

Nonlinear Model Predictive Control for Mobile Medical Robot using Neural Optimization

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Abstract—Mobile medical robots have been widely used in various structured scenarios, such as hospital drug delivery, public area disinfection, and medical examinations. Considering the challenge of environment modeling and controller design, how to achieve the information from the human demonstration in a structured environment directly arouse our interests. Learning skills is a powerful way that can reduce the complexity of algorithm in searching space. This is especially true when naturally acquiring new skills, as mobile medical robot must learn from the interaction with a human being or the environment with limited programming effort. In this paper, a learning scheme with nonlinear model predictive control (NMPC) is proposed for mobile robot path tracking. The learning-by-imitation system consists of two levels of hierarchy: in the first level, a multi-virtual spring-dampers system is presented for imitation of the mobile robot's trajectories; and in the second level, the NMPC method is used in the motion control system. The NMPC strategy utilizes a varying-parameter one-layer projection neural network to solve an online quadratic programming optimization via iteration over a limited receding horizon. The proposed algorithm is evaluated on a mobile medical robot with an emulated trajectory in simulation and three scenarios used in the experiment.

Index Terms—Imitation Learning, Multi Virtual Spring-Dampers, Model Predictive Control, Varying-Parameter One-Layer Projection Neural Network.

I. INTRODUCTION

Over the past decade, applications of mobile medical robots have attracted attention in our daily life. It is a powerful assistant platform that helps to reduce the workloads of operators [1]. Besides, mobile medical robots have unparalleled advantages regarding quarantine infectious disease scenarios, such as SARS and COVID-19, and thus can significantly reduce the infection rate [2]. How to control the mobile medical robot in a structured environment with a simple way and low computational complexity is a challenging issue [3].

Machine learning-based robotic system has been a popular research theme. With limited programming effort and almost no *ad hoc* modeling required, learning-by-imitation serves as a promising paradigm by which an agent learns a control

policy for a complex system via observing demonstration in a dynamic environment. Learning-by-imitation consists of a demonstrating expert that provides examples of the optimal behaviors, and a learning agent reproduces similar behaviors by imitating the expert [4]. This paradigm, because of its promising prospect, has been widely used by a variety of robots such as service robots and industrial robots in our daily life [5], [6]. The development of learning methodology has become an important tool in robotics [7]. In [8], the authors proposed the Dynamic Movement Primitives (DMP) algorithm fused with machine learning methods to learn desired trajectories from the demonstration successfully. In [7], a DMP-based reinforcement learning framework is proposed for learning pick-and-place tasks to improve the robustness of robot movements. These works demonstrate the power of the motion encoding process on humanoid-like manipulators and inspire our interests in combining a better encoding algorithm with machine learning methods to further improve motor-skills learning with a mobile robot. In this paper, a novel multi virtual spring-dampers method (MVSD) consisting of two VSD systems is proposed to learn motor skills, which has an extra radial VSD system and better convergence ability to the attraction factor point. Therefore, it not only inherits the characteristics of DMP but also shows a stronger learning ability with lower mean squared error (MSE).

In [9] and [10], the robot learning paradigm is utilized for manipulators or human-like robots. However, none of these works focus on the mobile robot. Motivated by [11] and [12], we propose a novel motor-skills learning system involving demonstration of imitation learning and the motion control for mobile robots.

For a mobile robot control, the traditional nonlinear sliding mode control is proposed for quickly switching to an inverted pendulum mode with a small turning radius [13]. However, physical constraints have not been considered when using these traditional control methods, such as velocity limits and velocity incremental limits, which hampers the full exhibition of the imitation-learning power of mobile robots. In [14], linear MPC is employed for the mobile robot model considering the obstacles. However, the linear model is inadequate to describe the object model. Due to the properties of the nonlinear model predictive control (NMPC) algorithm such as input-output non-linearity along a limited horizon [15], NMPC is widely applied to path tracking of the mobile robot under kinematic constraints [16], [17]. Moreover, it makes sure that the control sequences are limited physically, which is important for the motion control of a mobile robot [18].

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However, NMPC suffers from computational complexity as it highly relies on numerical optimization. In [19], a proposed neural-dynamic MPC method prevented some major pitfalls drew by conventional numerical optimization of NMPC, with advantages of global convergence and computational simplicity. Actually, neural network methods can solve real-time optimization problem better than numerical methods under physical conditions [20], [21]. In [18] and [22], a projection neural network (PNN) method and a primal-dual neural network (PDNN) were presented, respectively. Both methods have achieved good performance in solving quadratic programming (QP) problems. However, the convergence rate cannot be further improved for some special cases.

Inspired by the aforementioned works, a novel neural network method called the varying-parameter one-layer projection neural network (VP-OneLPNN) method is proposed to solve QP problems of MPC in this paper. Compared with the PNN and PDNN methods, the proposed VP-OneLPNN model has only one layer, the least number of neurons, and the fastest convergence rate which can ensure optimal solutions for real-time QP problems.

The main objective of this paper is to find a simple and effective method for the robot's repetitive movement, especially passing through a structured environment like a human. Therefore, a learning-control based scheme is designed for a medical mobile robot, which can imitate the motor skills from human demonstration. Thus, it can reduce the complexity of the control algorithm in searching space and guarantees that a mobile robot can run in a structured environment without complicated path planning, such as moving in a hospital. The whole framework that integrates the proposed demonstration learning component and motion control method, consists of two phases: the learning phase and the motion control phase.

In the former phase, MVSD is employed to model and recreate complicated trajectories, with the nonlinear term of MVSD. In the latter phase, VP-OneLPNN based MPC method is applied to manipulate the mobile robot in line with the reproduced trajectory. The scheme of learning-based MPC for mobile robot from human demonstrations is shown in Fig. 1.

