Importance Sampling in Life-Cycle Seismic Fragility and Risk Assessment of Aging Bridge Networks

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Abstract. Life-cycle structural reliability of deteriorating systems and multi-hazard risk assessment of aging infrastructure networks involve complex time-variant processes characterized by impactful uncertainties. Simulation methods are frequently the only viable tools to accurately estimate time-variant failure probabilities and risk metrics. However, simulation-based techniques are time-consuming and might be computationally inefficient and unfeasible in practice, particularly when analysis of large-scale systems are required to assess numerically sensitive performance indicators.
This paper proposes a novel computational approach based on Importance Sampling to efficiently estimate the time-variant seismic risk of aging road networks. In the proposed methodology, the seismic capacity of deteriorating structural systems is efficiently simulated to account for the time-variant model uncertainties typical of life-cycle structural reliability problems.
The possible improved trade-off in terms of sample size and estimate accuracy is tested in comparison with traditional Monte Carlo simulation approaches based on a practical application concerning the life-cycle seismic risk assessment of a road network with spatially-distributed deteriorating vulnerable bridges.

Keywords: Seismic Risk, Life-Cycle Assessment, Bridge Networks, Structural Deterioration, Simulation Techniques.

1 Introduction

Risk assessment of infrastructure lifelines must be carried out considering aging and deterioration processes that may adversely affect the performance of vulnerable system components, accounting for the mechanical deterioration mechanisms that critical structures in the network may suffer throughout their lifetime [1–4]. Despite the recognized high impact of long-term damage phenomena, their incorporation in large-scale risk assessment may be challenging [5].

In this paper, a novel computationally efficient methodology is proposed to estimate with a simulation-based approach the life-cycle seismic risk of spatially-distributed aging bridges in transportation road networks. The proposed procedure is based on Importance Sampling. This efficient variance reduction technique is herein improved to account for the time-variant modeling uncertainties typical of life-cycle reliability analysis by efficiently selecting a set of sample structural systems to be analyzed. In the proposed framework for seismic risk assessment, the uncertainties on regional seismic hazard, time-variant bridge vulnerability, and traffic detour-based exposure analysis are aggregated to numerically estimate the mean annual rate of occurrence of a prescribed daily users' cost threshold. The methodology is introduced and applied in direct comparison with traditional Monte Carlo simulation to highlight the possible advantages in terms of computational effort and estimate accuracy for multi-hazard risk assessment of complex systems.

2 Seismic Risk Assessment of Aging Bridge Networks

2.1 Regional Seismic Hazard

Decades of observations of seismic signals in regions prone to seismic hazard allowed seismologists to calibrate predictive models for the recurrence of major seismic events. Probabilistic Seismic Hazard Analysis (PSHA) allows identifying the exceedance probability of large seismic intensities i_b at the b-th vulnerable structure site. In classical PSHA approaches, earthquakes are described as stationary Poisson processes, i.e., earthquake occurrences are mutually independent events [6]. In Poissonian processes, the occurrence probability of at least one event in a prescribed time interval is identified based on the mean annual rate of occurrence v.

Among macroseismic measures of earthquake intensity, the moment magnitude M_w is widely recognized to be representative of sizeable seismic events [7]. The Gutenberg–Richter law is a commonly adopted model to capture the recurrence of major earthquakes in the region relying on the empirically-based assumption of proportionality between magnitude m and rate of occurrence v_m of earthquakes with magnitude larger than m. Based on this premise, it is possible to define the truncated exponential Cumulative Distribution Function (CDF) of the moment magnitude M_w given the occurrence of a seismic event [8]:

$$F_{M_w}(m) = \frac{1-10^{-b(m-m_{min})}}{1-10^{-b(m_{max}-m_{min})}} \quad (1)$$

where $m_{\min}$ and $m_{\max}$ are lower and upper bounds and b is a shape parameter, all calibrated based on historical and geological data.

