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Flyby design in the Circular Restricted Three Body Problem

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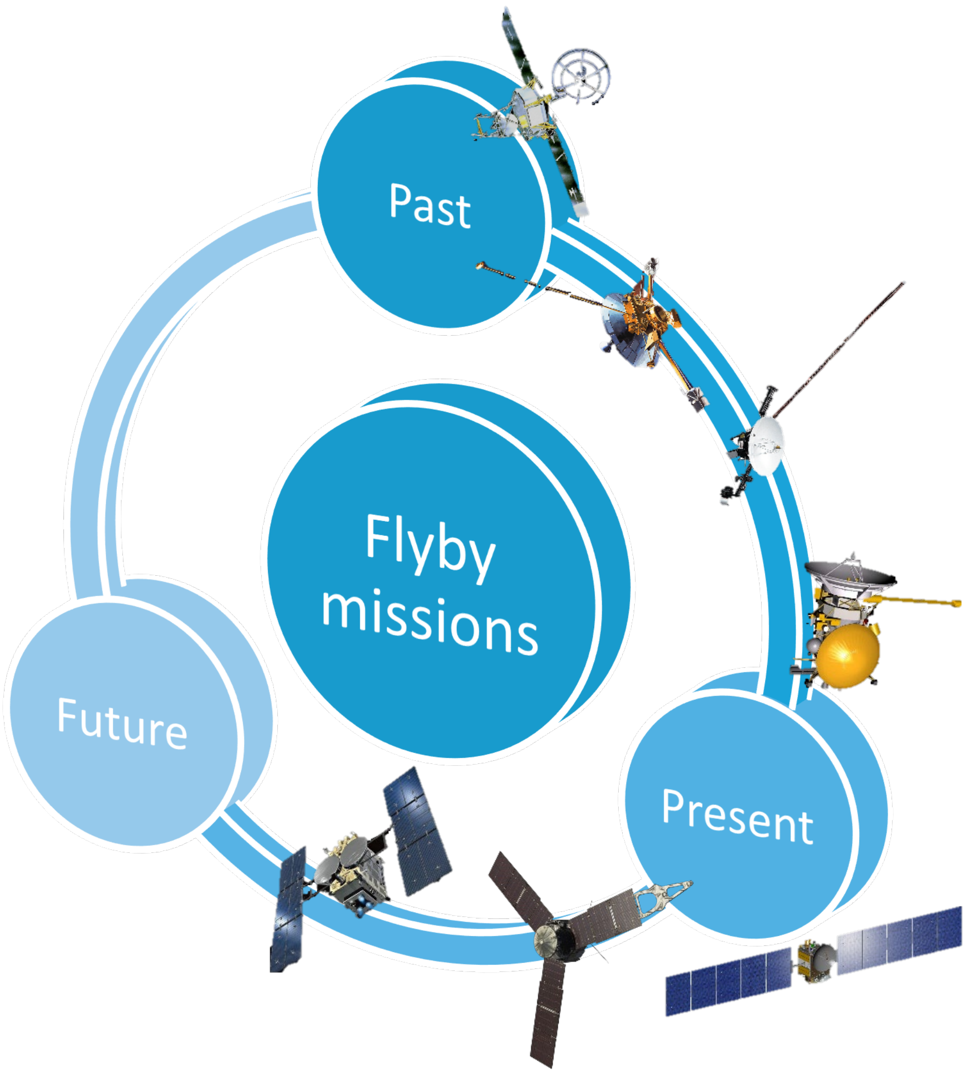
SDSM 2017, San Martino al Cimino (VT)

