- This is the accepted version of Mor, A., and M. G. Speranza. "Vehicle routing problems over time: a survey." 4OR 18 (2020): 129-149.

- Cite as:

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@article{mor2020vehicle,
 title={Vehicle routing problems over time: a survey},
 author={Mor, A and Speranza, MG},
 journal={4OR},
 volume={18},
 pages={129--149},
 year={2020},
 publisher={Springer}
}
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- Published Journal Article available at: https://doi.org/10.1007/s10288-020-00433-2

Vehicle routing problems over time: a survey

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January 7, 2025

Abstract

In Vehicle Routing Problems (VRPs) the decisions to be taken concern the assignment of customers to vehicles and the sequencing of the customers assigned to each vehicle. Additional decisions may need to be jointly taken, depending on the specific problem setting. In this paper, after discussing the different kinds of decisions taken in different classes of VRPs, the class where the decision about when the routes start from the depot has to be taken is considered and the related literature is reviewed. This class of problems, that we call VRPs over time, includes the periodic routing problems, the inventory routing problems, the vehicle routing problems with release dates, and the multi-trip vehicle routing problems.

Keywords: Routing, inventory routing problem, periodic routing problem, routing with release dates, multi-trip routing.

1 Introduction

Vehicle routing problems are among the most studied problems in the area of combinatorial optimization. Although the Traveling Salesman Problem (TSP) could be seen as belonging to the class, the most basic problem is considered to be the capacitated vehicle routing problem, where customers, each with a given demand, have to be served with a fleet of identical vehicles (Dantzig and Ramser [1959]). Each vehicle performs **at most one route**. All routes start and end at the depot. The decisions to be taken concern the assignment of customers to vehicles and the sequencing of the customers assigned to each vehicle in such a way that the total routing cost is minimized. In general, the expression *vehicle routing problems* (VRPs) is used when the customers to be visited are modeled through the nodes of a graph (see Toth and Vigo [2014]), while the problems where customers are modeled through arcs are usually called *arc routing problems* (ARPs) (see Corberán and Laporte [2013]). VRPs and ARPs are models usually aimed at supporting decision-making concerning the routing of commercial or service vehicles, typically trucks or similar kinds of transportation means. Despite the long tradition and the number of published papers, the field keeps attracting scientific and practical interest. The knowledge gained on these problems and the technological advancements allow researchers to reduce the gap between scientific literature and real-life applications. More realistic variants, more efficient and effective solution methods, and complex problems that include routing decisions are studied.

Taxonomies and surveys have been published over the years aimed at classifying the VRPs, overviewing the literature, and identifying recent trends. Recent surveys include Golden et al. [2008], Laporte [2009], and Toth and Vigo [2014]. A recent taxonomy of VRPs is provided in Braekers et al. [2016] that updates and expands the work of Eksioglu et al. [2009]. The authors classify the papers on the basis of five macro-categories: type of study, scenario and physical characteristics, information and data characteristics.

The basic decisions to be taken in VRPs concern the assignment of customers to vehicles and the sequencing of the customers assigned to each vehicle. If the assignment is made beforehand, then a VRP reduces to the solution of multiple TSPs. The assignment and sequencing decisions characterize the class. VRPs are an extremely broad class of problems and the literature on VRPs shows a clear trend towards the study of more complex problems to reduce the gap with real world applications. Problems may differ because of additional features a route must satisfy, such as a constraint on time duration, the respect of time windows or of pickup and delivery sequences of operations. The so called class of rich VRPs is a recently studied and rapidly growing class of multi-constrained problems (see Lahyani et al. [2015] and Caceres-Cruz et al. [2015]). Problems may also differ because decisions may need to be taken in addition to the assignment and sequencing. Such additional decisions may imply a considerable change in the mathematical programming formulation, with different sets of variables requested, and in the design of solution approaches. This is the reason why, in this paper, the VRPs are classified according to the decisions to be taken. To the best of our knowledge, this is the first paper that introduces this classification. We start from the most classical VRPs, where only assignment and sequencing decisions are taken and then consider the classes of the VRPs with profits, where the choice of customers to be served is also made, the VRPs with split deliveries, where the quantities are also decided, the VRPs with multiple commodities, where the commodity must also be chosen and the VRPs over time, where the starting time of a route must also be decided. This classification allows us to identify the latter class of the *VRPs over time*. In fact, more decisions have been considered in the so called class of integrated VRPs (see Bektaş et al. [2015]) which include, for example, the location-routing problems, the multi-echelon routing problems, the routing problems with loading constraints. It is beyond the scope of this paper to explore all possible extensions, in terms of additional decisions, of the most classical VRPs.

The VRPs over time include the periodic vehicle routing problems (PRPs), the inventory routing problems (IRP), the routing problems with release dates, and the multi-trip routing problems. In this paper we review the literature on this class of problems. While surveys have appeared on the VRPs with profits (see Archetti et al. [2014b]) and on the VRPs with split deliveries (see Archetti and Speranza [2012]), no survey has been published on the VRPs with multiple commodities. The class of VRPs over time has not been surveyed as such, but surveys have appeared on the PRPs, the IRPs, and on the multi-trip routing problems, separately. In this paper we present the VRPs over time as a single class with the aim, on one hand, of clarifying the definitions of the basic problems of the various subclasses, and, on the other hand, of identifying differences and commonalities that may help in the design of models and solutions approaches in future research.

The paper is organized as follows. In Section 2 the classes of VRPs that arise when different decisions are considered are presented. Section 3 is devoted to the class of the VRPs over time. In Sections 4 and 5 the periodic and the inventory routing problems are surveyed, respectively, while the contributions to the routing problems with release dates and the multi-trip routing problems are reviewed in Sections 6 and 7, respectively. Conclusions are drawn in Section 8.

2 Classes of vehicle routing problems

In this section the VRPs are structured according to the decisions that have to be taken to solve them. In the following, for the sake of simplicity, if not specified otherwise, we will refer to distribution problems where customers are delivery customers, although in most cases we could also refer to collection problems where customers are pick-up customers.