Regional seismic risk assessment should also account for the uncertainties in the epicenter location and the relative distance with respect to the site where vulnerable facilities are built. Localization of vulnerable structural facilities $\{x,y\}_b$ and earthquake epicenter location $\{X,Y\}_e$ leads to the definition of the source-to-site distance R_{eb} for the set of n_b network components, such as bridges. The seismic intensity measure at each b-th bridge site I_b [g] is defined in probabilistic terms by means of Ground Motion Prediction Equations (GMPEs). These predictive models describe in probabilistic terms the ground shaking severity at any given location in terms of seismic hazard parameters generally associated with earthquake magnitude, source-to-site distance, and other structural properties and geological features of the site [9]. The statistical description of attenuation laws typically relies on joint lognormal models characterized by within- or inter-event and between- or intra-event variability. Predictive models of spatial correlation of seismic intensities at different sites are often calibrated based on the geological features of the investigated area [10].

2.2 Bridge Fragility Curves and Damage-induced Traffic Limitations

The seismic capacity of bridges is given in terms of fragility curves, which express the probability of exceedance of a prescribed limit state s_b conditioned on the measure of seismic intensity at the bridge site i_b. Fragility curves are often associated with lognormal models as follows:

$$P[S_b(t) \geq s_b | i_b] = \Phi\left[\frac{\ln i_b - \lambda_b(t)}{\zeta_b(t)}\right] \tag{2}$$

where λ_b and ζ_b are the fragility curves parameters and S_b is a discrete random variable representing the bridge damage state. In particular, the median seismic capacity with respect to structural collapse $i_{b,m}$=exp(λ_b) may severely decay in time due to progressive deterioration induced by aging and deterioration.

Seismic events may lead to bridge closure due to extensive damage up to structural collapse, impairing the safety of road users and, in turn, limiting the bridge traffic functionality. The post-event damage state of the b-th bridge in the road network is defined based on a discrete random variable S_b. In the following applications, the bridge state is described by a Boolean random variable with the following possible states: undamaged and fully operational bridge (i.e., S_b=0) or collapsed and fully closed bridge (i.e., S_b=1).

2.3 Traffic Analysis and User Cost-based Life-cycle Seismic Risk Measure

Daily trips of road users in the network are generated from Origin nodes O and attracted to Destination nodes D. Traffic flow analysis allows to evaluate travel times t_{OD} and travel distances l_{OD} for each Origin-to-Destination traffic demand f_{OD}. Therefore, it is possible to assess Total Travel Time TTT and Total Travel Distance TTD given the combination of bridge damage states, collected in a vector **S**:

$$\begin{aligned} TTT(\mathbf{S}) &= \textstyle\sum_O \sum_D f_{OD} \cdot t_{OD}\,(\mathbf{S}) \\ TTD(\mathbf{S}) &= \textstyle\sum_O \sum_D f_{OD} \cdot l_{OD}\,(\mathbf{S}) \end{aligned} \tag{3}$$

The loss of network functionality induced by bridge closure leads to indirect economic losses that can be quantified in monetary terms based on the concept of user costs [4]. In this paper, two cost items proportional to Total Travel Delay $\Delta TTT(\mathbf{S})$ and Total Detour Distance $\Delta TTD(\mathbf{S})$ are considered:

$$\Delta TTT(\mathbf{S}) = TTT(\mathbf{S}) - TTT_0$$
$$\Delta TTD(\mathbf{S}) = TTD(\mathbf{S}) - TTD_0 \quad (4)$$

where TTT_0=TTT(**S**=**0**) and TTD_0=TTD(**S**=**0**) refer to unrestricted conditions. The related cost items are the Driver Delay Costs *DDC* and the Vehicle Operating Costs *VOC*, which are defined in terms of unitary costs Q_{DDC} and Q_{VOC} as follows:

$$DDC(\mathbf{S}) = Q_{DDC} \cdot \Delta TTT(\mathbf{S})$$
$$VOC(\mathbf{S}) = Q_{VOC} \cdot \Delta TTD(\mathbf{S}) \quad (5)$$

The lifetime monetary costs can be assessed based on the discount rate γ_c at the observation time *t* as follows:

$$C(t) = \frac{DDC+VOC}{(1+\gamma_c)^t} \quad (6)$$

Finally, the mean annual rate of exceedance of a cost threshold can be quantified based on the probability of exceedance of a prescribed cost threshold $P[C>c]$ and the mean annual rate of occurrence of seismic events v_m larger than the minimum moment magnitude of engineering interest $m_{\min}$:

$$v_c = v_{M_w \geq m_{\min}} \cdot P[C > c] \quad (7)$$

The simulation-based methodologies proposed in this paper allow estimating such time-variant probability encompassing all dimensions of life-cycle seismic risk assessment (i.e., regional seismic hazard, time-variant bridge vulnerability, and network cost-based exposure).