INTRODUCTION

Background

Motivation



Objectives and targets

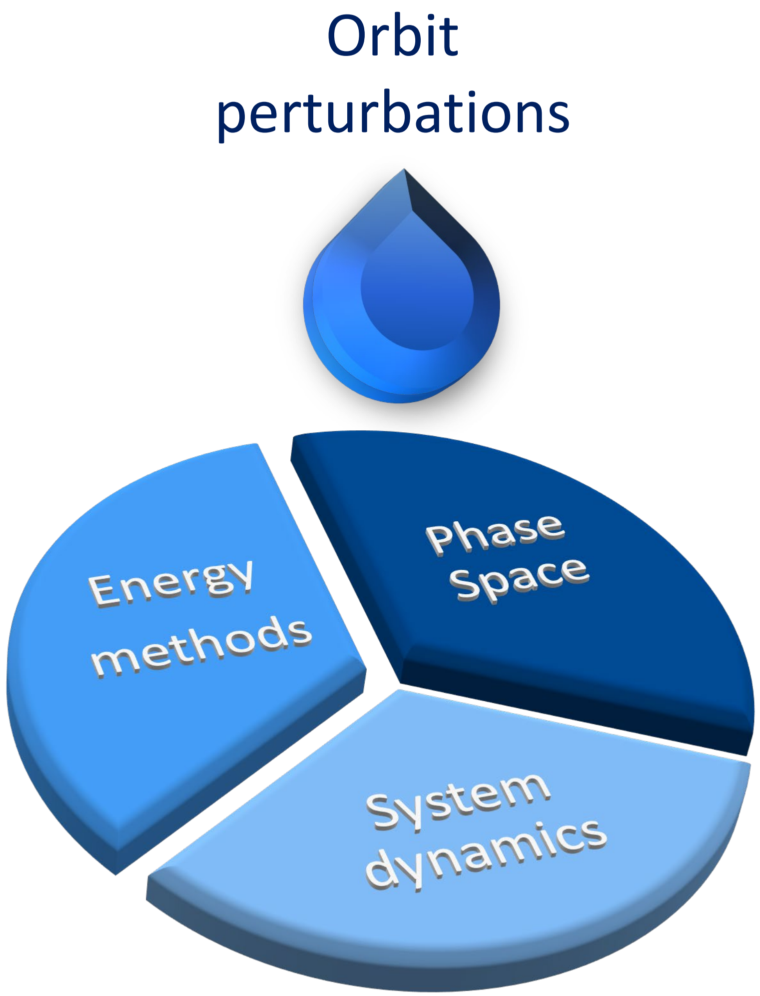
- Colonisation:
  - Mars (♂)
- Search for forms of life
  - Europa's ocean (♃)
  - Titan's methane seas (♄)
  - Encedalus' cryovolcanism (♄)
  - Triton's cryovolcanism (♆)
- Study of planet formation/asteroid deflection and exploitation
  - Main Asteroid Belt
  - Kuiper-belts

Problem:

Endgame to Mars

- Unmanned vs manned mission
- Optimal phasing:
  - one every 26 months
- Objective:
  - Safety of the crew
  - Containment of the cost
    - Fuel
    - Weight
- Constraints:
  - Heath threat:
    - Physiological (radiation dose)
    - Psychological (confinement)
- Solution:
  - flyby at Venus

Motivation



- Increase the understanding of the effect of perturbations on the dynamics of the system
- Develop optimisation strategy based on dynamics evolution and on stronger mathematical foundation

Outline

- 2-body design strategy:
  - Classical Lambert problem
  - Rotated and synodic frame
  - Powered gravity assist and legs patching
  - Preliminary results
- Refinement in the CR3BP:
  - CR3BP model
  - Classical and Modified Tisserand parameter
  - Optimisation algorithm
  - Preliminary results
- Conclusion and future work

2-BODY DESIGN STRATEGY

2-body design strategy

Classical Lambert problem

Design planet-to-planet trajectory is a targeting problem usually tackled through Lambert's algorithm*

- Assumptions:
  - Two-body gravitation
- Formulation:
  - Inertial reference
  - $r_1(t_1) \rightarrow r_1(t_2)$
- Input:
  - $r_1, r_2, \Delta t$

Strategy

Combining time law and Kepler's equation:

$$\sqrt{\frac{\mu}{a^3}}\Delta t = E_2 - e\sin E_2 - \left(E_1 - e\sin E_1\right)$$

and adopting the trasformation scheeme:

$$\left(E_1, E_2\right) \rightarrow \left(E_M, E_P\right) \rightarrow \left(E_M, \xi\right) \rightarrow \left(\alpha, \beta\right)$$

Solution is found resolving the non-linear eq.

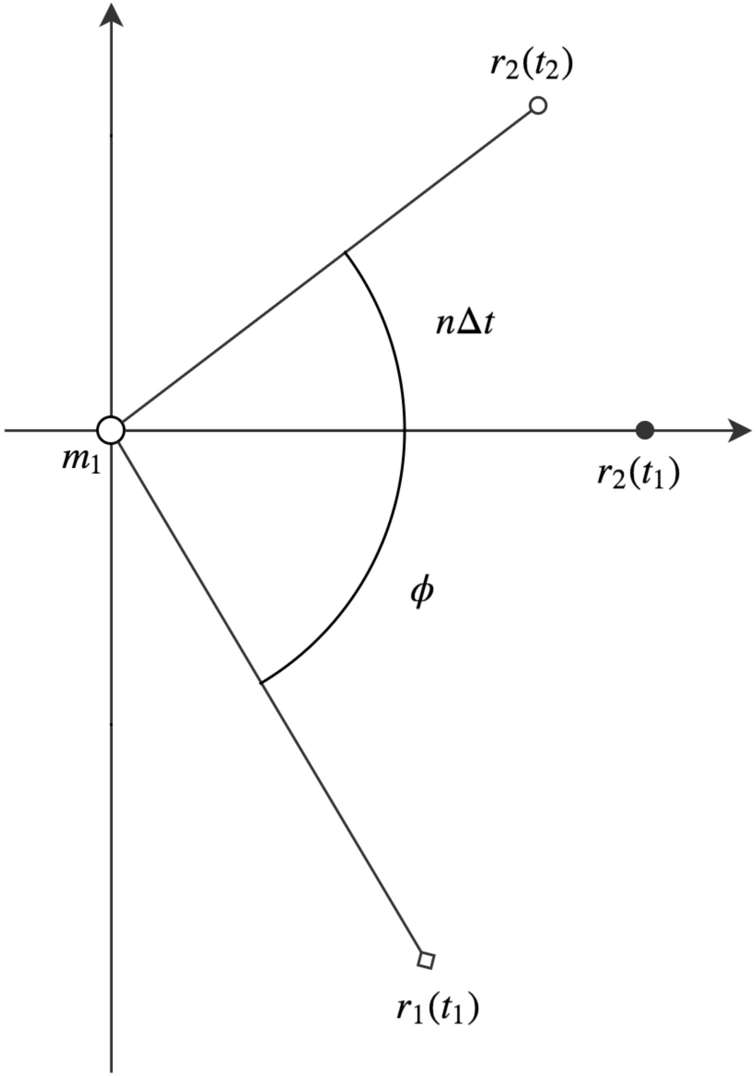
$$\sqrt{\frac{\mu}{a^3}}\Delta t = \alpha - \sin\alpha - \left(\beta - \sin\beta\right)$$