In summary, our main contribution is to develop an integrated learning-by-imitation system that includes:

- 1) A novel MVSD approach is proposed for trajectory learning of a mobile robot. This method has good learning performance to ensure high precision of imitation.
- 2) A new VP-OneLPNN algorithm is presented for QP optimization in MPC, in which all state variables can converge rapidly to the optimal one with physical constraints. Compared with the traditional methods, the proposed VP-OneLPNN model has only one layer, the least number of neurons, and the fastest convergence rate.
- 3) The learning system is applied to mobile robots for complex motor skills learning from demonstration, and it's suitable for human-machine interaction in the environment, such as delivering food or medicine to patients.

The remainder of this paper is organized as follows. The imitation learning scheme with our proposed MVSD model is presented in Section II. The kinematic model of the mobile robot and the NMPC system are demonstrated in Section III.

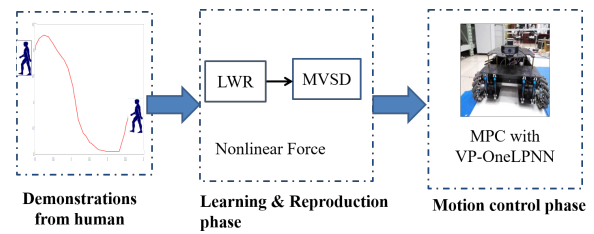


Fig. 1. The scheme of learning-based MPC for the mobile robot from human demonstrations.

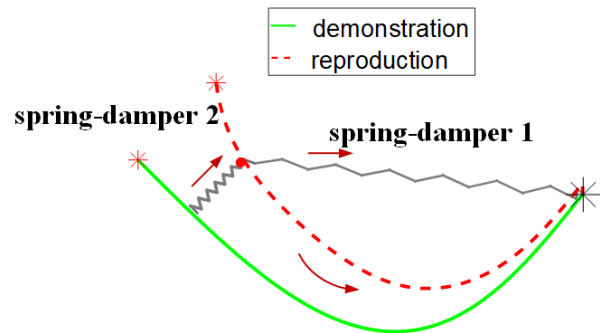


Fig. 2. The model of Multi Virtual Spring-Dampers.

The VP-OneLPNN method aims at real-time QP problem-solving in MPC is introduced in Section IV. Simulation and experimental results are conducted and discussed in Section V and Section VI, respectively. Finally, Section VII concludes this paper.

II. IMITATION LEARNING WITH MULTI-VIRTUAL SPRING-DAMPERS SCHEME

MVSD is a novel second-order dynamical system involving two virtual spring-dampers (VSD) modules, defined as $vsd1$, $vsd2$. A complex dynamical system can be represented by MVSD fusion with a nonlinear forcing term. Hence, the dynamical system of MVSD is modeled as,

$$\ddot{y}_t = K^{p1}(g - y_t) - K^{v1}\dot{y}_t + K^{p2}(y_{\perp} - y_t) - K^{v2}\dot{y}_t + F_t \quad (1)$$

where K^{p1} , K^{p2} and K^{v1} , K^{v2} are the corresponding stiffness matrices, damping matrices for $vsd1$ and $vsd2$; F_t is the nonlinear forcing term of a dynamical system which can be approximated with supervised learning, such as locally weighted regression (LWR); y_t , \dot{y}_t , \ddot{y}_t denote the position, velocity, and acceleration generated by MVSD, respectively; g is the attractor point; y_{\perp} represents the minimum distance from the end-effector to the reference path. The structure of MVSD is shown in Fig. 2.

$$F_t = s_t f_t \quad (2)$$

$$\dot{s}_t = \alpha_s s_t \quad (3)$$

The equation (2), as the nonlinear item, can be employed to model complicated trajectories. The equation (3) represents the canonical system of DMP, where α_s is the constant value and s_t is the asymptotic decay from the initial value to zero.

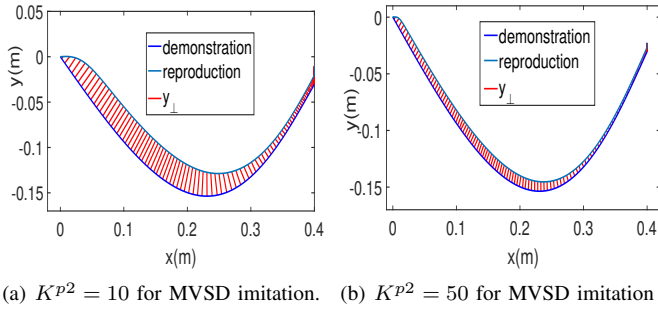


Fig. 3. Imitation results of different K^{p2} for MVSD.

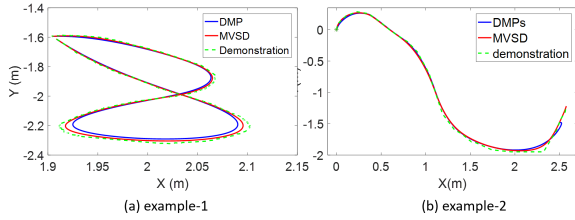


Fig. 4. Comparison between DMP and MVSD.

By multiply the canonical system, F_t will converge to zero ultimately, which indicates that the MVSD model will degrade into a linear multi spring-dampers system. To evaluate the performance of the trajectory estimation, the index called Mean Squared Error (MSE) is adopted, which is denoted as:

$$MSE = \frac{1}{N_{len}} \sum_{i=1}^{N_{len}} (y_{learn}^i - y_{demon}^i)^2.$$

where y_{learn} denotes the reproduction trajectory by learning; y_{demon} is the collection trajectory from human demonstration; N_{len} is the length of y_{demon} . The imitation results with different K^p of MVSD are shown in Fig. 3. In Fig. 4, MSE of Example-1 for MVSD and DMP are 0.0071 and 0.013, respectively, and MSE of Example-2 for MVSD and DMP are 0.0038 and 0.0636, respectively, which indicates MVSD achieves better performance than DMP. Moreover, trajectories with MVSD can converge to the end attractor g even with the different initial positions are shown in Fig. 5.