3 Simulation-based Life-Cycle Seismic Risk Estimates

3.1 Time-variant Monte Carlo Simulation

In life-cycle structural reliability and risk assessment, uncertainties vary in time due to mechanical and environmental deterioration processes suffered by vulnerable structural systems. The occurrence probability of a failure event $P[E]=p_E$ associated with the exceedance of a prescribed limit state can be formulated based on the failure domain $g(\mathbf{Z})\leq 0$ as a function of time-variant basic Random Variables (RVs) $\mathbf{Z}(t)$:

$$p_E = \int_{\mathbf{z}} I(\mathbf{Z}) \cdot f_{\mathbf{Z}(t)}(\mathbf{z}) d\mathbf{z} \quad (8)$$

where the indicator function $I(\mathbf{Z})=I[g_E(\mathbf{Z})\leq 0]$ is a Heaviside step function with unit value in the failure domain and zero value in the safe domain, whilst $f_{\mathbf{Z}(t)}(\mathbf{z})$ is the time-variant joint PDF of the basic random variables $\mathbf{Z}(t)$.

Traditional Monte Carlo Simulation (MCS) is a versatile tool for the numerical solution of multidimensional integrals based on the artificial generation of n_j samples $\mathbf{z}_j$ with $j=1,\ldots,n_j$ of the basic random variables $\mathbf{Z}$. In the context of reliability analysis, the sample mean of the indicator function is adopted as estimator of the failure probability:

$$\hat{p} = \frac{1}{n_j} \sum_{j=1}^{n_j} I_j(t) \tag{9}$$

where $I_j=I[g_E(\mathbf{z}_j)\leq 0]$. It is worth noting that each sample of the indicator function can be considered as a Boolean RV characterized by a Bernoulli distribution with failure probability equal to the failure probability p_E to be estimated. Thus, $\hat{p}$ is an unbiased estimator of the failure probability and its Coefficient of Variation (CoV) can be analytically defined in terms of failure probability p_E and sample size n_j [11]:

$$\delta_{MCS} = \sqrt{\frac{1-\hat{p}}{n_j \cdot \hat{p}}} \tag{10}$$

This statistical descriptor is a quantitative measure of accuracy of the MCS estimator.

3.2 Efficient Simulation by Importance Sampling

In structural reliability and risk assessment, sampling the indicator function often involves the analysis of complex models of large-scale structures and infrastructure systems, which may lead to prohibitive computational effort. Nonetheless, life-cycle analysis problems inherently require evaluating the evolution in time of the failure probability estimates. Most advanced simulation techniques rely on alternative formulations of the failure probability integral to reduce the variance of the reliability estimator.

In structural engineering and seismic assessment at infrastructure scale, Importance Sampling techniques have been successfully applied in risk analysis of spatially distributed infrastructure systems, specifically in the generation of ground motion maps of seismic hazard [12–14]. The accuracy can be largely improved by reformulating the failure probability in terms of an alternative sampling distribution $\psi_{\mathbf{Z}}(\mathbf{z})$, also referred to as proposal distribution or Importance Sampling (IS) distribution [11]:

$$p_E = \int_{\mathbf{z}} I[g_E(\mathbf{Z}) \leq 0] \cdot f_{\mathbf{Z}(t)}(\mathbf{z}) \cdot \frac{\psi_{\mathbf{Z}}(\mathbf{z})}{\psi_{\mathbf{Z}}(\mathbf{z})} d\mathbf{z} \tag{11}$$

The IS failure probability estimator is defined as the sample mean of the indicator function scaled by a suitable time-variant weighting coefficient $w_j(t)$:

$$\hat{p}_{IS}(t) = \frac{1}{n_j} \sum_{j=1}^{n} I_j \cdot w_j(t) \tag{12}$$

The j-th time-variant IS weight $w_j(t)$ is the ratio between the actual and sampling joint PDFs evaluated at each generated sample of the basic RVs $\mathbf{z}_j$:

$$w_j(t) = \frac{f_{\mathbf{Z}(t)}(\mathbf{z}_j)}{\psi_{\mathbf{Z}}(\mathbf{z}_j)} \tag{13}$$

