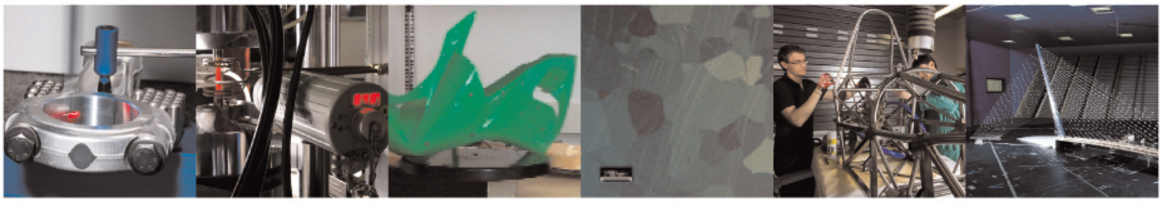

A cointegration-based approach for automatic anomalies detection in large-scale structures

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This is a post-peer-review, pre-copyedit version of an article published in Mechanical Systems and Signal Processing. The final authenticated version is available online at: https://doi.org/10.1016/j.ymssp.2021.108483

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Rev. 0

Mechanical Systems and Signal Processing

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--Manuscript Draft--

Manuscript Number:	MSSP21-1088R1
Article Type:	Standard Research Article
Keywords:	Structural health monitoring; environmental and operational variation; novelty detection; large structures; non-stationary time series; cointegration.
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Abstract:	In recent years, the development of structural health monitoring (SHM) solutions for the automatic evaluation of the health state of engineering structures is continuously growing. However, when considering real-world applications, structures are highly influenced by meteorological variations or human activities (like temperature, wind and traffic loading) which can overwhelm the changes induced by a damage. Thanks to its ability to remove the long-term trends from a set of variables of the same process, cointegration, a technique born in the field of econometrics, has been introduced about ten years ago in SHM applications as a valid method to project out the confounding influences, such as environmental and operational variations. Because of the few examples of implementation currently available, this paper provides an in-depth review of all the relevant aspects to consider when cointegration is used as damage detection strategy and data are acquired from real-world structures of large dimensions. The methodology is applied for the first time on a complex structure of a singular nature, i.e. the steel roof of the G. Meazza stadium in Milan, which consists of multiple modular elements referred to as rafts. The time series which measures the rotations of the rafts are used as input data for the development of the cointegration-based method. Then, Johansen procedure is adopted to create a unique feature from the multivariate dataset, namely cointegrating residual, in which the effects of environmental and operational variables are suppressed, while the effects due to damage remain evident. The obtained residual is therefore used for novelty detection by means of a control chart, demonstrating its effectiveness into identifying early signs of anomalies or modifications in the structure.

A cointegration-based approach for automatic anomalies detection in large-scale structures

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Abstract

In recent years, the development of structural health monitoring (SHM) solutions for the automatic evaluation of the health state of engineering structures is continuously growing. However, when considering real-world applications, structures are highly influenced by meteorological variations or human activities (like temperature, wind and traffic loading) which can overwhelm the changes induced by a damage. Thanks to its ability to remove the long-term trends from a set of variables of the same process, cointegration, a technique born in the field of econometrics, has been introduced about ten years ago in SHM applications as a valid method to project out the confounding influences, such as environmental and operational variations. Because of the few examples of implementation currently available, this paper provides an in-depth review of all the relevant aspects to consider when cointegration is used as damage detection strategy and data are acquired from real-world structures of large dimensions. The methodology is applied for the first time on a complex structure of a singular nature, i.e. the steel roof of the G. Meazza stadium in Milan, which consists of multiple modular elements referred to as rafts. The time series which measures the rotations of the rafts are used as input data for the development of the cointegration-based method. Then, Johansen procedure is adopted to create a unique feature from the multivariate dataset, namely cointegrating residual, in which the effects of environmental and operational variables are suppressed, while the effects due to damage remain evident. The obtained residual is therefore used for novelty detection by means of a control chart, demonstrating its effectiveness into identifying early signs of anomalies or modifications in the structure.

Keywords: Structural health monitoring; environmental and operational variation; novelty detection; large structures; non-stationary time series; cointegration.

1 Introduction

Structural Health Monitoring (SHM) describes a set of activities that can be followed to evaluate the health status of a structure or a system. These methods have been widely applied in many engineering branches due to its ability to automatically identify structural changes, improving structure reliability and life cycle management. Into details, a SHM procedure involves to continuously measure the system response through a sensor network and then to extract features that are directly linked to the physical properties of the structure itself. By means of a statistical analysis of these features, a damage detection strategy can be implemented, aimed at identifying anomalies or deviations from the normal behaviour [1].

One of the major technical issues that SHM must tackle when considering real-world structures under operational conditions is that measured signals are significantly affected by a set of external factors, commonly known as environmental and operational variables (EOVs). Among them, temperature is often found to be the dominant element on the structural response, as it directly affects both stiffness and boundary conditions [2], [3]. A practical example can be found in [4], where the authors demonstrated that temperature effects can account for 1-3% shifts in the natural frequencies of different grandstands of a stadium, a percentage shift which has the same order of magnitude of any potential structure degradations. Given that the most common monitored features are sensitive to real system variations as well as to changes of environmental and operational conditions, a successful SHM system must be able to distinguish between them, avoiding to generate false positive or false negative alarms of damage. To deal with this problem, data-based approaches for damage detection usually rely on two steps. A first approach looks for the variability induced by environmental and operational variables on structural response when operating under normal conditions.

Modelling and removing these effects from measured data is a process known as data normalization [5]. Only after this preliminary step, statistical classification tools can be adopted to identify the existence of any new fact which can be linked to the presence of damage [6].

One of the most diffused approaches in literature for data normalization relies on the use of regression models to establish a relationship between the monitored system parameters and those environmental or operational factors considered to have a relevant effect on the output data [7]. If the model properly fits the evaluated parameters, then the error between model predictions and observed values can be used as a damage sensitive feature. The main drawback these techniques is that to obtain good predictions all the influencing factors must be monitored and properly included in the model; although temperature measurements are relatively easy to perform, the optimal locations of temperature sensors may be difficult to determine. In practice, this increases system costs and complexity, especially as structure dimensions become significant. Another important aspect to consider is that regression models can be used only with stationary time series. As often SHM data from in-service structures are non-stationary [8], spurious regression may occur, leading to non-consistent Ordinary Least Squares (OLS) regression model parameter estimates.

To deal with these problems, other approaches where the measurement of the EOVs is not strictly required have also been studied. A simple method dating back to the end of 90s [9] provided data collection over a long time record (i.e. over a year) where all ranges of environmental or operational conditions have occurred, to define the normal behaviour of a system. Then, new measurements can be directly compared with the predetermined reference conditions. However, this requires the storage of large amount of data and the definition of normal conditions using such a long-time frame may reduce feature sensitivity to damage. Among the different approaches to filter out the effect of environmental and operational conditions on damage sensitive features [10], Principal Component Analysis (PCA) and Factor Analysis (FA) have attained considerable popularity. These multivariate statistical techniques are based on the identification of a linear subspace to which EOVs belong, in order to remove their effect on the monitored features. Using this idea, the methods proved to be very effective to identify the presence of damage in various SHM applications [11]–[13]. These projection approaches belong to a larger class of algorithms, associated with the concept of cointegration. Cointegration is defined as a property of some sets of non-stationary time series which share common trends, where a linear combination of them forms a stationary residual purged from those trends. This well-established method in econometrics, has recently been adopted for SHM purposes [14], [15], demonstrating that it can effectively remove long term trends from time series of monitored features. Moreover, the residual produced by cointegration can be used as an effective damage sensitive feature. In fact its stationarity allows the use of a control chart to identify any kind of anomalies. The parameters of the linear combination, referred to as a cointegrating vector, are often determined following methods from econometrics, like the Engle-Granger [16] or the Johansen procedures [17].

Thanks to the benefits described above, the interest of the scientific community for cointegration is quickly increasing. Many researchers agree on the possibility of applying cointegration for the removal of the confounding effects introduced by environmental and operational factors from SHM data and nowadays it is possible to find several examples. However, most of them refers to synthetic data, laboratory prototypes in controlled conditions or small components like composite plates [18]–[20].