Remark 1. *MVSD is a novel second-order dynamical system involving two virtual spring dampers modules which has an extra radial VSD modules. There exist two forces acting on the moving object: one acting towards the goal point and the other towards the immediate trajectory point. However, DMP has only one virtual spring dampers. The learning results are shown in Fig. 4, which shows that MVSD has a lower MSE. Therefore, MVSD has a better convergence performance to the desired trajectory than DMP.*

III. MODEL PREDICTIVE CONTROL SCHEME

In this part, a neural network-based MPC control scheme is presented. The proposed neural network methods can optimize the MPC problem on line for the mobile-robot system with physical constraints.

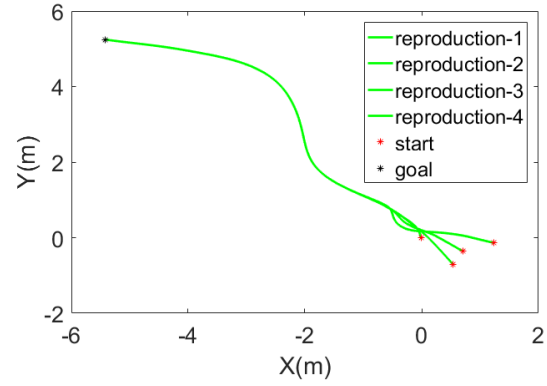


Fig. 5. Imitation learning from random initial position. The trajectory can converge to the goal g from the different initial positions.

A. Mobile Robot Control System

The kinematic model of the two-wheeled mobile robot is depicted in Fig. 6. Considering the relation between the robot velocity v and the two-driving-wheel velocity (v_l, v_r), we describe the robot velocity $v = (v_l + v_r)/2$ and the angle velocity $\omega = (v_l - v_r)/B$ respectively, where B is the distance between the wheels. The kinematic model of the mobile robot is described as follows:

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} u \quad (4)$$

where (x, y) is the Cartesian coordinates of the mobile robot, and θ donates the orientation angle; $u = (v, \omega)^T$ represents the control input, with the state vector X . Based on the representation of (4), the kinematic model of the reference trajectory can be acquired with state vector $X_r = (x_r, y_r, \theta_r)^T$ and control input $u_r = (v_r, \omega_r)^T$, which is expressed as:

$$\dot{X}_r = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} v_r \cos \theta_r \\ v_r \sin \theta_r \\ \omega_r \end{bmatrix} = \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{bmatrix} u_r \quad (5)$$

Consequently, the kinematic errors of the mobile robot can be computed as:

$$X_e = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (6)$$

where $X_e = [x_e, y_e, \theta_e]^T$. According to the error state in (6), the derivative of X_e can be expressed as follows:

$$\begin{cases} \dot{x}_e = \omega y_e - v + v_r \cos \theta_e \\ \dot{y}_e = -\omega x_e + v_r \sin \theta_e \\ \dot{\theta}_e = \omega_r - \omega \end{cases} \quad (7)$$

Then, the control input is reformulated as:

$$u_e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} v_r \cos \theta_e - v \\ \omega_r - \omega \end{bmatrix} \quad (8)$$

Substituting (8) into (7), one can rewrite the kinematic model for X_e as

$$\dot{X}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} + \begin{bmatrix} u_1 \\ v_r \sin \theta_e \\ u_2 \end{bmatrix} \quad (9)$$

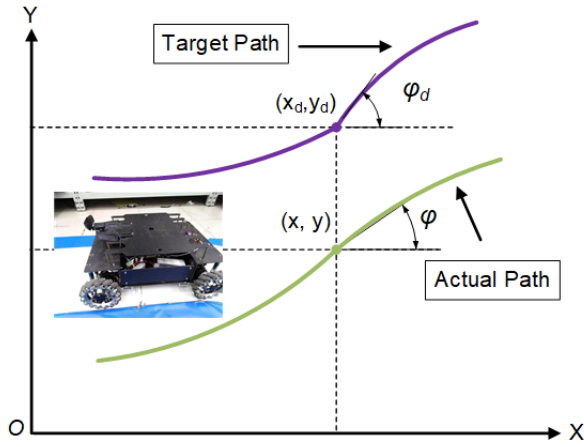


Fig. 6. The kinematic scheme of the two-wheeled mobile robot.

Reshaping the equation (9), one may obtain:

$$\dot{\bar{X}}_e = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & v_r \frac{\sin \theta_e}{\theta_e} \\ 0 & 0 & 0 \end{bmatrix} \bar{X}_e + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u_e. \quad (10)$$

where $\bar{X}_e = [x_e, y_e, \theta_e]^T$. Ultimately, a stabilization issue in the nonlinear affine system of (10) is transformed from the motion control issue with the mobile robot in (4).

B. Nonlinear Model Predictive Control

Considered a closed-loop optimal control matter along with the finite-time domain, the kinematic equation can be defined as the following nonlinear affine system:

$$X(j+1) = \psi_1(X(j)) + \psi_2(X(j))u(j) \quad (11)$$

where $\psi_1(\cdot)$ and $\psi_2(\cdot)$ are the continuous functions with the input vector u and the state vector X as follows:

$$\begin{aligned} X(j) &\in R^X, j = 1, 2, \dots, N, \\ u(j) &\in R^U, j = 1, 2, \dots, N_u. \end{aligned}$$

For the nonlinear affine system (11), both $\psi_1(\cdot)$ and $\psi_2(\cdot)$ symbolize nonlinear continuous functions [23], [24], with initial constraints as $\psi_1(0) = 0$ and $1 \leq N \leq N_u$, which marks the estimation horizon.

