Low-cycle fatigue life prediction and reliability evaluation of turbine blades with distributed collaborative LSSVR

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Abstract: Low-cycle fatigue (LCF) of turbine blades involves several disciplines with multi-uncertain factors and is a high-nonlinear complex problem. To improve prediction accuracy and computational efficiency, this paper develops DC-LSSVR approach, integrating least squares support vector regression (LSSVR) into the distributed collaborative (DC) strategy, for the LCF life prediction and reliability evaluation of turbine blades. Considering the influence of the uncertain factors, i.e., design sizes, applied loads and material properties, the reliability assessment framework is constructed. Through the integration of the DC-LSSVR reliability method with the theoretical models, including the Smith-Watson-Topper (SWT) mean stress correction and linear cumulative damage (LCD) rule, the LCF life is predicted and reliability evaluation is completed. Finally, the DC-LSSVR is proved to be a promising approach for the reliability assessment of complex structures.

Keyworks: Turbine blade, reliability analysis, fatigue life prediction, uncertain factors, DC-LSSVR.

Acronyms and Abbreviations

LCF	Low-cycle fatigue.
FORM	First-order reliability method.
SORM	Second-order reliability method.
RSM	Response surface method.
MCM	Monte Carlo method.
DCRSM	Distributed collaborative RSM.
LSSVR	Least squares support vector regression.
LSSVM	Least squares support vector machine.
DC-LSSVR	Distributed collaborative LSSVR.
SWT	Smith-Watson-Topper mean stress correction.
LCD	Linear cumulative damage.

Notations

σ_{max}	Maximum stress.
ε_a	Strain amplitude.
N_f	Number of cycles to failure.
D_f	LCF failure damage.
R	Reliability degree.
P_f	Failure probability.

1. Introduction

Turbine blades are important components for a gas engine and suffer from multi-failure causes, in which LCF seriously affects structure performance and reliability (Mishra 2018). Hence, LCF life prediction and reliability assessment is critical for the design and maintenance of turbine blades. However, in the process, there are multi-uncertain factors in design sizes, applied loads and material properties. The uncertainties should be carefully considered for accurate LCF life prediction and reliability evaluation (Zhu 2018).

Numerical simulation-based probability analysis approaches are the most applied for fatigue life prediction and reliability evaluation. The probability analysis methods mainly include first-order reliability method (FORM) (Cornell 1970), second-order reliability method (SORM) (Fu 2018), response surface method (RSM) (Pan 2015), Monte-Carlo method (MCM) (Alessandri 2018) and distributed collaborative response surface method (DCRSM) (Gao 2015, 2016, 2018 and 2019). However, the primary differences among the approaches are numerical accuracy and computation efficiency. These methods have some limitations for high-nonlinear probabilistic analyses due to the insufficient accuracy and efficiency.

In recent years, machine learning algorithms have been successfully applied to the field of the simulation and solution of structural reliability. LSSVM is applied to convert training model into solving a set of linear equations, in order to reduce the computational complexity and improves running speed (Guo 2009). In order to improve the prediction accuracy and computational efficiency, this paper integrates the distributed collaborative (DC) strategy and least squares support vector regression (LSSVR) for the LCF life prediction and reliability evaluation of turbine

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blades with multi-uncertain factors.

The distributed collaborative LSSVR (DC-LSSVR) reliability approach is proposed in Section 2. Considering the randomness of multi-uncertain factors, the reliability evaluation framework is constructed through the integration of the DC-LSSVR reliability method with theoretical models, including the Smith-Watson-Topper (SWT) mean stress correction and the linear cumulative damage (LCD) rule, in Section 3. Section 4 completes the LCF life prediction and reliability evaluation of turbine blades to verify the rationality and feasibility of the proposed DC-LSSVR. Some conclusions on this work are summarized in Section 5.

2. Basic approach

2.1. *LSSVR*

For a given data $\{(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \cdots, (\boldsymbol{x}_l, y_l)\} \in R^n \times R$, $\boldsymbol{x}_i$ is the input vector and y_i is the corresponding output response. The goal of modeling is to construct a regression function $y = f(\boldsymbol{x})$ to predict the outputs $\{y_i\}$ given a new set of samples and make the structure risk minimize. In the feature space, the linear regression function can be expressed as follow:

$$f(\boldsymbol{x}) = \boldsymbol{\omega}^T \varphi(\boldsymbol{x}) + b \tag{1}$$

where $\boldsymbol{\omega}$ is the weight vector, $\varphi(\cdot)$ denotes the nonlinear mapping function that can map nonlinear regression in the original feature space into linear regression in the high-dimensional feature space, and b is the bias term.

