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Uncertainty of coordinate measurement of geometrical deviations

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Abstract

At University of Bielsko-Biala, simplified models for the coordinate measurement of different geometrical characteristics (size, distance, angle and geometrical deviations) have been developed. The most important assumption for measurement model is taking into account the obvious fact that coordinate measurement is indirect measurement. To develop the models of different geometrical characteristics the mathematical minimum number of required points was assumed. Models use information on CMM accuracy only in the form of formula for maximum permissible error of length measurement $E_{L,MPE}$ verified by actual acceptance or reverification test results. The presented uncertainty budgets enable analysis of influence of measurement uncertainty of particular coordinates' differences on the total measurement uncertainty.

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Keywords: coordinate measurement; uncertainty evaluation; type B evaluation; task-specific uncertainty

1. Introduction

Research on uncertainty evaluation of coordinate measurements last for many years [1] and recently become particularly important not only for metrologists [2, 3]. The research carried out at University of Bielsko-Biała in this field was presented in some publications [4, 5]. The publications bring a lot of attention to possibility of simplifying the procedure of uncertainty evaluation in the way it's possible to apply also in the industrial conditions. Moreover, special attention is paid to the fact that the models are unambiguously compatible with the principle of the coordinate measuring technique.

Comparing to the previous publications, the examples in this paper additionally present uncertainty budgets which enable detailed analysis of the main components influencing the calculated uncertainty. The examples can be easily adopted for calculations for different data.

The developed models consist of the formula expressing the geometric deviation l as a function of <u>differences of coordinates</u> of characteristic points x_i of the workpiece.

$$l = f(x_i) \tag{1}$$

Nomenclature

CMM – coordinate measuring machine
u – standard uncertainty; index denotes deviation e.g. ST – straightness
$E_{L,\,MPE}$ – maximum permissible error of length measurement
x, y, z – coordinates
A, B ... – characteristic points
AB, AS ... – vector of coordinates' differences for points A and B, A and S etc.
ab_1, ab_2, ab_3 – respective coordinates (x, y, z) of the vector AB, i.e. coordinates' differences for points A and B
l – general indication of geometrical deviation as function of respective coordinates' differences of points x_i
$\partial l/\partial x_i$ – sensitivity coefficients
u_{xi} – standard uncertainties of particular input quantities

Thus uncertainty of measurement can be evaluated by means of the formula for uncertainty of indirect measurement [6]. Due to use of <u>differences of coordinates</u> of characteristic points one can assume lack of correlation between input quantities x_i.

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$$u(l) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial l}{\partial x_i}\right)^2 u_{xi}^2} \quad (2)$$

Standard uncertainties u_{xi} of measurements of particular differences of coordinates are evaluated by type B method assuming that biggest possible error is equal to maximum permissible error of length measurement of the CMM and that normal distribution applies with $k = 3$.

$$u_{xi} = E_{L,MPE}/3 \quad (3)$$

In all examples the uncertainty budget is composed for measurement on CMM with $E_{L,MPE} = 2 + 4L/1000)$ μm, L - in mm.

2. Reasons for assuming *k* = 3

Normal distribution with $k = 3$ corresponds to the probability $P = 0{,}9973$, that all indication errors of CMM are less than $E_{L,\ MPE}$. Here we assume that considered CMM conforms the requirements. This means that evaluated indication errors during verification procedure are smaller than $E_{L,\ MPE}$ for all 105 measurements of the calibrated lengths. Thus the probability that indication errors are less or equal to the permissible value is greater than (1 – 1/105).

If necessary different value of k may be assumed or it can be calculated from the verification results. Fig. 1 depicts example (quite typical) of CMM verification results for CMM with $E_{L,\ MPE} = 2 + 4L/1000$ μm.