With which vehicle? In which order? The most classical VRP is the Capacitated VRP (CVRP). Each vehicle starts from a depot and returns to the same depot. The number of homogeneous vehicles available is given and coincides with the number of routes as each vehicle performs **at most one route**. All routes can start at the same time. All customers must be served. The demand of each customer is entirely served by one vehicle. A single commodity is considered. Routes must be created, that is, customers must be assigned to vehicles and the customers assigned to each vehicle must be ordered. The objective is the minimization of the routing cost subject to the following constraints: each vehicle performs **at most one route**, the total demand of the customers served in the same route does not exceed the vehicle capacity, no customer is visited more than once.

We call *classical VRPs* the problems where the decisions to be taken are the assignment of customers to routes and the sequencing of the customers assigned to each route. These problems extend the CVRP by adding features to the vehicles or to the customers, and include, for example, the CVRP with time windows and several variants of the CVRP with pick-up and delivery.

Which customers to serve? In some cases it may be possible to decide whether to accept a potential customer or to outsource the service of an accepted customer when, for example, the customer is not conveniently located or the capacity of a vehicle is not satisfactorily exploited. In the classical VRPs, where all customers must be served, it is assumed that the decision about which customers to serve has already been taken.

In the class of the so called *VRPs with profits* the decision about which customers to serve is considered, assuming that a profit is associated with each customer. Examples of VRPs with profits are the orienteering problem and the team orienteering problem (see Archetti et al. [2014b] for a survey).

How much to a customer? In the classical VRPs all customers are visited once. This implies that the demand of a customer does not exceed the capacity of a vehicle. No decision about the quantity to deliver to a customer has to be taken as the entire demand is served by one vehicle. While this is often a realistic assumption, it may be beneficial to visit customers more than once, even when the demand of each customer does not exceed the capacity of a vehicle (see Archetti et al. [2006]).

The Split Delivery Vehicle Routing Problem (SDVRP) is defined as the CVRP with the difference that the quantity to be delivered to a customer by a vehicle has to be decided, which implies that a customer may be visited by multiple vehicles. A survey on the class of *VRPs with split deliveries* can be found in Archetti and Speranza [2012].

What commodity? A single commodity is considered in the classical VRPs. This does not mean that only one commodity is delivered but rather that one commodity is sufficient to model the entire set of real commodities. One commodity is sufficient if each vehicle can deliver anything, the depot contains all the goods, and the total demand of the customers - possibly of multiple real commodities - and the capacity of the vehicles are measured with the same unit of measure, typically weight or volume.

There are interesting and relevant vehicle routing applications that can be modeled only by explicitly considering multiple commodities. This happens, for example, when the vehicles have multiple compartments, each with limited capacity, to keep multiple commodities separated (see Yahyaoui et al. [2018] for a recent review of the literature). Multiple commodities have to be explicitly modeled also when the vehicles start from multiple locations where each commodity is available in limited, possibly location dependent, quantities. In these cases, an additional decision to be taken concerns the commodity to be loaded on a vehicle. To the best of our knowledge, no survey is available on this class of problems that we call *VRPs with multiple commodities*.

When does a route start? In the classical VRPs routes usually start from the depot at the same time unless specific constraints, for example time windows, prevent this. In several real-life situations there is some degree of flexibility in the choice of the day of service of a customer, for example in the cases where there is a service contract between customer and supplier and the customer must be served within a certain number of days from the order. There are also applications in which multiple routes of the same vehicle are possible because a vehicle may return to the depot multiple times in the same day as its traveling time is only limited, for example, by the duration of the shift of the driver. In such cases, an additional kind of decision has to be taken, that is when a route should start. We call *VRPs over time* the class of problems which consider the decision of when to start a route. In these problems the same vehicle can perform multiple routes.

3 Vehicle Routing Problems over time

In the classical VRPs a fleet of vehicles is available **and all vehicles** are all available at the depot when the planning of the routes is made. While it may happen that different routes start at different times from the depot, due to the presence of time windows or of a time-based objective function or of other characteristics of the problem, normally each vehicle performs one route. One vehicle is identified with one route and the two words – vehicle and route – are used interchangeably.

In the VRPs over time, one vehicle is expected to perform multiple routes and the starting time of a route becomes a decision to be taken, jointly with those about the assignment of customers to routes and the order of visit in each route. When the planning horizon is discretized in a finite number of time periods, we will refer to a period as day. In this case, a vehicle will perform multiple routes because it is used in multiple days, and the day or days in which to serve a customer will have to be decided. A vehicle may also perform multiple routes in the same day. In this case, the starting time of a route of a vehicle will have to follow the ending time, that is the arrival time to the depot, of the previous route.

In this section we overview the contributions to the VRPs over time and structure the class according to the characteristics of the specific problems studied. The literature on Periodic Routing Problems (PRPs) will be reviewed first (see Section 4), followed by the literature on Inventory Routing Problems (IRPs) (see Section 5). These two types of problem cover the majority of the contributions. In both cases, a homogeneous fleet of capacitated vehicles is available. The time is discretized in days and the same vehicle may be used in multiple days. A customer may be served in one or multiple days, to be decided. In the PRPs, possible sequences of days of visit (schedules) are predefined for each customer. Moreover, given a sequence of days for a customer, the quantity to be delivered to that customer is given. In the IRPs, different solutions may have the same day of visit for all customers but different quantities delivered. While in the PRPs the decision is when to visit and this decision implies the quantity to deliver, in the IRPs both decisions about when and how much to deliver have to be taken. Two other types of VRPs over time will then be discussed, namely the VRPs with release dates (see Section 6) and the multi-trip VRPs (see Section 7). In these two types of problem a single period, that is a day, is considered. Each vehicle may perform multiple routes in a day. In the VRPs with release date, a release date is associated with each customer. The release date of a customer is a lower bound on the time a vehicle can start its route to visit that customer. A vehicle may perform multiple routes because it may be convenient for it to start from the depot to serve a set of customers before the release date of some other customers, that will be served in a following route. In the multi-trip VRPs, the same vehicle can perform multiple routes, typically because a time duration is set on each route and on the total time a vehicle is available in a day.

