Statistical Analysis of the Environmental Impact of Satellite Constellations

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Summary

An analytical model for collision probability assessments between de-orbiting or injecting space objects and satellite constellations is presented in this study. Considering the first to be subjected to a tangential acceleration, its spiral motion would result in a series of close approaches in the proximity of a constellation; in this work, a model for the evaluation of the mean collision probability related to this type of event has been developed. The collision probability distribution with each orbital plane of the constellation is modelled with Chan's algorithm and the mean value of the probability is computed as the integral of this distribution over the ranges of true anomaly phase and radial distance between the crossing object and the constellation's satellites. The miss distance vector at the instants of nominal closest approach will be evaluated with respect to the Minimum Orbit Intersection Distance points such that an analytical solution can be derived. Two different expressions will be obtained: the first covers most orientations between crossing and constellation's planes, whereas the second is specifically designed for head-on collisions.

Keywords: *Analytical Model, Collision Probability, Low-Thrust, Satellite Constellation, Shell Crossing Event.*

1 Introduction

In the last years, a huge effort has been put in the realisation of satellite constellations. However, they require a large number of satellites, and if every constellation already approved is deployed, the active satellite population will quintuple within the next decade, posing new challenges for Low Earth Orbit (LEO) traffic management.

The main focus of the studies about the environmental impact of satellite constellations is related to their interaction with space debris [3], [5], showing that non-trackable debris pose a great threat to Space sustainability. Nonetheless, the radical change that LEO will undergo will drastically increase both the collision probabilities and the complexity of risk assessments, not only with space debris, but also with other active satellites, thus highlighting the need for efficient tools. In this context, a statistical model for risk evaluation of shell crossing events has been developed.

2 Statistical model

A shell crossing event involves the disposal or injection of a satellite subjected to a low-thrust tangential acceleration, which has to cross various constellations' shells in order to reach its final destination. The tangential acceleration results in a spiral trajectory which can be discretised into a sequence of circular orbits. For this reason, both the constellation's and crossing satellites of a shell crossing event are assumed to be on circular orbits. Moreover, the satellites are modelled as spheres, the error ellipsoid of each satellite is assumed to be diagonal in its respective RSW reference frame [1], [2] and the collision probabilities of each close approach are assumed to be independent from one another.

2.1 Collision probability distribution

A shell crossing event is in general characterised by a large number of close approaches and the collision probability distribution is modelled with Chan's algorithm [2] truncated at the first term:

$$P = P_{\odot} \exp\left(-\frac{1}{2}\frac{\mu_{\hat{x}}^2}{\sigma_{\hat{x}}^2}\right) \exp\left(-\frac{1}{2}\frac{\mu_{\hat{z}}^2}{\sigma_{\hat{z}}^2}\right) \qquad (1)$$

where $\sigma_{\hat{x}}$ and $\sigma_{\hat{z}}$ are semi-axis of the combined error ellipse on the encounter plane, $\mu_{\hat{x}}$ and $\mu_{\hat{z}}$ are the radial and along-track components of the miss-distance vector at the instant of nominal closest approach and:

$$P_{\odot} = 1 - \exp\left(-\frac{1}{2}\frac{r_a^2}{\sigma_{\hat{x}}\sigma_{\hat{z}}}\right) \qquad (2)$$

$$\sigma_{\hat{x}}^2 = \sigma_{R,1}^2 + \sigma_{R,2}^2 \qquad (3)$$

$$\sigma_{\hat{z}}^2 = (\sigma_{S,1}^2 + \sigma_{S,2}^2)\cos\frac{\varphi}{2} + (\sigma_{W,1}^2 + \sigma_{W,2}^2)\sin\frac{\varphi}{2} \qquad (4)$$

where r_a is the combined hard-sphere radius of the satellites, σ_R^2, σ_S^2 and σ_W^2 are the variances along the principal directions in the RSW frame of each satellite and φ is the angle between the angular momenta of the osculating orbits of the two approaching objects [1].
The radial and along-track components of the miss-distance vector at the instant of nominal closest approach can be defined as:

$$\mu_{\hat{x}} = \delta a \qquad \mu_{\hat{z}} = 2a_C \sin\frac{\delta\omega}{2}\cos\frac{\varphi}{2} \qquad (5)$$

where δa is the difference in semi-major axis between the crossing objects and the constellation satellite, a_C is the semi-major axis of the constellation satellite and $\delta\omega$ is the angular distance between the constellation satellite and Minimum Orbit Intersection Distance (MOID) at the instant in which the crossing object is at the MOID, as shown in figure 1. Substituting equations 3, 4 and 5 into equation 1 yields:

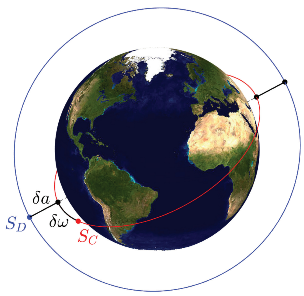

Figure 1: Miss distance vector when the crossing satellite is at the MOID.

$$P = P_{\odot} \exp\left(-\frac{1}{2}\frac{\delta a^2}{\sigma_r^2}\right) \exp\left(-\frac{1}{2}\frac{\delta z^2}{\sigma_\theta^2}\right) \qquad (6)$$

where $\sigma_r = \sigma_{\hat{x}}$, $\sigma_\theta = \sigma_{\hat{z}}/\cos\frac{\varphi}{2}$ and $\delta z = 2a_C \sin\frac{\delta\omega}{2}$.