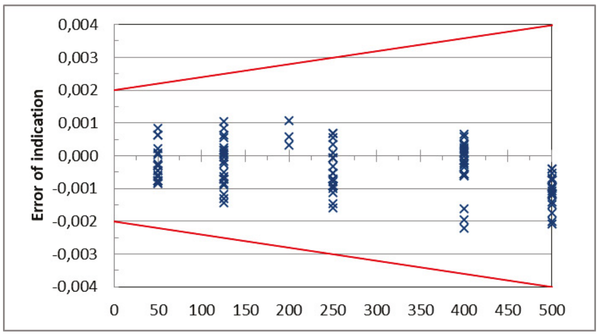

Fig. 1. Example of actual CMM verification results (errors of indication for different measured calibrated lengths in mm)

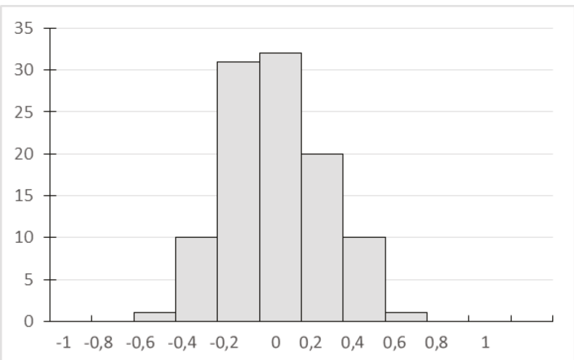

Fig. 2. Histogram of normalized error values E_{st} for $E_{L,MPE} = 2 + 4L$

Fig. 2 presents histogram of normalized indication errors E_{st} for the range (-1, 1) according to formula.

$$E_{st} = E/E_{L,MPE} \quad (4)$$

The standard deviation (in regard to 0) calculated for the normalized errors $s = 0{,}268$ which gives the coefficient $k = 1/s = 3{,}74$.

3. Measurement uncertainty of straightness

Fig. 3 depicts an example of tolerance specification and modelling of straightness deviation measurement. The model refers to a simplified method of classical measurement of straightness. The point S is to lay near the line connecting points A and B and usually is close to the middle of the line segment AB, where the sag arrow is the largest.

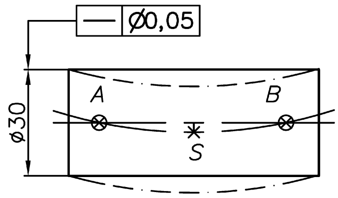

Fig. 3. Measurement model of deviation of axis straightness

The tolerance zone is the cylinder of the diameter equal to the tolerance value [7]. The straightness deviation l in this case is modelled as the distance between point S and line AB.

$$l = AS \times \frac{AB}{|AB|} \quad (5)$$

Two vectors occur in the formula and each consists of differences of three coordinates: $AB(ab_1, ab_2, ab_3)$ and $AS(as_1, as_2, as_3)$, therefore l is function of 6 variables denoted generally as x_i, $i = 1..6$.

$$l = \frac{\sqrt{e^2 + f^2 + g^2}}{m} \quad (6)$$

where: $e = as_2 \cdot ab_3 - as_3 \cdot ab_2, f = as_3 \cdot ab_1 - as_1 \cdot ab_3$, $g = as_3 \cdot ab_1 - as_1 \cdot ab_3, m = ab_1^2 + ab_2^2 + ab_3^2$.

The uncertainty of measurement of straightness equals the uncertainty of measurement of distance between point S and straight-line AB and is function of coordinate differences for pairs of points AS and AB [8].

$$u_{ST} = u(AS, AB) \quad (7)$$

The uncertainty of measurement of distance between a point and a straight-line is analysed depending on the location of the point S relative to the points A and B as well as the distance AB.

In presented example the characteristic points are centre points and not directly probed points. Such simplification leads to insignificant overestimation of the uncertainty.

In the example, the element is oriented parallel to the x axis and its length is 100 mm – the analyses is performed for the following coordinates of characteristic points of the tolerated element: A(100, 100, 100) and B(200, 100, 100).

For the position of the point S symmetrically with respect to points A and B, the standard uncertainty is the highest and amounts to 0.75 µm. The uncertainty budget for this position is presented in the table 1.