For each sampling interval, the solution of the optimal input vector can be obtained via online optimization using a cost function, and the NMPC method will accordingly be formed through the iterative solution to the optimal control problem. Therefore, the cost function is defined as:

$$S(X, u) = \sum_{i=j}^{j+N-1} L_1(X(i), u(i)) + L_2(X(j+N)) \quad (12)$$

where $L_1(X, u)$ is the immediate cost which satisfies the $L_1(0, 0) = 0$; $L_2(X(j+N))$ is the terminal cost. Besides, the cost function $S(X, u)$ can also be expressed in the following quadratic form:

$$S(j) = \sum_{i=1}^N \|X(j+i|j)\|_Q^2 + \sum_{i=0}^{N_u-1} \|\Delta u(j+i|j)\|_R^2 \quad (13)$$

where $X(j+i|j)$ is the following horizontal state to be estimated; the increment of the input vector is formulated as $\Delta u(j+i|j) = u(j+i|j) - u(j-1+i|j)$; both the constant parameters R and Q represent the designed weight matrices; $\|X\| = \sqrt{X^T X}$ denotes as its Euclidean norm based on the related vector. Then, we reshape the formulation (11):

$$\bar{X}_e(j+1) = \psi_1(\bar{X}_e(j)) + \psi_2(\bar{X}_e(j))u_e(j) \quad (14)$$

$$\text{s.t.} \quad u^- \leq u_e(j) \leq u^+ \quad (15)$$

$$\Delta u^- \leq \Delta u(j) \leq \Delta u^+ \quad (16)$$

$$X^- \leq \bar{X}_e(j) \leq X^+ \quad (17)$$

with

$$\begin{aligned} \psi_1(\bar{X}_e) &= \begin{bmatrix} 1 & \omega T & 0 \\ -\omega T & 1 & v_r \frac{\sin x_3}{x_3} T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ \psi_2(\bar{X}_e) &= T \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

where $\bar{X}_e = [x_1, x_2, x_3]^T = [x_e, y_e, \theta_e]^T$ denotes the state vector with the sampling interval T ; the input vector is described as $u_e = [u_1, u_2]^T$ with $u_1 = v_r \cos \theta_e - v$ and $u_2 = \omega_r - \omega$; (u^-, u^+) , $(\Delta u^-, \Delta u^+)$ and (x^-, x^+) are respectively corresponding to the lower and upper boundaries of the input variable, the input increment variable, and the state variable.

Some vectors are introduced for a compact presentation of the QP problem in online optimization:

$$\bar{X} = [\bar{X}_e(j+1|j), \dots, \bar{X}_e(j+N-1|j)]^T \quad (18)$$

$$\bar{u}(j) = [u_e(j|j), \dots, u_e(j+N_u-1|j)]^T \quad (19)$$

$$\Delta \bar{u}(j) = [\Delta u_e(j|j), \dots, \Delta u_e(j+N_u-1|j)]^T \quad (20)$$

Based on (14) and (18)–(20), the output is estimated as:

$$\bar{X}(j) = A \Delta \bar{u}(j) + \hat{\psi}_1 + \hat{\psi}_2 \quad (21)$$

where

$$\begin{aligned} A &= \begin{bmatrix} \psi_2(\bar{X}_e(j|j-1)) & \dots & 0 \\ \psi_2(\bar{X}_e(j+1|j-1)) & \dots & 0 \\ \vdots & \ddots & \vdots \\ \psi_2(\bar{X}_e(j+N-1|j-1)) & \dots & \psi_2(\bar{X}_e(j+N-1|j-1)) \end{bmatrix} \\ \hat{\psi}_1 &= \begin{bmatrix} \psi_1(\bar{X}_e(j|j-1)) \\ \psi_1(\bar{X}_e(j+1|j-1)) \\ \vdots \\ \psi_1(\bar{X}_e(j+N-1|j-1)) \end{bmatrix} \\ \hat{\psi}_2 &= \begin{bmatrix} \psi_2(\bar{X}_e(j|j-1)u(j-1)) \\ \psi_2(\bar{X}_e(j+1|j-1)u(j-1)) \\ \vdots \\ \psi_2(\bar{X}_e(j+N-1|j-1)u(j-1)) \end{bmatrix} \end{aligned}$$

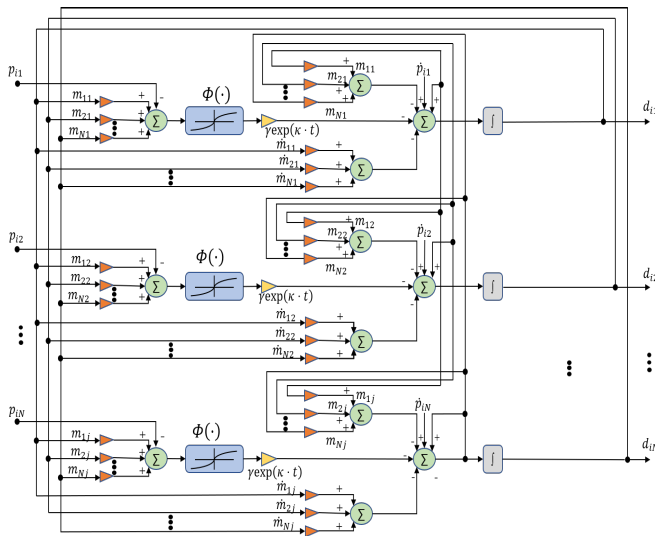


Fig. 7. Structure of VP-OneLPNN, where $p_i = (c + \xi \nabla G(d))_i$.

with $A \in R^{3N \times 2N_u}$, $\hat{\psi}_1 \in R^{3N}$ and $\hat{\psi}_2 \in R^{3N}$. Afterwards, one can redescribe the optimization problem of (13) as:

$$\min \left\| A\Delta\bar{u}(j) + \hat{\psi}_1 + \hat{\psi}_2 \right\|_Q^2 + \|\Delta u(j)\|_R^2 \quad (22)$$

$$\text{s.t.} \quad \Delta\bar{u}^- \leq \Delta\bar{u}(j+1) \leq \Delta\bar{u}^+ \quad (23)$$

$$\bar{u}^- \leq \bar{u}(j-1) \leq \bar{u}^+ \quad (24)$$

$$\bar{u}^- \leq \bar{u}(j-1) + H\Delta\bar{u}(j) \leq \bar{u}^+ \quad (25)$$

$$X^- \leq \hat{\psi}_1 + \hat{\psi}_2 + A\Delta\bar{u}(j) \leq X^+ \quad (26)$$

$$\text{where } H = \begin{bmatrix} I & 0 & 0 & 0 \\ I & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \dots & I \end{bmatrix} \in R^{2N_u \times 2N_u}.$$

