Field-to-Wire Coupling in Bundles of Wires: Comparison Between Different Models

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Abstract

In this paper, the electromagnetic disturbances induced by a plane-wave field in random wire bundles routed in close proximity with a ground plane is investigated by adopting a statistical approach. Accurate geometrical description of the random trajectories of the wires is used in combination with a multiconductor transmission line model of the bundle involving fine segmentation into a sequence of uniform cascaded sections. Repeated-run analysis is then adopted for reproduction of the frequency response of the voltages induced across the terminal loads. The outcomes of this analysis are compared versus a modeling approach previously developed and based on a more approximate representation of the wire trajectories. It is shown that the ability to reproduce smooth wire trajectories (mimicking the shape of wires in real bundles) is important in order to avoid artifacts in the frequency response, at specific frequencies, and lead to induced voltages which, at high frequencies, show probability density functions with different shape.

1 Introduction

Uncertainty quantification in Electromagnetic (EM) Compatibility modeling of hand-assembled wiring harnesses is a challenging task owing to the inherent stochastic nature of these complex bundles [1]. Precisely, physically-sound representation of the bundle geometry as well as effective modeling of the involved EM phenomena are important to estimate the noise at the terminal units, with the aim to design proper mitigation techniques for reliable operation of the whole system.

Several geometrical modeling techniques for random wire-bundles can be found in the literature [1-5], though many of them do not intentionally reproduce the finest geometrical details of random wire paths, possibly leading to non-smooth wire transition, overlapping of different wires, etc. [4], [5]. In recent works, a statistical radiated-susceptibility (RS) study of random wire bundles was presented [1], [2]. Specifically, the low-frequency behavior (i.e., for a cable length much shorter than the wavelength) was analyzed resorting to analytical expressions [1], whereas electrically long bundles were investigated through simulations [2]. Due to the low-fidelity geometrical representation adopted in [1], [2] (whose complexity is intentionally limited to lower the computational burden), significant artifacts and prediction errors may appear at high frequency [6].

In this work, these limitations are solved by resorting to a recently developed iterative bundle-generation process [6], based on the following features: a) accurate mathematical representation of random wire trajectories inside the bundle with high-order polynomial functions, b) incorporation of physical constraints (nonoverlapping, continuity and compactness) in the algorithmic process. This geometrical model is then used as an effective tool for investigating the statistics of voltage induced at terminal loads by an external plane-wave field, by means of repeated-run (Monte Carlo) simulations. In contrast to the previous models, the geometry obtained in paper preserves the smooth shape of random wire bundles, and, hence, avoids unphysical spurious resonances in the high-frequency region due to unrealistic mutations of wire positions. By such an accurate representation and a suitable procedure for the calculation of p.u.l. parameters, the feasibility of a conventional multiconductor transmission line (MTL) solution (that is, the Uniform Cascade Section (UCS) technique [7]) is extended to a higher frequency range even in the presence of strong line non-uniformity, thus avoiding the tremendous computational burden of full-wave solvers.

This paper is organized as follows. In Sec. 2, the basic geometric model of random wire bundle and the MTL solution are briefly discussed. Sec. 3 applies the technique to the study of the RS of wires inside the bundle, and the previous rough model [1] is used for comparison, to illustrate the significance of accurate modelling. Several statistical properties of wire bundles are also demonstrated. Sec. 4 draws concluding remarks.

2 Wire Bundle Modeling and Solution Technique

2.1 Summary of Wire Bundle Modeling

With respect to the reference framework in Fig. 1, [6], a single wire i inside a random bundle consisting of N wires can be expressed by its center trajectory, f(x_i, y_i), where x_i, y_i are sets of polynomial functions in the x- and y-directions, respectively. The analytic representation of the bundle is generated by using (a) a mature algorithm based on Graph Theory [1] for the generation of reference cross-sections, and (b) a suitable polynomial interpolation algorithm, together with (c) an ad hoc heuristic algorithm to assure non-overlapping constraints for wires in the bundle. After an iterative process, the polynomial function representation of wires is obtained retaining all the physical constraints of realistic wire bundles [6].
It is worth noting that, owing to closed-form polynomial expressions of wire trajectories, accurate RS predictions could be easily obtained by any full-wave EM solver (by importing wire geometry using 3D analytic curve functionalities, suitably setting up terminal and boundary conditions, and solving the Maxwell’s equations). In this work, a less accurate though more computationally efficient MTL solution is proposed.

2.2 Solution of Field-to-Wire Coupling

The MTL model (UCS method) is used in combination with the simplified field-to-wire coupling model [7, Sec. III-B] for evaluating an equivalent-circuit representation of the bundle. Specifically, the bundle is discretized and approximated by a large number of uniform MTLs, and the circuit model of the entire bundle is constructed by cascading the equivalent-circuit models of each uniform MTL section, then solved for terminal induced voltages. The parameters of the plane-wave incident field are defined in Fig. 1. Each wire in the bundle is connected to ground through an impedance.

![Figure 1. Parameters of the incident plane-wave field. Each wire is terminated to ground through an impedance (just one is sketched for simplicity) at both ends.](image)

This efficient UCS solution was already validated in [6] by comparison with a full-wave solution. Hence, this method can be applied in the proposed statistical study.