4 Periodic routing problems

The PRPs are characterized by the fact that each customer must be visited multiple times, that is, in multiple periods (e.g., example days), of a planning horizon (e.g., one week), and that there are options on the periods of visit. Such options can be specified in different ways but typically are either the result of a given frequency for the visits (e.g., every two days), of a given number of visits within the planning horizon (e.g., twice), or of a fixed set of periods specified by the customer (e.g., Monday and Thursday or Tuesday and Friday). We call each possible set of periods (e.g., Monday and Thursday) a visiting option. The most common objective is the minimization of the routing cost. The decision about the visiting option for each customer is taken jointly with the decisions about the assignment

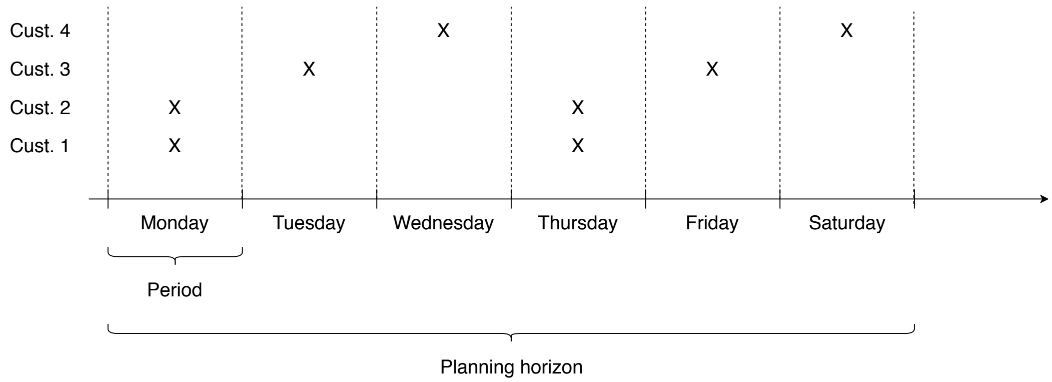

Figure 1: Periodic Routing Problems: an example.

of customers to routes and the sequencing of customers in each route. The quantity to be delivered to customers is not a decision to be taken, i.e., is implicit and fixed.

The *basic PRP*, as defined in Campbell and Wilson [2014], is the problem of assigning to each customer one of its feasible visiting options over a planning horizon, with the following requirements: the route of each vehicle starts and ends at the same depot, the amount of goods to be delivered to each customer is known and is entirely served by one vehicle, the size of the available fleet is given, and the total travel time of each vehicle is limited. Formally, let $G = (V, A)$ be a directed graph, where the set of vertices V is composed of the vertex 0, denoting the depot, and the set of vertices $N = \{1, \ldots, |N|\}$, denoting the customers. Over a discretized planning horizon $T = \{1, \ldots, |T|\}$, each customer $i \in N$ is characterized by a set of feasible visiting options, where each visiting option is a subset of T. The demand of customer $i \in N$ at each visit is indicated as q_i. The set of vehicles available for the distribution is denoted by $K = \{1, \ldots, |K|\}$. The vehicles are homogeneous and have capacity Q. The cost of traveling from i to j is denoted by c_{ij}. One visiting option for each customer must be chosen and routes for each period must be created. The objective is to minimize the total routing cost over the planning horizon. An example of schedule for the visiting of customers for a PRP is shown in Figure 1.

The introduction of the first routing problem with a periodic nature is attributed to Beltrami and Bodin [1974], where the study of the problem was motivated by an application in the context of waste collection. The paper also introduces heuristics for the problem. The definition of the problem is extended in Russell and Igo [1979]. The first paper to identify the problem as a PRP is Christofides and Beasley [1984]. The instances proposed therein have become part of the standard benchmark instance set that newly devised algorithms are tested on.

The most recent survey on PRPs is provided in Campbell and Wilson [2014]. The authors discuss the advancements since the paper of Beltrami and Bodin [1974].

Several papers have been published after the survey in Campbell and Wilson [2014], mostly on variants of the basic PRP. A well studied variant of the PRP is the PRP with time windows (PRPTW), where customers are allowed to be served within a specific time interval of each period. A genetic algorithm is proposed for the PRPTW in Nguyen et al. [2014]. A study of the performance of a particle swarm optimization approach is presented in Norouzi et al. [2015] for a PRPTW defined in a competitive environment, in which a fraction of the customer demand is served by the first competitor to arrive to the customer location.

Another variant that has received considerable interest is the multi-depot PRP in which the customers are allowed to be served from multiple depots, where the vehicle returns at the end of a route. A heuristic that combines elements derived from the mechanics of electromagnetism and simulated annealing is investigated in Mirabi [2014]. The fleet sizing problem faced when dealing with a multi-depot PRPTW is investigated in Rahimi-Vahed et al. [2015].

In the basic PRP the problem faced is either to deliver goods or to pickup goods at customer locations. A variant of the basic PRP is introduced in Jayakumar et al. [2016], where the pick-up and the delivery phases are jointly considered.

In Archetti et al. [2015b] the multi-period VRP with due dates is studied. The problem considers the case in which a set of customers is characterized by a release and a due date that are the first and last period in which each customer must be served, respectively. The problem can be seen as a PRP as each period between the release and the due date is a visiting option and the amount of goods to be delivered to the customers is given. The multi-depot periodic VRP with due dates and time windows is considered in Cantu-Funes et al. [2018]. The work is inspired by a real case of a brewing company. Heterogeneous vehicles are used to serve a distribution center from multiple depots. Each vehicle is allowed to perform multiple routes in each period. Deliveries must be performed within a time window, specified for each customer, which is the same in all periods. The demand of each customer must be satisfied within a due date. Also in this case, each period before the due date can be seen as a visiting option.