The linear regression problem can be expressed as an optimization problem with equality constraint according to the principle of structural risk minimization, fully considering the minimization of the Vapnik-Chervonenkis (VC) dimension and empirical risk. The objective function of the optimization problem is given as follows:

$$\begin{cases} \min\limits_{\boldsymbol{\omega}, b, e} & \frac{1}{2}\|\boldsymbol{\omega}\|^2 + \frac{1}{2}C\sum\limits_{i=1}^{l} e_i^2 \\ \text{subject to} & e_i = y_i - \boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) - b \end{cases} \tag{2}$$

where C is the regularization parameter to determine the trade-off between the training error minimization and the smoothness of the estimated function. e_i is the error between the actual output and predictive output of the ith sample.

In order to solve the above optimization problem, the Lagrange multiplier α_i is introduced. Further, Eq. (2) is converted into an optimization problem without constraint given by Eq. (3):

$$L(\boldsymbol{\omega}, b, e_i, \alpha_i) = \frac{1}{2}\|\boldsymbol{\omega}\|^2 + \frac{1}{2}C\sum_{i=1}^{l} e_i^2 - \sum_{i=1}^{l} \alpha_i[e_i - y_i + \boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) - b] \tag{3}$$

Based on Karush-Kuhn-Tucker (KKT) conditions, the following equations are obtained:

$$\begin{cases} \frac{\partial L}{\partial \boldsymbol{\omega}} = 0 \to \boldsymbol{\omega} = \sum\limits_{i=1}^{l} \alpha_i \varphi(\boldsymbol{x}_i) \\ \frac{\partial L}{\partial b} = 0 \to \sum\limits_{i=1}^{l} \alpha_i = 0 \\ \frac{\partial L}{\partial e_i} = 0 \to \alpha_i = Ce_i \\ \frac{\partial L}{\partial \alpha_i} = 0 \to \boldsymbol{\omega}^T\varphi(\boldsymbol{x}_i) + b + e_i - y_i = 0 \end{cases} \tag{4}$$

Finally, the regression decision function can be written as the following explicit expression:

$$f(\boldsymbol{x}) = \sum_{i=1}^{l} \alpha_i K(\boldsymbol{x}, \boldsymbol{x}_i) + b \tag{5}$$

where α and b follow from Eq. (4), and $K(\boldsymbol{x}, \boldsymbol{x}_i)$ is a kernel function with Mercer condition.

2.2. *Distributed collaborative LSSVR*

For the probability analysis problem involving l level interrelated structure responses, we assume that $\boldsymbol{y}^l \in R^p$ and $\boldsymbol{x}^l \in R^n$ respectively denote the l-th level output responses and corresponding input variables. According to the basic principle of LSSVR, the k-th output response $y_k^1 (k = 1,2,\cdots,p_1)$ of the first-level probability analysis can be expressed as Eq. (6):

$$y_k^1 = f_k^1(\boldsymbol{x}_k^1) = \sum_{i_1=1}^{l_1} \alpha_{k_{i_1}}^1 K_k^1\left(\boldsymbol{x}_k^1, \boldsymbol{x}_{k_{i_1}}^1\right) + b_k^1 \tag{6}$$

where $\boldsymbol{x}_k^1$ is the corresponding input variable vector of the k-th output response y_k^1. α_k^1, b_k^1 and K_k^1 are corresponding parameters. Similarly, we can establish all the first-level distributed LSSVR models.

Considering the specified failure dependence that the same samples are selected for the same variables, the first-level output responses $\boldsymbol{y}^1 = [y_1^1, y_2^1, \cdots, y_{p_1}^1]$ are regarded as the input variables of the second-level output responses $\boldsymbol{y}^2$. The s-th output response $y_s^2 (s = 1,2,\cdots,p_2)$ of the second-level probability analysis given a new set of samples $\boldsymbol{x}_s^2$ can be expressed as Eq. (7):

$$y_s^2 = f_s^2(\boldsymbol{y}_s^1) = \sum_{i_2=1}^{l_2} \alpha_{s_{i_2}}^2 K_s^2\left(\boldsymbol{y}_s^1, \boldsymbol{y}_{s_{i_2}}^1\right) + b_s^2 \tag{7}$$

where p_2 is the number of the output responses of the second-level probability analysis. $\boldsymbol{y}_s^1 \in \boldsymbol{y}^1$ is the input variable vector corresponding to the s-th output response y_s^2. Eq. (7) is called the second-level distributed LSSVR. In the same way, all the distributed LSSVR models can be established.