3 Statistical Analysis of RS

3.1 Simulation Configuration

The wire bundle is divided into a small number \( N_b \) (here \( N_b=10 \)) of sections driving the change of wire locations along the path of a hand-assembled cable. In [1], these sections had not only this geometrical meaning, but also a major role in the EM solution, since they were roughly modeled as uniform MTLs in the UCS method. Conversely, in the proposed bundle model, the slow movement of wires inside each section is finely represented and, several subsections are defined for the UCS solution. Three different bundles with increasing number of wires (\( N_b=7, 13, 19 \)) are exemplified. The initial cross section is compact with pseudo-circular shape (see Fig. 2 for \( N_b=19 \)).

Different bundle samples are generated to obtain a collection of approximately 2,000 curves in each case (see the caption of Fig. 4 for more details). For all the bundles, the wire radius is 0.5 mm with conductor radius 0.25 mm, and air insulation is considered; the initial wire separation/radius ratio (see [6]) is set to be 3. The bundle runs at an average height of 3.5 mm above a perfect metallic ground plane (note that strong line nonuniformity arises due to this low height configuration), and its axial length is 1 m. Each wire in the bundle is connected to ground by an impedance \( Z=150 \, \Omega \), both at the left and right terminals. In the radiated-susceptibility test setup, the structure is illuminated by an external electromagnetic field (see Fig. 1) characterized by E-field strength \( E_0=1 \, \text{V/m} \) and incidence angles \( \theta=50^\circ, \eta=60^\circ, \psi=20^\circ \). As illustrative examples, the 3D geometries of three bundles with the above parameters are generated and illustrated in Fig. 3 (for better visualization, half of the axial length, i.e. 0.5 m is shown).

![Figure 2. Initial crosssection for a wire bundle composed of 19 wires.](image)

![Figure 3. Generated random bundle samples with 0.5 m axial length consisting of (a) 7 wires, (b) 13 wires and (c) 19 wires.](image)

3.2 Frequency Response of Terminal Voltage

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Voltages induced at left termination of different bundle structures are plotted in Fig. 4. Two regions of the frequency response can be readily observed in all the cases: (a) +20 dB/decade slope at low frequencies and (b) complex behavior with resonances at high frequencies. Specifically, in the low-frequency range, the frequency response is more condensed, with an envelope larger than the average value by a few decibels; at high frequencies, the envelope and average values are much more separated, indicating a large sensitivity to different bundle configurations.

For each generated bundle sample, two different prediction models are used: a) the model proposed here and in [6], b) the constant-section model in [1]. At low frequencies where the bundle is electrically short and analytical expressions can be derived, the results obtained by both models are fully consistent. In the frequency range of few megahertz, the enlarged discrepancies start to contribute to the different statistical behaviors (average value and envelope); with the further increase of frequency up to above 1.5 GHz, the model in [1] completely diverges from the real case and even spurious spikes appear (see Fig. 4) due to unphysical wave reflections introduced by abrupt wire changes between nearby sections. Hence, it results that accurate geometry modeling is of paramount importance in analyzing the statistical behaviors.

3.3 Statistical Distributions

The statistical behavior of field-to-wire induced voltage can be better studied by the light of the probability density function (pdf) in different cases (see Fig. 5). For bundles with 7 wires and 10 initial sections, the bundle is well-randomized and the pdf at 1 MHz converges to a Normal density by collecting together the voltages of different wires (see Fig. 5(a) for the normalized histogram of the voltage induced at the left termination); at high frequency, a good approximation of Rayleigh distribution is observed (see Fig. 5(b)). These observations, coherent with [1], can also be predicted by theoretical analysis [2].

Due to the shielding effect of wires, with the increase of wire number, the distributions in both low- and high-frequency ranges changes (see Fig. 5, (c)-(f)). This phenomenon can be better visualized by comparing the induced terminal voltages of inner and outer wires (see Fig. 1 for classification of inner and outer wires). Though the wire position alters along the axial length, the inner wires at the initial cross-section are more likely to keep in the inner layer within a limited number of \( N_i \) sections and similarly for the outer wires. This shielding effect results into an obvious lower induced voltage for the inner wires and higher voltage for the outer wires (see Fig. 6). For the 7-wire bundle, however, since the number of wires is small, the wire position mutation along the bundle will sufficiently randomize the bundle behavior and mitigate the shielding effect, contributing to well-behaved Normal/Rayleigh distributions at low/high frequencies.

4 Conclusion

In this paper, the statistical analysis for voltages induced at terminal loads in random bundles of wires is performed with a precise bundle geometrical description. For well-randomized bundles, the introduced low- and high-frequency terminal voltages yield Normal and Rayleigh distributions respectively, as expected [2]. With a constant wire section number \( N_i \) and increased wire number \( N \), the shielding effect becomes dominant, lowering the induced...
voltage levels of inner wires. Besides, comparison of the different results between the proposed geometrical model and a previous rough model [1] proves the significance of accurate shape modeling in RS prediction of bundles, especially in the high frequency range to avoid unphysical spurious peaks.

5 References