Other works, although related to real structures, present some limitations. In [21] authors applied different statistical analysis techniques, including cointegration, to data acquired from a permanently monitored long-span arch bridge. However, the performance of the data detection methods are on a structure where a numerically simulated damage is imposed. He et al. [22] developed a “frequency-temperature-humidity” model based on cointegration theory to eliminate the effect of temperature and humidity on the fundamental frequency of a three-span concrete bridge. In this case, the accuracy of the model strongly depends on the availability of reliable measurements of the environmental parameters, which requires the use of multiple transducers distributed along the structure and the study of complex phenomena like thermal inertia or solar radiation. Also, although the effect of temperature and humidity on modal frequency is studied, it would be necessary to consider also the effect of other parameters, i.e. wind speed or traffic loading, which may have a relevant influence on a structure like a bridge. Finally, while authors effectively remove trends due to temperature and humidity, they did non test the sensitivity of the proposed method to identify changes caused by a damage. Shi et al. [23] presents a situation where an intact structure undergoes a well-defined and abrupt damaging event, something that is not always feasible for in-service structures. Finally, the majority of other research studies used the Engle-Granger procedure to determine the cointegrating relationship among variables, which works only for bivariate datasets [24]–[27].

In light of the few existing representative examples for large structures [28], [29], the major contribution of this paper is to provide a complete reference for the use of cointegration and statistical process control as a monitoring strategy for this type of structures. In such a situation, system complexity increases together with the number of features to be monitored and the number of external factors affecting the features, which is usually not known a-priori. At the same time, the number of sensors adopted for the measurements is typically very limited to allow low-cost systems for continuous monitoring. The use of multivariate cointegration therefore represents a powerful data normalization tool to effectively remove the environmental effects from the time histories of the measured features and to reveal even small variations due to other causes, like a damage. The overall methodology is applied on real datasets acquired by the recently developed SHM system of the steel roof of the G. Meazza stadium in Milan. The acquired signals measuring the tilt angles of the rafts in which this massive, one-of-a-kind structure is divided are directly used as input data for the cointegration approach. This considerably reduces the computation time and the uncertainty in the estimates associated to other kinds of damage sensitive features typically used for cointegration, above all natural frequencies or mode shapes. Johansen method for cointegration is therefore applied on a dataset of 10 months from a group of contiguous and equal rafts showing similar long-term response, mainly due to the influence of EOVs. This maximum-likelihood multivariate technique utilizes the input signals to construct the cointegration relationship from training samples, producing a stationary residual where common trends present into the original variables are suppressed. When considering the case of structures of considerable dimensions, the latter aspect offers the advantage of having a unique reference signal that intrinsically contains information about the health status of the different subcomponents or, equivalently, of the different parts of the structure itself. After the removal of the effects induced by EOVs using cointegration analysis, residual behavior is monitored through a control chart. Data from the following two months acquired by the SHM system are used to evaluate the ability of the proposed methodology for damage detection, demonstrating its effectiveness into identifying early signs of anomalies or structural modifications.

The structure of the paper is built as follows. First, the monitoring system of roofing structure of the G. Meazza stadium, together with the main characteristics of the acquired data, are described in Section 2. Section 3 provides an introduction to the mathematical formulation of the presented method, based on the concepts of stationarity of signals and cointegration theory. In Section 4, the algorithm is applied on the experimental data and the performance of this real scenario are evaluated, then paper concludes after some discussion.

2 Case study: the steel roof of the G. Meazza stadium

The suitability of the cointegration approach for the removal of the common trends due to environmental and operational conditions is assessed here through the application to a realistic case study, the monitoring of the steel roof of the G. Meazza stadium.

2.1 Description of the monitoring system

The G. Meazza Stadium is one of the most iconic football temples worldwide. Its peculiar structure has been modified in occasion of the 1990 Italy's World Cup, by adding a third level of grandstands and a steel roof.

To guarantee the safety of people attending events, a permanent SHM system has been progressively set-up during the years. Originally, a network of accelerometers was installed for the vibration measurements on the grandstands of the second and third tier [30]. More recently, a monitoring system for the roof of the stadium was developed too [31]. As visible from Figure *1*, this structure is composed by an upper steel truss supporting, by means of bolted joints called hangers, a lower steel truss made of modular structures referred to as "rafts". Potential sources of damage for this type of structure may be related to the failure of one or multiple bolted connections due to hidden corrosion phenomena. If this occurs, the weight of the raft redistributes over the remaining hangers causing a variation of the original trim. Therefore, a widespread network of tiltmeters has been installed to detect early signs of damage through the long-time monitoring of the inclination levels assumed by the structure.

Figure 1 The steel roof of the G. Meazza stadium (left) and a detail of bolted joints connecting upper and lower trusses (right)

To obtain a detailed picture of such a complex structure and at the same time maintain a reasonable number of sensors, 37 MEMS biaxial tiltmeters have been installed, one close to the centre of each raft. These sensors are characterized by a measuring range of ±10°, a resolution of 0.01% FS (0.002°) and a mechanical bandwidth of 18 Hz. The layout of the adopted system is shown in Figure *2*, where the tiltmeters are represented with coloured circles while the acquisition units are represented with coloured boxes (sensors are acquired by the acquisition unit having the same colour). Each tiltmeter has two measurement directions, referred to as channel A and channel B. The former measures the rotations around the X axis, while the latter measures the rotations around the Y axis, as indicated in Figure *2*. Positive rotations are shown according to by the right-hand rule.

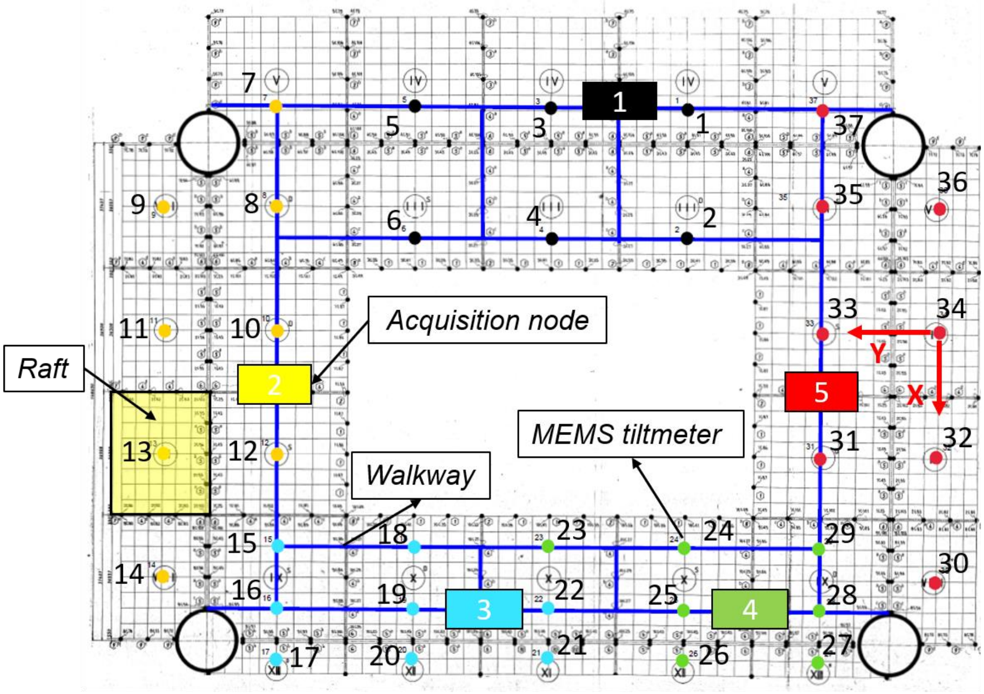

Figure 2 Layout of the measurement network of the roof

2.2 Main characteristics of the acquired data

The monitoring system of the roof of the G. Meazza stadium became fully operative by the beginning of the 2020. Due to the recent realization, the experimental data used in this work refers only to a period of one year, ranging from February 1st, 2020 to January 31st, 2021. To increase the confidence of detecting true novelties due to structural changes, cointegrating relations should be estimated, wherever possible, from a comprehensive time span and resolution of healthy training data under different environmental and operational conditions [23]. In this way, all the possible sources of variability characterizing the normal conditions of the in-service structure are covered. Accordingly, 80% of the entire dataset is used for training (corresponding to the first ten months of data) to capture a wide range of behaviours induced by EOVs. Simultaneously, the remining 20% of the full dataset (corresponding to the last two months of data) are used for testing, allowing to verify the goodness of the model on a statistically significant sample.