In traditional MCS, samples are deemed to be generated based on a time-variant multivariate distribution and the sampling likelihood progressively shifts within the sample space throughout the system lifetime. Conversely to traditional MCS, the proposed IS methodology allows adopting a time-invariant sampling distribution $\psi_{\mathbf{Z}}(\mathbf{z})$, which leads to an efficient sample selection to effectively maps the sample space. Whilst traditional MCS in a life-cycle framework would require one simulation per observation time instant t, the proposed IS strategy requires a single simulation to evaluate the failure probabilities at time instants t by weighting each failed sample based on its time-variant likelihood of occurrence. Finally, the quantitative measure of accuracy of the IS reliability estimator can be evaluated based on the variance of the failure probability:

$$\mathrm{Var}\left[P_{E,IS}\right] = \frac{1}{n_j}\left(\int_{\mathbf{z}} \left\{I[g_E(\mathbf{Z}) \leq 0] \cdot \frac{f_{\mathbf{Z}(t)}(\mathbf{z})}{\psi_{\mathbf{Z}}(\mathbf{z})}\right\}^2 \cdot \psi_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z} - p_f^2\right) \tag{14}$$

The time-variant integral in this definition can be quantified numerically based on the sample samples variance based on the simulated samples of the basic RVs:

$$\mathrm{Var}\left[P_{E,IS}\right] \approx \frac{1}{n_j - 1}\left\{\frac{1}{n_j} \sum_{j=1}^{n}\left[I_j \cdot w_j^2(t)\right] - \hat{p}_{IS}^2(t)\right\} \tag{15}$$

The choice of a suitable proposal distribution is a key issue, which may allow mitigating the computational effort required to obtain sufficiently accurate estimates.

4 Application

4.1 Highway Network with Aging Bridges

The road network with three spatially-distributed vulnerable aging bridges shown in Figure 1 is investigated. The network includes three nodes originating and attracting daily road users' trips connected by main highways characterized by speed limits, each one with a single bridge, and secondary detour roads. Highways and detours are characterized by lengths l and maximum speed limits of v_{lim} reported in Figure 1. The OD modes of the daily traffic demands f_{OD} are herein modeled as symmetric triangular distribution over the range $f_{OD}^{mode} \pm 1500$ cars/hour, with $f_{12}^{mode} = 3500$, $f_{13}^{mode} = 3000$, and $f_{13}^{mode} = 4000$ cars/hour. Traffic demands are assumed to be perfectly correlated for a given OD pair (e.g., f_{12} and f_{21}) and statistically independent for different OD pairs (e.g., f_{12} and f_{13}). The aleatory uncertainties in the cost model are considered adopting positive-truncated normal distributions for the unitary users' costs Q_{DDC} and Q_{VOC} with CoV=0.30 and mean values 30.0 €/(cars×hour) and 0.20€/(cars×km), respectively. A symmetric triangular distribution for the discount rate γ_c with mode 2.0% and extremal values 2.0%±1.0% is considered.

Seismic events may lead to bridge closure due to structural collapse and the post-event damage state of the b-th bridge in the network is defined accordingly as a Boolean random variable S_b, i.e. S_b=0 for undamaged bridge and S_b=1 for collapsed bridge. The bridge damage states S_b are collected in a column vector $\mathbf{S}$. Travel Time TTT and Total Travel Distance TTD are obtained based on shortest path analysis.