where α and β are function of s, c and a alone

* J.E.Prussing, B.A.Conway. Orbital Mechanics, 1993

2-body design strategy

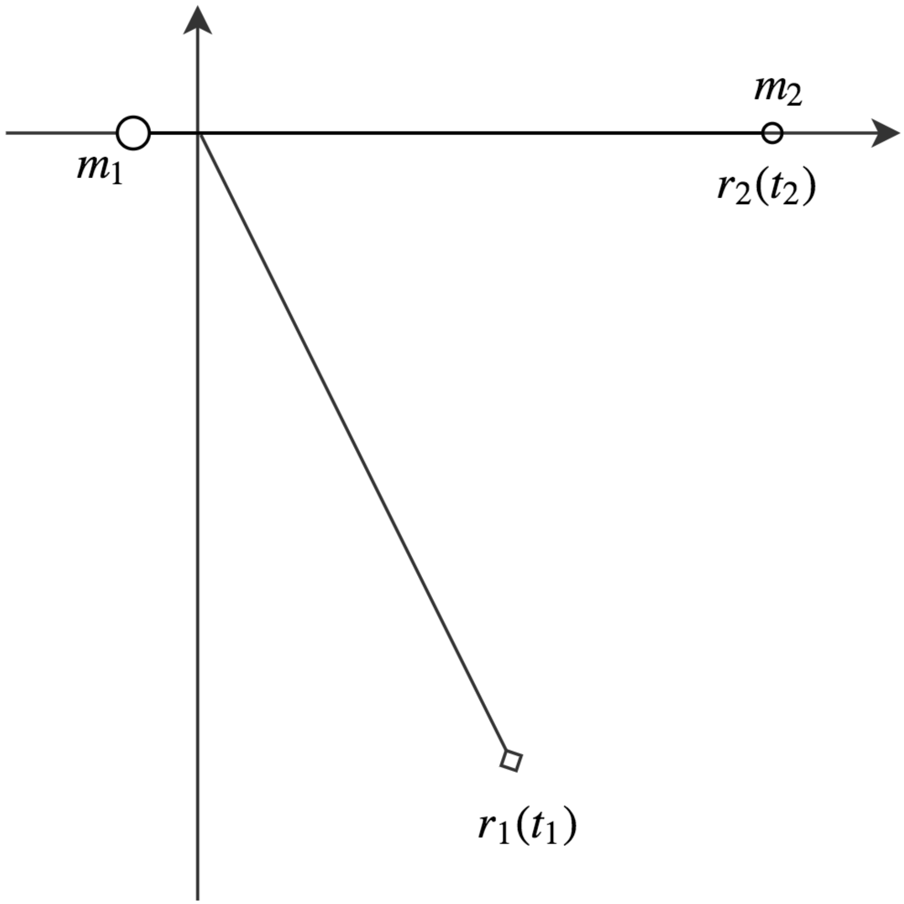
Lambert problem and rotated frame



- Geometric relations and Lambert
  - $a = f(s, c, \Delta t)$
- Initial conditions defined in the rotated frame:
  - $\Delta\theta = \phi + \mathrm{n}\Delta t$
  - ϕ, phasing between planets at given t
  - $\mathrm{n}\Delta t$, angular distance travelled by the arrival planet during the flight of the probe