The tiltmeters raw signals are collected by the acquisition system every ten minutes. The daily mean of each channel is used as input time history for the cointegration procedure that will be illustrated in Section 4. The use of the daily mean of signals as time basis for the analysis is twofold. First, it allows to alleviate the short-term variability caused by daily cycles, which is the main source of heteroskedasticity in the data. As demonstrated in [23], this phenomenon can have a negative impact on damage detection algorithms based on statistical process control of residual series from regression, like linear cointegration. In fact, if the residual series retains seasonal changes in variance, this can severely compromise the effectiveness of damage identification by means of a control chart. Second, the long-term relationships between signals are still preserved, which are expected to be the ones which contain the most relevant information. Indeed, cointegration projects out components of data that correspond to these long-term trends, i.e., non-stationarity. As environmental variations usually manifest themselves on longer timescales than the dynamics of the structure that are sensitive to damage, the method appears to be well suited to SHM purposes.

Figure 3 shows the data acquired by all the sensors for the considered training period. Due to the unavailability of direct measures that allow to estimate the previous values of elastic deformation of the structure, the initial condition of the structure is set as a reference, i.e. the rotation at the beginning of the monitoring campaign is set equal to 0° for all the measurement channels. This aims for all the variables to be compared from the same conditions. Empty values in the data refer to periods where the acquisition system has been shut down due to maintenance activities. At a first glance it is possible to say that sensor-to-sensor and channel-to-channel differences exist, which are related to multiple aspects. First, the placement of each sensor is not exactly the same for all the rafts, due to problems of accessibility during the mounting phase. Also, only some rafts are very similar under a design point of view, implying different geometries and boundary conditions; furthermore, due to the huge size of the structure, different areas of the roof may be subjected to different environmental and operational conditions. At the same time, it is possible to observe that most of the signals shows common features, like a marked seasonality and a common long-time trend.

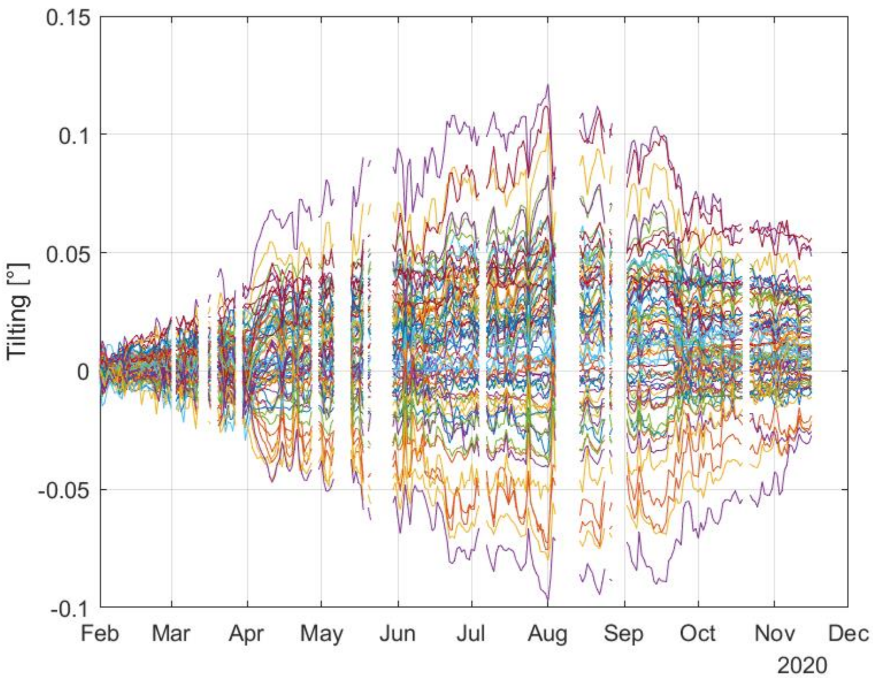

Figure 3 Tiltmeter signals recorded over the considered training period

3 Background theory on cointegration

3.1 Stationarity, non-stationarity and unit-root tests

The theoretical foundations behind the concept of cointegration are based on a clear understanding of the ideas of stationarity and non-stationarity of time series. Some important definitions extracted from [32] and helpful for a better comprehension of the cointegration procedure adopted in this work, are reported in the followings. A ‘strictly’ stationary process x_t is which the parameters to describe a statistical distribution remain the same as time passes by; that is, the multivariate distribution of the random variables x_{t1+h}, x_{t2+h}, …, x_{tn+h} is independent of h. Here, t_1, t_2, …, t_n represent any finite set of parameter values. However, this mathematical definition does not find application in engineering, since it is framed in terms of multivariate density functions, something that is difficult to be estimated from measured data. In case of signals obtained from SHM, the concept of stationarity should be reformulated in terms of quantities that can be estimated more easily. In such a case, it is preferable to refer to the definition of stationarity in ‘weak sense’, where a signal can be considered as ‘weakly’ stationary if its mean and autocovariances are not dependent on time. When this condition is not met, the signal is said to be non-stationary.

Time series models are used in econometrics to explain stationarity and non-stationarity of signals. By considering the common example of a first-order autoregressive process (AR(1)), a time series y_t is defined as:

$$y_t = a y_{t-1} + \varepsilon_t \tag{1}$$

Where ε_t is an independent Gaussian white noise process with zero mean and variance σ^2, also known as error or residual. Depending on the value assumed by the parameter a, three different scenarios can be distinguished. If $|a|<1$, the process oscillates around the mean value, therefore it is stationary. If $|a|>1$, the variance of y_t increases with time and approaches infinity; the time series becomes non-stationary and explosive. Finally, if $a=1$ (unit root), y_t moves up and down, behaves like a non-stationary process, but slowly. This is known as ‘random walk’ model. For this specific case, Eq.(1) becomes:

$$y_t = y_{t-1} + \varepsilon_t \rightarrow \Delta y_t = y_t - y_{t-1} = \varepsilon_t \tag{2}$$

Although the process is non-stationary, its first difference is stationary; in econometrics jargon, it is said to be integrated of order one, denoted as $y_t \sim I(1)$. From the above considerations, one can understand how the verification of stationarity simply reduces to test whether the value of $|a|$ is strictly less than 1. This is what is typically done in unit root tests [33], [34]. Among them, the Augmented Dickey-Fuller (ADF) test [35] has become one of the most popular. This test involves fitting the data to a the following time series model:

$$\Delta y_t = \rho \Delta y_{t-1} + \sum_{j=1}^{p-1} b_j \Delta y_{t-j} + \varepsilon_t \tag{3}$$

Here, it is assumed that data are generated by an AR(p) process and a parametric correction for higher-order correlations is introduced by adding p lagged difference terms $\Delta y_{t-j}=y_{t-j} - y_{t-j-1}$. To pass from the traditional AR(p) model to the current one, the following substitutions should be made: $a_1=1+\rho+b_1$, $a_n= -b_{n-1}+b_n$, for n=2, …, p-1 and $a_p= -b_{p-1}$, where a_j are the AR(p) model coefficients.

It can be demonstrated [14] that, for y_t to be integrated of order one, it is necessary that $\rho=0$ in Eq.(3). Therefore, the ADF procedure provides to estimate the parameters of Eq.(3) by the least-squares methods and then tests the null hypothesis of $\rho=0$. The test statistic:

$$t_\rho = \frac{\hat{\rho}}{s(\rho)} \tag{4}$$

should be compared with the critical values from Dickey-Fuller distribution. Again, if $|t_\rho| < |t_{cr,a}|$ the null hypothesis is accepted, meaning that y_t has a unit root and is I(1). If the null is rejected, the test is repeated for Δy_t, if the hypothesis is then accepted y_t is a non-stationary series, integrated of order two (I(2)). This procedure can be continued until the integrated order of the process is obtained.