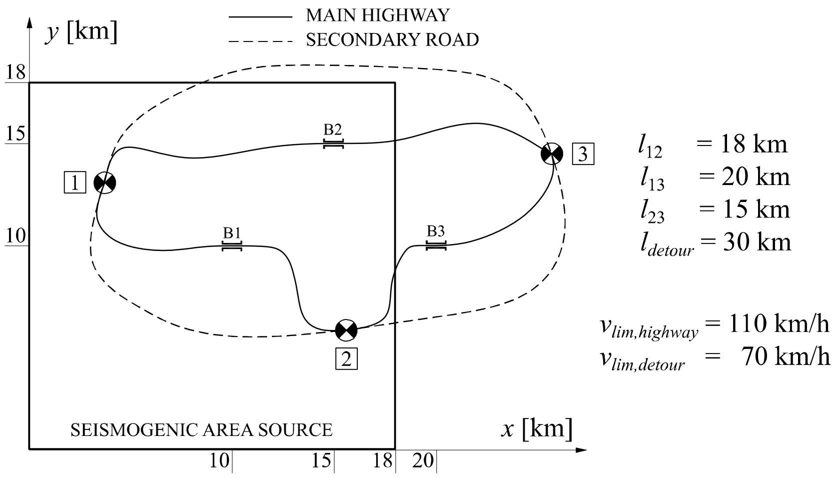

Fig. 1. Road network topology and seismic hazard scenario.

A time-variant lognormal model is considered to reproduce the life-cycle seismic fragility of each bridge, considering a constant standard deviation of the logarithm of the seismic capacity ζ=0.60 and statistical independency between seismic capacities of different bridges. The median seismic capacity $i_{b,m}$ with respect to structural collapse varies in time due to the progressive deterioration induced by environmental aging based on the following degradation law [4, 15]:

$$i_{b,m}(t) = i_{b,m0} - k \cdot t^2 \tag{16}$$

with median of the pristine structure $i_{b,m0}$=0.70g, 0.80g, and 0.90g and decay rate parameter k=15, 18, and 25×10^{-5} g/years^2 for bridges B1, B2, and B3, respectively.

Active faults in seismically hazardous region can be described by refined seismogenic models as linear faults or via area sources that account for distributed seismicity. In the applications of this paper, the occurrence probability of the epicentral distance R_{eb} is defined by assuming the earthquake epicenters are equally likely to occur at all locations of the prescribed squared area source represented in Figure 1. The recurrence of seismic events is modelled as a Poisson process with mean annual rate ν=0.05. The occurrence probability of earthquakes with moment magnitude M_w is governed by a truncated Gutenberg-Richter recurrence law with $m_{\min}$=4.3, $m_{\max}$=5.5, and b=1.242. The seismic intensity measure at each b-th bridge site I_b [g] is defined in probabilistic terms based on the GMPE in [16] with soil of type C based on Eurocode and normal faulting type. Finally, correlation between residuals of bridge pairs b_l and b_k depend on their relative distance:

$$\rho = \exp\left(-\frac{r_{b_h b_k}}{r_{ref}}\right) \tag{17}$$

with r_{ref}=10 km.

4.2 Life-Cycle Seismic Risk Estimates and Time-variant Accuracy

The mean annual rate ν_c of cost threshold exceedance C>750k€/day is estimated via traditional Monte Carlo Simulation carried out from 0 to 50 years every five years, with a total number of simulations per time instant n_t=11. An alternative simulation strategy is carried out assuming time-invariant lognormal fragility curves statistically independent for each bridge with median seismic capacity $i_{b,m}$ =0.30g and standard deviation of the logarithm of the seismic capacity ζ=1.00.

Figure 2a collects the results for traditional MCS (gray dots) and proposed IS strategy (continuous line) with a sample size of n_j=10^6 per simulation. Both sampling procedures lead to consistently similar results over the infrastructure lifetime, showing that the risk metric increases of about one order of magnitude in 50 years due to the progressive deterioration of the seismic capacity of each vulnerable network component. The convergence rate can be qualitatively assessed for each simulation based on Figure 2b, which shows the evolution of the risk estimates for traditional MCS (gray lines) and proposed IS strategy (black lines) in pristine conditions (thin lines) and at 50 years of network lifetime (thick lines).