While the basic PRP and the above mentioned variants are defined on graphs where customers are represented by vertices, recently problems where customers are represented by arcs have received some interest. The Periodic Capacitated Arc Routing Problem (PCARP) is introduced in Lacomme et al. [2005] as extension to a periodic setting of the capacitated arc routing problem, as the PRP is an extension to a periodic setting of the VRP. The authors describe several versions of the PCARP, present a classification scheme and a memetic algorithm based on a crossover operator considering both planning and scheduling decisions. In recent years, the PCARP has been investigated in Zhang et al. [2017], where a memetic algorithm together with a route decomposition operator is proposed, and in Riquelme-Rodríguez et al. [2014]. In the latter paper an adaptive large neighborhood search is presented for the PCARP with inventory constraints. The problem arises in open-pit mines, where unpaved roads have to be watered to prevent the dust from damaging the equipment. A maximum capacity is considered for the water depot and water demand is modeled as a function of time and humidity level.

Trends and future research After the first PRP was introduced in the literature to model a waste collection problem, for some years the contributions have focused on extensions still motivated by the context of waste collection. Only more recently, the PRPs have been used to model other routing applications, where one among alternative visiting options has to be chosen for each customer. It appears that no contribution to the exact solution of the classic PRP has been brought after Baldacci et al. [2011]. Toth and Vigo [2014] highlights how no exact algorithm has been proposed for the PRPTW. Useful results in the advance of the research in this area might come from the study of the single vehicle case. A common subject of the literature has been the study of new variants inspired by applications. To this regard, we point out that only one paper in this survey considers the PRP with pickup and delivery operations (see Jayakumar et al. [2016]). In general, efforts in bridging the gap with real life problem can also be made with respect to the expansion of the benchmark instances to include real or real-like instances, for instance using data from real maps.

5 Inventory routing problems

The IRPs are problems characterized by the fact that deliveries take place over time without predefined schedules and the quantity to be delivered to any customer at any time is a decision variable. The quantity delivered to a customer must satisfy the inventory capacity of the customer. In the IRPs several decisions are simultaneously taken: when to visit customers, how much to deliver to customers, how to assign customers to routes and how to order customers in each route.

In the terminology of the IRP often a supplier has the role of the depot and retailers the role of customers. We will use both terms in this section. The *basic IRP* is defined on a directed graph, where a vertex represents a supplier and the other vertices the retailers. A discretized planning horizon is given. The quantity available at the supplier and demanded by each retailer in each period of the horizon is known. Each retailer has an inventory holding capacity that cannot be exceeded. Inventory holding costs may be charged at the supplier and at the retailers. Stock-out situations are not allowed, that is, in each period the inventory at each retailer must be sufficient to satisfy the demand. The objective is to minimize the total distribution cost which includes the routing cost and, if relevant, the inventory holding cost. The basic IRP with a single vehicle was introduced in Archetti et al. [2007]. The problem is formally defined as follows. Let $G = (V, A)$ be a directed graph, where the vertex 0 represents the common supplier and the set of vertices $N = \{1, \ldots, |N|\}$ the retailers. The set of vehicles available for the distribution is denoted by $K = \{1, \ldots, |K|\}$. The vehicles are homogeneous and have capacity Q. The cost of traveling from i to j is denoted by c_{ij}. Each retailer i has a maximum inventory level and an initial inventory level. At each period t over the planning horizon $T = \{1, \ldots, |T|\}$ the quantity r_{0t} is made available at the supplier and the quantity r_{it} is consumed at retailer $i \in N$. The initial inventory level of the supplier is known. A unit inventory holding cost is defined for both the supplier and the retailers. The customers to be visited in each period must be chosen, together with the quantity to be delivered in each visit, and the routes must be created. The objective is the minimization of the inventory and routing costs. An example of the schedule for the visit of customers and the quantity to be delivered for an IRP is shown in Figure 2.

The first paper to use the expression *inventory routing problem* was Bell et al. [1983],

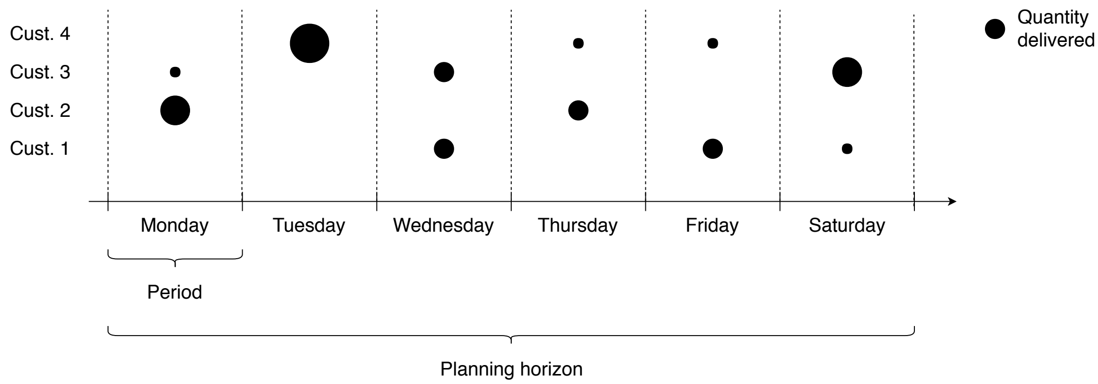

Figure 2: Inventory Routing Problems: an example.

where the authors discuss the integration of the inventory management at retailers in an industrial gas supply chain. The study of the IRPs is usually motivated by applications aimed at integrating inventory management and distribution problems.

A few surveys on the IRPs have recently appeared. In Andersson et al. [2010] the industrial aspects of integrating inventory management and routing are analyzed. In Bertazzi and Speranza [2012] and Bertazzi and Speranza [2013] tutorials on the IRPs are provided, with a classification of the characteristics that make the IRPs different from each other, starting from the case where there is one retailer only. A survey of the papers studying IRPs is presented in Coelho et al. [2013], where problems are classified with respect to their structural variants and the availability of information on retailer demand. We review here only the papers published after these surveys.