Table 1. Uncertainty budget for measurement of point – straight line distance (Fig. 3) for the case when point S is lying over the line AB – S(150, 100, 100.01)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
as_1	50	0	0.73	0
as_2	0	0	0.67	0
as_3	0.01	1	0.67	0.67
ab_1	100	0	0.80	0
ab_2	0	0	0.67	0
ab_3	0	-0.50	0.67	0.33
			u =	0.75

It should be noted that one of the uncertainty components occurs with a weight of 1 and one with a weight of 0.5, and both concern a small measured value. The components related to large measured values and related to the length of the measured object occur in the budget with a weight equal zero, which means that the length of the measured object does not affect the measurement uncertainty of straightness.

4. Measurement uncertainty of flatness

Fig. 4 depicts an example of tolerance specification and modelling of flatness deviation measurement. The model refers to a simplified method of classical measurement of flatness. The point S is to lay near the gravity centre of points A, B and C.

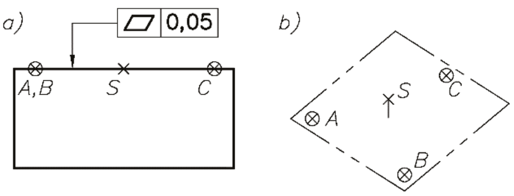

Fig. 4 Measurement model of deviation of flatness

The tolerance zone is limited by pair of parallel planes apart by the tolerance value [7]. The uncertainty of measurement of flatness equals the uncertainty of measurement of distance between point S and plane ABC and is function of coordinate differences for pairs of points AS, AB and AC [9].

$$u_{FL} = u(AS, AB, AC) \tag{8}$$

Table 2 presents the uncertainty budget of flatness measurement. The measured surface is a square with side length 400 mm. The points A(5, 5), B(395, 5) and C(200, 395) define the reference plane. Point S can be located in any place near the plane but usually around the central point of the plane. The result do not differ significantly over the plane - the largest value of the measurement uncertainty is 0.74 µm.

Table 2. Uncertainty budget of the flatness measurement for S(200,150,0)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
as_1	150	0	0.87	0
as_2	100	0	0.80	0
as_3	0.01	1	0.67	0.67
ab_1	300	0	1.07	0
ab_2	0	0	0.67	0
ab_3	0	-0.33	0.67	0.22
ac_1	150	0	0.87	0
ac_2	300	0	1.07	0
ac_3	0	-0.33	0.67	0.22
			u =	0.74

It can be seen that one of the uncertainty components occurs with a weight of 1 and two with a weight of 0.33, and all concern a small measured value. The components related to large measured values and related to the size of the measured object occur in the budget with a weight equal 0.

5. Measurement uncertainty of parallelism

The parallelism tolerance may be applied to straight line or plane.

5.1. Parallelism of a line related to a datum plane

Fig. 5 depicts an example of tolerance specification and measurement modelling of parallelism of axis related to a datum plane. The point S is to lay near the end of toleranced axis.

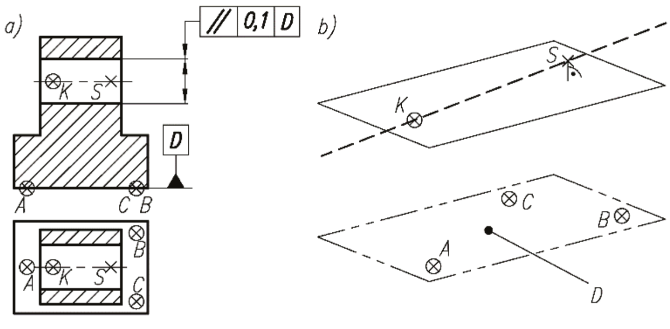

Fig. 5. Measurement model of deviation of parallelism of axis and plane

The tolerance zone is limited by two parallel planes the distance equal tolerance value apart and parallel to the datum [7]. The uncertainty of measurement of parallelism equals the uncertainty of measurement of distance between point S and plane containing point K and parallel to plane ABC [9].

$$u_{PRL} = u(KS, AB, AC) \tag{9}$$

Table 3 presents the uncertainty budget of parallelism measurement. The points A(50, 200, 10), B(350, 350, 10) and

C(350, 50, 10) define the datum plane and point K(50, 200, 200) the toleranced axis. The largest value of the uncertainty is 0.82 µm for the point S(350, 200, 200).