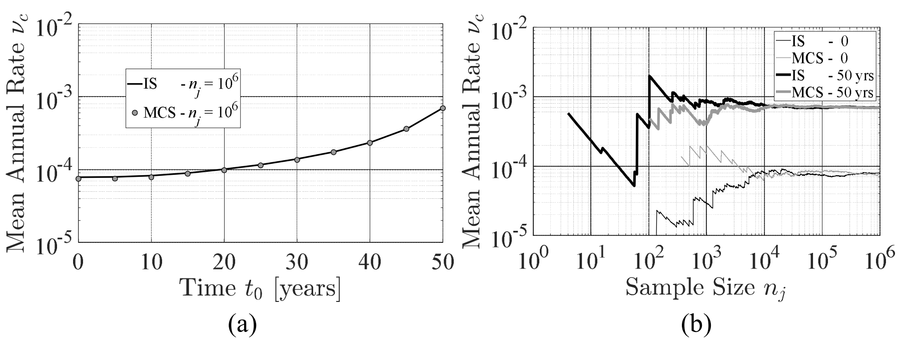

Fig. 2. Mean annual rate of cost threshold exceedance estimated by traditional MCS and proposed IS. (a) Time-variant risk estimates by MCS every five years (gray markers) and IS (black line). (b) Risk estimates at t_0=0 (thin lines) and 50 years (thick lines)

A quantitative measure of the convergence rate is also given by the numerical estimate of the CoV of the risk metric δ_ν, that is shown in Figure 3a versus the sample size. The accuracy of the estimates is directly proportional to the sample size n_j and the mean annual rate of cost threshold exceedance ν_c to be estimated. Both procedures show good convergence (i.e., δ_ν<0.30) with n_j>10^4. The actual computational cost of traditional MCS applied for life-cycle analysis problems should not only account for the sample size n_j of each simulation at discrete observation time instants t_0, but also the total number of observation time instants n_t. On the other hand, the proposed IS strategy relies on a single simulation with sample size n_j to obtain continuously in time the evolution of the risk metric based on suitable time-variant weighting coefficients. Figure 3b graphically proves the higher efficiency of the proposed IS procedure in terms of time-variant CoV of the risk metric with 10^5 total generated samples (continuous black line), in comparison with traditional MCS at n_t=11 observation times and sample size per simulation n_j=9091 (gray markers).

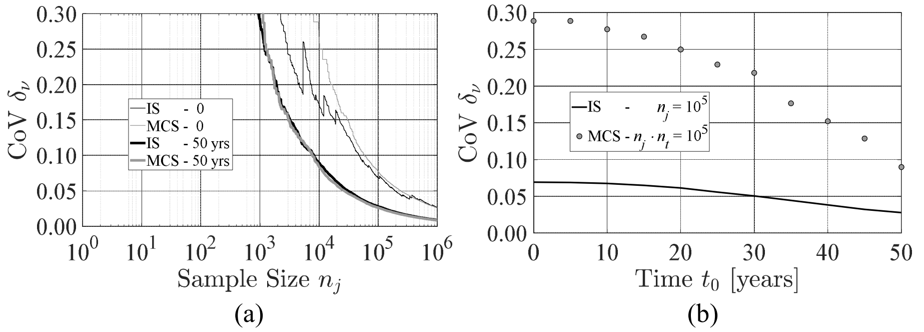

Fig. 3. CoV of mean annual rate of cost threshold exceedance estimated by traditional MCS and proposed IS. (a) CoV estimates at t_0=0 (thin lines) and 50 years (thick lines). (b) Time-variant CoV estimates by traditional MCS every five years (gray markers) and proposed IS (black line).

5 Conclusions

The paper presents a novel computational approach to efficiently estimate the life-cycle seismic risk of bridge networks based on Importance Sampling (IS). In the proposed approach, structural systems are efficiently simulated to account for the time-variant model uncertainties typical of life-cycle structural reliability problems. The potentialities of the proposed efficient sampling method emerge when time-consuming analyses are required to define the structural response and capacity of network components characterized by constitutive parameters that evolve in time due to deterioration processes. Potentially fruitful fields of application of the proposed method include the development of parametric analyses on the random variables involved in environmental hazard and other phenomena affecting in time the structural behavior, such as different climate change scenarios that may affect the deterioration rate and the rate of occurrence of extreme detrimental events. It is worth mentioning that one of the major drawbacks of the IS methodology is the tendency to produce inefficient probability estimates when a large set of basic random variables are involved in the sampling procedure [17]. Further research should be devoted to optimal selection strategies of the proposal distribution and assessment of feasibility limitations for large-scale problems with complex deterioration processes.

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