When testing for a unit root it is important to properly specify the null and the alternative hypothesis depending on the trend properties assumed by the considered data. The model of Eq.(3) could be extended to also include constants (drifts), deterministic trends, or both:

$$y_t = \mu + \delta t + \rho \Delta y_{t-1} + \sum_{j=1}^{p-1} b_j \Delta y_{t-j} + \varepsilon_t \tag{5}$$

In this case the null hypothesis for the time series to be integrated of order one should be modified to include $\mu=\delta=0$. Additional considerations on these specific cases can be found in [35].

One final issue associated to the specification of test regression models is the number of lagged difference terms p to be added. If p is too small, the remaining serial correlation among residuals could bias the test results. At the same time, if p is too large, the power of the test reduces. In virtue of this, the procedure employed in the paper for the optimal lag selection, that involves the use of the Akaike Information Criterion (AIC) [36] as preferential method for ADF test and for cointegration, has been implemented following the guidelines suggested in [37]. Successfully applications of this approach can be found also in other SHM researches [18], [38]. Here, the data-dependent lag selection procedure results in stable size for both ADF and cointegration tests and minimal power loss. In addition, although the theory behind AIC might seem not strong enough to indicate which lag value is the optimal, when this indicator is used for small samples size cases, as in the present study, results become more accurate [39].

The AIC statistic can be estimated by AIC $=n\log(\sigma_{\varepsilon t}^2) + 2p$, where $\sigma_{\varepsilon t}^2$ is the variance of the error term εt, p is the lag length examined and n is the number of observations of the series under consideration. The procedure of lag-length specification starts with the estimation of the maximum lag number (p_{max}), according to Schwert formula [40], $p_{max}= [12 (n/100)]^{0.25}$, where n indicates the number of observations. Next, the ADF model is employed and run for multiple lag values, from zero to p_{max}. During each run, the AIC statistic is calculated and the lag length corresponding to the lowest AIC is employed as the most appropriate one. Resuming, ADF test checks if the null hypothesis of a unit root (non-stationary, I(1) time series) can be rejected in favour of the alternative (stationary, I(0) time series). This occurs when the t-statistic exceeds the critical value defined for a given significance level α. The assessment about stationarity or non-stationarity of signals constitutes the first step for the cointegration procedure illustrated in the next section.

3.2 Cointegration basics

For a better comprehension of the theoretical principles behind the cointegration method adopted in this work, some references, drawn from the practical-oriented texts [14],[41], are reported in the followings. Readers who are interested in more mathematically rigorous discussions could refer to [42].

Let $Y_t=(y_{1t}, y_{2t}, \ldots, y_{nt})^T$ be an (n x 1) vector of non-stationary, I(1) series. The series are cointegrated if there exists a (n x 1) vector $\beta=(\beta_1, \beta_2, \ldots, \beta_n)^T$ such that:

$$\beta^T Y_t = \beta_1 y_{1t} + \beta_2 y_{2t} + \cdots \beta_n y_{nt} = u_t \sim I(0) \tag{6}$$

If this is the case, vector β^T is called cointegrating vector and the linear combination $\beta^T Y_t$ describes the long-run equilibrium relationship between time series. Ideally, if all the variables of Y_t are in equilibrium, the specific linear constraint $\beta^T Y_t = 0$ occurs, i.e. the error term u_t is zero. In more practical situations, variables could deviate from the long-term equilibrium relationship for many reasons and this disequilibrium condition is reflected by the quantity u_t. It should be noted that multiple cointegrating relations could also exist. If Y_t includes a set of n variables, there may be r linearly independent cointegrating vectors, where $0<r<n$ such that:

$$B^T Y_t = U \rightarrow \begin{pmatrix} \beta_1^T Y_t \\ \beta_2^T Y_t \\ \vdots \\ \beta_r^T Y_t \end{pmatrix} = \begin{pmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{rt} \end{pmatrix} \sim I(0) \tag{7}$$

Where B is a (n x r) matrix forming a basis for the space of r cointegrating vectors. The r stationary linear combinations $u_{rt} = \beta_r^T Y_t$ represent the cointegrating residuals which are obtained by projecting the variables space Y_t on the cointegrating space B.

Tests for cointegration essentially determine if one or more stationary linear combinations among the considered variables exist. However, before running any kind of tests, two major constraints should be respected: the considered time series must share common trends and must be integrated of the same order. While the first condition is usually met for variables from the same process or system (they are driven by the same set of influencing factors), the order of integration must be checked with a stationarity test. Augmented Dickey-Fuller (ADF) test is typically used for this purpose. Once it has been ascertained that these two requirements are fulfilled, the cointegrating vector has to evaluated, capable of producing the most stationary combination of the variables. Two common approaches could be adopted. The first is the Engle-Granger two-step estimation procedure [16], usually employed when there are only two process variables, while the second is the Johansen procedure [17], a maximum likelihood multivariate estimation procedure able to determine the existence of cointegration and the relative number of cointegrating relationships. Due to the solid theoretical foundations and due to the possibility of handling multiple variables, the Johansen procedure is adopted in the present work.

3.3 Johansen procedure

The Johansen procedure uses a maximum-likelihood approach for finding stationary combinations of non-stationary I(1) variables. The variables of interest are arranged in a vector error-correction model (VECM), that may be seen as an extension of the well-known Vector Autoregressive (VAR) model [42]. Consider a VAR(p) model for the (n x 1) vector Y_t:

$$Y_t = \phi D_t + \Pi_1 Y_{t-1} + \cdots + \Pi_p Y_{t-p} + \varepsilon_t \tag{8}$$

where ϕD_t contains deterministic terms (i.e. no constant, constant only, constant plus time trend). As discussed for the univariate case of section 3.3, the VAR model is stable if:

$$\det\left(I_n - \Pi_1 z - \cdots - \Pi_p z^p\right) = 0 \tag{9}$$

has all the roots outside the complex unit circle. If Eq.(9) has unit roots, then some or all the variables of Y_t are I(1) and may also be cointegrated. To make the cointegrating relations more visible, the VAR(p) model is transformed to the vector error correction model (VECM):

$$\Delta Y_t = \phi D_t + \Pi Y_{t-1} + \sum_{j=1}^{p-1} B_j \Delta Y_{t-j} + \varepsilon_t \tag{10}$$

where ΔY_t is the vector of the first differences, $\Pi = \Pi_1 + \ldots + \Pi_p - I_n$ is the long-run impact matrix and describes the long-term equilibrium between variables, B_j matrix is the short-run impact matrix and provides short-run adjustments that return the process to equilibrium if any drift occurs and ε_t is a normally distributed noise process $\varepsilon_t \sim N(0,[\Sigma])$. Eq.(10) represents the multivariate analogue of Eq.(3).

In the VECM formulation, ΔY_t and its lagged terms ΔY_{t-j} are assumed to be I(0). The term ΠY_{t-1} is the only one which potentially accounts for I(1) series. Therefore, for ΔY_t to be I(0), ΠY_{t-1} should be also I(0), i.e. ΠY_{t-1} must contain the cointegrating relations if they exist.

If the VAR(p) model has unit roots (z=1), then from Eq.(9) $\det(\Pi) = 0$, namely the matrix Π becomes singular. This means that it has a reduced rank, say $\text{rank}(\Pi) = r < n$. There are two distinct cases to consider:

- $\text{rank}(\Pi) = 0$, which implies that $\Pi = 0$, Y_t is I(1) and no cointegrating relationships between the variables of Y_t exist. This leads to the reduction of the VECM to a VAR(p-1) model in first differences.

- $0 < \text{rank}(\Pi) = r < n$, which implies that Y_t is I(1) with r linearly independent cointegrating vectors and n - r common stochastic trends.