Table 3. Uncertainty budget of the parallelism measurement for S(350, 200, 200)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
ks_1	300	0	1.07	0
ks_2	0	0	0.67	0
ks_3	0	-1	0.67	0.67
ab_1	300	0	1.07	0
ab_2	150	0	0.87	0
ab_3	0	0.50	0.67	0.33
ac_1	300	0	1.07	0
ac_2	-150	0	0.87	0
ac_3	0	0.50	0.67	0.33
			u =	0.82

One can see that one of the uncertainty components occurs with a weight of 1 and two with a weight of 0.5, and all concern a small measured value. The components related to large measured values and related to the size of the measured object occur in the budget with a weight equal to zero, which means that the size of the measured object nor the distance of the toleranced element from the datum do not affect the uncertainty of the parallelism measurement.

5.2. Parallelism of planes

Fig. 6 depicts an example of tolerance specification and measurement modelling of parallelism of planes. The point S is to lay anywhere on the toleranced plane but usually near the opposite corner to location of point K.

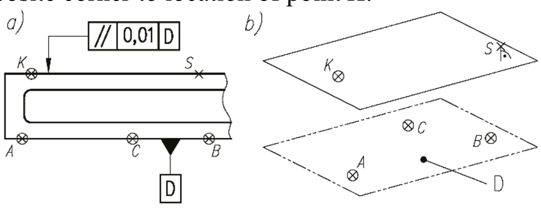

Fig. 6. Measurement model of parallelism of planes

The tolerance zone is limited by two parallel planes the distance equal tolerance value apart and parallel to the datum [7]. The uncertainty of measurement of parallelism equals the uncertainty of measurement of distance between point S and plane containing point K and parallel to plane ABC [8].

$$u_{PRL} = u(KS, AB, AC) \tag{10}$$

Table 4 presents the uncertainty budget of parallelism measurement. The points A(50, 200, 10), B(350, 350, 10) and C(350, 50, 10) define the datum plane and point K(50, 200, 200) the toleranced axis. The largest value of the uncertainty is 1.0 µm for the point S(350, 350, 200).

Table 4. Uncertainty budget of the parallelism measurement for S(350, 350, 200)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
ks_1	300	0	1.07	0
ks_2	300	0	1.07	0
ks_3	0	1	0.67	0.67
ab_1	300	0	1.07	0
ab_2	150	0	0.87	0
ab_3	0	1	0.67	0.67
bc_1	0	0	0.67	0
bc_2	-300	0	1.07	0
bc_3	0	0.50	0.67	0.33
			u =	1.00

The budget analysis shows that two of the uncertainty components occurs with a weight of 1 and one with a weight of 0.5, and all concern a small measured value. The components related to large measured values and related to the size of the measured object occur in the budget with a weight equal to zero, which means that the size of the measured object nor the distance of the toleranced element from the datum do not affect the uncertainty of the parallelism measurement.

5.3. Parallelism of axes

Fig. 7 depicts an example of tolerance specification and measurement modelling of parallelism of axes. The datum axis is represented by two points A and B. The toleranced axis is given by points K and S. The uncertainty of measurement of parallelism equals the measurement uncertainty of distance of a point S from the straight line parallel to the axis AB and going though point K. The point S is to lay anywhere near the toleranced axis but usually near the end of it.

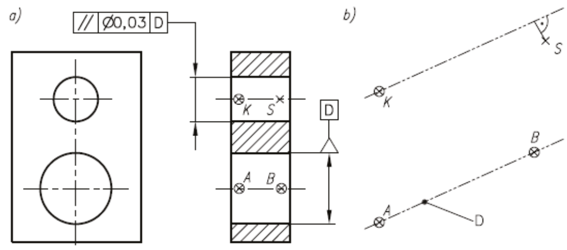

Fig. 7. Measurement model of parallelism of axes

The tolerance zone is limited by a cylinder of diameter equal tolerance value and parallel to the datum axis [7]. The uncertainty of measurement of parallelism equals the uncertainty of measurement of distance between point S and straight-line passing through point K and parallel to line AB [8].

$$u_{PRL} = u(KS, AB) \tag{11}$$

Table 5 presents the uncertainty budget of parallelism measurement. The points A(50, 50, 10) and B(250, 50, 10) define the datum axis and point K(50, 350, 10) defines the toleranced axis. The largest value of the uncertainty is 1.0 µm for the point S(250, 350, 10.01).