In Archetti et al. [2014a] a comparison of different formulations for the basic IRP with multiple vehicles is presented. It is shown that a vehicle-indexed formulation performs better than more compact formulations. Exact approaches to the solution of the basic IRP with multiple vehicles are described in Coelho and Laporte [2014a] and Desaulniers et al. [2015]. The former paper introduces new valid inequalities for the problem. The authors also assess how input ordering affects the solution. The latter paper presents a branch-and-price-and-cut algorithm to solve a new formulation, providing the optimal solution for additional 54 instances over the standard benchmark instance set of 640 instances introduced in Archetti et al. [2007]. An analysis of the benefits of integrating inventory management and routing in supply chain management is provided in Archetti and Speranza [2016] through the comparison of the solutions obtained by considering inventory management and routing separately and those obtained by solving the basic IRP.

A number of new variants of the basic IRP have been recently introduced. The IRP where lost sales are allowed for customers is defined in Park et al. [2016]. A genetic algorithm is proposed for the solution of the problem. In Niakan and Rahimi [2015] a multi-objective IRP arising in the medicinal drug distribution to healthcare facilities is presented. Together with inventory and routing costs and greenhouse gas emissions, the objective function considers product shortage and expired drugs minimization. The IRP with pick-up and delivery (IRP-PD) is considered in Archetti et al. [2018a], where a many-to-many single-vehicle IRP-PD is studied. A model with various classes of valid inequalities is presented together with a branch-and-cut method. Iassinovskaia et al. [2017] studies the IRP-PD arising in supply chains where reusable packets are used. Products are delivered in reusable packets and, simultaneously, empty packets are collected. In Van Anholt et al. [2016], the IRP-PD arising in the context of ATM replenishment in the Netherlands is discussed. The authors propose a decomposition into tractable sub-problems that are solved by a branch-and-cut algorithm.

The IRP with perishable goods is considered in Coelho and Laporte [2014b], while a single perishable product is considered in Azadeh et al. [2017] where the IRP with transshipment is studied. The authors propose a genetic algorithm for the problem where the parameters of the algorithm are tuned using the Taguchi method.

The so called Flexible Periodic Vehicle Routing Problem (FPVRP) is presented in Archetti et al. [2017]. In this problem, each customer has a total demand that must be served within a planning horizon and a limit is defined on the maximum quantity that can be delivered in each visit. No predefined schedules are given and the quantity to be delivered is a decision variable. While the problem is called periodic by the authors, it shares the basic features of an IRP.

The multi-product variant of the basic IRP is studied in Cordeau et al. [2015] where a three-phase heuristic approach is presented which is based on the decomposition of the decision process. In the first phase, a Lagrangian based method is used to plan customer replenishment. In the second phase, the routing among customers is obtained with possible split deliveries. Finally, the solution is improved by means of a feedback model. In the multi-product variant studied in Mjirda et al. [2014] vehicles visit suppliers to collect products to be delivered to assembly plants. Each supplier provides one product and can be visited multiple times during a period. A two-phase variable neighborhood search is proposed. In the first phase an initial solution is built by solving a CVRP for each period and in the second phase this solution is iteratively improved minimizing the routing and inventory costs. In Shaabani and Kamalabadi [2016] a population-based simulated annealing heuristic is proposed for the multi-product multi-retailer IRP of perishable goods. Its performance is assessed by comparing the algorithm with a simulated annealing and a genetic algorithm. A multi-product IRP is also considered in Laganà et al. [2015] in the context of the supermarket distribution industry. Two mixed integer problem formulations are presented and a decomposition approach is proposed.

The multi-depot IRP is studied in Bertazzi et al. [2017] in a city logistic environment. A formulation for the problem is presented together with a branch-and-cut algorithm. Moreover, a three-phase matheuristic is presented, composed of a clustering, a routing construction and an optimization phase. Benchmark instances for the multi-depot IRP are proposed in Noor and Shuib [2015] which are generated by applying clustering techniques to single-depot IRP benchmark instances.

In the context of maritime inventory routing, Papageorgiou et al. [2014a] presents an approximate dynamic programming approach for the deterministic maritime IRP with a long planning horizon. Two decomposition algorithms for a single product maritime IRP are presented in Papageorgiou et al. [2014b]. The IRP of liquefied natural gas is introduced in Andersson et al. [2016]. The peculiarity of the problem lies in the fact that a constant rate of the cargo evaporates in the tanks each day and part of the load is used as fuel during transportation. The authors present a path flow formulation that is solved with a decomposition algorithm.

In the periodic IRP (PIRP) a replenishment schedule must be identified with the additional constraint that the initial inventory levels must be equal to those at the end of the planning horizon, allowing for the schedule to be repeated. A heuristic is proposed in Qin et al. [2014] for the problem where the inventory and routing components are solved by means of a local search and a tabu search, respectively, with the two components iteratively executed. Liu et al. [2016] introduces a hybrid heuristic consisting of an iteration of a particle swarm algorithm, a local search improving each particle found in the previous step and a large neighborhood search to escape from local optima. The algorithm is shown to outperform the one proposed in Qin et al. [2014] by more than 10% on average over a set of 10 instances. Finally, the selective and periodic inventory routing problem (SPIRP) is studied in Aksen et al. [2014], where an adaptive large neighborhood search is proposed. Montagné et al. [2018] extends the work of Aksen et al. [2014] for the real case of reusable waste oil collection in Canada, and presents a constructive heuristic to solve instances with up to 3000 customers in a 30-day time horizon.