Furthermore, a QP problem can be reformulated from the optimization issue of the equations (22)-(26):

$$\min \quad \frac{1}{2} \Delta\bar{u}^T M \Delta\bar{u} + c^T \Delta\bar{u} \quad (27)$$

$$\text{s.t.} \quad E \Delta\bar{u} \leq r \quad (28)$$

with

$$M = 2(A^T Q A + R) \in R^{2N_u \times 2N_u}$$

$$c = -2A^T Q(\hat{\psi}_2 + \hat{\psi}_1) \in R^{2N_u}$$

$$E = \begin{bmatrix} -H \\ H \\ -A \\ A \\ I \\ -I \end{bmatrix} \in R^{n_1}, r = \begin{bmatrix} -u^- + u(j-1) \\ u^+ - u(j-1) \\ -X^- + \hat{\psi}_1 + \hat{\psi}_2 \\ X^+ - \hat{\psi}_1 - \hat{\psi}_2 \\ \Delta\bar{u}^+ \\ \Delta\bar{u}^- \end{bmatrix} \in R^{n_2}$$

where $n_1 = (6N_u + 6N) \times 2N_u$ and $n_2 = 6N_u + 6N$.

IV. VARYING-PARAMETER ONE-LAYER PROJECTION NEURAL NETWORK (VP-ONELPNN)

In this section, the QP problem is supposed to be rewritten as:

$$\min \quad \frac{1}{2} d^T M d + c^T d \quad (29)$$

$$\text{s.t.} \quad g(d) = E d - r \leq 0 \quad (30)$$

with $d = \Delta\bar{u}$. Considering the Lagrangian function:

$$L(d, l) = \frac{1}{2} \Delta\bar{u}^T M \Delta\bar{u} + c^T \Delta\bar{u} + l^T g(d) \quad (31)$$

According to Karush-Kuhn-Tucker (KKT) conditions in [25], if d^* is a solution to (29)–(30), then there exists $l^* \in R^m$ such that $(d^*, l^*)^T$ satisfies the following conditions:

$$M d^* + c + \sum_{i=1}^{6N_u+6N} l_i^* \nabla g_i(d^*) = 0 \quad (32)$$

$$l^* \geq 0 \quad (33)$$

$$l_i^* g_i(d^*) = 0 \quad (34)$$

The equivalent equation of (32)–(33) is expressed as [26]:

$$M d + c + \nabla G(d) l = 0 \quad (35)$$

$$G(d) = \sum_{i=1}^{6N_u+6N} \max\{0, g_i(d)\}$$

Similar to [27], a penalty function is defined for the problem (29)–(30) as follows:

$$E(d) = \frac{1}{2} d^T M d + c^T d + \xi G(d) \quad (36)$$

where $\xi > 0$ is a penalty parameter. We can further obtain the gradient of (36):

$$\partial E(d) = M d + c + \xi \nabla G(d) \quad (37)$$

Actually, the residual errors in (37) exist in practice, so we define the following error function:

$$e(t) = M d + c + \xi \nabla G(d) \quad (38)$$

To ensure that the error converges to zero, a negative time derivative of (38) is defined as:

$$\dot{e}(t) = -\gamma \exp(\kappa \cdot t) \Phi(e(t)) \quad (39)$$

where $\Phi(e(t))$ is the active function; κ and γ are positive parameters that are employed to scale the convergence speed. The error in (39) is converged to zero. The active function is defined as:

$$\Phi(e_i(t)) = \frac{(1 + \exp(-\beta)) (1 - \exp(-\beta e_i(t)))}{(1 - \exp(-\beta)) (1 + \exp(-\beta e_i(t)))}$$

where $\beta \geq 2$ which makes $0 \leq |e_i(t)| \leq 1$.

The extended form of the (39) is expressed as:

$$\begin{aligned} \dot{M}d + M\dot{d} + \dot{c} + \xi \nabla \dot{G}(d) \\ = -\gamma \exp(\kappa \cdot t) \Phi \{M d + c + \xi \nabla G(d)\} \end{aligned} \quad (40)$$

Then, a new neural network called VP-OneLPNN is proposed to solve the QP problem in (29)–(30) as follows:

$$\begin{aligned} M\dot{d} = - \left(\dot{M}d + \dot{c} + \xi \nabla \dot{G}(d) \right) \\ - \gamma \exp(\kappa \cdot t) \Phi \{M d + c + \xi \nabla G(d)\} \end{aligned} \quad (41)$$

The equation can be further reformulated in the following form:

$$\begin{aligned} \dot{d} = (I - M) \dot{d} - \left(\dot{M}d + \dot{c} + \xi \nabla \dot{G}(d) \right) \\ - \gamma \exp(\kappa \cdot t) \Phi \{M d + c + \xi \nabla G(d)\} \end{aligned} \quad (42)$$

The structure of VP-OneLPNN for the QP problem in MPC is shown in Fig. 7. For comparison purposes, the traditional

TABLE 1
COMPARISON OF DIFFERENT NEURAL NETWORK MODELS

Neural network type	number of layers	number of neurons	number of connections	objective function
RNN [18]	2	$n + m$	$n^2 + 2mn$	strictly convex
PDNN [22]	2	$n + m$	$3n^2 + 3mn$	convex
The proposed	1	n	$2n^2$	convex

gradient descent-based recurrent neural network is denoted as [28],

$$\dot{d} = -\gamma \{Md + c + \xi \nabla G(d)\} \quad (43)$$

Compared with other neural network models, the VP-OneLPNN has the fastest convergence rate and involves the least neurons and layers in the structure shown in Table.1.