2-body design strategy

From rotated to synodic frame



- Lambert's solution in the synodic frame

$$\vec{r}_1 = \overrightarrow{r_1(t_1)} - \mu^* \overrightarrow{r_2(t_1)}$$

$$\vec{r}_2 = (1-\mu^*)\overrightarrow{r_2(t_1)}$$

$$\vec{\dot{r}}_1 = \vec{v}_1 - \omega \times \overrightarrow{r_1(t_1)}$$

$$\vec{\dot{r}}_2 = R\vec{v}_2 - \omega \times \overrightarrow{r_2(t_1)}$$

where

$$R = R(n\Delta t)$$

$$\mu^* = \frac{m_2}{m_1 + m_2}$$

2-body design strategy

Powered flyby: zero-SOI vs patch conic approximation

Zero-Sphere of Influence

- Dynamic:
  - single attractor, m_1
- Flyby:
  - at interception of the planet
  - instantaneous change of relative velocity:

$$\delta = \widehat{v_2} \measuredangle \widehat{v_1}$$

- Patching:
  - manoeuver at infinite

$$\Delta v = \left| v_2 - v_1 \right|$$

Patched conic approximation

- Dynamic:
  - single attractor per domain
- Flyby:
  - at crossing of the SOI
  - two hyperbolic trajectories sharing periapsis, r_p

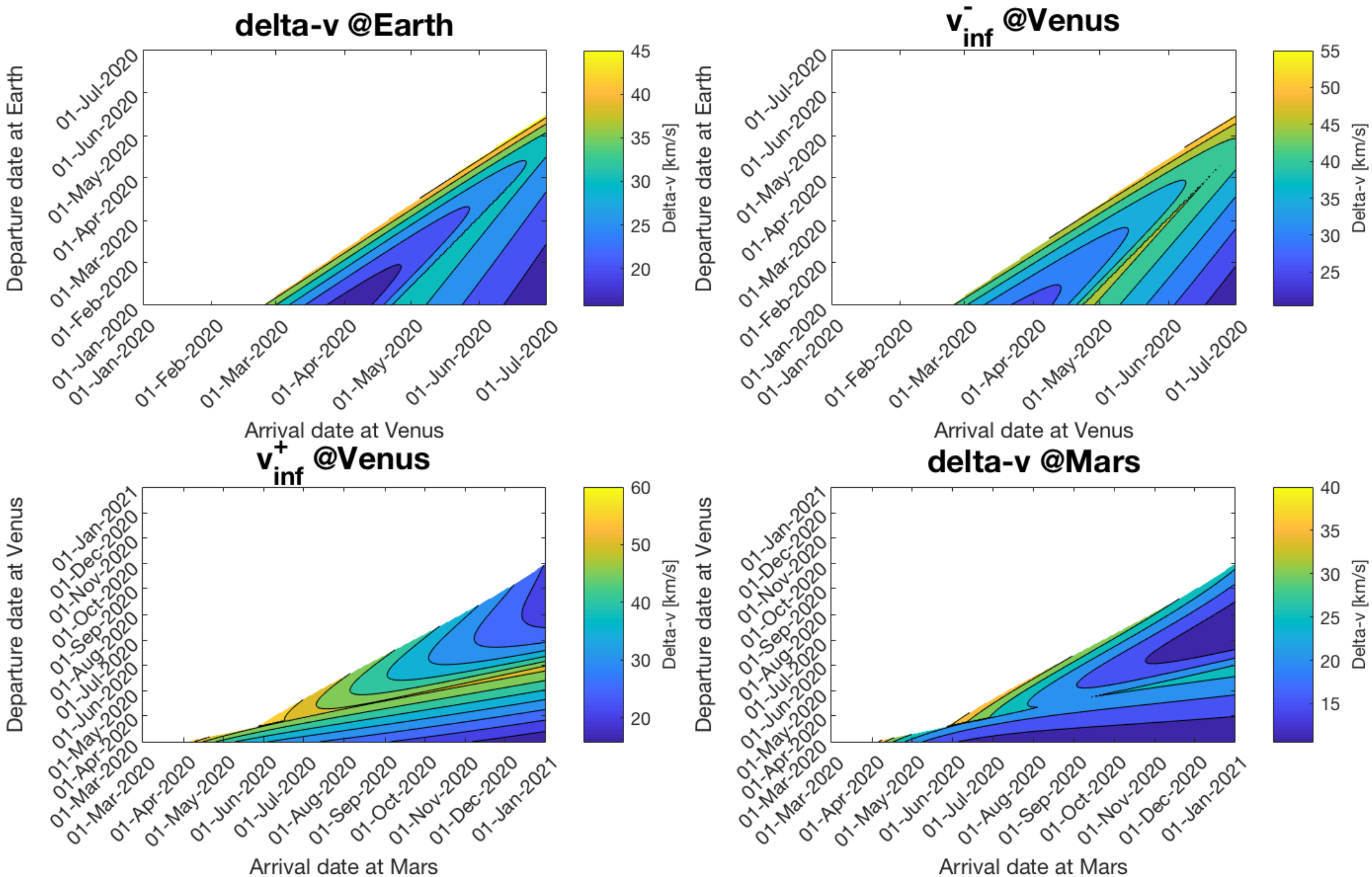
$$\delta = \sin^{-1} \frac{\mu_2}{\mu_2 + r_p v_2^2} + \sin^{-1} \frac{\mu_2}{\mu_2 + r_p v_1^2}$$