The literature presented so far models the IRP as a problem on a planning horizon of a finite set of periods. As reported in Andersson et al. [2010], the other options considered in the literature for the horizon over which the IRP is studied are: an instant horizon, when the planning horizon is so short that at most one visit per customer is needed; and an infinite horizon, when the decision focuses on the distribution strategies rather than on the schedules. An instant horizon is considered in Li et al. [2014] where the IRP arising in the petrochemical industry and the related specific constraints, like hours-of-service of the vehicles, is studied. A measure of workload balance, namely the maximum route travel time, is minimized. A single period horizon is considered in Juan et al. [2014] where a simheuristic, a solution algorithm combining simulation and heuristics, is proposed for the solution of the IRP with stochastic demands and possible stock-out. An infinite planning horizon is considered in the IRP-PD studied in Van Anholt et al. [2016], mentioned above.

Stochastic settings for IRPs have also been recently studied. In Coelho et al. [2014] some heuristics are proposed for the dynamic stochastic IRP. The paper shows that considering stochastic information is beneficial for the quality of the solution at the expense of an increase of the computational time. It is also shown that considering a longer rolling horizon step does improve the solution, and allowing consistent solutions is more beneficial in a static setting than in a dynamic one. The IRP with transportation procurement and stochastic demand is studied in Bertazzi et al. [2015]. The authors show the benefit of considering the complete stochastic information with respect to the average value of the demand. A stochastic dynamic programming formulation is proposed and a matheuristic is presented. The IRP with stochastic stationary demand rates is proposed in Abdul Rahim et al. [2014] where a Lagrangian relaxation approach is proposed. Mes et al. [2014] describes the IRP arising in waste collection from underground containers equipped with sensors. The amount of waste to be collected is assumed to be stochastic and dynamic. Collection costs are minimized together with a function of customer satisfaction. Stochastic demand is also considered in Crama et al. [2018] that studies the IRP with perishable products. Four solution methods are presented and compared. Perishable products are also considered in the IRP variant studied in Soysal et al. [2018] where collaboration is considered among multiple suppliers. The benefits of collaboration are investigated while accounting for demand uncertainty. Expected inventory costs, waste costs, fuel and driver costs are minimized. Within the context of IRPs with stochastic information, papers on IRPs with multiple objectives have also recently appeared. In Nolz et al. [2014a] the authors extend their previous work on the stochastic IRP for medical waste collection (see Nolz et al. [2014b]) to discuss a bi-objective IRP in the context of waste disposal. Together with the routing and inventory costs, the authors consider the quality of service as part of the objective function. A multi-objective IRP is studied in Rahimi et al. [2017] where, in addition to the inventory and routing costs, the authors consider service level and the environmental footprint as part of the objective function. In Yadollahi et al. [2017] a chance-constrained formulation is proposed for an IRP with stochastic demand. A safety stock-based deterministic optimization model is used to determine near-optimal solutions. Different safety stock models are investigated and insights on the setting of the safety stock level are obtained.

Trends and future research The first optimization models presented in the literature as inventory routing problems were motivated by real case applications, where the decisions about when to serve customers and how much to deliver in each visit had to be taken. Goods may be delivered to customers in advance with respect to consumption. Clearly, rare deliveries optimize the transportation cost while frequent deliveries optimize the inventory cost. The search for the optimal trade-off between transportation and inventory cost was at the basis of all the pioneering papers in the area. The basic IRP has started a stream of methodological papers oriented to the mathematical formulations, the design of exact and heuristic methods. To date, the only papers investigating the single vehicle IRP remain Archetti et al. [2007] and Archetti et al. [2018a]. Further work on this variant could lead to insightful results for the multiple vehicle case. Recent papers present insights on the formulation of the problem and new sets of valid inequalities. Desaulniers et al. [2015] presents a new formulation for the IRP including new sets of valid inequalities, and describes a branch-and-price-and-cut algorithm, whereas Archetti et al. [2018a] proposes a formulation for the IRP-PD and different classes of valid inequalities. These works show that the formulation and the exact solution of IRPs is still a prolific topic for future research.

Future developments may include theoretical analysis of special cases but also further extensions of the problem. For instance, it would be interesting to extend popular variants of the CVRP to the case of a IRP, e.g., routing with split deliveries and routing with profits. As mentioned in Section 4 for the PRP, the literature could benefit from the introduction of real or real-like instances, in the effort of reducing the gap with real life problems.

6 Vehicle routing problems with release dates

The definition of the classical VRPs implies that the goods to be delivered to the customers are ready for delivery at the depot at the beginning of the planning period. This is, however, not always the case in practical applications. For example, in consolidation and distribution centers, goods to be distributed in the city center arrive during the distribution period while the delivery of other goods, that have previously arrived to the distribution center, is taking place. In this case, over time, a decision to be taken is whether it is better to start a route that delivers goods to customers or to wait for more goods to reach the distribution center and better routes to be built. Similar problems arise in the context of cross docking and same-day delivery problems. The former is a practice in logistics of unloading goods from incoming vehicles and loading the goods directly into outbound vehicles, with little or no storage in between. In the latter, customers place orders dynamically during the same-day the orders have to be fulfilled. Same-day delivery problems are faced by many distributors with online purchases. Distributors must quickly react to incoming orders to deliver goods on time.

We define the *basic VRP with release dates* (VRP-rd) as follows. Let $G = (V, A)$ be a directed graph. The set V is composed of the vertex 0, identifying the depot, and the set $N = \{1, \cdots, |N|\}$, representing the customers. The demand of customer i is defined by a quantity $q_i \geq 0$ and a release date $r_i \geq 0$ is associated with customer i. A set $K = \{1, \ldots, |K|\}$ of homogeneous vehicles with capacity Q is available to perform the

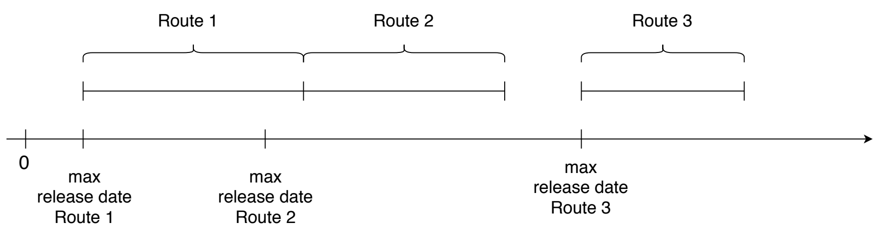

Figure 3: Vehicle Routing Problems with release dates: an example.

distribution. Each arc of the graph is characterized by a traveling time and a distance, which are assumed to be identical. The time and distance from i to j is denoted by c_{ij}. Routes must be created and assigned to each vehicle in such a way that the starting time of a vehicle cannot be earlier than the maximum release date of the customers assigned to it. The objective function is the minimization of the completion time.