Remark 2. *The proposed VP-OneLPNN has the simplest network structure and least neurons and layers among other models in Table.1. However, the proposed method has slower computational efficiency, because the equation in (41) involves more matrix operations, including \dot{M} , \dot{l} , etc. Noted that, the proposed method only involves matrix operations, which still satisfy the requirements of computational efficiency.*

V. SIMULATION

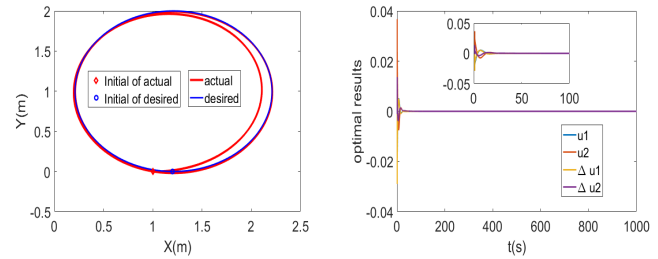
In this part, the proposed VP-OneLPNN based MPC scheme is demonstrated in simulation. To demonstrate the validity of the proposed algorithm, two different neural networks are also tested and the experimental results are compared with the proposed one. One kind of scenario is built based on a periodic-moving mobile robot with constant linear and angular velocity for the circle-shape movement. In the simulation experiment, we design the initial position and orientation of the mobile robot that is different from the desired trajectory, which is convenient for us to observe the convergence performance of the proposed method. The robot is supposed to track the trajectories with the VP-OneLPNN based MPC method in the scenario. We analyze the result in terms of tracking errors and convergence. The sampling period is $\Delta t = 0.1s$. The details of the parameters refer to Table.2.

A. Circle-Shape Movements with Constant Velocity for Robot

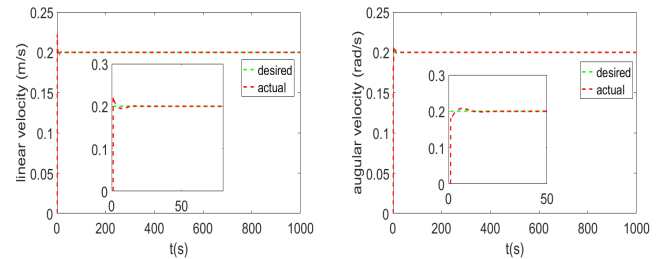
To track the circle-shape trajectory within the periodic motion, as the task for the mobile robot, in this case, is supposed to be executed with constant linear and angular velocities. The initial position and orientation of the mobile robot are randomly chosen but different from the ideal trajectory.

The linear and angular velocities are respectively set to $v = 0.2m/s, w = 0.2rad/s$ as reference values. As for the initial position of the reference trajectory, it refers to $X_r(0) = [1.2; 0; 0]$, while that of the mobile robot denotes as $X(0) = [1; 0; 0]$.

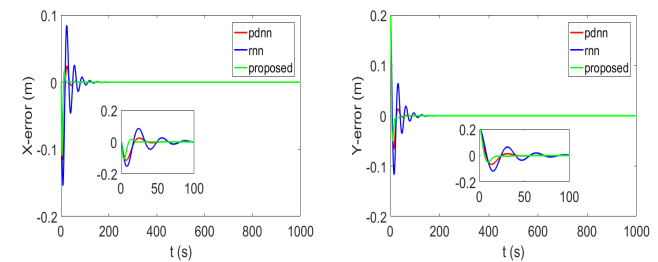
As shown in Fig. 8(a), it's clear that the trajectories of the mobile robot rapidly converge to the reference one, despite the mobile-robot's initial positions. The optimization result of the circle-shape trajectory is shown in Fig. 8(b), where all the state variables quickly reach optimal solutions. As displayed in Figs. 8(c)–8(d), the linear and angular velocities of the



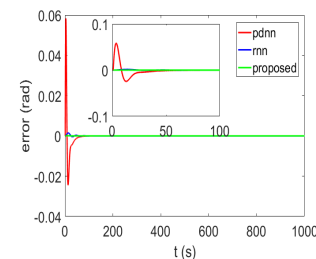
(a) The tracking result of the circle shape trajectory for the periodic circle shape trajectory with VP-OneLPNN. (b) The optimization result of the circle shape trajectory with VP-OneLPNN.



(c) The tracking result of the mobile robot velocity v . (d) The tracking result of the angular velocity ω .



(e) The tracking error X_e . (f) The tracking error Y_e .



(g) The tracking error θ_e .

Fig. 8. The simulation result of the circle shape trajectory using NMPC based on VP-OneLPNN.

TABLE 2
PARAMETERS OF THE LEARNING-CONTROL SYSTEM

Name	Parameters
MVSD	$K^{p1} = \text{diag}[50, 50], K^{v1} = \text{diag}[10, 10], K^{p2} = \text{diag}[10, 10], K^{v2} = \text{diag}[\sqrt{20}, \sqrt{20}]$
MPC	$N = 3, N_u = 2, Q = I, R = 2I$
VP-OneLPNN	$\kappa = 0.1, \gamma = 10^2$
Dimension of the robot movements	$D = 2$

mobile robot can approach the respective reference ones quite well. The comparison results of the recurrent neural network, the primal-dual neural network, and our proposed method are shown in Figs. 8(e)–8(g), where all the tracking errors of the mobile robot can converge to zero, but the proposed method has the fastest convergence rate.

B. Discussion

Successful trajectory-tracking execution of the mobile robot is guaranteed for the circle-shape trajectory scenario. The rapid convergence of the proposed method is quite significant for a mobile robot, especially for the real-time optimization of the mobile robot. The optimal solution to the trajectory-learning problem can be performed by the mobile robot regardless of variable position and orientation initializations. Besides, the effect and speed of the tracking-errors convergence are satisfactory. The effectiveness and robustness of our proposed technique can be demonstrated by the aforementioned results. Furthermore, from the comparison results, it is observed that the proposed VP-OneLPNN based MPC control scheme has an overall promising performance for real-time robot running, in terms of convergence speed, and tracking errors.