2.2 *Mean collision probability with one constellation's orbital plane*

The mean collision probability with one constellation's orbital plane is modelled by assuming that the spiral trajectory of the crossing object results in a sequence of N_{CA} close approaches, all characterised by the same average collision probability $\overline{P}$ and the overall collision probability is given by:

$$P_{plane} = 1 - (1 - \overline{P})^{N_{CA}} \qquad (7)$$

The mean collision probability $\overline{P}$ is the integral of the probability distribution over the the annulus $\mathfrak{D}$ centred at the constellation's orbital plane, represented in figure 2, divided by its area. The width of the annulus $\mathfrak{D}$ is set to be equal to $2\alpha\sigma_r$, where α is a parameter. As the mean probability is evaluated within $\mathfrak{D}$, a close

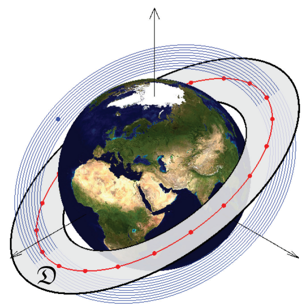

Figure 2: Probability evaluation domain $\mathfrak{D}$.

approach is considered if both the crossing object and a constellation's satellite are in $\mathfrak{D}$ at the same time instant. Let N_S be the number of constellation's satellites per orbital plane, then N_S close approaches will occur every time the crossing object intersects the domain $\mathfrak{D}$. Two intersections per revolution shall occur, thus the number of close approaches is equal to:

$$N_{CA} = 2N_S N_{\cap}^{\mathfrak{D}} \qquad (8)$$

where $N_{\cap}^{\mathfrak{D}} = 2\alpha\frac{\sigma_r}{|\Delta a|}$ is the number of revolutions of the crossing object that intersects $\mathfrak{D}$ and Δa is the semi-major axis rate of change per revolution of the crossing object, which is assumed to be constant during the shell crossing event.
Finally, the collision probability with one orbital plane is evaluated by imposing that $\mathfrak{D}$ has an infinite radial extension and the following expression is obtained:

$$P_{plane} = 1 - \exp\left[-2\sqrt{2\pi}\frac{P_{\odot}N_S\sigma_r}{|\Delta a|}\exp\left(-\frac{a_C^2}{\sigma_\theta^2}\right) I_0\left(\frac{a_C^2}{\sigma_\theta^2}\right)\right] \qquad (9)$$

where I_0 is the modified Bessel function of the first kind of order zero.
Moreover, for most cases, the values of $\delta\omega$ that lead to appreciable collision probabilities are small and the arc-length, rather than the chord-length, can be considered for the evaluation of the along-track distance. It follows that $\delta z \approx a_C \delta\omega$, and:

$$P_{plane} = 1 - \exp\left[-2\frac{P_{\odot} N_S \sigma_r \sigma_\theta}{|\Delta a| a_C}\right] \quad (10)$$

2.3 Shell crossing event collision probability

The overall mean collision probability of a shell crossing event can be evaluated by considering the probabilities of collision with each one of the N_P orbital planes which compose the constellation's shell:

$$P_{shell} = 1 - \prod_{i=1}^{N_P}(1 - P_{plane,i}) \quad (11)$$

where $P_{plane,i}$ is given by equation 9 if $\varphi \approx \pi$ and by equation 10 otherwise.
The expression for the general case which does apply for most values of the angle can be further simplified. For most satellite applications, indeed $2\sigma_{\hat{x}}\sigma_{\hat{z}} \gg r_a^2$ and $P_{\odot} \approx \frac{r_a^2}{2\sigma_{\hat{x}}\sigma_{\hat{z}}}$ and if the shell crossing event is characterised by the absence of head-on collisions, equation 11 becomes:

$$P_{shell} = \frac{N_S r_a^2}{|\Delta a| a_C} \sum_{i=1}^{N_P} \frac{1}{\cos\frac{\varphi_i}{2}} \quad (12)$$

which does not depend anymore on the position errors of the satellites.

3 Results

The model hereby developed has been used for risk assessments of end-of-life constellation satellites replacement at different engine input powers, also considering the presence of space debris. 12 approved constellations that are already, are being or to be deployed have been considered, for a total of 33 shells and 18,348 satellites. Results are shown in figure 3. The consequences of an in-orbit catastrophic collision have also been investigated by means of the probability of a collision between constellations satellites and the generated fragments cloud.

4 Conclusion

This new model constitutes an efficient tool for preliminary analyses in the context of LEO traffic management. Using an i7-4720 at 2.6 GHz with 8 GB

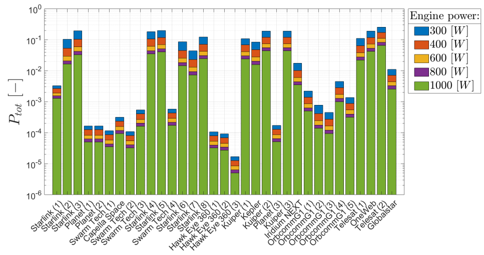

Figure 3: Collision probability related to the replacement of every shell considered.

of RAM, a million of shell crossing events can be assessed in less than 6 seconds, providing the collision probability P_{shell} averaged over every possible true anomaly phase between the constellation's and crossing satellites. The evaluation of P_{shell} has been validated with a propagation model, considering different orbits orientations, error ellipsoids, engine input powers and number of satellites involved [4].

5 Aknowledgements

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