Finally, according to the specified failure criterion, the r-th global response $y_r^l (r = 1,2,\cdots,p_l)$ of the l-th level probability analysis can be expressed as Eq. (8):

$$y_r^l = f_r^l(\boldsymbol{y}_r^{l-1}) = \sum_{i_l=1}^{l_l} \alpha_{r_{i_l}}^l K_r^l\left(\boldsymbol{y}_r^{l-1}, \boldsymbol{y}_{r_{i_l}}^{l-1}\right) + b_r^l \tag{8}$$

where p_l is the number of the global output responses.

$\boldsymbol{y}_r^{l-1} \in \boldsymbol{y}^{l-1}$ is the input variable vector corresponding to the r-th global response y_r^l. Eq. (8) is the r-th collaborative LSSVR. Similarly, all the collaborative LSSVR models can be established $\boldsymbol{y}^l = [y_1^l, y_2^l, \cdots, y_{p_l}^l]$ to predict the required samples.

This approach is called as distributed collaborative LSSVR (DC-LSSVR) surrogate modeling. If DC-LSSVR is applied to the probabilistic analysis of complex structures, this method is more efficient based on the previous studies (Gao 2019).

3. Theoretical framework

3.1. *Preparation*

High-pressure turbine blades are selected to perform the LCF life prediction and damage assessment, to validate the availability of the DC-LSSVR approach. The detailed explanation about the finite element (FE) model, loading mode and constraint scheme for the gas turbine blade with firtree-tenon can be found in the references (Gao 2018). The surface-to-surface contacts is applied to consider the effect of turbine disks on the turbine blade in the LCF life prediction and damage assessment.

In this paper, the considered random variables mainly include the blade-tip width w_1, the blade-root width w_2, the maximum blade-tip thickness t_1, the maximum blade-root thickness t_2, the blade-tip torsion angle δ_1, the blade-root torsion angle δ_2, the blade height H, the upper leading-edge temperature T_1, the lower leading-edge temperature T_2, the upper trailing-edge temperature T_3 and the lower T_4, the blade-root temperature T_5, the lower disk-edge temperature T_a, the first tenon-rim temperature T_b, the angular velocity ω, the elastic modulus E, the density ρ, the Poisson ratio μ, the heat conductivity λ, the specific heat $\acute{c}$ and the linear expansion coefficients α. The random variables are assumed to be mutually independent and follow the Gaussian distributions, as shown in Table 1 (Gao 2020).

Table 1. Basic random variables and stochastic models.

Variable	Mean	Std. Dev.	Variable	Mean	Std. Dev.	Variable	Mean	Std. Dev.
w_1, mm	31.7472	0.6349	T_1,℃	803.5136	16.0703	E, MPa	161000	3220
w_2, mm	41.1602	0.8232	T_2,℃	782	15.64	ρ, t/mm^3	8.56×10^{-9}	1.712×10^{-10}
t_1, mm	3.6416	0.07283	T_3,℃	750.5	15.01	μ	0.3172	0.006344
t_2, mm	8.1968	0.1639	T_4,℃	714.8	14.296	λ, N/(s·℃)	16.92	0.3384
δ_1, radian	0.5458	0.01092	T_5,℃	660	13.2	$\acute{c}$, mm^2/(s^2·℃)	4.94×10^{8}	9.88×10^{6}
δ_2, radian	0.3397	0.006794	T_b,℃	500	10	α, /℃	1.285×10^{-5}	2.57×10^{-7}
H, mm	100	2.0	T_a,℃	230	4.6	ω, radian/s	1 016.4	20.328

3.2. *Reliability assessment framework*

Smith, Watson and Topper suggested that the product of the maximum tensile stress σ_{max} and strain amplitude ε_a in strain-life model controls the influence of both the mean stress and strain amplitude. The Smith-Watson-Topper (SWT) model (Smith 1970) was proposed in Eq. (9), which assumes that the product $\sigma_{max}\varepsilon_a$, at a given life, remains constant for different combination of the maximum stress σ_{max} and strain amplitude ε_a. The SWT mean stress correction gives good agreement with test data in long life regime, while it is conservative in LCF region (Ince 2011).

$$\sigma_{max}\varepsilon_a = \frac{(\acute{\sigma}_f)^2}{E}(2N_f)^{2b} + \acute{\sigma}_f\acute{\varepsilon}_f(2N_f)^{b+c} \quad (9)$$

which N_f the number of cycles to failure. b is the fatigue strength exponent, $\acute{\sigma}_f$ the fatigue strength coefficient, c the fatigue ductility exponent, $\acute{\varepsilon}_f$ the fatigue ductility coefficient. E is the elastic modulus.