In the latter case, matrix Π is rank-deficient and it can be written as the product of two matrices $\Pi = AB^T$, where A and B are (n x r) matrices with rank(A) = rank(B) = r. The rows of B^T form a basis for the r cointegrating vectors and the elements of A distribute the impact of the cointegrating vectors to the evolution of ΔY_t. The VECM of Eq.(10) then becomes:

$$\Delta Y_t = \phi D_t + AB^T Y_{t-1} + \sum_{j=1}^{p-1} B_j \Delta Y_{t-j} + \varepsilon_t \tag{11}$$

Where $\beta^T Y_{t-1} \sim I(0)$ since B^T is the matrix of r cointegrating vectors. As anticipated in section 4.1, the stationary linear combinations $u_{rt} = \beta^T Y_t$ represent the cointegrating residuals obtained by projecting the (n x 1) vector of non-stationary time series in Y_t on the cointegrating vectors.

Since the rank of the long-run impact matrix Π gives the number of cointegrating relationships in Y_t, Johansen formulates two likelihood ratio (LR) statistic tests based on the rank of matrix Π, namely trace statistic and maximum eigenvalue statistic [17]. These tests are based on the estimated eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ of the matrix Π. These eigenvalues can be interpreted as the squared canonical correlations between ΔY_t and ΔY_{t-1} corrected for lagged ΔY_t and D_t and so they lie between 0 and 1.The rank of Π is equal to the number of non-zero eigenvalues of Π.

Johansen's LR statistic tests the nested hypothesis $H_0(r)$: $r = r_0$ vs $H_1(r)$: $r > r_0$. The LR statistic called trace statistic is the most used one and is given by:

$$LR_{trace}(r_0) = -T \sum_{i=r_0+1}^{n} \ln(1 - \lambda_i) \tag{12}$$

Where T stands for the effective sample size. Therefore:

- If rank(Π) = r_0 then $\lambda_{r0+1},\ldots,\lambda_n$ should be all close to zero and $LR_{trace}(r_0)$ should be small since $\ln(1-\lambda_i) \approx 0$ for $i > r_0$.
- If rank(Π) > r_0 then some of $\lambda_{r0+1},\ldots,\lambda_n$ will be non-zero (but less than one) and $LR_{trace}(r_0)$ should be large since $\ln(1-\lambda_i) << 0$ for some $i > r_0$.

The asymptotic null distribution of $LR_{trace}(r_0)$ is not chi-square but a multivariate version of the Dickey-Fuller unit root distribution which depends on the dimension $n–r_0$ and the specification of the deterministic terms. Critical values for this distribution are tabulated in [43] for $n–r_0 = 1,\ldots,10$.

The test uses a sequential procedure to determine the number of cointegrating vectors, whose relevant passages are explained here.

- First, test H_0:r=0 against H_1:r>0. If the null is not rejected, this means that there is no cointegration among the variables of Y_t. If the null is rejected, it is concluded that there is at least one cointegrating vector.
- Proceed to test H_0:r=1 against H_1:r>1. If the null is not rejected, this means that there is only one cointegrating vector. If the null is rejected, it is concluded that there are at least two cointegrating vector.
- Repeat the sequential procedure until the null is not rejected.

Once the relevant cointegrating vectors have been found, the choice of the 'best cointegrating' vector is evaluated by running an ADF test for each of the cointegrating residuals. The more the magnitude (in absolute terms) of the t-statistic, the most stationary the residuals will be. Selecting the most stationary cointegration residual, it is likely that the common trends shared between series are successfully purged.

3.4 Summary of the main steps to apply cointegration to SHM applications

As a conclusion of this chapter, the main steps for the application of the cointegration procedure to SHM problems are summarized below.

a. Define a set of measured variables Y_t which belong to the same process and share common trends.

b. Run an ADF test for each signal to determine the order of integration.
c. For the I(1) variables, fit a VAR(p) model and determine the most suitable model order p using the Akaike information criterion (AIC) or others similar. Then, convert the VAR(p) model in a VECM(q) model with lagged difference terms, where q=p-1.
d. Perform likelihood ratio (LR) tests (trace statistic and/or maximum eigenvalue statistic) for the rank of the long-run impact matrix Π to determine the number of linearly independent cointegrating vectors (r).
e. Obtain the r cointegrating residuals u_{rt} by projecting the monitored variables on the cointegrating vectors, $u_{rt}=B^TY_t$. In addition, the remaining variables for the resulting cointegrated VECM of Eq.(11) can be estimated by maximizing the likelihood of observing the correct noise error sequence $\varepsilon_t \sim N(0,[\Sigma])$.
f. The cointegration vector providing the most stationary cointegrating residual is determined by running an ADF test; then it can be employed for damage detection purposes. This is achieved within cointegration by establishing the cointegration vector based on the undamaged data and then by projecting the remaining/future data on the cointegration vector.

4 Common trends removal and anomalies detection using SHM data

The methodology used to implement a damage detection strategy based on cointegration for the inclination signals acquired by the monitoring system of the roof of the G. Meazza stadium is described here. As stated in Section 2.2, this represents a very challenging task, which involves a large number of monitored structures having different geometries, boundary conditions and external influences.

In order to decrease the complexity of a problem characterized by many sources of variability that are effectively impossible to be controlled individually, the following analysis have been conducted on a reduced group of measured signals which came from rafts located in contiguous positions and which are very similar under a design point of view. In this way, inclination data are measured from similar structures, influenced by the same set of environmental and operational (EO) conditions. This could make it easy to establish cointegrating relations between the variables which are expected to share common trends. In a similar fashion, the procedure can be eventually extended to other groups of rafts showing similar behaviours. The inclination signals in question, representing the daily measurements recorded over the monitoring period of ten months, are reproduced in Figure *4*. The sensor positions and channel names are the same as described in section 2.1. It can be observed that all the channels exhibit the same pattern over time, independently from measurement direction. Signals follow an increasing trend up to summer and then there is an inversion to the original reference position; they can therefore be considered as non-stationary signals. The only exception is represented by the channel referred to as ‘05B’, which shows the same characteristics of the previous, but with an inverse pattern. The reasons behind the out-of-trend behaviour of channel ‘05B’ lie primarily in the construction features that characterize the rafts. Indeed, although they might seem similar under a geometrical point of view, there exist some differences in terms of boundary conditions, connections and constraint positions, which may induce different responses in terms of rotation angles. Another potential cause is the influence of the upper bearing structure, made of four massive trusses, which are not perfectly symmetric and do not provide a uniform response along their length.

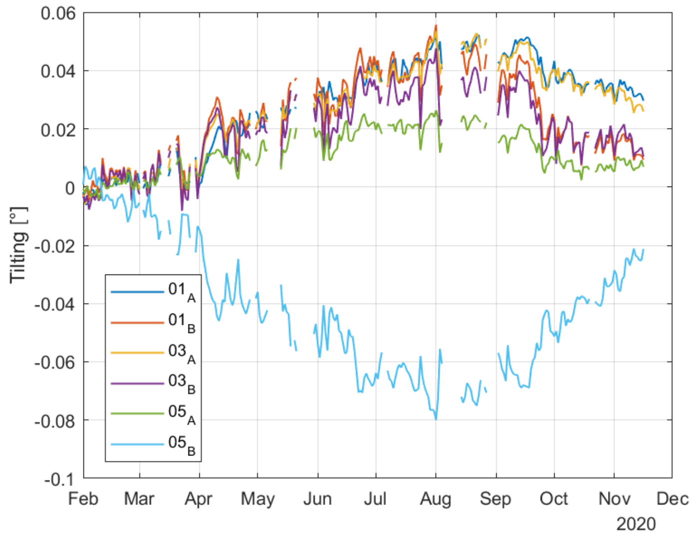

Figure 4 Tiltmeter signals from a set of contiguous and similar rafts selected for cointegration analysis

4.1 Influence of temperature on tiltmeters data

As signals are non-stationary, the first step is to identify the probable sources behind this non-stationarity. A previous study on the roof of the stadium [31] demonstrated that the nature of such a behaviour is mainly related to the influence of temperature, while other variables as wind speed or relative humidity have a minor effect. Since temperature data are available from a weather station located close to the stadium and directly managed by the Regione Lombardia authority, the influence of temperature on tilt measurements is also investigated here as a potential cause driving the non-stationary behaviour of the signals. In order to do this, inclination data are compared to temperature ones. The example of channels '01B', '05A' and '05B' is presented here for illustrative purposes. More specifically, Figure *5* shows the time records of tiltmeters data plotted against temperature. A marked linear dependence between inclination (Y) and temperature (X) can be assumed. Similar relations are also observed for the remaining channels. These are resumed in Table *1*, where the linear correlation coefficient ρ (or Pearson's coefficient) provides evidence of a strong linear correlation ($\rho > 0.80$) between temperature and inclination for all the considered channels. In addition, Figure *6* provides a plot of the previous tiltmeters channels, plotted against each other. Once again, pairwise relationships are characterized by a pronounced linearity.

