynamics of the whole system, that comprises the vehicle but also the rider. This will allow to improve vehicle performances at a reduced cost, since prototypes testing can be avoided. In particular the model should allow to define the most effective suspension geometries and also to tune their parameters to improve both comfort and road holding. Moreover, for comfort enhancement purposes, the model allows also to define some specific insulation layers between the frame and the rider at proper interface points.

References

- [1] V. Cossalter, R. Lot, A Motorcycle Multi-Body Model for Real Time Simulations Based on the Natural Coordinates Approach, *Veh. Syst. Dyn.* 37 (2002) 423–447. <https://doi.org/10.1076/vsd.37.6.423.3523>.
- [2] V. Cossalter, R. Lot, F. Maggio, A Multibody Code for Motorcycle Handling and Stability Analysis with Validation and Examples of Application, SAE Tech. Pap. (2003). <https://doi.org/10.4271/2003-32-0035>.
- [3] L. Leonelli, N. Mancinelli, A multibody motorcycle model with rigid-ring tyres: formulation and validation, *Veh. Syst. Dyn.* 53 (2015) 775–797. <https://doi.org/10.1080/00423114.2015.1014820>.

- [4] A.A. Popov, S. Rowell, J.P. Meijaard, A review on motorcycle and rider modelling for steering control, *Veh. Syst. Dyn.* 48 (2010) 775–792. <https://doi.org/10.1080/00423110903033393>.
- [5] A. Polanco, E. Marconi, L. Muñoz, D. Suárez, A. Doria, Effect of Rider Posture on Bicycle Comfort, in: *Proc. ASME Des. Eng. Tech. Conf.*, Anaheim, CA, USA, 2019. <https://doi.org/10.1115/DETC2019-97763>.
- [6] A. Doria, E. Marconi, P. Cialoni, Modal Analysis of a Utility Bicycle From the Perspective of Riding Comfort, in: *Proc. ASME Des. Eng. Tech. Conf.*, Anaheim, CA, USA, 2019. <https://doi.org/10.1115/DETC2019-97277>.
- [7] D.A. Winter, *Biomechanics and Motor Control of Human Movement*, 4th ed., Wiley, Hoboken, NJ, USA, 2009. <https://doi.org/10.1002/9780470549148>.
- [8] W. Blajer, K. Dziewiecki, Z. Mazur, Multibody modeling of human body for the inverse dynamics analysis of sagittal plane movements, *Multibody Syst. Dyn.* 18 (2007) 217–232. <https://doi.org/10.1007/s11044-007-9090-2>.
- [9] J.A.C. Ambrósio, A. Kecskeméthy, Multibody Dynamics of Biomechanical Models for Human Motion via Optimization, in: J.C. García Orden, J.M. Goicolea, J. Cuadrado (Eds.), *Multibody Dyn.*, Springer Netherlands, Dordrecht, 2007: pp. 245–272. <https://doi.org/10.1007/978-1-4020-5684-0>.
- [10] N. Nawayseh, S. Hamdan, M. Bernardo-Filho, R. Taiar, Modelling the apparent mass of the standing human body under whole-body vibration training conditions, *Proc. Inst. Mech. Eng. Part H J. Eng. Med.* 234 (2020) 697–710. <https://doi.org/10.1177/0954411920917311>.
- [11] L. Wei, M.J. Griffin, Mathematical Models for the Apparent Mass of the Seated Human Body Exposed to Vertical Vibration, *J. Sound Vib.* 212 (1998) 855–874. <https://doi.org/10.1006/jsvi.1997.1473>.
- [12] Y. Matsumoto, M.J. Griffin, Mathematical models for the apparent masses of standing subjects exposed to vertical whole-body vibration, *J. Sound Vib.* 260 (2003) 431–451. [https://doi.org/10.1016/S0022-460X\(02\)00941-0](https://doi.org/10.1016/S0022-460X(02)00941-0).
- [13] N. Nawayseh, M.J. Griffin, A model of the vertical apparent mass and the fore-and-aft cross-axis apparent mass of the human body during vertical whole-body vibration, *J. Sound Vib.* 319 (2009) 719–730. <https://doi.org/10.1016/j.jsv.2008.05.030>.
- [14] M. Tarabini, S. Solbiati, B. Saggin, D. Scaccabarozzi, Apparent mass matrix of standing subjects exposed to multi-axial whole-body vibration, *Ergonomics*. 59 (2016) 1038–1049. <https://doi.org/10.1080/00140139.2015.1108459>.
- [15] M. Tarabini, S. Solbiati, G. Moschioni, B. Saggin, D. Scaccabarozzi, Analysis of non-linear response of the human body to vertical whole-body vibration, *Ergonomics*. 57 (2014) 1711–1723. <https://doi.org/10.1080/00140139.2014.945494>.
- [16] M.G.R. Toward, M.J. Griffin, Apparent mass of the human body in the vertical direction: Inter-subject variability, *J. Sound Vib.* 330 (2011) 827–841. <https://doi.org/10.1016/j.jsv.2010.08.041>.
- [17] N. Nawayseh, S. Hamdan, Apparent mass of the standing human body when using a whole-body vibration training machine: Effect of knee angle and input frequency, *J. Biomech.* 82 (2019) 291–298. <https://doi.org/10.1016/j.jbiomech.2018.11.003>.
- [18] Y. Matsumoto, M.J. Griffin, Dynamic Response of the Standing Human Body Exposed to Vertical Vibration: Influence of Posture and Vibration Magnitude, *J. Sound Vib.* 212 (1998) 85–107. <https://doi.org/10.1006/jsvi.1997.1376>.
- [19] P. Lugner, H. Pacejka, M. Plöchl, Recent advances in tyre models and testing procedures, *Veh. Syst. Dyn.* 43 (2005) 413–426. <https://doi.org/10.1080/00423110500158858>.
- [20] R. Lot, A Motorcycle Tire Model for Dynamic Simulations: Theoretical and Experimental Aspects, *Meccanica*. 39 (2004) 207–220. <https://doi.org/10.1023/B:MECC.0000022842.12077.5c>.
- [21] M. Asperti, M. Vignati, F. Braghin, Modelling of the Vertical Dynamics of an Electric Kick Scooter [IN PRESS], *IEEE Trans. Intell. Transp. Syst.* (2021).
- [22] International Organization for Standardization, *Mechanical vibration — Road surface profiles — Reporting of measured data (ISO Standard No. 8608:2016)*, (2016). <https://www.iso.org/standard/71202.html>.
- [23] F. Koutny, Load-deflection curves for radial tyres, *Appl. Math. Model.* 5 (1981) 422–427. [https://doi.org/10.1016/S0307-904X\(81\)80025-X](https://doi.org/10.1016/S0307-904X(81)80025-X).
- [24] S. Rapoport, J. Mizrahi, E. Kimmel, O. Verbitsky, E. Isakov, Constant and variable stiffness and damping of the leg joints in human hopping, *J. Biomech. Eng.* 125 (2003) 507–514. <https://doi.org/10.1115/1.1590358>.
- [25] International Organization for Standardization, *Mechanical vibration and shock — Mechanical impedance of the human hand-arm system at the driving point (ISO Standard No. 10068:2012)*, (2012). <https://www.iso.org/standard/53714.html>.