This paper will apply the SWT model to predict LCF life of the turbine blade. Because the number of cycles to failure (LCF life) N_f in Eq. (9) can be expressed as

$$N_f = N_f(\sigma_{max}, \varepsilon_a, E, \acute{\sigma}_f, b, \acute{\varepsilon}_f, c) \quad (10)$$

according to the basic principle of DC-LSSVR, the maximum stress σ_{max} and strain amplitude ε_a are regarded as first-level output responses; The relationships $\sigma_{max}(\boldsymbol{x})$ and $\varepsilon_a(\boldsymbol{x})$ between the maximum stress σ_{max}, strain amplitude ε_a and input variables $\boldsymbol{x} = [w_1, w_2, t_1, t_2, \delta_1, \delta_2, H, T_1, T_2, T_3, T_4, T_5, T_a, T_b, \omega, E, \rho, \mu, \lambda, \acute{c}, \alpha]$ are the first-level distributed LSSVR models. The LCF life N_f is the second-level output response; Eq. (9) is regarded as the second-level distributed LSSVR model.

The linear cumulative damage (LCD) rule is applied to assess the LCF failure damage D_f (Miner 1945), as expressed in Eq. (11) under constant-amplitude cyclic loads.

$$D_f = \frac{n_0}{N_f} \quad (11)$$

In which D_f is fatigue damage and n_0 is the number of applied cycles. The LCF damage D_f is considered as the global output responses and the LCD rule in Eq. (11) is the collaborative LSSVR model.

The limit state function is defined as follows:

$$Z = a - D_f \quad (12)$$

where a is the damage strength parameter and is usually set to $a = 1$. In this paper $Z \leq 0$ defines the failure domain.

The reliability degree R/failure probability P_f can be estimated by:

$$R = 1 - P_f = 1 - \frac{1}{M}\sum_{j=1}^{M} I_F[Z_j] = 1 - \frac{m}{M} \quad (13)$$

where $I_F[\cdot]$ is the indicator function in the failure domain,

m the number of sample points in the failure domain and M the number of the total sample points Z.

4. Case study

4.1. *First-level output responses*

Through some required simulations with the random variables $\boldsymbol{x}$, 253 sets of sample points at the dangerous point (the first firtree-tenon of the suction surface) for the turbine blade are extracted by the Latin hypercube method. These sample points are adopted to establish the first-level distributed LSSVR models $\sigma_{max}(\boldsymbol{x})$ and $\varepsilon_a(\boldsymbol{x})$. The sampling history and frequency distribution histograms of 10000 sets of first-level output responses σ_{max} and ε_a are drawn in Fig. 1 and Fig. 2, respectively. Their statistical characteristics are listed in Table 2. As shown in Fig. 1, Fig. 2 and Table 2, the maximum stress σ_{max} and strain amplitude ε_a at the dangerous point follow normal distributions for the turbine blade.

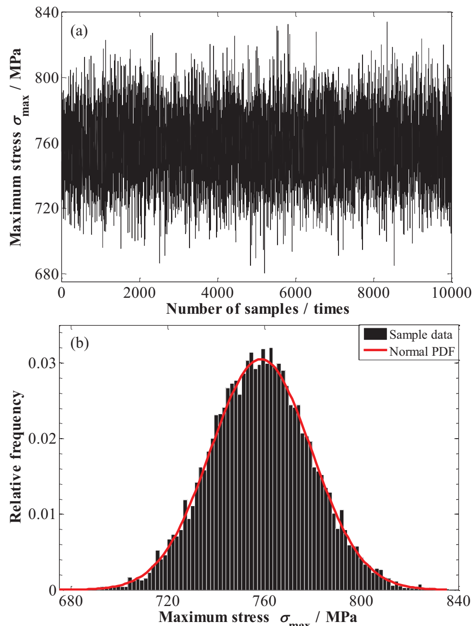
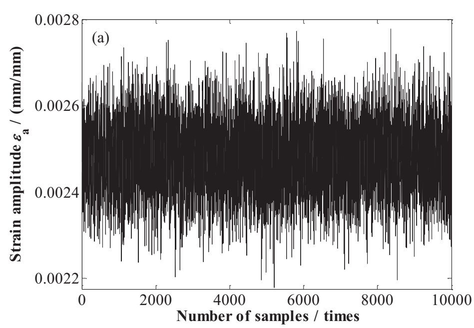


Fig. 1. (a) Sampling history and (b) frequency distribution histogram of the maximum stress σ_{max}.