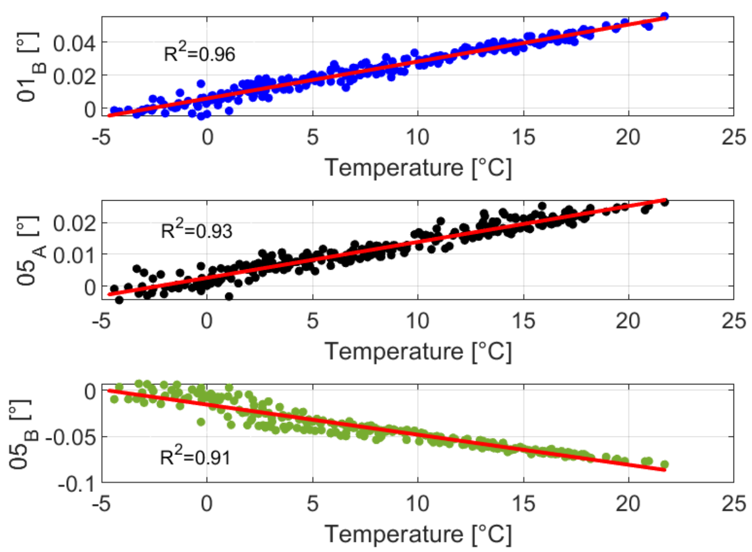

Figure 5 Inclination data for channels '01B','05A' and '05B' plotted with respect to temperature. The red line provides the best linear fit based on OLS.

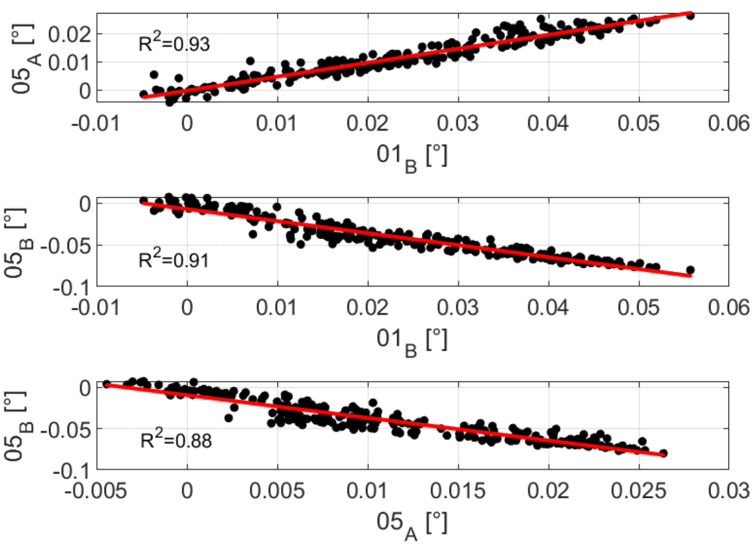

Figure 6 Inclination data for channels '01B','05A' and '05B' plotted against each other. The red line provides the best linear fit based on OLS.

	01A	**01B**	**03A**	**03B**	**05A**	**05B**
Temp	0.807	0.981	0.840	0.955	0.967	-0.952

Table 1 Pearson's linear correlation coefficient between temperature and tiltmeter data

It is quite evident that series share common linear trends, mainly associated with the long-term effect that temperature imposes on inclination signals. However, the use of linear regression models to describe the behaviour of physical quantities from complex structures may be inaccurate due to the following aspects. First, although air temperature plays a dominant role, it may not be the only factor affecting the variability of tilting angles. This aspect cannot be investigated here due to the lack of available data from additional environmental and operational variables. Then, temperature can show significant differences from point to point due to local sun radiation and exposure to environmental conditions, making the creation of a reliable model a challenging issue. In this case, air temperature can be used only to provide a rough estimation of the long-term relationships. Moreover, a critical aspect in the application of linear regression (LR) methods has to be found in the distribution of the time series, which is required to be stationary. If signals are non-stationary, the sample mean and variance cannot be used to infer the distribution characteristics of random variables at each data point. Therefore, the linear regression analysis of non-stationary series has the problem of spurious regression, which may lead to unacceptable reliability of the regression model (non-consistent OLS estimates). On the contrary, if a cointegration relationship exist between the signals, despite each of them is non-stationary and has statistical moments which change over time, there always be a combination which is stationary, i.e. having stable statistical moments. Therefore, cointegration produces super-consistent OLS estimates even in case of non-stationary processes. Last but not least, the development of a monitoring strategy based on LR methods would require defining as many models as the number of involved variables, which could be not very straightforward as the system complexity increases.

4.2 Cointegration approach on tiltmeters data

Cointegration analysis is adopted in this work as a response to the major limitations of linear regression methods illustrated above. More specifically, the aim here is to use cointegration to create a feature that becomes insensitive to the EOVs-induced variations, but still sensitive to damage or abnormal modifications in the properties of the structure. This technique offers many advantages for SHM applications [15], since it

can be performed without including measured EOVs (it is an unsupervised method) and it allows one to merge multiple signals to create a single feature which is representative of the system under observation.
Another aspect worthy of consideration is that, being the relationships between cointegrated variables (i.e. tiltmeter series) as well as the relationships between these variables and the temperature linear, the adoption of linear cointegration for the following analysis is adequate. Moreover, as anticipated in section 2.2, the choice of using the daily mean of signals as time basis produces slowly varying trends and reduces heteroskedasticity in the data, making the use of non-linear cointegration unnecessary.
The procedure applied in this example provides to test the input variables for cointegration. Then, only those which are cointegrated can be combined through the Johansen procedure to create a stationary residual. This stationary linear combination should be established using a training set of data able to cover as well as possible all the different behaviours assumed by the structure under normal operation. The projection of new data onto the linear combination should produce a feature that remain stationary as soon as the structure remains undamaged (even though EOV fluctuations occur) but should become non-stationary in presence of damage.

4.2.1 ADF test on level signals

From the analysis performed in the previous section, it was shown that the main driver of tilting variability is temperature. In addition, as measured signals share common trends which are directly or inversely proportional, it is likely to expect that they could be combined through cointegration to form a stationary linear combination. To satisfy cointegration assumptions, all variables included in the model must be I(1). For this, the six tiltmeter channels that have been selected for the analysis are tested for the presence of a unit root using the ADF test. For each of them, the regression model of Eq.(3) is applied and maximum lag length p_{max} is originally set to 16, according to Schwert formula [40]. Then, the ADF model is run for different lag values, from 0 to $p_{\max}$, and AIC statistic is calculated at each iteration. The minimum value for AIC is reached with a lag equal to 2 for all the series. Results from the ADF test with optimal lag value p=2 are summarized in Table *2*. As can be seen, since the ADF t-statistic never exceed, in absolute terms, the 5% critical value for all the time series, the test fails to reject the null of a unit root at a 5% significance level. That is, all signals are non-stationary and integrated of order one.

Variables	**ADF t-stat**	**5% crit value**	**Stationarity**	**Integration order**	**DW statistic**
01A	-0.097	-1.942	NO	1	2.055
01B	-0.848	-1.942	NO	1	2.048
03A	-0.226	-1.942	NO	1	2.059
03B	-1.082	-1.942	NO	1	2.052
05A	-0.929	-1.942	NO	1	2.057
05B	-0.456	-1.942	NO	1	2.047

Table 2 ADF test for tiltmeter series

To check the goodness of the proposed outcomes, residuals analysis is carried out. The Durbin-Watson (DW) statistic is calculated to test for residuals serial correlation, and it is presented as the last column of Table *2*. The DW statistics are all close to 2, indicating the absence of any kind of correlation. This is further confirmed by the graphs of Figure 7 where, for the representative example of channel ‘01B’, the autocorrelation function (ACF) and the plot of residuals assumes the typical behaviour of an independent Gaussian white noise process.