Table 5. Uncertainty budget of the parallelism measurement for S(250, 350.01, 10.01)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i}u_{xi}$, µm
as_1	200	0	0.93	0
as_2	0	0	0.67	0
as_3	0.01	1	0.67	0.67
ab_1	200	0	0.93	0
ab_2	0	0	0.67	0
ab_3	0	-1	0.67	0.67
			u =	0.94

It should be noted that two non-zero uncertainty components occur with a weight of 1 and both concern a small measured value. The components related to large measured values and related to the size of the measured object occur in the budget with a weight equal to zero, which means that the size of the measured object nor the distance of the toleranced element from the datum do not affect the uncertainty of the parallelism measurement.

6. Measurement uncertainty of runout

In the following the examples of circular and total runout are discussed (Fig. 8 and 9).

The tolerance zone of total radial runout is limited by two coaxial cylinders with a difference in radii equal to the tolerance value, the axes of which coincide with the datum [7].

The runout deviation is the smallest zone between radii of the coaxial circles or cylinders coinciding with the datum and surrounding all points of actual cross-section or surface (toleranced element), in this case these are the points S_1 and S_2. The uncertainty of runout measurement equals the geometric sum of uncertainties of two deviations of point-line distance.

It turns out that neglecting the insignificant differences in uncertainty associated with different angular positions and assuming for further calculations only the largest uncertainty of point-line distance measurement one can find that the uncertainty of the runout is $\sqrt{2}$ times greater than the uncertainty of the distance between the point and the line.

$$u_{RNT} = \sqrt{2} \cdot u(AS, AB) \quad (12)$$

This applies to both runout examples described below.

6.1. Circular radial runout

Fig. 8 depicts an example of tolerance specification and measurement modelling of circular radial runout. The datum axis is represented by two points A and B. The toleranced cylindrical surface is given by points S_1 and S_2 representing the extreme radii.

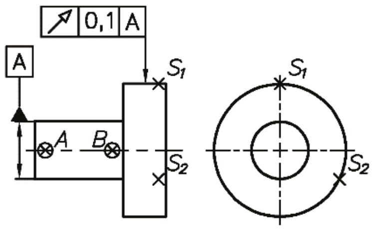

Fig. 8. Measurement model of circular radial runout

Table 6 presents the uncertainty budget of circular runout measurement. The points A(100, 100, 100) and B(130, 100, 100) define the datum axis. The diameter of the toleranced element is 50 mm. The largest value of the uncertainty is 1.31 µm for the point S(180, 100, 125).

Table 6. Uncertainty budget of the circular runout measurement for S(180, 100, 125)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,\,MPE}/3$, µm	$\frac{\partial l}{\partial x_i}u_{xi}$, µm
bs_1	50	0	0.73	0
bs_2	0	0	0.67	0
bs_3	25	1	0.70	0.70
ab_1	30	0	0.71	0
ab_2	0	0	0.67	0
ab_3	0	-1.67	0.67	1.11
			u =	1.31

Analysis of the budget shoes that the uncertainty component connected with the diameter of the toleranced element occurs in the budget with the weight of 1 and the weight factor of the second non-zero uncertainty component arises from the ratio of distance between the toleranced element to the datum and width of the datum. In this example 50 mm / 30 mm = 1.67.

6.2. Total radial runout

Fig. 9 depicts an example of tolerance specification and measurement modelling of total radial runout. The datum axis is represented by two points A and B. The toleranced feature is given by points S_1 and S_2 representing the extreme radii and lying anywhere between the points A and B.

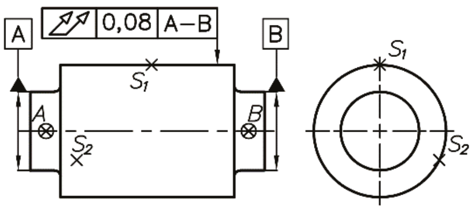

Fig. 9. Measurement model of total radial runout

Table 7 presents the uncertainty budget of total runout measurement for following data: A(100, 100, 100), B(200, 100, 100), S(150, 100, 125)). The largest value of the uncertainty is 0.78 µm for the point S(180, 100, 125).