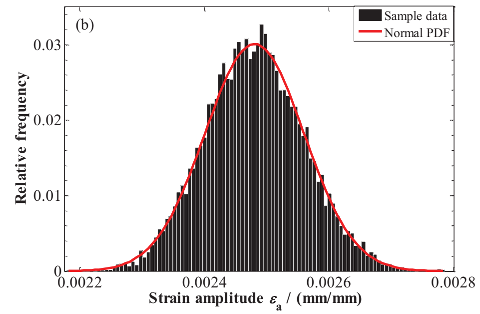


Fig. 2. (a) Sampling history and (b) frequency distribution histogram of the strain amplitude ε_a.

Table 2. Statistical characteristics of first-level output responses.

Response	Mean	Std. dev.	Distribution
σ_{max}, MPa	757.69	20.352	Normal
ε_a, mm/mm	2.4806×10^{-3}	3.9718×10^{-5}	Normal

4.2. *LCF life prediction*

In this section, the maximum stress σ_{max} and strain amplitude ε_a at the dangerous point are considered as two random variables of the LCF life N_f. In the second-level distributed LSSVR model, the statistical characteristics of the other random variables are as follows:

Table 3. Distribution characteristics of the material parameters.

Variable	ε_f'	c	σ_f', MPa	b
Mean	0.2634	-0.5041	1388.1	-0.1106
Std. dev.	0.005268	0.010082	27.762	0.002212
Distribution	Lognormal	Normal	Normal	Normal

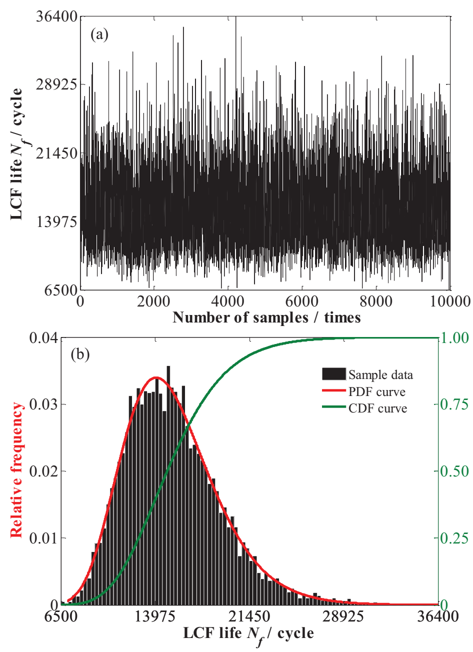


Fig. 3. (a) Sampling history and (b) distribution characteristics of the LCF life N_f.

The sampling history and distribution characteristics of 10000 sets of the LCF life N_f are drawn in Fig. 3, where the red and green curves represent the probability density function (PDF) and cumulative density function (CDF) of a fitted lognormal distribution, respectively. The statistical properties are shown in Table 4. From Fig. 3, we can see that the LCF life N_f follows lognormal distribution for the turbine blade.

Table 4. Statistical characteristics of the LCF life N_f.

Parameter	Estimate	Std. Err.	Distribution
Mean, cycle	9.61162	0.00243161	Lognormal
Std. dev., cycle	0.243161	0.00171954	

4.3. *Reliability evaluation*

In this section, the LCD rule is applied to assess the LCF failure damage D_f with the number of applied cycles n_0 for the turbine blade, as shown in Fig. 4.

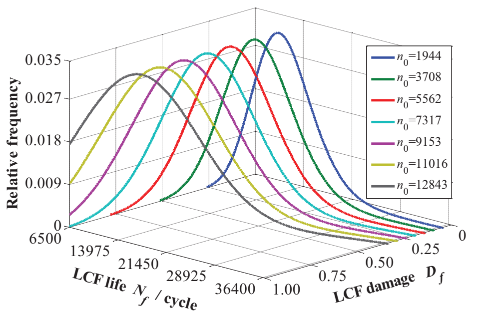


Fig. 4. Distribution characteristics of the LCF damage D_f.