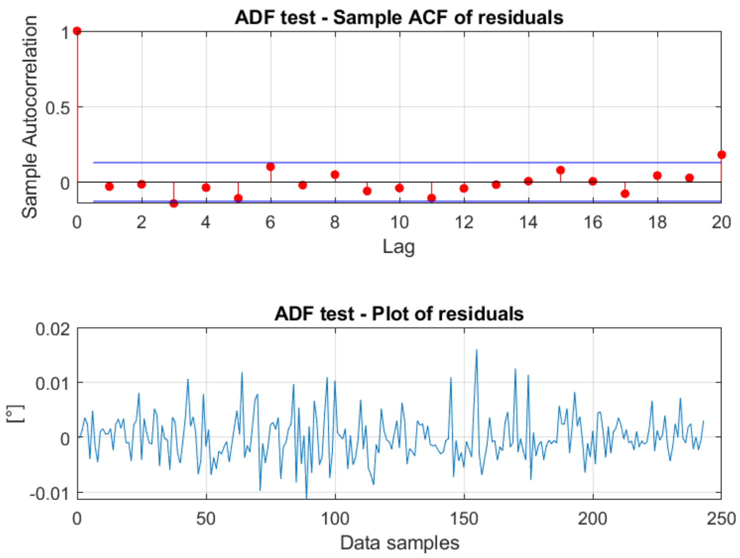

Figure 7 ACF and plot of residuals for channel 01B

4.2.2 Johansen's procedure to cointegration

According to the steps described in section 3.4, Johansen's procedure requires to fit the variable in question to a VAR model. Since a deterministic trend could not be clearly observed in the experimental data, the adopted model is specified like the one of Eq.(10), but with the term ϕD_t omitted. As for the ADF test, the most appropriate lag length p for the VAR(p) model is identified using the AIC statistic, running multiple least squares fittings for different lag numbers. In this case, it turns out that the minimal lag length p is equal to 3. Then, to make the cointegrating relations explicit, the VAR model is transformed into the VECM(q) model of Eq.(11), where q=p-1=2.

Given the time series arranged into VECM notation, the likelihood ratio (LR) test, using trace statistic, is carried out to indicate the number of possible cointegrating vector for that set of variables. The test depends upon the rank of the long-run impact matrix Π and assesses the null hypothesis H_0 of cointegration rank equal to r among the time series in Y_t against the alternative H_1 of cointegration rank higher than r. The results of the sequential procedure implemented for determining the number of cointegrating vectors are reported in Table *3*. The summary shows that the test rejects (h=1) a cointegration rank of 0, 1 and 2, but fails to reject (h=0) a cointegration of rank 3. So, the inference is that there are 3 cointegrating vectors.

r	h	Trace stat	5% crit value	pVal	Eigval
0	1	158.95	83.94	0.001	0.23
1	1	96.23	60.06	0.001	0.19
2	1	45.16	40.17	0.015	0.09
3	0	21.57	24.27	0.106	0.05
4	0	9.80	12.32	0.127	0.02
5	0	3.58	4.13	0.069	0.01

Table 3 Summary of sequential procedure for determining the number of cointegrating vectors

Cointegrating residuals can be obtained by simply projecting the original series on the cointegrating vectors just found. Figure 8 shows the linear combinations created by the Johansen's procedure from the training dataset. No immediate trend can be observed in all the signals that resemble white noise processes. This means that common induced trends, especially those due to temperature, have been successfully removed.

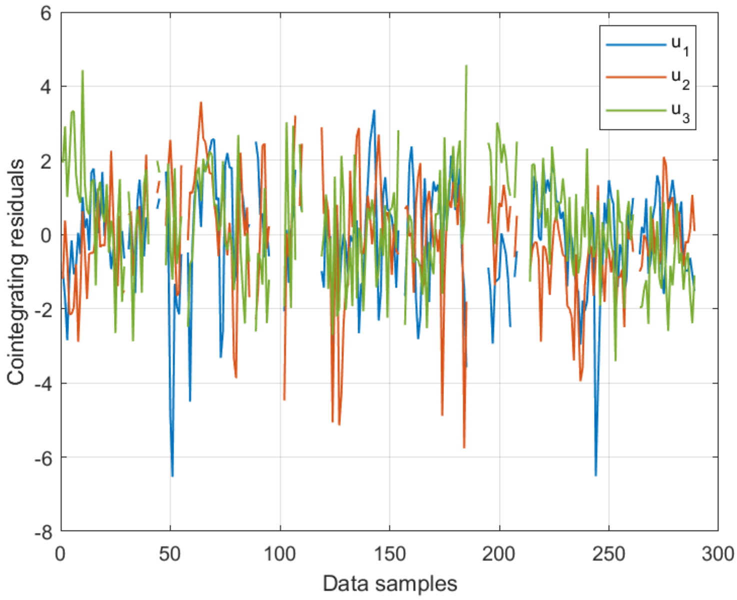

Figure 8 Cointegrating residuals created by Johansen's procedure

To assess stationarity also in a quantitative way, then ADF test is performed on the residuals obtained from the three projection cases. Again, the degree of stationarity is evaluated using the ADF t-statistic. Table *4* shows test results when a lag p=2 is used. In this case, even though all the series provide strong evidence for stationarity, the first cointegrating residual displays the largest negative value of the t-statistic and so it represents the most stationary combination. This is also in agreement to what has been found in [14]. In such a case, the cointegrating vector was $\beta_1 = [80.94\ -154.26\ -242.70\ 598.53\ -404.70\ -17.09]^T$.

Variables	ADF t-stat	5% crit value	Stationarity
u1	-7.25	-1.942	YES
u2	-6.13	-1.942	YES
u3	-5.05	-1.942	YES

Table 4 ADF test on cointegrating residuals

4.2.3 Cointegrating residuals and novelty detection

To demonstrate the effectiveness of Johansen's procedure for cointegration within SHM applications, the most stationary linear combination obtained above is used as a damage sensitive feature to infer the health status of the involved structures. Indeed, the continuing stationarity of the combination when new data is projected on it can be used as an indicator that the structure continues to operate in normal conditions. Figure 9 shows the cointegrating residual obtained by projecting onto β_1 the inclination signals from both training and testing data. A control chart method in this case was used to detect the presence of abnormalities more efficiently. The confidence interval was set to 99%, which corresponds to the range $\mu \pm 2.58\sigma$, where μ and σ are the mean and the standard deviation of the cointegrating residual obtained from training data.

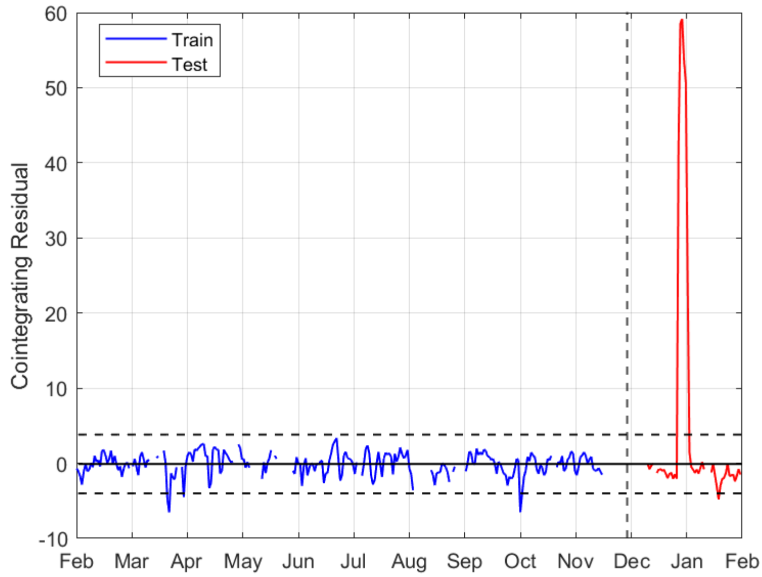

Figure 9 Cointegrating residual of tiltmeter signals. Training data are in blue, while test data are in red colour. Horizontal black dashed lines refer to the upper and bottom thresholds, which are $\pm 2.58\sigma$ from residual mean.