VI. EXPERIMENTAL DEMONSTRATION

In this section, the proposed learning-by-imitation scheme is tested using real-world trajectories that are collected by the Xsens mti-310 sensor. We build three scenarios to explore the mobile-robot learning skills abilities including the trajectory generation from a demonstration by human and trajectory tracking with VP-OneLPNN based MPC. The parameter settings of MVSD and VP-OneLPNN based MPC can refer to Table.2. The experimental procedures are listed as:

- The human holds the Xsens mti-310 sensor and walk through the structured environment, where the trajectories are collected.
- Then, the motor skills are modeled and learned by MVSD.
- Finally, the robot is driven to perform the learning skills in a structured environment using neuro-dynamic optimization.

In our experiments, the tasks in different scenarios vary from simple to complex, involving to learn the straight line-shape, 'S'-shape, and 'U'-shape trajectories from human demonstration, which are shown in Fig. 9.

A. Learning the Straight Path Tracking

In the first experiment, the mobile robot is expected to learn a simple task of a straight line-shape path tracking from

demonstration, which is shown in Fig. 10. In this scenario, a human subject walks along a straight path. There are no obstacles placed on any side along the path. The human subject walks along the path repeatedly and the trajectory of the human subject is collected with an Xsens mti-310 sensor. This scenario is quite simple since the mobile robot only needs to learn how to move in the forward direction.

B. Learning the 'S'-Shape Path Tracking

Then in the second experiment, it's supposed for the mobile robot to learn a relatively complicated task of an 'S'-shape path tracking, as Fig. 11 exhibits. In this scenario, a human subject walks along an 'S'-shape path. We randomly set obstacles on the two sides along with the 'S'-shape path ('◇' denotes the obstacle). The human subject is instructed to follow the path without any collision with obstacles and repeats walking along the path to obtain several trajectories. The robot's task in this scenario is more difficult than Scenario VI-A as the mobile robot needs to learn to go forward, turn left and right.

C. Learning the 'U'-Turn Path Tracking

As for the last experiment, it is essential for the mobile robot to learn the more complicated task of a 'U'-shape path tracking based on demonstrations, as Fig. 12 shows. In this scenario, a human subject walks along with a 'U'-turn path. We randomly drop obstacles on the two sides along with the 'U'-turn path again('◇' denotes the obstacle). The human subject is instructed to follow the path without any collision with obstacles. The robot's task in this scenario is much more difficult than Scenario VI-B as the mobile robot needs to learn to make a 'U'-shape turn within quite confined space.

D. Discussion

In Scenario VI-A, the learning results are shown in Fig. 10. The trajectory reproduced is plotted with a green line, while the tracking trajectory acquired with the proposed method is in red. It can be seen that both the linear and angular velocities coincide with the desired ones in Figs. 10(b) and 10(c). Moreover, the proposed method proves to be capable to quickly converge to the global optimal solution. Fig. 10(d) displays the trajectory tracking errors in Cartesian space. It shows that all the errors are limited to a small scope.

It is not surprising that we get similar results from Scenarios VI-B even the task is more challenging, and the experimental results are recorded in Fig. 11. In Scenario VI-B, the trajectory and the mobile robot's skills are very complex. The results of the learning-based MPC system for an 'S'-shape path tracking are shown in Figs. 11(a)–11(d). At the motion control stage,

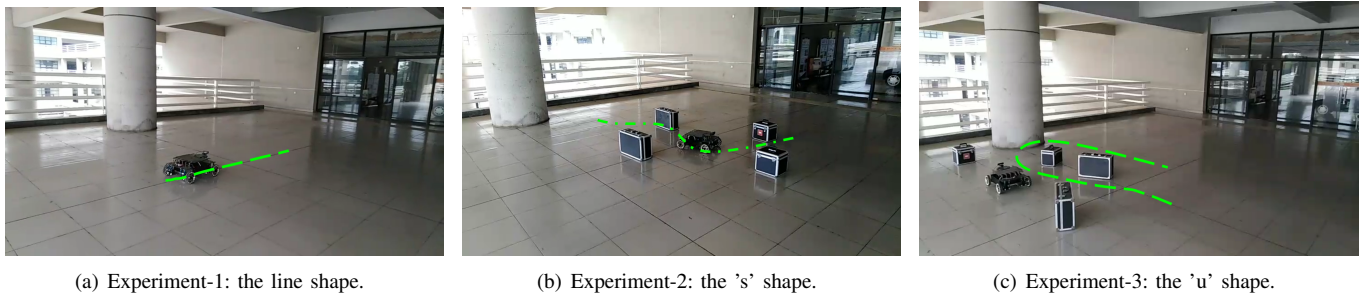


Fig. 9. Experiment demonstration: from simple to complex scenarios.