- Patching:
  - optimal manoeuver at r_p

$$\Delta v = \left| \sqrt{v_2^2 + 2\frac{\mu_2}{r_p}} - \sqrt{v_1^2 + 2\frac{\mu_2}{r_p}} \right|$$

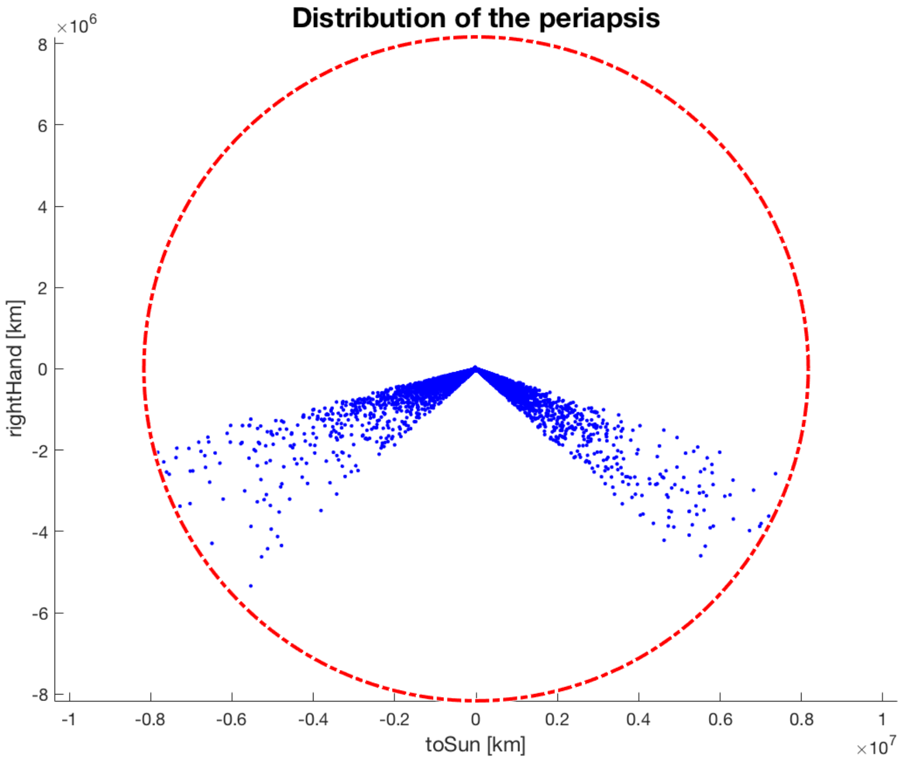
2-body design strategy

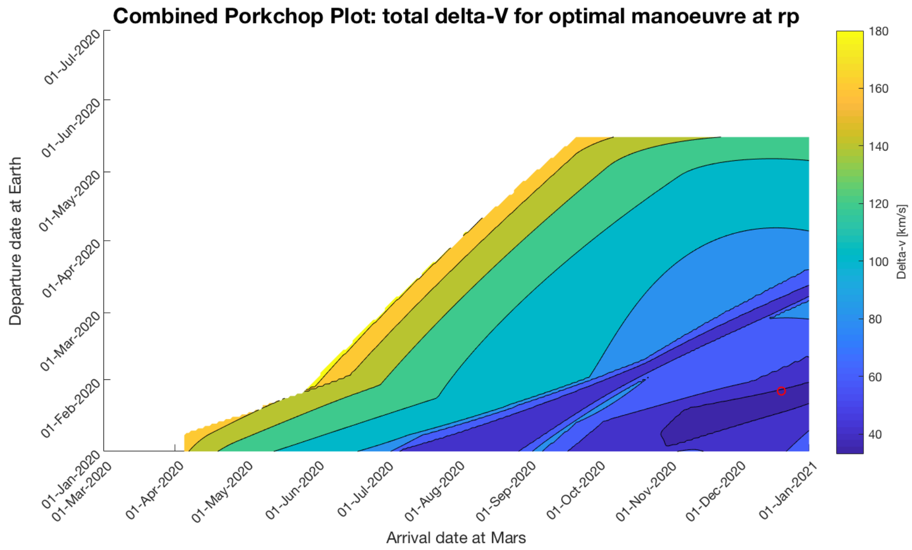
Preliminary results for an hypothetic future Earth-Venus-Mars mission



Design strategy

Preliminary results for an hypothetic future Earth-Venus-Mars mission





REFINEMENT IN THE CIRCULAR RESTRICTED THREE-BODY PROBLEM

Refinement in the CR3BP

2BP vs CR3BP

Lambert solutions propagated in the CR3BP don’t result in a unique trajectory.