The budget shows that the uncertainty component connected with the diameter of the toleranced element occurs in the budget with the weight of 1 and the weight factor of the second non-zero uncertainty component arises from the ratio of distance between the toleranced element to the datum and width of the datum. In this example 50 mm / 100 mm = 0.5.

Table 7. Uncertainty budget of the total runout measurement for *S*(150, 100, 125)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
bs_1	-50	0	0.73	0
bs_2	0	0	0.67	0
bs_3	25	1	0.70	0.70
ab_1	100	0	0.71	0
ab_2	0	0	0.67	0
ab_3	0	0.50	0.67	0.33
			u =	0.78

7. Measurement uncertainty of perpendicularity

Fig. 10 depicts an example of tolerance specification and measurement modelling of perpendicularity of axis to the plane. The datum plane is represented by points *A, B* and *C*. The toleranced axis is given by points *K* and *S* which lay on the opposite ends of the toleranced element.

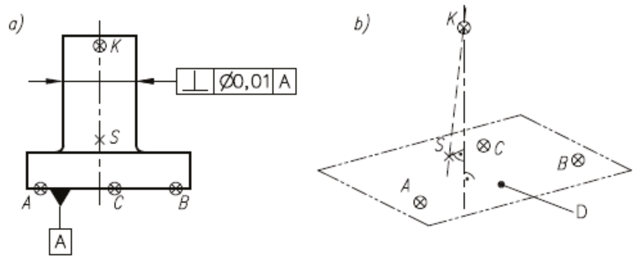

Fig. 10. Measurement model of perpendicularity

The tolerance zone is limited by a cylinder of diameter equal tolerance value and perpendicular to the datum plane [7, 10]. The uncertainty of measurement of perpendicularity equals the uncertainty of measurement of distance between point *S* and straight-line passing through point *K* and perpendicular to plane *ABC*.

$$u_{PRP} = u(KS, AC, BC) \tag{13}$$

Table 8 presents the uncertainty budget of perpendicularity measurement. The points *A*(10, 50, 10), *B*(10, 350, 10) and *C*(10, 200, 190) define the datum plane and point *K*(350, 200, 60) defines the toleranced axis. The largest value of the uncertainty is 1.09 µm for the point *S*(20, 200, 60.01).

It can be seen that one of the non-zero uncertainty components occurs with a weight of 1 and two additional with weight of 0.92 and all of them concern a small (zero or almost zero) measured value. The components related to large measured values and related to the size of the measured object occur in the budget with a weight equal 0, which means that the size of the measured object does not affect the uncertainty of the parallelism measurement.

Table 8. Uncertainty budget of the perpendicularity measurement for *S*(20, 200, 60.01)

Component	x_i, mm	$\frac{\partial l}{\partial x_i}$	$u_{xi} = E_{L,MPE}/3$, µm	$\frac{\partial l}{\partial x_i} u_{xi}$, µm
ks_1	-330	0	1.11	0
ks_2	0	0	0.67	0
ks_3	0.01	1	0.67	0.67
ac_1	0	0.92	0.67	0.61
ac_2	150	0	0.87	0
ac_3	180	0	0.91	0
bc_1	0	0.92	0.67	0.61
bc_2	-150	0	0.87	0
bc_3	180	0	0.91	0
			u =	1.09

8. Conclusions

The presented measurement models and respective uncertainty budgets enable analysis of influence of measurement uncertainty of particular coordinates' differences on the total measurement uncertainty.

It can be seen that in case of proper orientation of the workpiece in relation to the CMM coordinate system the analysis of the resulting uncertainty budget can help to draw some conclusions on the factors influencing measurement uncertainty. The presented examples concern a few example cases of deviations of form, orientation and run out. It is worth noting that in the case of deviations of form and orientation, the components of uncertainty related to the dimensions of the workpiece do not have an impact on the uncertainty budget and have sensitivity coefficients equal zero.

Authors started some experimental validation and plan comparison with other uncertainty evaluation software.

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