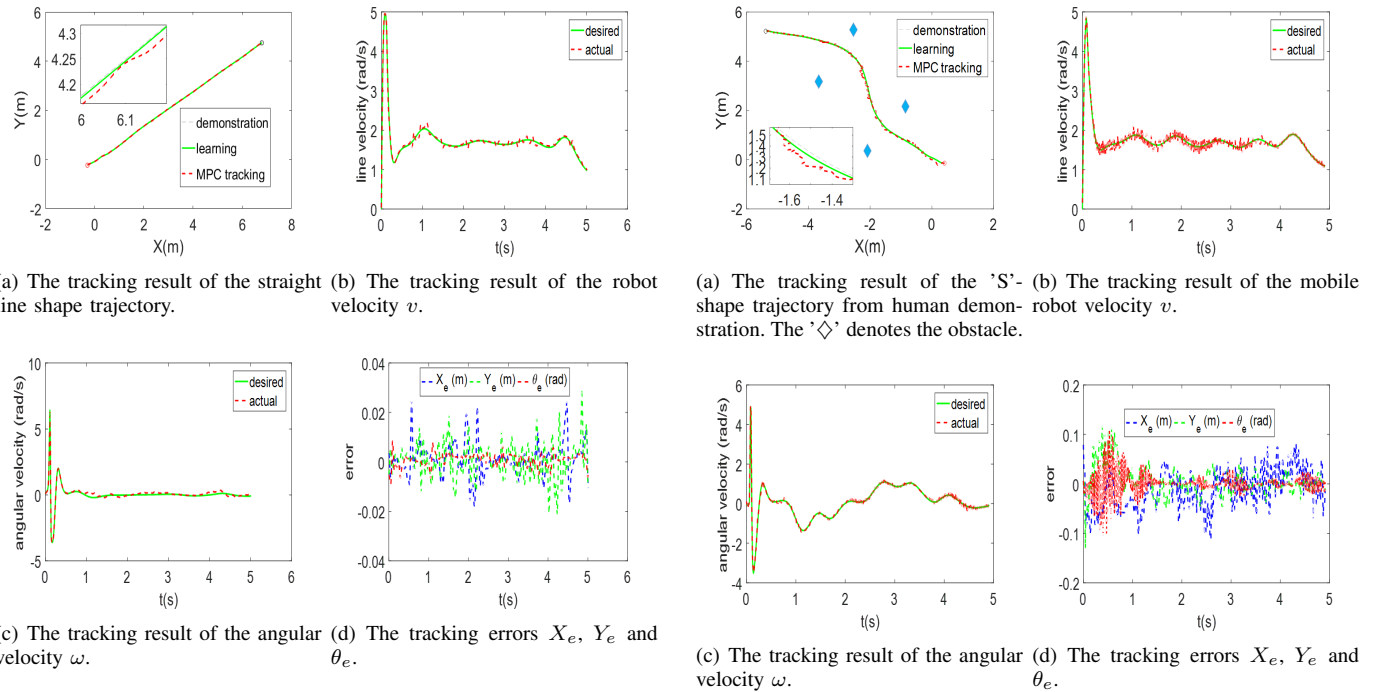


Fig. 10. The tracking result of the straight line-shape trajectory using NMPC based on VP-OneLPNN. The red '*' indicates the initial position, while the black '*' denotes the end attractor.

Fig. 11. The tracking result of the 'S'-shape trajectory using NMPC based on VP-OneLPNN. The red '*' indicates the initial position, while the black '*' denotes the end attractor.

the mobile robot succeeds in crossing the path without any obstacle collision.

In Scenario VI-C, there is no obvious degradation of the overall performance compared with other scenarios. The results of the learning-based MPC system for the 'U'-turn path tracking are shown in Figs. 12(a)-12(d). Experimental results show that our learning-by-imitation system is independent of any specific scenes, and it is a powerful way to provide a mobile robot with motor skills in a natural way.

VII. CONCLUSION

In this paper, the Multi-Virtual Spring-Dampers method is presented to reproduce the trajectory from the demonstration. The proposed imitation learning strategy with MVSD can promote motor-skills learning for the mobile robots by imitation learning.

For trajectory tracking, the NMPC is used to control the mobile robot along with the finite-time domain under input

and state limitations. The new network of VP-OneLPNN is also applied to the online QP optimization in the MPC. So, the mobile robot can cross obstacles from human demonstration and complete more complex tasks by using our method, which reduces the complexity of motion planning in a structured environment.

To test the proposed learning-by-imitation system of mobile robots, we build one challenging scenario in simulation and two real-world scenarios. The tasks of these scenarios are designed to examine the aforementioned learning abilities of motor skills. The results confirm that the mobile robot can learn the motor-skills from human demonstration with our method. To sum up, the mobile robot can successfully cross the path with no obstacle collision in all the test scenarios. Besides, small tracking errors and quick convergence to the global optimal solution emerge with the VP-OneLPNN based MPC. With the above characteristics, the proposed method is guaranteed to be a mighty and efficient solution to complex

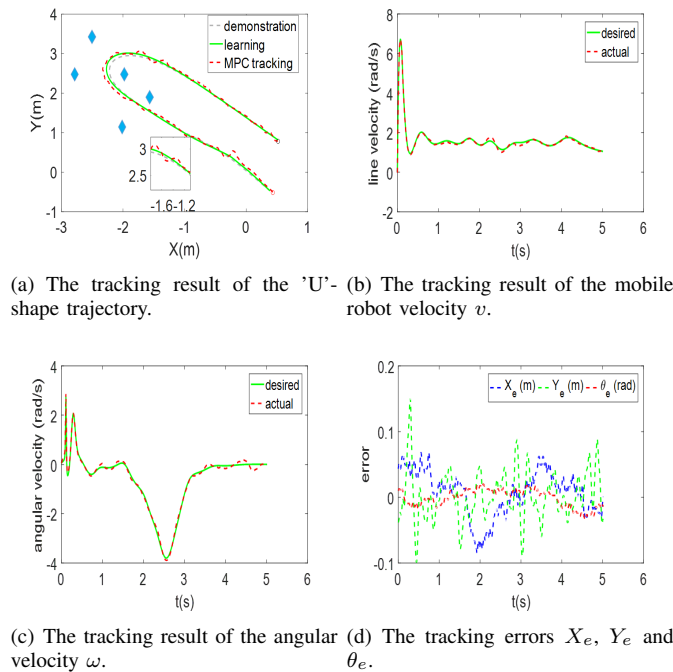


Fig. 12. The tracking result of the 'U'-shape trajectory using NMPC based on VP-OneLPNN from human demonstration. The red '*' indicates the initial position, while the black '*' denotes the end attractor.

motion planning and control tasks.

Future works will involve different imitation learning methods, such as kernelized movement primitives (KMP) for dynamic obstacle avoidance. Additionally, we will explore the more complex model and reinforcement learning in MPC.

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