- Single attractor per domain vs mixed attraction

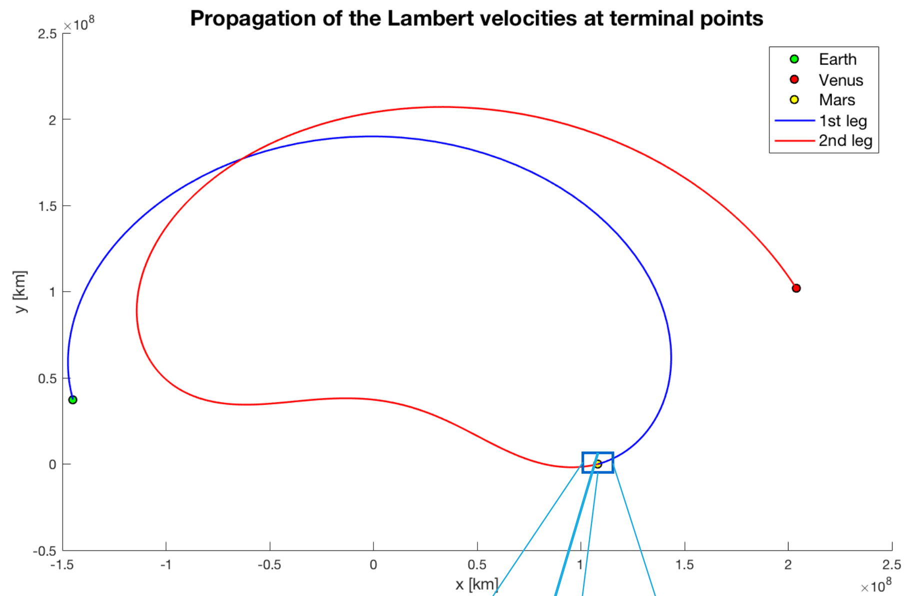
$$F = \frac{n^2}{2}\left(x^2 + y^2\right) + G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

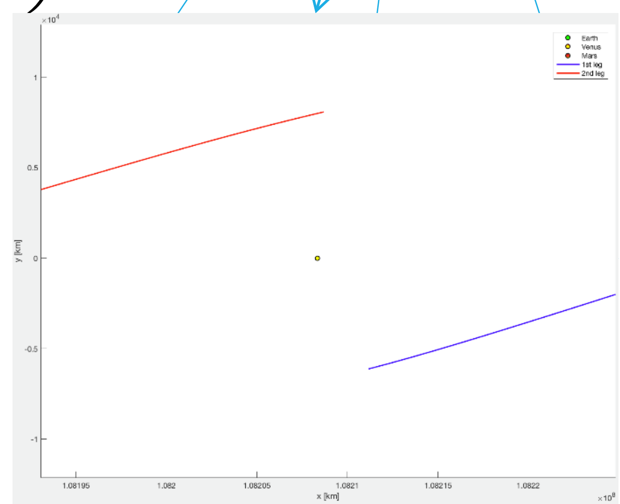
$$\begin{cases} \dfrac{d^2x}{dt^2} - 2n\dfrac{dy}{dt} = \dfrac{\partial F}{\partial x} \\[2ex] \dfrac{d^2y}{dt^2} + 2n\dfrac{dx}{dt} = \dfrac{\partial F}{\partial y} \end{cases}$$

$$r_1 = \sqrt{\left(x+b\right)^2 + y^2}$$

$$r_2 = \sqrt{\left(x-a\right)^2 + y^2}$$

$$n = \sqrt{\frac{G\left(m_1 + m_2\right)}{l^3}} \qquad a = \frac{m_2}{m_1 + m_2}l \qquad b = \frac{m_1}{m_1 + m_2}l$$





Refinement in the CR3BP

Optimisation strategy

Local optimisation is used to refine the two-body rotating solution in the CR3BP dynamics.

MATLAB® *fmincon* is used. Common set up requires:

- the selection of:
  - a control variable and an initial guess
- the definition of:
  - an objective function which takes into account the cost of the mission and is what the optimisation algorithm tries to minimise
  - a constraints function which the solution shall satisfy

- Campagnola ad Russel* showed that the optimisation problem allows quasi-ballistic solutions, irreproducible in the two-body design, and thus not comparable with the previous ones.

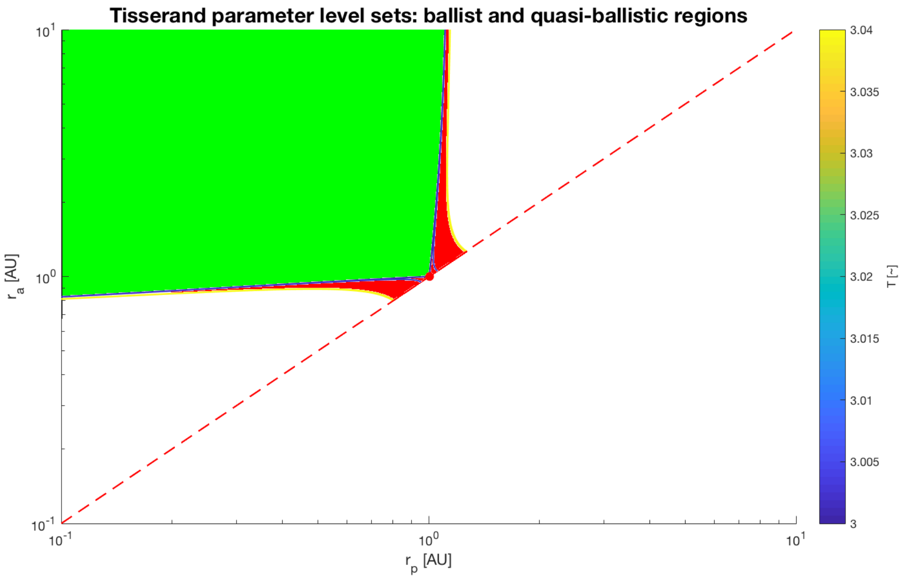
* S. Campagnola, R. Russel. Endgame Problem part 2: multibody technique and the T-P graph