An example of solution of a basic VRP-rd is provided in Figure 3 for a single vehicle version. Route 1 can start when the last of the parcels to be delivered in that route reaches the depot. As the vehicle is traveling when all the parcels to be delivered in Route 2 have reached the depot, that route starts as the vehicle becomes available again, i.e., when it returns to the depot. Finally, the vehicle waits for all the parcels to be delivered in the last route, Route 3, to reach the depot and performs its final route.

The possibility that customers have a release date has only recently been introduced in VRPs. To the best of our knowledge, the first paper that considers release dates for parcel delivery is Cattaruzza et al. [2016a]. Archetti et al. [2015a] investigates the complexity, on special topologies of the graph, of both the TSP and the uncapacitated VRP with release dates in the cases of completion time and total distance minimization with a deadline on the completion time. In Reyes et al. [2018] the study of the complexity of the problem is extended to consider a service guarantee for the deliveries to the customers. In Archetti et al. [2018b] the authors provide a formulation for the TSP with release dates and completion time minimization. Two variants of an iterated local search are presented and compared. Cattaruzza et al. [2016a] introduces the multi-trip VRP with time windows and release dates. A hybrid genetic algorithm is proposed which makes use of a route decomposition technique for chromosome decoding and a local search. In Shelbourne et al. [2017] the

VRP with release and due dates is considered. A due date, that is the time by which the order should be delivered to the customer, is also associated with each customer. A convex combination of operational costs and customer service level is considered. The former is expressed as the total traveled distance and the latter as the total weighted delivery tardiness. The authors present a path relinking algorithm for the solution of the problem. Liu et al. [2017] describes the capacitated vehicle routing problem with order available times in e-commerce industry. In this problem, a fleet of homogeneous vehicles deliver orders that are available for transportation only after they have been processed by the warehouse. A granular tabu search is presented for the problem and shown to have a small gap to a Lagrangian relaxation algorithm.

In the above discussed literature known customers have a deterministic release date. Some contributions have appeared that assume that probabilistic information is available on potential customers. Various papers investigate the routing problems arising in the logistic of same-day deliveries. The information on customers is assumed to be stochastic either in time or both in space and time. Klapp et al. [2016] studies the dynamic dispatch waves problem of a single vehicle on a line. The distributor has to decide whether to dispatch a vehicle to serve known customers or to wait for potential requests that may arrive later, with the objective to minimize expected vehicle operating costs and penalties for unserved requests. In Klapp et al. [2018] the authors extend their previous work to a general network. A deterministic model is used to find an optimal a priori solution to the stochastic variant and two dynamic policies are developed. The trade-off between minimizing operational costs and maximizing the total order coverage is studied. Voccia et al. [2017] investigates the same-day delivery problem, where a fleet of vehicles is used to serve requests characterized by time windows or a delivery deadline. Probabilistic information about the arrival rate of future requests at known locations is available. The authors identify the circumstances that make waiting at the depot beneficial to maximize the number of requests that are served on time. Ulmer et al. [2016] considers the same-day delivery problem where vehicles are allowed to return to the depot before having completed their distribution to load the parcels of new customers. Unknown customers are characterized by a probability distribution. Ulmer et al. [2017] introduces the restaurant meal delivery problem of picking up meals at a restaurant chosen by the customer and delivering the meal. The probability distributions on the time and location of meal requests are known. Before the delivery, the selected vehicle has to pickup the meal at the restaurant. The meal preparation time is random and therefore the vehicle may wait at the restaurant to pickup the meal. Archetti et al. [2020] studies the dynamic traveling salesman problem with stochastic release dates. The problem is represented as a Markov Decision Process and is solved through a reoptimization approach. Two models are proposed for the problem to be solved at each stage. The first model is stochastic and exploits the entire probabilistic information available for the release dates. The second model is deterministic and uses an estimation of the release dates.

Routing problems with release dates have also been addressed when considering routing jointly with order picking. Among the works published in recent years, Schubert et al. [2018] studies the order assignment and sequencing, and the vehicle routing problem with due dates of a supermarket chain where deliveries for supermarkets are picked in a central warehouse and Moons et al. [2018] studies order picking and vehicle routing problems jointly in a B2C e-commerce context, showing how considering the two problems together results in an average cost saving of 14% and in the ability of customers to place orders at later times while still having the same delivery time windows.

Trends and future research Routing problems with release dates are relatively new to the literature. As such, there are many interesting aspects that are open for investigation. On the methodological side, the exact solution of the problems could benefit from stronger formulations or different solution approaches such as column generation algorithms. Following a general trend in the routing literature, extensions could be proposed to narrow the gap with real-life problems, for instance, considering information coming from connected vehicles. Within the logistics of the depot, manufacturing and loading times could be taken into account to better represent the factors affecting the starting time of the vehicles. The focus of the literature considering release dates is frequently the distribution in city centers arising from e-commerce. Specific characteristics of this area of application could be considered. For instance, traffic and time-dependent travel times, restricted traffic areas and loading and unloading areas.

7 Multi-trip vehicle routing problems

The general assumption in the classical VRPs is that each vehicle only performs one route. Considerations on the multiple use of a vehicle might be made once a solution is obtained, with the underlying assumption that non-overlapping routes can be performed by the same vehicle. In the multi-trip VRPs (MTVRPs) each vehicle is explicitly allowed to perform multiple trips during its service time.