Afterwards, the limit state function in Eq. (12) is applied to predict the probabilistic distribution of the structural response Z, as shown in Fig. 5. To graphically illustrate the LCF damage reliability R with the number of applied cycles n_0, the acquired samples of the structural response Z are employed to plot the relationship between the number of applied cycles n_0 and LCF damage reliability R, as drawn in Fig. 6. As shown in Figs. 4~6, the LCF damage reliability R of the turbine blade gradually reduces with increasing number of applied cycles n_0.

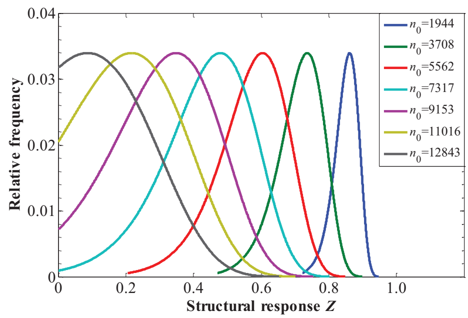


Fig. 5. Distribution characteristics of the structural response Z.

4.4. *DC-LSSVR validation*

In this section, the prediction accuracy and computation efficiency of the proposed DC-LSSVR are validated through the comparison of the LCF reliability and running time with MCM, RSM, LSSVR and DC-LSSVR, in which the numerical results of MCM are considered as the reference for the comparison of RSM, LSSVR and DC-LSSVR listed in Table 5. From Table 5, we can see that the proposed DC-LSSVR can improve the prediction accuracy and computational efficiency, due to higher-accurate reliability and less time than RSM and LSSVR.

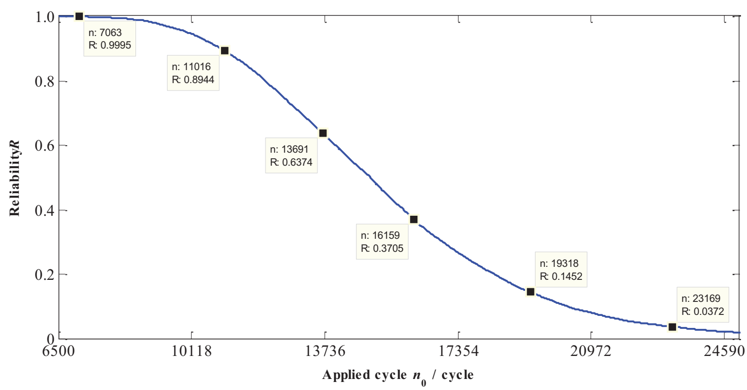


Fig. 6. Distribution of the LCF damage reliability R with the number of applied cycles n_0.

Table 5. Running time and reliabilities with the number of applied cycles for different methods.

Method	Running time, s	$n_0 = 1944$	$n_0 = 3708$	$n_0 = 5562$	$n_0 = 7317$	$n_0 = 9153$	$n_0 = 11016$	$n_0 = 12843$
MCM	275375	1.0000	1.0000	1.0000	0.9993	0.9797	0.8945	0.7262
RSM	157579	1.0000	1.0000	0.9979	0.9744	0.8833	0.7087	0.5155
LSSVR	7685	1.0000	1.0000	1.0000	0.9957	0.9576	0.8398	0.6527
DC-LSSVR	2830	1.0000	1.0000	1.0000	0.9982	0.9737	0.8798	0.7064

5. Conclusions

Through the integration of least squares support vector regression (LSSVR) and distributed collaborative (DC) strategy, DC-LSSVR reliability approach is proposed for the low-cycle fatigue (LCF) life prediction and reliability evaluation of turbine blades with multi-uncertain factors. Through the case study, the proposed DC-LSSVR method has been proved to improve prediction accuracy and computational efficiency. In addition, this approach provides a new thinking for the reliability problems of complex structures.

Through the integration of the DC-LSSVR reliability method with the theoretical models, including the Smith-Watson-Topper (SWT) mean stress correction and linear cumulative damage (LCD) rule, the reliability assessment framework is constructed for the LCF life prediction and reliability evaluation of the turbine blade. Considering the relevant uncertain factors, the LCF life and damage are predicted, and the reliability assessment is launched for the turbine blade. The numerical results show that the LCF life follows lognormal distribution and the reliability reduces with increasing number of applied cycles for the turbine blade.

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