Refinement in the CR3BP

The Tisserand parameter

Jacobi integral

$$C = 2G\left(\frac{m_1}{r_1}+\frac{m_2}{r_2}\right)+2n\left(\varepsilon\dot{\eta}-\eta\dot{\varepsilon}\right)-\left(\varepsilon^2+\eta^2+\zeta^2\right)$$



Assuming

- Large distance from m_2
- Small mass parameter

through

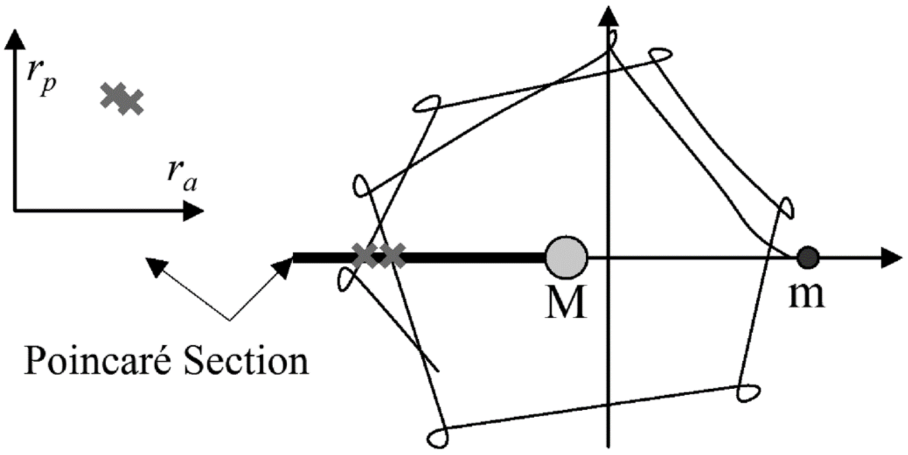
- Vis-viva equation
- Angular momentum definition

$$\overline{T} = \frac{1}{\overline{a}_P} + 2\sqrt{\overline{a}_P\left(1-e_P^2\right)}\cos i_P$$

Which has a direct relation with relative velocity the encounter: $\overline{T} = 3 - \overline{v_\infty^2}$

Refinement in the CR3BP

The Modified Tisserand Parameter



- Assumptions and applicability
  - Con vs Un solution
- Novelty:
  - $r_2 \nrightarrow \infty$
- Implications:
  - Barycentric (B) vs heliocentric (P)
  - Additional term

$$\bar{T} = \frac{1}{\bar{a}_B} + \sqrt{\bar{a}_B\left(1-\bar{e}_B^{\,2}\right)}\cos\bar{i}_B + \mu^* \frac{\bar{r}_2 - \bar{r}_1}{\bar{r}_2 \bar{r}_1}$$

Refinement in the CR3BP

Optimisation algorithm

Propagate back and forward the state vector at departure and arrival and optimises initial conditions to statisfy constraints and minimize cost.

where:

- x_0 is the initial guess: $\left[v_{dep@E}^{lambLeg1}, v_{arr@M}^{lambLeg2}\right]$
- $[C, Ceq]$ define the non-linear inequality constraints:
  1. imposing patch of the legs: $\left\| r_{dep@V}^{Leg2} - r_{arr@V}^{Leg1} \right\| < tol1$
  2. forcing ballistic encounter: $T_{leg} - 3 < 0$
- $[J, Jeq]$ represent the cost function that wants to be minimised:
  1. $\left\| v_{dep@E}^{opt} - v_{dep@E} \right\| + \left\| v_{arr@M}^{opt} - v_{arr@M} \right\| + \left\| v_{arr@V}^{opt} - v_{dep@V}^{opt} \right\|$
  2. $\left\| v_{dep@E}^{opt} - v_{dep@E} \right\| + \left\| v_{arr@V}^{opt} - v_{dep@V}^{opt} \right\| + \left| T_{Leg1} - T_{Leg2} \right| \frac{\mu}{l}$

* S. Campagnola, R. Russel. Endgame Problem part 2: multibody technique and the T-P graph