The *basic MTVRP*, as defined in Cattaruzza et al. [2016b], is a CVRP where each vehicle performs a set of routes in such a way that the total demand of customers served in each route does not exceed its capacity and that its last route is completed within a given deadline. The total routing time is minimized. Formally, the problem is defined on a directed graph $G = (V, A)$, where the set V is composed of the vertex 0, representing the depot, and the set N of customers is defined as $N = \{1, \ldots, |N|\}$. The set of arcs is defined as $A = \{(i, j)|i, j \in V\}$ with the cost and time required to travel on the arc $(i, j) \in A$ being denoted as c_{ij}. The set of vehicles available for the distribution is denoted by $K = \{1, \ldots, |K|\}$. The vehicles are homogeneous and have capacity Q. Each vehicle is available at time 0 and has to complete all deliveries and return to the depot within time T_H. The demand of customer i is indicated as $q_i, q_i \geq 0, i \in N$. Routes must be created and assigned to a vehicle with the objective of minimizing the total routing time, subject to the following constraints: each route starts and ends at the depot, each customer is visited exactly once, the sum of the demands of the customers in each route does not exceed the capacity of the vehicle, the sum of the duration of the routes assigned to the same vehicle does not exceed T_H.

An example solution of a MTVRP is presented in Figure 4 for a single vehicle version. At the beginning of the distribution the vehicle is fully loaded and departs for its first route, serving three customers. After serving the three customers, the vehicle returns empty to the depot. A second route serving three customers is performed, with the vehicle leaving the depot fully loaded. The final route serves only one customer with the vehicle loaded only to serve its demand.

A recent review of the literature on the MTVRP is provided in Cattaruzza et al. [2016b]. Various contributions have been introduced after this survey. In François et al. [2016] a large neighborhood search is proposed to solve a relaxed version of the problem where the deadline

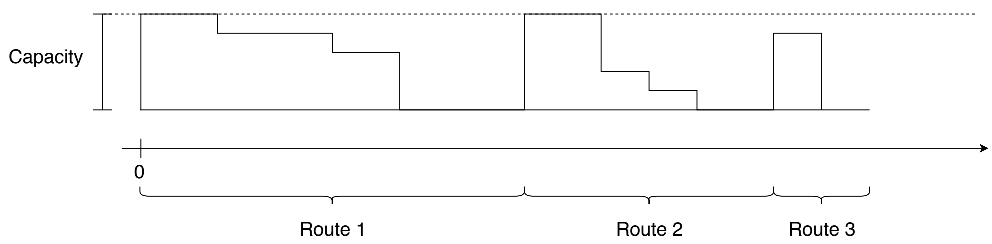

Figure 4: Multi-trip Vehicle Routing Problems: an example.

on the service time of the vehicles is not considered as a constraint and overtime is penalized in the objective function. In Liu et al. [2018] the multi-trip repairmen problem with time windows is introduced. The problem arises from a real-life application of a repair company facing the additional costs of the allowances that are paid to the repairmen when stationed at customer locations. The trade-off between returning to the depot (increasing the routing costs) and waiting at the customer location (increasing the allowance costs) is investigated. A branch-and-price method is proposed for the problem. The MTVRP with backhauls is introduced in Wassan et al. [2017] where trips must be constructed under the condition that, in each route, backhaul customers must be visited after all linehaul customers have been served. A formulation is presented and a variable neighborhood search is proposed. The multi-trip pick-up and delivery problem with time windows and synchronization (MT-PDTWS) is introduced in Nguyen et al. [2017]. A tabu search heuristic is proposed for the problem and its performance is assessed. In Tirkolaee et al. [2018] the multi-trip capacitated arc routing problem arising in waste collection is studied where depots and disposal facilities are in different locations. A formulation is presented for the problem and an ant colony algorithm is proposed.

Stochastic information is considered in two papers. Chu et al. [2017] studies a variant of the MTVRP where split deliveries are allowed and soft time windows are considered to achieve customer inventory replenishment. Stochastic travel times are assumed. A two-stage heuristic is proposed. In the first stage a solution for the problem is built and in the second stage the solution is improved by balancing the load of the vehicles to reduce delays, penalties and idling. In Tirkolaee et al. [2017] the robust MTVRP of perishable products is introduced with customers demand uncertainty, time windows and intermediate depots.

A formulation is presented and results assessing its robustness are illustrated.

Trends and future research While multi-trip routing problems are not new in the literature, only in recent years an effort has been observed to define a research stream (see Cattaruzza et al. [2016b]), with the consequent highlighting of promising research directions. Given the limited amount of papers addressing the problems studied after the reported survey, we find the indications for future research discussed therein to be still relevant. In particular, given the strong relevance of the issues arising in the last-mile delivery problems, the dynamic aspects involved in multi-trip routing problems in urban areas and the integration with other related problems are promising topics.

8 Conclusions and future research directions

In this paper a classification of vehicle routing problems based on the decisions that have to be taken has been proposed. Then, the literature on vehicle routing problems over time has been surveyed. In this class of problems, in addition to the classical decisions about the assignment of customers to vehicles and the sequencing of customers in each route performed by vehicles, the decision about when a route starts from the depot has to be taken. Besides the most studied problems in this class, periodic routing problems and inventory routing problems, we survey the literature on recently studied problems, namely vehicle routing problems with release dates and multi-trip vehicle routing problems.

The decision about when each route starts increases the computational complexity of the problems and implies the need of additional variables in the mathematical programming formulations, with respect to the more classical problems, and specific solution methods. At the same time, the vehicle routing problems over time model more integrated problems whose solution allows savings with respect to sequential approaches.

Future research directions include the study of the most appropriate formulations and solution methods of already studied problems and the investigation of deterministic and stochastic variants. Dynamic vehicle routing problems over time would also deserve attention, especially considering the technological evolution towards the use of digital devices that allow continuous generation and transmission of data.

Acknowledgments The authors would like to thank the anonymous reviewers for their suggestions and comments which helped improving a previous version of this paper.

Compliance with Ethical Standards We hereby state that we hold no conflicts of interest with respect to the submission of the text to 4OR.

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