Starting from the training region, the residual remained stationary, with approximately 95% of the observations inside the confidence bounds, something pointing out a process under control. Outliers were only a few and they did not hide any real alarm source. Moving to the testing region, it can be observed that the residual remains within the predetermined statistical thresholds, except for a few days (from December 28th to January 2nd, 2021) where an important deviation from the stationary behaviour is observed. This is further confirmed by the plot of level signals in Figure 10.

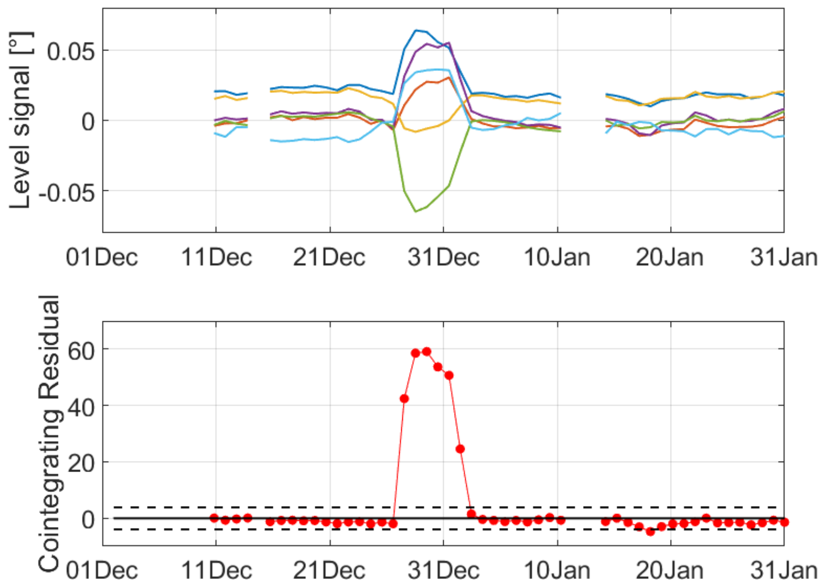

Figure 10 Time window around the snowfall of December 28th: original tiltmeters data (up) and cointegrating residual (down).

This phenomenon is related to the fact that during the night of December 28th there was a snowfall in Milan, which continued throughout the day. As a result, about 20 centimetres of snow progressively accumulated on the roof and the resulting load induced a change in the static trim of the structure. Given that the rotation of each raft was influenced in a different manner depending on the height of the snow, the cointegrating relation established from training data was no more respected and, consequently, residuals were projected far from the other observations. Nevertheless, as soon as the snow melted away, the process returned within the confidence limits in the following days and behaved like a stationary signal, meaning that the structures have gone back to their normal conditions.

This example demonstrates how cointegration, thanks to its ability to generate a stationary residual, helped to better identify a change in the behaviour of the structure with respect to the case of just applying a control chart on raw inclination signals. In fact, still referring to Figure 10, the values assumed by the cointegrating residuals during the snowfall were about 60 times higher than the ones assumed during normal operating conditions. On the contrary, if one consider only original tilt data, the sensitivity to abnormal conditions reduced, being this difference only 2-3 times higher and therefore comparable to the values assumed throughout the year due to environmental and operational variations (see Figure *4*).

Due to the promising results and given the authors willingness to test the flexibility and the repeatability of the proposed method, the same procedure has been applied to other four channels (namely ‘32A’,’32B’,’34A’,’34B’) still coming from two similar and contiguous rafts, but located in a different area of the roof. Readers can refer to the layout of Figure *2* to identify the position of the rafts within the roof of the stadium. In this case, starting from the four original inclination signals, Johansen’s procedure identifies the presence of two cointegrating vectors (and thus two cointegrating residuals), meaning that there exist two common stochastic trends that has been subsequently removed. The most stationary residual is then selected as damage sensitive feature and a control chart is used for anomaly detection purposes, as shown in Figure 11. Again, it is possible to observe the effectiveness of the cointegration-based method, independently from the number of input variables and the number of identified common trends, to promptly detect the presence of a modification in the properties of the structure with respect to its normal conditions.

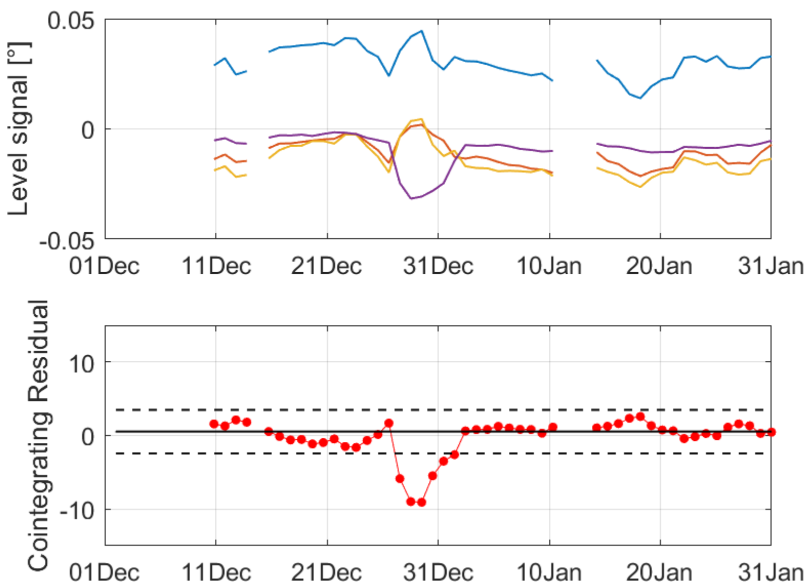

Figure 11 Cointegration-based method applied on channels ‘32A’,’32B’,’34A’,’34B’: original tiltmeters data (up) and cointegrating residual (down).

At this point, authors believe it is important to provide a discussion on the existing relationship between the number of variables to be cointegrated (n), the number of common trends (t), and the number of cointegrating residuals (r). From cointegration theory, it is known that the number of common stochastic trends can be derived as $t = n - r$. Dealing with large steel structures as the ones analysed in this work, while it is evident that air temperature has a similar impact on tiltmeter signals, the direct effect of other parameters that cause the presence of common trends is difficult to identify. For example, it is well-known from past studies that other factors like solar radiation or wind speed may be significant, though less important than temperature. However, the establishment of a relation between these parameters and the presence of common trends in the signals is hardly feasible, due to the vast dimensions of the structure and to the different level of exposure between the different parts of the structure itself. This is precisely the reason why cointegration can be of great help. Even though there is not the possibility to certainly identify or to directly measure the "benign" causes of variability in the monitored features of a structure during its normal conditions, the number of common trends are identified and consequently removed from the original signals, increasing then the possibility of identifying other variations due to "malign" causes, i.e. a damage.

5 Conclusions

In this paper, a monitoring strategy for the automatic anomalies detection in real-world large scale structures is presented. The proposed method relies on the use of cointegration to eliminate the variability of the monitored features induced by environmental and operational effects. A stationary residual is obtained from the linear combination of the previous variables, where all the confounding effects are removed and is considered as a damage sensitive feature. The presence of changes in the structure are associated to deviations from stationary conditions of the cointegrating residual. Due to its large dimensions and due to its considerable exposure to atmospheric agents, the steel roof of the G. Meazza stadium is employed as realistic case study to verify the feasibility of the cointegration-based technique. Time series associated to the tilt angles of a group of contiguous and equal rafts, which show non-stationarity and a similar long-term behavior, are used for the analysis. Johansen method is then applied to create a cointegrating residual able to successfully eliminate the common trends induced by the influence of EOVs and identify the health status of the structure.

Acknowledgements

The work presented in this paper was partially supported by the PRIN grant 2017 - Life-long optimized structural assessment and proactive maintenance with pervasive sensing techniques, sponsored by the Italian Ministry of University and Research (MIUR).

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