Refinement in the CR3BP

Preliminary results for same hypothetic future mission Earth-Venus-Mars

Given initial/final condition at departure/arrival:

$$r_0 = \left[[-1.4484, 0.3742, 0], [2.0395, 1.0178, 0]\right] 1e8$$

$$v_0 = \left[[4.6605, 18.0399, 0], [22.1662, -44.4195, 0]\right]$$

the initial guess, provided by the Lambert:

$$x_0 = \left[[-7.4811, 31.5044, 0], [28.7419, -53.8860, 0]\right]$$

and a tolerance of 50 km for the patching

Three optimisation strategy:

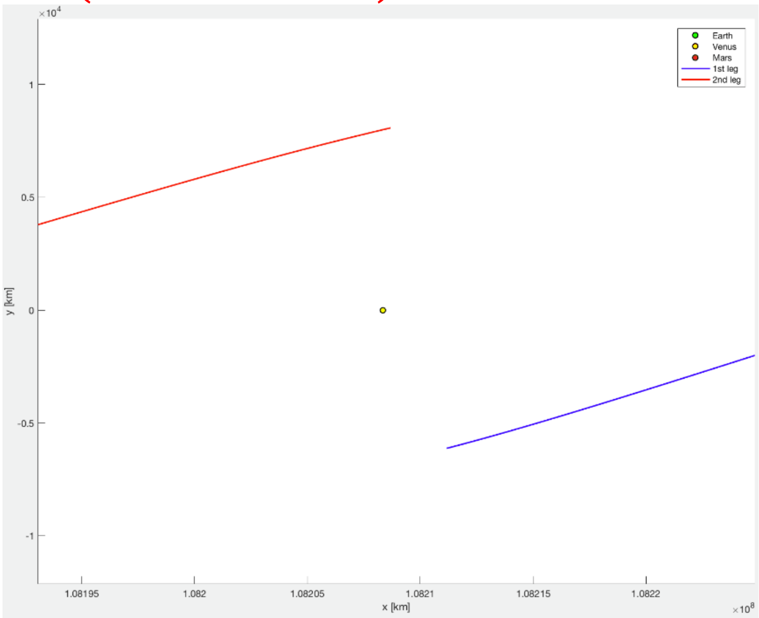
- Classical (C1-J1)
- Energy (C1-C2-J2)

Refinement in the CR3BP

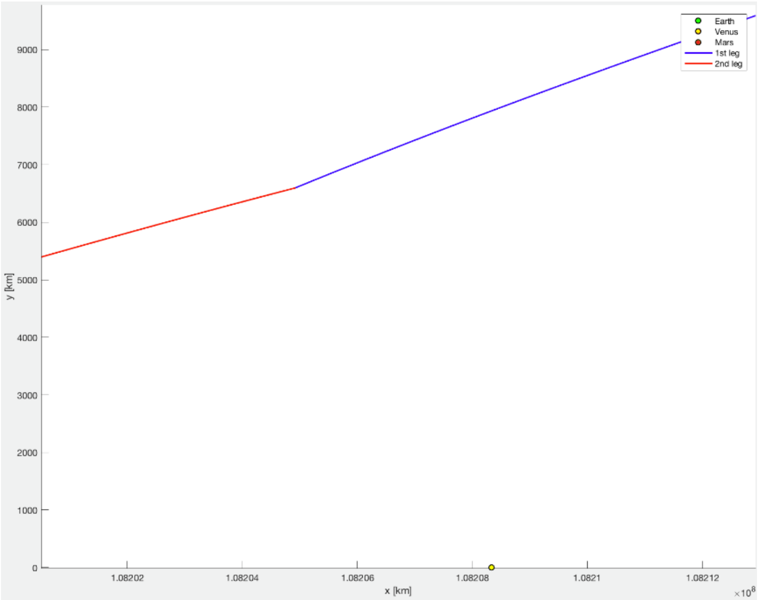
Preliminary results for same hypothetic future mission Earth-Venus-Mars

Method	Convergence	Iterations	Eval-function	Delta-v [km/s]
Classical C1-J1	✓	42	343	22.0465
Energetic C1-C2-J2	✓	27	249	25.5841

$$d\left(r_{dep@V}^{Leg2}, r_{arr@V}^{Leg1}\right) = 1.4419e4\ km$$



$$d\left(r_{dep@V}^{Leg2}, r_{arr@V}^{Leg1}\right) = 49.3651\ km$$



CONCLUSION

Conclusion and future work

- Fly-by in the CR3BP can be designed refining Lambert solution via shooting method exploiting energetic constraints and objective function
- Preliminary results shows that the modified Tisserand parameter appears more stable and seems to ensure more efficient constraint and more correct objective function

Future work

- Exploit calculus of variations in the resolution of free time of flight/phasing problem
- Advance the optimisation scheme through the implementation of the Hamilton-Jacobi-Bellman equations for low-thrust manoeuvres
- Improve preliminary design including third body effect through the Kick-map method

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Thank you for your attention!
Any questions?

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